METCS 248 HW#4

1) Fill in the following table so that ({x,y,z}, *) forms a group.

*	x	y	z
x		y	
y			
z	z		

Identify the identity element, and the inverse element of x, y, and z.

2) Does [{0000, 0010, 1101, 1111}, +2] form a subgroup of [B4 , +2] ? If yes find all the cosets. (+2: represents addition modulo 2, B4 = BxBxBxB , where x is the Cartesian product, and B={0,1}).

3) Check if the following is a homomorphism or an isomorphism:

f: [B3 , +2] ------> [Z4 , +4] f(x) = w(x), w(x) is the weight of x i.e. the number of ones (1's) in the binary number x. [e.g. if x = 101, then w(x) = 2, if x = 111 then w(x)=3 .....etc.), Z4 = { 0, 1, 2, 3 }, +4: addition modulo 4, (1 +4 2 = 3, 3 +4 3 = 2 ...etc.)

4) Let G be the set of all-nonzero real numbers and let a*b= (a.b)/2. Show that [G,*] is an abelian group.

5) Let S = {a, b, c} and T = {x, y, z}, and let [S,*] and [T,#] be defined as:

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

#	x	y	z
x	z	x	y
y	x	y	z
z	y	z	x

Can you find an isomorphism between [S,*] and [T,#]?

6) Check if the following mappings form a homomorphism or an isomorphism from [Z,+] to [Z,+].

a) f(x) = 5x

b) g(x) = x + 1

7) Can you find an isomorphism from [5Z , +] to [12Z , +]?