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Unique Implementation of Incentive Contracts with Many Agents

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In this paper we consider the problem of implementation when a principal hires many agents and is not able to monitor their actions. We distinguish two cases: (i) when actions are mutually observable among agents, (ii) when actions are not observable at all. In (i), there is a mechanism in which the first-best arises as a unique perfect equilibrium. In (ii), we show by two examples that typically there are multiple equilibria if the principal merely offers a set of optimal sharing rules. However, we prove that the principal can use these optimal sharing rules as a starting point and construct a multi-stage mechanism that has a unique second-best perfect Bayesian equilibrium.

1. INTRODUCTION

Several papers have studied the principal-many agent problem: Holmström (1982), Demski and Sappington (1984), Mookherjee (1984) and Malcomson (1986). The major focus of these papers is the structure and welfare properties of the optimal incentive schemes (sharing rules) the principal offers agents who take unobservable actions in a correlated environment. These sharing rules are intended to implement a vector of agents' actions as an equilibrium whose outcome maximizes the principal's (second-best) welfare. However, the implementation problem has never been adequately addressed. Optimal sharing rules are in general not sufficient to guarantee the principal's desired actions as the only equilibrium in the agents' game. In other words, the incentive schemes may fail to implement uniquely the principal's preferred actions. This problem is most explicit when agents are assumed to use Nash equilibrium strategies, since typically there are multiple Nash equilibria. In many examples, as we show below, the Nash equilibrium the principal wishes to implement can involve agents choosing weakly-dominated strategies, or there may be other Pareto superior (from the agents' point of view) Nash equilibria. It is then not at all clear that the principal's preferred equilibrium will arise.

There have been attempts to avoid multiple equilibria, see Demski and Sappington (1984) and Mookherjee (1982, 1984). The previous approach has been to strengthen the set of incentive constraints so that the principal's preferred equilibrium is in some sense more stable. In other words, the principal is supposed to solve a more constrained maximization problem. Apart from the fact, as Mookherjee (1982) has shown, that this may be more awkward to solve, the principal's welfare, due to extra constraints, invariably decreases. However, as we demonstrate in this paper, there is an alternative way to solve the implementation problem: by considering game forms with larger strategy sets, the principal, exploiting features of the second-best incentive contracts, may be able to knock out unwanted equilibria. Such a method has proved useful in a "hidden information" model of Demski and Sappington—see Ma, Moore and Turnbull (1987). In that paper, besides a pair of incentive contracts, the principal also allows an agent to have more strategies. By choosing the extra strategies and outcome function judiciously, we can prove that the extended mechanism has a unique second-best Bayes–Nash equilibrium. Here we are interested in Mookherjee's model. This is an extension of Grossman and Hart (1983). To provide a solution to the multiple equilibria problem of this "hidden action" model, in addition to strategies we also put stages in the mechanism. Under some conditions and by appealing to sequential rationality in the sense of Kreps and Wilson (1982), we are able to implement actions as a unique perfect Bayesian equilibrium whose outcome is the same as the desired equilibrium outcome if the principal had offered just a set of incentive contracts. We therefore argue that there is no need for the principal to incur additional cost by resorting to strengthening incentive contracts. In fact, he can use the second-best contracts as a starting point to construct a "bigger" game to determine a unique (and still) second-best equilibrium outcome.

One other possibility that the recent literature has not explored is that even though the principal does not observe agents' actions, the agents nevertheless may be able to. It is not hard to imagine situations in which agents operate together and take mutually observable actions while there is no other party who can monitor them. In this case, the analysis proceeds quite differently. We are able to establish a first-best result: the principal may extract all relevant information about actions from agents. Indeed, under a mild condition, we present a multi-stage mechanism to implement any desired actions as a unique perfect equilibrium with first-best outcomes. In a recent paper, Moore and Repullo (1986) also study multi-stage mechanisms in a general model and provide necessary and sufficient conditions for sub-game perfect implementation of social choice correspondences. We note that the main distinction is that in their paper, the framework is "hidden information", while in ours, it is "hidden action".

The paper proceeds as follows. In Section 2, Mookherjee's many agent model is summarized. We discuss first-best contracts in a multi-stage mechanism when actions are mutually observable in Section 3. Section 4 gives two examples to illustrate the consequences of multiple equilibria in the case of privately observable actions. We introduce another multi-stage mechanism that (sequentially) uniquely implements second-best outcomes in Section 5. Last, conclusions are given in Section 6.

2. THE MODEL

For most of the paper we shall concentrate on the case of one principal contracting with two agents, although the results derived generalize to any finite number of agents and we shall point out the necessary extensions as we go along. A principal owns two production processes, $f^g(a, b, \theta^g)$, $g = A, B$, where (θ^A, θ^B) is a vector of discrete random variables which may be correlated, and (a, b) is a pair of actions to be decided by agents A and B respectively. For a pair of actions and a realization of the random variables, each production process generates one of a (finite) range of possible outputs. Let $X^A = \{x_1^A, \ldots, x_I^A\}$ and $X^B = \{x_1^B, \ldots, x_J^B\}$ be the possible outputs for f^A and f^B respectively. The sets of actions available to agents A and B are also finite and denoted by $A = \{a_1, \ldots, a_K\}$ and $B = \{b_1, \ldots, b_L\}$ respectively. It is more convenient to work with probability distributions on X^g, $g = A, B$ induced by a pair of actions and the random variables. Hence, we let $\pi_{ij}(a_k, b_l)$ be the probability of output pair (x_i^A, x_j^B) in $X^A \times X^B$ under actions (a_k, b_l) in $A \times B$. It is assumed that for each pair (x_i^A, x_j^B) there is at least one action pair (a_k, b_l) such that $\pi_{ij}(a_k, b_l) > 0$. With (a_k, b_l), the principal, who is risk neutral, derives a benefit $F(a_k, b_l) \equiv \sum_{ij} \pi_{ij}(a_k, b_l)\tilde{F}(x_i^A, x_j^B)$, for some function $\tilde{F}$. Agents have von Neumann-Morgenstern utility functions that are additively separable in income and action. Thus A's utility function is $V^A(Y^A) - G^A(a_k)$, where Y^A is income received from the principal. We shall assume $V^A(\cdot)$ to be strictly increasing and concave. For convenience, we do not put bounds on the utility function V^A, but none of the results in this paper relies on imposing arbitrarily heavy penalties on an agent. The utility function of B is analogously defined; however, to ease notation, we shall make the inessential assumption that A and B have identical utility functions, thereby dropping the superscripts. Each agent has a reservation utility $\bar{U}$. The random variables (θ^A, θ^B) are not observable to anyone. This completes the basic structure of the model.

We are concerned with the situation where the principal cannot directly monitor actions chosen by agents. Nevertheless, we distinguish two cases: (i) actions are mutually observable among agents, and (ii) actions are only privately observable. As mentioned in the introduction, there exists a mechanism in case (i) whereby the principal can effectively eliminate all incentive problems and achieve the first-best (perfect monitoring) welfare. This will be developed fully in the next section. Under case (ii), which is otherwise called a second-best situation, the principal must provide incentives for agents to perform according to his wish. This is accomplished by a departure from optimal risk sharing. In particular, payments may be contingent on both (random) outcomes, i.e. if (x_i^A, x_j^B) occurs, agent g is paid Y_{ij}^g, $g = A, B$, $i = 1, \ldots, I$ and $j = 1, \ldots, J$.

