RAND Journal of Economics
Vol. 26, No. 1, Spring 1995
pp. 1–19

# Regulating a dominant firm: unknown demand and industry structure

**Gary Biglaiser***

**and**

**Ching-to Albert Ma****

*In this article, we study the optimal regulation of a dominant firm when an unregulated firm actively competes. Generally, the existence of an active rival imposes new and binding constraints on regulatory problems. We characterize optimal policies both when demands are known (complete information) and unknown (incomplete information) to the regulator. Optimal policies under complete information may set the price at the dominant firm above or below its marginal cost. Optimal policies under incomplete information may be either pooling or separating, constant over a range of the prior distribution of the firm's private information, and leave no information rent to the firm.*

## 1. Introduction

■ Most of the previous theoretical regulation literature concentrates on the study of monopoly; see, for example, Baron and Myerson (1982), Laffont and Tirole (1986), and Lewis and Sappington (1988) and the references they cite. Although the monopoly model is a convenient and sometimes realistic model for analysis, most regulated firms do face rivals. In many major industries, such as telecommunication, energy, and express mail, a dominant, regulated firm has to compete with unregulated firms; in addition, the entry of unregulated firms may be encouraged by a process of deregulation. For example, before and after its divestiture, AT&T was a regulated, dominant firm in the long-distance telephone market and was competing vigorously against unregulated rivals such as MCI and Sprint.

In this article, we investigate the optimal regulation of a dominant firm that faces an unregulated rival. Our model embodies two central assumptions. First, we assume that the regulated firm possesses private information about the relative demands of the firms, but that the firms' cost information is common knowledge. Second, we suppose that the

* University of North Carolina.
** Boston University.

We thank Michael Riordan, Jacques Thisse, and Ingo Vogelsang for very helpful discussions, and we are grateful to James Friedman, Robert Rosenthal, and David Sappington for their comments on an earlier version. We also thank Coeditor Michael Whinston and two referees for their detailed and useful suggestions. The Industry Study Program at Boston University provided research support. The second author also thanks the Management Science Group at the Veterans Administration, Bedford, MA, for financial support.

unregulated firm possesses some market power, acting as a Stackelberg follower by choosing its price after the dominant firm's price is set via a regulatory mechanism.

Our focus on an asymmetry of information about relative demands is particularly appropriate in light of the recent wave of deregulation. Very often when competitors are allowed to enter previously foreclosed markets, the incumbent firm is still subject to regulation. With the entry of a new firm and the introduction of a new or differentiated product, previous history about demand need not give the regulator sufficient information about relative demands of the dominant firm and its new rival. On the one hand, it is natural to assume that firms possess better information about consumer preferences and proclivity to choose one product over another because of their contact with consumers from past transactions or market surveys. On the other hand, it is plausible that firms' private information about costs is of second-order importance because the regulator has seen past cost realizations.

The assumption that the unregulated firm possesses some market power is a significant departure from the existing literature on regulation of a firm facing unregulated rivals. In this literature, the usual assumption has been that the unregulated firm is a competitive fringe that either produces outputs to satisfy residual demand until the point at which price is equal to the fringe firm's marginal cost of production (Cournot competition), or sets a price equal to its marginal cost (Bertrand competition); see Caillaud (1990) and Lewis and Sappington (1989).

In the case of the regulation of a monopolist, Lewis and Sappington (1988) demonstrate that, when a monopolist's marginal costs are nondecreasing, its private information about demand is inconsequential for the design of optimal regulatory mechanisms. That is, the regulator will implement the same allocations as if the monopolist's knowledge about demand were public information. Thus, none of the incentive constraints is binding; the monopolist earns no information rent and will supply socially efficient levels of outputs.

The information structure of our formal model resembles that of Lewis and Sappington (1988), and we assume constant marginal costs of production. However, we obtain strikingly different results. The cause of these differences can be traced to the market power exercised by the unregulated firm. Indeed, although the existence of the unregulated firm strictly enhances welfare, the design of regulatory policies must consider a tradeoff between the efficient distribution of consumers across firms and the welfare loss due to excess profits, even under complete information.

This tradeoff arises because the unregulated firm's profit-maximizing price is negatively related to the consumers' demand for the dominant firm's product. Furthermore, this price is always above marginal cost, thus enabling the unregulated firm to earn a strictly positive profit. Given this reaction, the regulator can implement the efficient distribution of consumers among the two firms only by setting the regulated firm's price above marginal cost. When the social welfare function puts more weight on consumer surplus than on profits, the regulator will wish to distort the distribution of consumers across firms to reduce profits. In fact, the higher the welfare weight for consumer surplus, the lower is the optimal price at the regulated firm and the more distortionary the allocation becomes. In the extreme, as the weights on firms' profits in social welfare approach zero, the dominant firm's price is set below marginal cost.

Hence, the complete-information benchmark already is a second best, and prices at the firms are not equal to marginal costs. As a result, an information rent for the regulated firm may exist when the regulator does not observe the firms' demand information. Thus, in addition to correcting for the misallocation due to the excessive profits and inefficient distribution of consumers across firms, the optimal (third-best) regulatory policy must limit the dominant firm's information rent.

