1) Fill in the following table so that \((x,y,z), *\) forms a group.

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Identify the identity element, and the inverse element of \(x, y,\) and \(z.\)

2) Does \([\{0000, 0010, 1101, 1111\}, +2]\) form a subgroup of \([B4, +2]\)? If yes find all the cosets. (+2: represents addition modulo 2, \(B4 = BxBxBxB,\) where \(x\) is the Cartesian product, and \(B = \{0,1\}\)).

3) Check if the following is a homomorphism or an isomorphism:

\[ f: [B3, +2] \longrightarrow [Z4, +4] \]

\(f(x) = w(x), w(x)\) is the weight of \(x\) i.e. the number of ones (1's) in the binary number \(x.\) [\(e.g.\) if \(x = 101,\) then \(w(x) = 2,\) if \(x = 111\) then \(w(x) = 3\) ... etc.], \(Z4 = \{0, 1, 2, 3\},\)

\(+4:\) addition modulo 4, \((1 +4 2 = 3, 3 +4 3 = 2 ... etc.)\)

4) Let \(G\) be the set of all nonzero real numbers and let \(a*b = (a.b)/2.\) Show that \([G,*]\) is an abelian group.

5) Let \(S = \{a, b, c\}\) and \(T = \{x, y, z\},\) and let \([S,*]\) and \([T,#]\) be defined as:

\[
\begin{array}{ccc}
* & a & b & c \\
\# & x & y & z \\
\end{array}
\]

Can you find an isomorphism between \([S,*]\) and \([T,#]?\)

6) Check if the following mappings form a homomorphism or an isomorphism from \([Z,+] \) to \([Z,+].\)

a) \(f(x) = 5x\)

b) \(g(x) = x + 1\)

7) Can you find an isomorphism from \([5Z,+] \) to \([12Z, +]?\)