Mookherjee (1984) has characterized the second-best incentive contracts. In his analysis, for any given action pair, say (a_k, b_l) the principal is content to find incentive schemes (Y_{ij}^A, Y_{ij}^B) under the requirement that (a_k, b_l) are Nash equilibrium strategies. Thus a pair of (optimal) contracts defines a game for agents A and B whose strategy sets are respectively A and B, and the payments are given by (Y_{ij}^A, Y_{ij}^B). To find optimal (second-best) incentive schemes for action pair (a_k, b_l), it is convenient to take contingent utilities as the principal's control variables. Using the notation $h = V^{-1}$ and $V(Y_{ij}^g) = v_{ij}^g$, $g = A, B$, we may represent optimal contracts as a solution to the following programme: Choose (v_{ij}^A, v_{ij}^B) to minimize

$$\sum_{ij} \pi_{ij}(a_k, b_l)[h(v_{ij}^A) + h(v_{ij}^B)]$$

subject to

$$\sum_{ij} \pi_{ij}(a_k, b_l)v_{ij}^A - G(a_k) \geqq \bar{U}$$

$$\sum_{ij} \pi_{ij}(a_k, b_l)v_{ij}^B - G(b_l) \geqq \bar{U}$$

$$\sum_{ij} \pi_{ij}(a_k, b_l)v_{ij}^A - G(a_k) \geqq \sum_{ij} \pi_{ij}(a_m, b_l)v_{ij}^A - G(a_m), \quad a_m \text{ in } A$$

$$\sum_{ij} \pi_{ij}(a_k, b_l)v_{ij}^B - G(b_l) \geqq \sum_{ij} \pi_{ij}(a_k, b_n)v_{ij}^B - G(b_n), \quad b_n \text{ in } B.$$

Throughout this paper, we assume the solution (v_{ij}^A, v_{ij}^B) of the above problem exists. We shall sometimes call the second-best mechanism, as defined above, the Mookherjee game, or simply Γ.

In this paper, we are not directly concerned with the structure of (v_{ij}^A, v_{ij}^B). Nor are we interested in the choice of action pairs, which certainly depends on the function $F(a_k, b_l)$ and the solution of the above programme. Without loss of generality, in later sections, we shall assume that the principal prefers to implement certain given action pairs. We do rule out the uninteresting case where the principal implements an action that minimizes utility cost in the set of actions. In the following analysis, we find it helpful to imagine that the principal pays the agent in utility units. We therefore use the term "utility-payment" to mean the utility of income an agent receives. Thus utility-payment v is equivalent to income $h(v)$.

3. MUTUALLY OBSERVABLE ACTIONS AND THE FIRST-BEST

In this scenario, the principal, who cannot act as a direct monitor, wants to know exactly what the agents' choices of actions are in order to levy the appropriate reward or punishment on them. We shall assume that agents pick actions simultaneously, and afterwards they (but no one else) observe each other's action. It is also natural to postulate that there is a lag between the choices of actions and the realizations of outputs; if (x_i^A, x_j^B) and (a_k, b_l) occurred contemporaneously, then the principal could not utilize the (random) outputs as monitoring devices. We suppose that the principal wants to implement (a_f, b_f). Since agents are risk averse, optimal risk sharing implies that under perfect monitoring, an agent's reward is independent of the stochastic outputs. Hence the first-best utility-payments are $G(a_f)+\bar{U}$ and $G(b_f)+\bar{U}$.

The important point to note is that when agents know all about the actual actions, a mechanism (designed by the principal) induces a game of *perfect information* between them. Clearly, if the principal contemplates exploiting agents' information, agents need to send messages after actions have been taken. We assume that messages are publicly sent and verification is costless.

It is perhaps not too surprising that the principal can design incentives for agents to select (a_f, b_f). In fact, it is relatively easy to construct a mechanism in which action pair (a_f, b_f) with utility-payments $G(a_f)+\bar{U}$ and $G(b_f)+\bar{U}$ form *one* perfect equilibrium. Consider the following, simple mechanism. At Stage 1, actions are taken. At Stage 2, each agent relates to the principal which action the other agent has chosen. If A says B has done b_f, then B is paid $G(b_f)+\bar{U}$. Otherwise, B is paid δ, where $\delta < \min_b G(b)+\bar{U}$. Payments to A are analogously defined. It is easy to verify that the strategies of (a_f, b_f) at Stage 1, and at Stage 2, each agent announcing the action that has been taken by the other form a perfect equilibrium with first-best payoffs. The unsatisfactory aspect in this mechanism is that agents picking least costly actions at Stage 1 and agent A (resp. B) always saying b_f (resp. a_f) at Stage 2 constitute another perfect equilibrium.[1] Notice both agents are better off in the latter equilibrium than in the former. The fact remains that implementing (a_f, b_f) *uniquely* is not a trivial matter.

The principal needs a more subtle way to extract information about actions. Notice that while an agent can provide false information to the principal, the accuracy of this information is known to the other agent. This suggests that once an agent has reported on an action pair, the principal may appeal to the other agent for verification. Also, utility-payments contingent on outputs may be designed for monitoring purposes. Obviously, the aim is to make sure that agents find (a_f, b_f) attractive and first-best utility-payments are awarded in a unique equilibrium.

In the following analysis, we shall refer to

Condition (C.1).

$$\Pi(a_k, b_l) \neq \Pi(a_m, b_n) \quad \text{whenever } (a_k, b_l) \neq (a_m, b_n),$$

where $\Pi(a_k, b_l) = (\pi_{ij}(a_k, b_l))_{i,j}$.

(C.1) implies that each pair of actions induces a distinct probability distribution on outputs. The way (C.1) is used will be clear below.

To see how the principal can use one agent's report to examine the authenticity of another's, suppose A reports $(\hat{a}, \hat{b})$ concerning a pair of actions taken. Subsequently, B is allowed an opportunity to "challenge" A's veractiy. If B challenges, then he announces an alternative pair of actions. On reporting $(\tilde{a}, \tilde{b})$, B is entitled to a lottery $\varepsilon(\hat{d}, \tilde{d}) = (\varepsilon_{ij}(\hat{d}, \tilde{d}))$ contingent on (x_i^A, x_j^B), where $\hat{d} = \Pi(\hat{a}, \hat{b})$, $\tilde{d} = \Pi(\tilde{a}, \tilde{b})$, and $\varepsilon(\hat{d}, \tilde{d})$ satisfies

$$\textstyle\sum_{ij} \varepsilon_{ij}(\hat{d}, \tilde{d})\hat{d}_{ij} < 0 \quad \text{and} \quad \sum_{ij} \varepsilon_{ij}(\hat{d}, \tilde{d})\tilde{d}_{ij} > 0. \tag{1}$$

Notice that (C.1) implies that for any (a_k, b_l) in $A \times B$, A cannot announce $(\hat{a}, \hat{b}) \neq (a_k, b_l)$ but with $\Pi(\hat{a}, \hat{b}) = \Pi(a_k, b_l)$. Suppose the actual action pair is (a_k, b_l). How will B behave? This depends on A's report $(\hat{a}, \hat{b})$. First, if A reports $(\hat{a}, \hat{b}) \neq (a_k, b_l)$, then B prefers to challenge: with announcement $(\tilde{a}, \tilde{b}) = (a_k, b_l)$, the contingent utility-payment $\varepsilon(\hat{d}, \tilde{d})$ gives (in expected terms) $\sum_{ij} \varepsilon_{ij}(\Pi(\hat{a}, \hat{b}), \Pi(a_k, b_l))\pi_{ij}(a_k, b_l)$, which by (1) is positive. Second, if A's report is $(\hat{a}, \hat{b}) = (a_k, b_l)$, (1) implies that the expected value of $\varepsilon(\hat{d}, \tilde{d})$ corresponding to any $(\tilde{a}, \tilde{b})$ of B is $\sum_{ij} \varepsilon_{ij}(\Pi(a_k, b_l), \Pi(\tilde{a}, \tilde{b}))\pi_{ij}(a_k, b_l) < 0$. Hence B will avoid (falsely) accusing A. The point is that B's "challenge" is a signal that A has been lying and deserves punishment.

The remaining question is, knowing that he can, in some sense, cross-examine agents, can the principal successfully induce them to perform (a_f, b_f)? The precise details as to how this is achieved are given in Theorem 1.

Theorem 1. *Suppose* (C.1) *holds. Suppose further that actions are mutually observable among agents. Then the principal can design a multi-stage mechanism with a unique perfect equilibrium. In this equilibrium,* (a_f, b_f) *are taken, and utility-payments to agents are first-best, namely* $G(a_f) + \bar{U}$ *and* $G(b_f) + \bar{U}$.

Proof. Consider the following mechanism:

Stage 1: Both agents take actions simultaneously.
Stage 1+: Agents observe each other's action.
Stage 2: Agent A announces a pair of actions $(\hat{a}, \hat{b})$, where $(\hat{a}, \hat{b})$ in $A \times B$.
Stage 3: Agent B can either "agree" or "challenge". If B "challeges" then he announces $(\tilde{a}, \tilde{b})$, where $(\tilde{a}, \tilde{b})$ in $A \times B$ but $(\tilde{a}, \tilde{b}) \neq (\hat{a}, \hat{b})$.

Utility-payments are defined in Tables 1 and 2.

TABLE 1

Agent A's utility-payments

A's announcement	$\hat{a} = a_f$	$\hat{a} \neq a_f$
B "agrees"	$G(a_f) + \bar{U}$	$\min_a G(a) + \bar{U} - \delta$
B "challenges"	$G(a_f) + \bar{U} - \gamma$	$\min_a G(a) + \bar{U} - \delta - \gamma$

TABLE 2

Agent B's utility-payments

A's announcement	$\hat{b} = b_f$	$\hat{b} \neq b_f$
B "agrees"	$G(b_f) + \bar{U}$	$\min_b G(b) + \bar{U} - \delta$
B "challenges"	$G(b_f) + \bar{U} + (\varepsilon_{ij}(\hat{d}, \tilde{d}))$	$\min_b G(b) + \bar{U} - \delta + (\varepsilon_{ij}(\hat{d}, \tilde{d}))$

In the Tables, $\delta > 0$, $\gamma > [G(a_f) - \min_a G(a) + \delta] > 0$ and $\hat{d} = \Pi(\hat{a}, \hat{b})$, $\tilde{d} = \Pi(\tilde{a}, \tilde{b})$. $\varepsilon(\hat{d}, \tilde{d})$ satisfies (1) for any $\hat{d}$ and $\tilde{d}$. (The notation $K + (\varepsilon_{ij}(\hat{d}, \tilde{d}))$ means that agent B receives a utility-payment $K + \varepsilon_{ij}(\hat{d}, \tilde{d})$ contingent on (x_i^A, x_j^B).)

We claim that the following strategies form the unique perfect equilibrium of the game. Agent A takes a_f at Stage 1, and at Stage 2 reports honestly—i.e. reports whatever action pair chosen at Stage 1. Agent B takes b_f at Stage 1 and "agrees" at Stage 3 if and only if A is honest at Stage 2.

First, consider Stage 3 and suppose actions (a_k, b_l) have been taken at Stage 1. What is B's optimal move? There are two cases to consider. First, A has announced $(\hat{a}, \hat{b}) = (a_k, b_l)$. From (1) and Table 2, B always prefers to "agree". Second, A has announced $(\hat{a}, \hat{b}) \neq (a_k, b_l)$. Then B always "challenges" since (1) implies that B can improve his welfare by reporting $(\tilde{a}, \tilde{b}) = (a_k, b_l)$. These two cases summarize B's best responses at Stage 3 depending on any combination of previous moves at Stages 1 and 2. Thus, A knows that B "agrees" if any only if he is honest at Stage 2.

Next, consider Stage 2. What is A's best choice, given that B "challenges" if and only if he lies? From the property of γ, we have $G(a_f) + \bar{U} - \gamma < \min_a G(a) + \bar{U} - \delta$. Then from Table 1, we know that A prefers the utility-payments when B "agrees" to those when B "challenges". Therefore A is always honest, since he always prefers B to "agree".

Finally, consider Stage 1. If A takes a_k, $k \neq f$, then, regardless of B's action, he obtains less than $\bar{U}$, because he reports $\hat{a} \neq a_f$ at Stage 2. However, by choosing a_f and at Stage 2 reporting the actual action pair, A's payoff is $\bar{U}$, again regardless of B's action. We conclude that A's unique choice at Stage 1 is a_f. It is also obvious that B's unique best choice is b_f. Since A is honest at Stage 2 and B "agrees" at Stage 3, B gets his maximum payoff $\bar{U}$ by performing b_f.

To conclude, in equilibrium, (a_f, b_f) are chosen, and utility-payments to agents are first-best, namely $G(a_f) + \bar{U}$ and $G(b_f) + \bar{U}$. ||

The intuition behind Theorem 1 should be clear by now. The principal simply elicits information from one agent, (here A), and uses agent B's reaction as a policing device. Incentives to B are provided by demanding that any accusation be supported by the acceptance of a lottery. However, the lottery is valuable (in expected terms) if and only if agent A has lied. Furthermore, the principal fills up the utility-payment matrices so that, given agents' optimal announcements at subsequent stages, the proposed actions are also attractive to agents.

Generalizing Theorem 1 (together with an appropriate definition of (C.1)) to the case of three or more agents is very easy. We have seen how the principal can make agent A's report reliable by requesting B to verify it. Now that A is honest, the principal can compensate other agents according to information revealed by A. For example, assume there are three agents and the principal wants agent C to perform c_f. The principal can keep the structure of the mechanism in Theorem 1 above, i.e. A reports $(\hat{a}, \hat{b}, \hat{c})$; if B "challenges", he reports $(\tilde{a}, \tilde{b}, \tilde{c})$, and utility-payments to A and B are given by Tables 1 and 2. In addition, according as A reports $\hat{c} = c_f$ or $\hat{c} \neq c_f$, agent C is paid $G(c_f) + \bar{U}$ or $\min_c G(c) + \bar{U} - \delta$. Then from C's point of view, agent A is effectively supervising him, and he therefore finds action c_f his best choice.

4. TWO EXAMPLES OF MULTIPLE EQUILIBRIA

When actions are unobservable, due to the conflict between incentives and risk sharing, implementation of the first-best actions (a_f, b_f) at first-best cost is generally impossible.

Furthermore, since agents never observe any state variable in the course of their relationship with the principal, allowing them to send messages does not increase the principal's welfare. One can formally prove the above statements using the Revelation Principle; we shall not bother with the details here. In fact, the Mookherjee game, Γ, discussed in Section 2, appropriates to the principal *the* second-best welfare, *if* agents follow the actions (Nash equilibrium) prescribed by the principal. It is however crucial to notice that the sole concern of the optimal contracts (v_{ij}^A, v_{ij}^B) is to guarantee a certain pair of (second-best) actions, say (a_s, b_s), as a Nash equilibrium in Γ. This is tantamount to filling *one* column and *one* row of a payoff matrix judiciously; yet, (v_{ij}^A, v_{ij}^B) also define payoffs of agents' various other actions. The loophole is that the incentive constraints in the programme for optimal contracts impose no restrictions on payoffs when *both* agents deviate from the principal's preferred actions. We shall show by two examples that this can give rise to other equilibria in actions which may, from agents' perspectives, dominate (a_s, b_s). (We will return to these two examples at the end of the next section.)

Example 1: *Binary actions and outputs*

Each agent has available two actions, $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$. The random outputs have binary supports, $X^A = \{x_1^A, x_2^A\}$, $X^B = \{x_1^B, x_2^B\}$. Write $X = (x_1^A, x_2^A, x_1^B, x_2^B)$. The utility costs of actions are $G(a_1) = 10$, $G(a_2) = 7{\cdot}5$, $G(b_1) = 10$, $G(b_2) = 8{\cdot}5$; also $\bar{U} = 20$. The probability distributions on X induced by different action pairs are $\Pi(a_1, b_1) = (1/8, 1/4, 1/2, 1/8)$, $\Pi(a_1, b_2) = (1/8, 3/8, 3/8, 1/8)$, $\Pi(a_2, b_1) = (1/4, 1/4, 3/8, 1/8)$ and $\Pi(a_2, b_2) = (1/8, 1/8, 3/8, 3/8)$. Utility functions of income are $V(I) = \sqrt{(2I)}$, or equivalently $h(v) = 0{\cdot}5v^2$. We assume that the principal wants to implement[2] (a_1, b_1). Routine computations show that the following contingent utility-payments (on X) are optimal contracts: $v^A = (14, 30, 34, 30)$, $v^B = (30, 22, 34, 30)$. Given (v^A, v^B), we can also calculate agents' payoffs corresponding to other action pairs; the full payoff matrix is in Table 3. (A's utility payoff is on the top left corner of each cell.)

TABLE 3

	b_1		b_2	
	20		19·5	
a_1				
		20		20
	20		22	
a_2				
		19·5		22

Clearly, the principal's preferred actions, namely (a_1, b_1), are, for the agents, weakly dominated strategies! Moreover, (a_2, b_2) gives both agents higher payoffs than (a_1, b_1). It is not plausible that (a_1, b_1) should be an outcome of the game, and hence the implementation of (a_1, b_1) by (v^A, v^B) is questionable. We will insist that this example is entirely non-pathological: since the programme for optimal contracts *only* guarantees that (a_1, b_1) is a Nash equilibrium, the calculations of v^A and v^B do *not* involve $\Pi(a_2, b_2)$; there indeed exist an open set of examples such that (a_1, b_1) remain weakly dominated strategies. Also, it is not to be expected that in more complex models, the problem of multiple equilibria will disappear.

Example 2: Mookherjee's rank-order tournament

This example was first provided by Mookherjee (1984). Here, the problem is that in Γ, the equilibrium the principal wants is, from agents' point of view, Pareto dominated by many other equilibria. The example is special in the sense that even though actions are unobservable, the first-best is still feasible.

Agents have the following sets of actions $A=\{a_1, a_2, \ldots, a_K\}$, $B=\{b_1, b_2, \ldots, b_K\}$. The sets of possible outputs are $X^g=\{x_1^g, \ldots, x_I^g\}$, $g=A, B$, with $x_i^A=x_j^B$ *iff* $i=j$, and $x_i^A<x_{i+1}^A$ for $i=1, 2, \ldots, I-1$. Let X^A (resp. X^B) denote the random variable for x_i^A (resp. x_j^B). Costs of actions are ordered as follows: $G(a_k)<G(a_{k+1})$, $G(b_k)<G(b_{k+1})$, for $k=1, \ldots, K-1$. The stochastic structure satisfies $\pi_{ij}(a_k, b_l)=0$ iff either

(i) $x_i^A<x_j^B$ and $k\geqq l$, or

(ii) $x_i^A>x_j^B$ and $k\leqq l$, or

(iii) $x_i^A=x_j^B$ and $k\neq l$.

We can then say that A wins whenever A puts in relatively more effort than B, because with probability one, $X^A>X^B$. Similarly for B. When both agents put in (relatively) the same amount of effort, a tie, $X^A=X^B$, occurs with probability one. We assume that the principal wants to implement action pair (a_t, b_t), $1<t\leqq K$. Mookherjee (1984, p. 442) has offered the following rank-order tournament: a prize $\bar{U}+G(a_t)$ to A if $X^A\geqq X^B$, otherwise A is paid $\bar{U}+G(a_1)-\delta$, with $\delta>0$; a prize $\bar{U}+G(b_t)$ to B if $X^B\geqq X^A$, otherwise B receives only $\bar{U}+G(b_1)-\delta$. In this game (a_t, b_t) is a Nash equilibrium, but so is every other (a_k, b_k), $1\leqq k\leqq t$![3] As Mookherjee has pointed out, (a_1, b_1) is a Nash equilibrium that Pareto dominates all other equilibria, but the outcome of the tournament gives no clear indication whether (a_1, b_1) or (a_t, b_t) has been chosen. Once again, there are multiple equilibria in agents' game. Moreover, the equilibrium the principal prefers does not seem attractive to agents.

In view of the interest in rank-order tournaments shown in recent literature of optimal contracting with many agents (see Green and Stokey (1983), Holmström (1982), Lazear and Rosen (1981) etc.), we here propose a solution to this example. First, note that (a_k, b_k), $k<t$, is an equilibrium because there is no extra utility for winning. Second, the principal cannot distinguish (a_k, b_k) from (a_t, b_t) because no agent has any incentive to tell him. Can we make use of these two properties of the tournament to construct another game with bigger strategy sets to implement (a_t, b_t) uniquely?

Consider the following "modified" rank-order tournament. Let A have a range of extra strategies; specifically, A can announce an integer N, where $1\leqq N\leqq t$. Agent A chooses N when he takes his action, and more importantly, agent B does not know A's announcement when he picks an action. The complete utility-payment matrices are expressed in Tables 4 and 5.

TABLE 4

If A announces $N=t$, then

A's utility-payment		B's utility-payment	
If $X^A\geqq X^B$	$G(a_t)+\bar{U}$	If $X^B\geqq X^A$	$G(b_t)+\bar{U}$
If $X^A<X^B$	$G(a_1)+\bar{U}-\delta$	If $X^B<X^A$	$G(b_1)+\bar{U}-\delta$

TABLE 5

If B announces $N=k$, $1 \leqq k \leqq t-1$, then

A's utility-payment		B's utility-payment	
If $X^A \geqq X^B$	$G(a_t)+\bar{U}+(\varepsilon^k_{ii})$	If $X^B > X^A$	$G(b_t)+\bar{U}$
If $X^A < X^B$	$G(a_1)+\bar{U}-\delta$	If $X^B \leqq X^A$	$G(b_1)+\bar{U}-\delta$

In Tables 4 and 5, $\delta>0$, ε^k_{ii} is an (extra) utility-payment to A contingent on outputs (x^A_i, x^B_i) and satisfies, for $k=1,2,\dots,t-1$,

$$\textstyle\sum_i \varepsilon^k_{ii}\pi_{ii}(a_k, b_k)>0 \quad \text{and} \quad \sum_i \varepsilon^k_{ii}\pi_{ii}(a_t, b_t)<0. \tag{2}$$

(Note that the principal is assumed to implement (a_t, b_t) and therefore for any $k<t$, $\Pi(a_k, b_k) \neq \Pi(a_t, b_t)$. This implies (ε^k_{ii}) satisfying (2) exists.) The quantities δ and (ε^k_{ii}) are chosen[4] such that

$$G(a_{t+1})-G(a_t)>\delta+\max_i|\varepsilon^k_{ii}| \quad \text{for } k=1,2,\dots,t-1 \tag{3}$$

and

$$G(b_{t+1})-G(b_t)>\delta. \tag{4}$$

In the "modified" tournament, the principal wants to obtain some indication from an agent, here A, that (a_t, b_t) are not being taken. This is achieved via (ε^k_{ii}) and (2) by motivating A to call out $N \neq t$. Notice that the extra utility-payment ε^k_{ii} is available only if a tie occurs. Now if $N \neq t$ is reported, B has to win to gather a prize $G(b_t)+\bar{U}$. When B increases his effort, A can do no better than increasing his effort in step. Happily, the "modified" rank-order tournament has a unique (first-best) Nash equilibrium, as the following proposition shows.

Proposition 1. *The "modified" rank-order tournament has a unique Nash equilibrium, namely A calls out $N=t$, performs a_t and B performs b_t. In equilibrium, each agent obtains $\bar{U}$.*

Proof. First note that agent A never takes any action a_{t+l}, where $l \geqq 1$ (if $t<K$, that is). Since in any Nash equilibrium, a strictly dominated move can never occur, it suffices to show that these actions are strictly dominated by some other. An upper bound for A's utility under action a_{t+l}, from Table 5, is $G(a_t)+\bar{U}+\max_{i,k}|\varepsilon^k_{ii}|-G(a_{t+l})$. A lower bound for A's utility under action a_1, from Table 4, is $G(a_1)+\bar{U}-\delta-G(a_1)$. However, (3) implies that action a_{t+l} is strictly dominated by a_1. By a similar argument, using (4), we know that B never takes b_{t+l}, where $l \geqq 1$. The proposition is proved in four steps.

Step 1. The strategies in the statement of the proposition constitute a Nash equilibrium.

Suppose B chooses b_t. If A takes a_k, $k<t$, then A always loses, obtaining utility $G(a_1)+\bar{U}-G(a_k)-\delta<\bar{U}$, independent of his announcement. However, if A takes a_t, then by calling out $N=t$, he has $\bar{U}$, since a tie occurs with probability one. Indeed, this is his unique best response, since, by (2), reporting $N=k<t$ merely decreases his (expected) utility. It is easy to see that B's unique best response against a_t and $N=t$ is b_t.

Step 2. A only takes action a_k, $k=1,2,\ldots,t$, if he believes b_k occurs with positive probability.

To see this, note that from Tables 4 and 5, A never strictly benefits from winning (as distinct from a tie). Moreover, losing for A, which carries a utility less than $\bar{U}$, is never optimal, since he can always guarantee $\bar{U}$ by a_t and $N=t$. First, suppose b_1 never occurs, then A never selects a_1, since a_1 always loses. Next, suppose b_k never occurs, $k>1$, then the probability of losing induced by action a_k is identical to that of a_{k-1}. Thus, A's (expected) utility-payment cannot decrease if he selects a_{k-1} rather than a_k. But a_k is more costly than a_{k-1}. Hence A never takes a_k.

Step 3. Whenever A takes a_t, it is optimal for him to announce $N=t$. But if A takes a_k, $k<t$, then he should never announce $N=t$.

Consider a_t. From Step 2, b_t occurs with positive probability. If A selects a_t as a best response, then the event of a tie must be due to (a_t, b_t), and from (2), each of (ε_{ii}^k), $k=1,2,\ldots,t-1$ would strictly decrease A's expected utility. Thus A optimally chooses $N=t$. Next, consider a_k, $k<t$. Using a similar argument, we know that a tie only results from (a_k, b_k). Hence reporting $N=k<t$ is (in expected terms) suprior to $N=t$—by (2) again.

Step 4. There are no other equilibria other than the one in the statement of the Proposition.

Suppose not. Suppose in some equilibrium, a_k, $k<t$, is the least costly action among those A chooses. Note that from Step 3, he also announces some $N\neq t$ together with a_k. From Step 2, we need only consider whether B will choose b_k optimally.

Will B choose b_k? First, observe that B can always have $\bar{U}$ by taking b_t. This is because b_t is never a losing strategy for B. And yet if a tie occurs, A must also be saying $N=t$, which makes Table 4 applicable. If b_t actually wins for B, then of course A's announcement is irrelevant. Second, is b_k optimal for B? The answer is no. By assumption, A never takes an action that is less costly than a_k, and hence b_k can never win. Now either a tie occurs, in which case Table 5 applies (since $N\neq t$), or B loses. In both instances, B obtains less than $\bar{U}$. Thus b_k cannot be optimal.

Step 2 then implies that A never selects a_k. Contradiction. ||

Example 2 and its solution perhaps lends some support to the conjecture that while the Mookherjee game is susceptible to multiple equilibria problems, one may still be able to use "nuisance" strategies to eliminate those undesirable ones. We now explain how the principal may achieve this.

5. UNIQUE SECOND-BEST IMPLEMENTATION

Our discussions in Section 4 suggest that the principal's problem, when he employs more than one agent, is qualitatively different from a simple, constrained optimization problem. The issue of multiple equilibria certainly deserves more attention. A tentative solution that has been proposed (see Mookherjee, 1982) is to strengthen the (incentive) constraints in the programme for optimal contracts to make the second-best equilibrium, in some sense, more stable. But this is not desirable for the principal since strengthening constraints is costly.

On the other hand, the result in Example 2 is more encouraging. It shows that while adding on strategies (asking agents to report) never improves welfare, it does help with uniquely implementing the desired action pair. Can the idea generalize? It must be noted that an appealing feature in Example 2 is that by injecting a relatively small number of nuisance strategies, we eliminate all unwanted equilibria in the tournament. This presumably is due to the simple stochastic structure on outputs with respect to action pairs. Furthermore, we have a very good understanding of all equilibria. In an abstract, second-best model, these helpful clues are not going to be there. Are there more powerful tools?

On top of nuisance strategies, we may also use stages. Notice however the sets of Nash equilibria of an extensive form game and the corresponding normal form game are identical. Therefore, if multi-stage mechanisms are to be more useful than normal form mechanisms, a refinement of Nash equilibrium must be applied. In our model, actions are privately known and no proper subgame exists if agents are playing a multi-stage game. It is then natural to adopt sequential equilibrium or perfect Bayesian equilibrium as the relevant implementation concept. The main result in this section is that under some conditions, we are able to extend the Mookherjee game to a multi-stage game with a unique second-best perfect Bayesian equilibrium. In other words, we knock out all undesirable equilibria.

Some more notation is needed before we go on. Let (a, b), *without subscripts*, denote random actions, i.e. $\Pi(a, b) \equiv \sum_{kl} \alpha_k \beta_l \Pi(a_k, b_l)$, a_k in A, b_l in B; α_k, $\beta_l \geqq 0$, $\sum_k \alpha_k = \sum_l \beta_l = 1$. a therefore denotes a probability distribution (α_k) on A. Similarly for b. Let $\bar{D} = \{\Pi(a, b) \mid (a_k, b_l) \text{ in } A \times B\}$, i.e. $\bar{D}$ is the set of all distributions obtained by combining all possible random action pairs. We shall assume that the second-best action pair to be implemented is (a_s, b_s). Let $D = \bar{D} - \{\Pi(a_s, b_s)\}$. Also (v_{ij}^A, v_{ij}^B) are optimal incentive schemes for (a_s, b_s).

The following conditions will be made use of:

Condition (C.2).

$$\Pi(a_s, b) \neq \Pi(a_s, b_s) \quad \text{for any given } b \neq b_s,$$

and

$$\Pi(a, b_s) \neq \Pi(a_s, b_s) \quad \text{for any given } a \neq a_s,$$

where a and b are random actions.

(C.2) asserts that if agent A (resp. B) is taking action a_s (resp. b_s), then whenever agent B (resp. agent A) deviates from b_s (resp. a_s), the probability distribution on outputs will never be that of $\Pi(a_s, b_s)$. The first part of (C.2) says that the following system has a *unqiue* solution in $\beta_1, \ldots, \beta_l, \ldots, \beta_L$. (The second part has a similar meaning.)

$$\begin{aligned} \textstyle\sum_l \beta_l \pi_{ij}(a_s, b_l) &= \pi_{ij}(a_s, b_s), \qquad && i = 1, \ldots, I \text{ and } j = 1, \ldots, J, \\ \beta_l &\geqq 0, && l = 1, \ldots, L. \end{aligned}$$

The system has $I \times J$ equations and L inequalities. Also, note that the fact that $\Pi(a_k, b_l)$'s are probability distributions implies $\sum_l \beta_l = 1$. Clearly, if $\Pi(a_s, b_l)$'s are affine independent, then this part of (C.2) is satisfied. Since the vectors $\Pi(a_s, b_l)$, $l = 1, \ldots, L$ are generically affine independent if $I \times J + 1 > L$, this part of (C.2) is generically satisfied if $I \times J + 1 > L$. Similarly, if $I \times J + 1 > K$, the second part of (C.2) is generically satisfied. Note, however, that (C.2) may still be true for $I \times J + I \leqq K$ or $I \times J + 1 \leqq L$.

Condition (C.3).

$$\text{For any } k \neq s, k = 1, \ldots, K, \qquad \Pi(a_k, b) \neq \Pi(a_s, b_s),$$

where b is any random action of agent B.

$$\text{For any } l \neq s, l = 1, \ldots, L, \qquad \Pi(a, b_l) \neq \Pi(a_s, b_s),$$

where a is any random action of agent A.

(C.3) says that whenever agent A (resp. B) does not take action a_s (resp. b_s), then the probability distribution on outputs will never be $\Pi(a_s, b_s)$. Notice that the first part of (C.3) says that for any $k \neq s$, the following system has no solution in $\beta_1, \ldots, \beta_l, \ldots, \beta_L$. (The second part of (C.3) has an analogous meaning.)

$$\textstyle\sum_l \beta_l \pi_{ij}(a_k, b_l) = \pi_{ij}(a_s, b_s), \qquad i = 1, \ldots, I \text{ and } j = 1, \ldots, J,$$

$$\beta_l \geqq 0, \qquad l = 1, \ldots, L.$$

(In this system, there are $I \times J$ equations and L inequalities. Also, $\sum_l \beta_l = 1$.) If $I \times J + 1 > L$, then generically the system has no solution. Hence, for $I \times J + 1 > L$, this part of (C.3) is generically satisfied. Similarly, if $I \times J + 1 > K$, then the second part of (C.3) is generically satisfied. We can easily find a necessary and sufficient condition for (C.3) in terms of probability distributions $\Pi(a_k, b_l)$'s. By Farkas Lemma (see e.g. Takayama (1985), p. 46), (C.3) is equivalent to the following:

Condition (C.3′). For every $k \neq s$, $k = 1, \ldots, K$, there exists a vector $P^k \equiv (p_{11}, \ldots, p_{ij}, \ldots, p_{IJ})$ such that

$$\Pi(a_k, b_l) \cdot P^k > 0, \qquad l = 1, \ldots, L \qquad \text{and} \qquad \Pi(a_s, b_s) \cdot P^k < 0.$$

For every $l \neq s$, $l = 1, \ldots, L$, there exists a vector $Q^l \equiv (q_{11}, \ldots, q_{ij}, \ldots, q_{IJ})$ such that

$$\Pi(a_k, b_l) \cdot Q^l > 0, \qquad k = 1, \ldots, K \qquad \text{and} \qquad \Pi(a_s, b_s) \cdot Q^l < 0.$$

One last point, we find it convenient to assume in the Theorem that the implementation of (a_s, b_s) involves genuine second-best considerations. In other words, in the game Γ, assuming B (resp. A) is taking b_s (resp. a_s), A (resp. B) is indifferent between a_s (resp. b_s) and some other action a_k (resp. b_l) (incentive constraints are binding). However, the game form to be constructed does not rely on this assumption: for the case of the first-best being feasible, see footnotes 7 and 8.

We are now in a position to state and prove our main Theorem.

Theorem 2. *Suppose* (C.2) *and* (C.3) *hold. Suppose actions are privately observed. Then the principal can design a multi-stage mechanism with a unique perfect Bayesian equilibrium. In this equilibrium,* (a_s, b_s) *are taken, and utility-payments are second-best, i.e. agent g gets* v^g_{ij} *contingent on* (x^A_i, x^B_j), $g = A, B$.

Proof. Recall that in the Mookherjee game Γ, agents have strategy sets A and B, with utility-payment (v^A_{ij}, v^B_{ij}) contingent on (x^A_i, x^B_j). Moreover, by construction (a_s, b_s) is a Nash equilibrium in Γ. Define a multi-stage game $\bar{\Gamma}$ as follows:

Stage 1: Actions are taken.
Stage 2: Each agent says[5] either "Yes" or "No". If either both agents say "Yes", or both say "No", then the second-best schemes (v^A_{ij}, v^B_{ij}) are enforced. If

one agent, say y, has said "Yes", while the other, say n, has said "No", Stage 3 applies. (Of course, $y \neq n$, $y, n = A, B$.)

Stage 3: The agent who has said "Yes" at Stage 2 (y) announces $Y = 0, 1, 2, \ldots$. The agent who has said "No" at Stage 2 (n) announces $N = 1, 2, 3, \ldots$.

There are three cases to consider:

(i) If $Y \geqq N$, then the agent who has said "Yes" at Stage 2 announces d in D. This agent then has contingent utility-payment $v_{ij}^{y} + \varepsilon_{ij}(d)$, and the other agent has utility-payment $v_{ij}^{n} - \delta$.

(ii) If $N > Y > 0$, then the agent who has said "No" at Stage 2 announces d in D. This agent then has contingent utility-payment $v_{ij}^{n} + \varepsilon_{ij}(d)$, and the other agent has utility-payment $v_{ij}^{y} - \delta$.

(iii) If $Y = 0$, (v_{ij}^{A}, v_{ij}^{B}) are enforced.

(Note that since actions are not observable, the distinction between Stage 1 and Stage 2 is semantic and is made for expositional convenience.)

In the above mechanism, $\delta > 0$, $(\varepsilon_{ij}(d))$ satisfies,[6] for d in D,

$$\textstyle\sum_{ij} \varepsilon_{ij}(d) d_{ij} > 0 \quad \text{and} \quad \sum_{ij} \varepsilon_{ij}(d) \pi_{ij}(a_s, b_s) < 0, \tag{5}$$

and, for some $\bar{k}$, $\bar{l}$, where $\bar{k}, \bar{l} \neq s$,

$$\textstyle\sum_{ij} \pi_{ij}(a_{\bar{k}}, b_s)[v_{ij}^{A} + \varepsilon_{ij}(\Pi(a_{\bar{k}}, b_s))] - G(a_{\bar{k}}) > \sum_{ij} \pi_{ij}(a_s, b_s) v_{ij}^{A} - G(a_s) \tag{6a}$$

and

$$\textstyle\sum_{ij} \pi_{ij}(a_s, b_{\bar{l}})[v_{ij}^{B} + \varepsilon_{ij}(\Pi(a_s, b_{\bar{l}}))] - G(b_{\bar{l}}) > \sum_{ij} \pi_{ij}(a_s, b_s) v_{ij}^{B} - G(b_s). \tag{6b}$$

Recall that we assume some incentive constraint binds, we can just pick $a_{\bar{k}}$ (resp. $b_{\bar{l}}$) to be an action binding on a_s (resp. b_s). Then (5) automatically implies (6).[7]

Figure 1 provides a sketch of the mechanism.

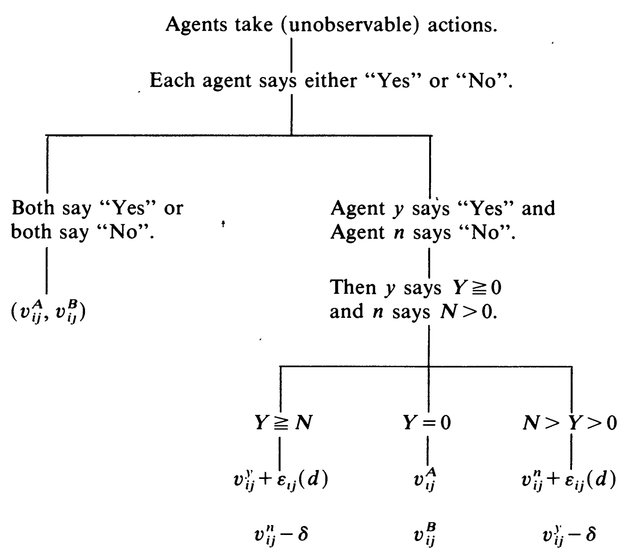

FIGURE 1

The Theorem is proved by the following three claims.

Claim 1. If (a, b) is a Nash equilibrium in Γ, then $(a, \text{Yes}, 0; b, \text{Yes}, 0)$ is a Nash equilibrium in $\bar{\Gamma}$.

(The notation $(a, \text{Yes}, 0)$ means that A adopts the strategy of action a at Stage 1, says "Yes" at Stage 2, and, if ever reached, announces $Y = 0$ at Stage 3. Similarly for $(b, \text{Yes}, 0)$.)

Claim 1 is fairly obvious. Suppose A says "Yes" and $Y = 0$, then (v_{ij}^A, v_{ij}^B) are enforced. B is therefore indifferent between all moves at Stages 2 and 3; he may as well choose (Yes, 0) always. If a is taken, then B cannot do better than b, since (a, b) is a Nash equilibrium in Γ. A symmetric argument establishes that $(a, \text{Yes}, 0)$ is a best response against $(b, \text{Yes}, 0)$.

Claim 2. In any Nash equilibrium of $\bar{\Gamma}$, *both* agents *always* say "Yes" and announce $Y = 0$.

We will first argue that if ever an agent (say A) is allowed to revise his reward scheme, (v_{ij}^A), by announcing some d after winning the integer game at Stage 3, he strictly prefers to do so. Note that an agent will never want to lose the integer game, since he can avoid the fine δ by announcing (Yes, 0). Thus in considering whether to announce some d, an agent will compare the expected payoff between (v_{ij}^A) and $(v_{ij}^A + \varepsilon_{ij}(d))$.

Assume that agent A believes B has picked action b. Suppose A takes an action a_k, where $k \neq s$. (C.3) says $\Pi(a_k, b) \neq \Pi(a_s, b_s)$. (5) implies that announcing d to revise the utility-payments to $(v_{ij}^A + \varepsilon_{ij}(\Pi(a_k, b)))$ is better than having (v_{ij}^A). Next, suppose that A takes a_s. There are two possibilities: either $b \neq b_s$ or $b = b_s$. In the former case, (C.2) says that $\Pi(a_s, b) \neq \Pi(a_s, b_s)$ so that by a similar argument we know that agent A strictly prefers to announce some d. Now consider the case $b = b_s$. (5) says none of the $(\varepsilon_{ij}(d))$ gives extra positive utility. However, agent A is indifferent between a_s and $a_{\bar{k}}$ under (v_{ij}^A). But because by (C.3) $\Pi(a_{\bar{k}}, b_s) \neq \Pi(a_s, b_s)$, action $a_{\bar{k}}$ and announcement $d = \Pi(a_{\bar{k}}, b_s)$ is superior to action a_s with no revision in the contingent utility-payments (see also (6a)). Hence, we conclude that A strictly wants to announce some d at Stage 3 if he is ever allowed to.

To establish Claim 2, suppose not, i.e. suppose that in some equilibrium there is at least one agent (say B) who says (Yes, $Y = 0$) with probability less than one. It suffices to show that there is no equilibrium in the "integer game" at Stage 3. Such will be the case when *both* agents want to win the "integer game" in order to announce some d in D.

Suppose B does not always say (Yes, 0). By the structure of Stage 3, (5) and (6a), agent A wants to force the game to Stage 3 as far as possible, and choose an integer big enough in order to announce some d at Stage 3. Therefore A never says (Yes, 0), because (Yes, 0) rules out any report of d at Stage 3. Now, given A never says (Yes, 0), by the structure of Stage 3, (5) and (6b), B prefers to win the integer game as well, regardless of A's choice of action. Hence there cannot be an equilibrium in which an agent announces (Yes, 0) with probability less than one.

To summarize, Claims 1 and 2 together prove that the sets of Nash equilibria of Γ and $\bar{\Gamma}$ are identical.[8] It remains to prove that only one Nash equilibrium in $\bar{\Gamma}$ is a perfect Bayesian equilibrium.

Claim 3. $(a_s, \text{Yes}, 0; b_s, \text{Yes}, 0)$ is the unique perfect Bayesian equilibrium in $\bar{\Gamma}$.

Proof. We begin by showing that $(a_s, \text{Yes}, 0; b_s, \text{Yes}, 0)$ is a perfect Bayesian equilibrium and then later show that it is the unique perfect Bayesian equilibrium.

In (a_s, Yes, 0; b_s, Yes, 0), Stage 3 is never reached. Are the prescribed moves at Stage 3 rational? Consider agent A first. Suppose A finds himself in the situation where B has said "No" at Stage 2. What does A conclude about B's action at Stage 1? We cannot apply Bayes' Rule directly since this occurs with zero probability. We specify that A believes B has taken b_s with probability one. Call this belief (*).

If Stage 3 is ever reached, assuming (*) and suppose A has taken a_s at Stage 1, A never wants to announce some d, since to him, each $(\varepsilon_{ij}(d))$ lowers his expected utility—by (5). On the other hand, he does not want to lose the integer game, since his utility-payment will decrease by δ. Thus, his unique optimal choice is to announce $Y=0$, independent of agent B's announcement. We conclude therefore (a_s, Yes, 0) is sequentially rational.

By a symmetric argument, (b_s, Yes, 0) is also sequentially rational. In sum, (a_s, Yes, 0; b_s, Yes, 0) is a perfect Bayesian equilibrium.

Now suppose $(a, b) \neq (a_s, b_s)$ is any other Nash equilibrium in Γ, is (a, Yes, 0; b, Yes, 0) a perfect Bayesian equilibrium in $\bar{\Gamma}$? The answer is no.

Without loss of generality, suppose $a \neq a_s$. Let $a_k \neq a_s$ be in the support of a (since a may be a random action). Suppose A discovered that B has said "No" at Stage 2. Given *any* belief, b^*, A has about B's action, (C.3) implies that $\Pi(a_k, b^*) \neq \Pi(a_s, b_s)$. For any $N(>0)$ announced by B at Stage 3, A would never optimally choose $Y=0$: by (5), A would strictly prefer to win the integer game at Stage 3 to revise the contract to $(v_{ij}^A + \varepsilon_{ij}(d))$, some d, so he must call out $Y>0$. This contradicts Claim 2. We therefore conclude that (a, Yes, 0; b, Yes, 0) is not a perfect Bayesian equilibrium. ||

It may be useful to provide the intuition behind Theorem 2. In the game $\bar{\Gamma}$, each agent has a "veto" (viz., calling out (Yes, $Y=0$)) to enforce the second-best schemes. Also, in $\bar{\Gamma}$, under (C.2) and (C.3) an agent always finds some $\varepsilon(d)$ (revising (v_{ij}^A, v_{ij}^B)) attractive, whenever he is allowed to play the integer game; but if one agent gains, the other agent loses. In equilibrium, such competing moves to revise (v_{ij}^A, v_{ij}^B) in the "integer game" at Stage 3 cannot happen. Hence in $\bar{\Gamma}$, each Nash equilibrium must involve agents' utility-payment being determind by (v_{ij}^A, v_{ij}^B). This requires that each agent always exercises his "veto" in equilibrium. To understand the importance of sequential rationality, suppose the distinction between Stage 2 and Stage 3 were dropped. Then all those nuisance strategies in $\bar{\Gamma}$ would *not* eliminate even one undesirable equilibrium in Γ—in all simple, Nash equilibria of $\bar{\Gamma}$, (v_{ij}^A, v_{ij}^B) are always enforced, making the sets of equilibrium actions in Γ and $\bar{\Gamma}$ identical. However, there *are* stages in $\bar{\Gamma}$, and plays *are* sequential. When it *actually* comes to Stage 3, according to sequential rationality, an agent must act *optimally*, basing on his belief about actions at Stage 1. Will the agent who has said "Yes" really say $Y=0$? In other words, is it "credible" that the "veto" is exercised? Under (C.3), $Y=0$ is optimal only when (a_s, b_s) have been chosen at Stage 1, because none of the $\varepsilon(d)$'s gives extra positive expected utility. Under any other actions, an agent strictly wants to revise (v_{ij}^A, v_{ij}^B) to benefit himself. Thus saying $Y=0$ cannot be optimal at Stage 3. By appealing to sequential rationality, we have then knocked out all unwanted equilibria.[9]

Theorem 2 can be generalized to the case of $T \geqq 3$ agents. In a T-agent second-best contract, agent t is offered a T-dimensional contingent utility-payment, v^t, $t=1, 2, \ldots, T$, that pays according to the occurrence of a T-dimensional (random) output vector. A T-vector of actions $(a^1, \ldots, a^T)$ generates a T-dimensional probability distribution $\Pi(a^1, \ldots, a^T) \equiv \Pi(a)$. Assume that the principal wants to implement $(a_s^1, \ldots, a_s^T)$, and (v^t) is a vector of optimal contracts. $\bar{D}$, D and Conditions (C.2) and (C.3) are defined similarly.

Theorem 2A. *Consider a T-agent model. Under Conditions* (C.2) *and* (C.3), *the following game has a unique perfect Bayesian equilibrium in which* $a_s = (a_s^1, \ldots, a_s^T)$ *are taken with second-best contingent utility-payments.*

Stage 1: *Agents take actions.*

Stage 2: *Each agent can say either "Yes" or "No". If either all T agents say "Yes", or all T agents say "No", then the second-best schemes* (v^t) *are enforced. Otherwise Stage* 3 *applies.*

Stage 3: *Those agents, say r, who have said "Yes" announce* $Y_r = 0, 1, 2, \ldots.$ *Those agents, say s, who have said "No" announce* $N_s = 1, 2, 3, \ldots.$

There are three cases to consider:

(i) If all the agents who have said "Yes" at Stage 2 say $Y_r = 0$, then the second-best schemes are enforced.

Otherwise, let $Y = \max_r (Y_r)$, $y = \arg\max_r (Y_r)$,

$$N = \max_s (N_s),\; n = \arg\max_s (N_s).$$

(If there are multiple y and/or n, select arbitrarily.)

(ii) If $Y \geqq N$ then agent y announces d in D and agent y has a utility-payment $v^y + \varepsilon(d)$.
All other agents have $v^r - \delta$ or $v^s - \delta$.

(iii) If $N > Y$ then agent n announces d in D and agent n has a utility-payment $v^n + \varepsilon(d)$.
All other agents have $v^r - \delta$ or $v^s - \delta$.

$$\varepsilon(d) \text{ satisfies } \varepsilon(d) \cdot d > 0 \text{ and } \varepsilon(d) \cdot \Pi(a_s) < 0, \text{ and}$$

$$\Pi(a_k^t, a_s^{-t}) \cdot [v^t + \varepsilon(\Pi(a_k^t, a_s^{-t}))] - G(a_k^t) > \Pi(a_s) \cdot v^t - G(a_s^t),$$

some $a_k^t \neq a_s^t$, $t = 1, \ldots, T$ and $a_s^{-t} = (a_s^1, \ldots, a_s^{t-1}, a_s^{t+1}, \ldots, a_s^T)$.

The proof of Theorem 2A is omitted since it so closely matches that of Theorem 2: Claims 1 and 2 go through directly. For Claim 3, it is not difficult to see that under action vector a_s, and some belief, those agents who have said "Yes" are indeed going to announce $Y_r = 0$.

Finally, we would like to note that the stochastic structure in Example 1 in the last section satisfies (C.2) and (C.3) and hence the multiple equilibria problem in that example can be solved. Also, in general the stochastic structure in Example 2 does not satisfy the conditions in Theorem 2 so that (C.2) and (C.3) are not necessary for unique implementation.

6. CONCLUSION

We have shown in this paper how the principal can achieve the first-best when actions are known to agents. When actions are unobservable, to guarantee the second-best as a unique equilibrium, the principal has to consider a more subtle game form than just offering agents a set of optimal sharing rules. We show in Theorems 2 and 2A how this game $\bar{\Gamma}$ can be constructed.

Perhaps, a drawback of $\bar{\Gamma}$ is that agents' strategy sets are quite large. This presumably is because $\bar{\Gamma}$ has to apply to any model (satisfying (C.2)+(C.3) of course). Indeed, in an abstract model, there is very little information about the set of equilibria of Γ. When we extend Γ to $\bar{\Gamma}$, the only useful property of Γ is that (a_s, b_s) is an equilibrium. However, we suspect in specific examples, as Proposition 1 demonstrates, the "nuisance" strategies required need not always be too complex.

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NOTES

1. In fact, in this game, with δ small enough, any pair of actions can be sustained as equilibrium strategies at Stage 1.
2. It is easy to find $F(a_k, b_l)$ so that (a_1, b_1) is optimal with respect to F and the costs of implementation.
3. With δ small enough, (a_{t+l}, b_{t+l}), $t+l \leqq K$, is also a Nash equilibrium. Also, there may be mixed strategy equilibria.
4. That is, only if $t < K$. If not, then (3) and (4) are unnecessary.
5. All announcements are public and costless. As in Section 3, we also assume that realizations of outputs and the choices of actions are not contemporaneous.
6. If one wants, one can let $\varepsilon_{ij}(d) \to 0$ as $d \to \Pi(a_s, b_s)$, all i, j.
7. If none of the incentive constraint binds, then (6) is still possible when ε's are chosen big enough.
8. When none of the incentive constraint binds, Claim 2 will be modified slightly, essentially because under (a_s, b_s), equilibrium moves at Stages 2 and 3 are not uniquely determined. There are other equilibria: agents choose (a_s, b_s) and randomize between (Yes, 0) and (No, N), or between (Yes, 0) and (Yes, $Y > 0$), with big enough probability on (Yes, 0); or one agent always chooses (Yes, 0) and the other agent randomizes between (Yes, 0), (Yes, $Y > 0$) and (No, N), with big enough probability on (Yes, 0). But such equilibria give rise to *identical* outcome paths as (a_s, Yes, 0; b_s, Yes, 0), i.e. utility-payments are (v_{ij}^A, v_{ij}^B). Notice that if (a_s, b_s) are equilibrium actions, no agent would like to announce d, because of (5). Also, no agent would want to lose the integer game, because of the penalty δ. When (a_s, b_s) are equilibrium actions, (Yes, 0) must be announced with sufficiently high probability. Otherwise, by (6), an agent will deviate from a_s to $a_{\bar{K}}$. It is easy to show that the moves described above are equilibrium moves at Stages 2 and 3. Moreover, such Nash equilibria *are* perfect Bayesian equilibria, since Stage 3 is reached with positive probability.
9. We wish to note that (C.2) and (C.3) are not the only (sufficient) conditions for unique implementation. In an earlier version of this paper, we also show that Theorem 2 is true under some other conditions. The interested reader is referred to Ma (1987).

REFERENCES

DEMSKI, J. and SAPPINGTON, D. (1984), "Optimal Incentive Contracts with Multiple Agents", *Journal of Economic Theory*, **33**, 152-171.

GREEN, J. and STOKEY, N. (1983), "A Comparison of Tournaments and Contracts", *Journal of Political Economy*, **91**, 349-364.

GROSSMAN, S. and HART, O. D. (1983), "An Analysis of the Principal-Agent Problem", *Econometrica*, **51**, 7-45.

HOLMSTRÖM, B. (1982), "Moral Hazard in Teams", *Bell Journal of Economics*, **13**, 324-340.

KREPS, D. and WILSON, R. (1982), "Sequential Equilibria", *Econometrica*, **50**, 863-894.

LAZEAR, E. and ROSEN, S. (1981), "Rank-Order Tournaments as Optimum Labour Contracts", *Journal of Political Economy*, **89**, 841-864.

MA, C. T. (1987), "Unique Implementation of Incentive Contracts with Many Agents" (STICERD Discussion Paper, 87/146, LSE).

MA, C. T., MOORE, J. and TURNBULL, S. (1987), "Stopping Agents From Cheating", *Journal of Economic Theory* (forthcoming).

MALCOMSON, J. (1986), "Rank-Order Contracts for a Principal with Many Agents", *Review of Economic Studies*, **53**, 807-817.

MOOKHERJEE, D. (1982), "Optimal Incentive Schemes in Multi-agent Situations" (STICERD Discussion paper, 82/57, LSE).

MOOKHERJEE, D. (1984), "Optimal Incentive Schemes with Many Agents", *Review of Economic Studies*, **51**, 433-446.

MOORE, J. and REPULLO, R. (1986), "Subgame Perfect Implementation", *Econometrica* (forthcoming).

TAKAYAMA, A. (1985), *Mathematical Economics, (2nd ed.)* (Cambridge: Cambridge University Press).