The third-best, incomplete-information policies exhibit a rich variety of properties. First, if the welfare weight on consumer surplus is low, the optimal policy can be either separating or pooling. Also, for a range of the regulator's prior beliefs, identical separating policies may be offered and the regulated firm may not earn information rent. In addition, in a separating equilibrium, a dominant firm with a high demand will be offered a price identical to the optimal price under complete information; in a pooling equilibrium, the price will be between the complete-information optimal prices for low- and high-demand dominant firms. Second, if the welfare weight on consumer surplus is sufficiently high, the optimal policies must be separating. The optimal price for a dominant firm with low demand will be the optimal price under complete information. But again, for a range of the regulator's prior beliefs, the same optimal policies will be offered, and the information rent is entirely extracted.

Thus, our policy recommendations differ markedly from those in the literature, such as Baron and Myerson (1982) and Laffont and Tirole (1986), in which separation is generally optimal and the optimal incentive scheme depends strictly monotonically on the regulator's prior beliefs. Also, although pooling mechanisms are known to be important because of countervailing incentives (Lewis and Sappington, 1989), or the regulator's lack of commitment ability (Laffont and Tirole, 1988), such problems are absent in our model. Here, pooling arises because the complete-information optimal prices may be decreasing with respect to the dominant firm's demand, whereas incentive-compatibility conditions require a nondecreasing relationship between prices and demand. Because the relationships between prices and demands implied by optimality and incentive compatibility are opposites of each other, optimal mechanisms may set identical prices for firms with different demands.[1]

The article that is closest in spirit to ours is Caillaud (1990). He found that in a Baron-Myerson (1982) class of regulation models of cost uncertainty, the existence of a set of competitive fringe firms that charge prices equal to (*ex post*) marginal cost can help limit a monopolist's informational advantage. Although fringe firms are passive in Caillaud's model, we think that his basic result will hold when they behave as Stackelberg followers (as in our model). A few articles in the public enterprise literature also consider the effects of strategic rivals on regulatory policies; see Harris and Wein (1980), Braeutigam (1984), and Ware and Winter (1986). In these models, the public firm is controlled directly by the regulator, who picks the product price at the public firm to maximize an index of social welfare. The public firm may have to compete with an unregulated private firm in an imperfectly competitive market. The main focus of these models has been the effect of competition on the Ramsey pricing formulas and social welfare. None of these models, however, allows any asymmetry of information. Furthermore, the regulated firm in our model remains private and seeks only to maximize profits.[2]

We present our model in Section 2. In Section 3, we derive the efficient planning benchmark and the optimal regulatory mechanism under complete information. In Section 4, we analyze the optimal policies under incomplete information. Finally, in Section 5, we draw conclusions. All proofs are found in the Appendix.

## 2. The model

■ We consider regulating a dominant firm, firm D, when regulatory policies cannot be imposed on a rival, firm R. Our concern is the design of optimal regulatory mechanisms

[1] Lewis and Sappington (1988) also show that, if a monopolist's marginal costs are decreasing and it possesses private information about demand, the optimal mechanism to regulate it must be pooling.

[2] Recently, a group of articles considered the direct regulation of industry structure. Auriol and Laffont (1992), Dana and Spier (1994), and McGuire and Riordan (forthcoming) allow the regulator to determine whether a market should be served by one firm or two firms when the regulator is uninformed about costs. Wolinsky (1993) also lets the regulator split the market unevenly between the regulated firms. Our model differs from these models in two ways: we consider demand uncertainty, and we do not allow the regulator to control the regulated firm's rival. Because of these differences, results here are not directly comparable to those in the above articles.

when firms possess private information about consumer demands. This private information about demands is characterized by a parameter $K$, commonly known to firms but unknown to the regulator. One interpretation of the asymmetry of information is that firms can predict relative demands more accurately than can the regulator.

Formally, $K$ represents consumers' valuation differentials between products of the dominant firm and its rival. Thus, the demand parameter $K$ may reflect consumers' perceived quality differences between, or proclivities for, the two firms' products. For simplicity, we assume that $K$ is a binary random variable, with support $\{K_L, K_H\}$, where $0 < K_L < K_H$. The probabilities on $K_L$ and $K_H$ are $\mu$ and $1 - \mu$, respectively; these probabilities are common knowledge. Let $\bar{K}$ denote $\mu K_L + (1 - \mu)K_H$. Even though both firms know the value of the demand parameter, it is convenient to describe the information structure only in terms of firm D's knowledge. A firm D who knows that the demand parameter is $K_i$ is called a type-$i$ firm D, often abbreviated firm to $D_i$, $i = L, H$.

Both horizontal and vertical product differentiations are present in the model. We assume a continuum of consumers with total mass $l$ and density normalized to one. Each of these consumers values the products sufficiently highly and will buy a good from either the dominant firm D or the unregulated firm R. Consumers' valuations in the horizontal product differentiation dimension are described by a uniform distribution. Let $x$ be uniformly distributed on the interval $[0, l]$. If the demand parameter is $K$, a consumer with index $x$ obtains a (gross) benefit of $K - tx$ if he buys from firm D, where $t > 0$ is a parameter. If a consumer with index $x$ buys from firm R, he obtains a benefit of $-t(l - x)$. Equivalently, the distribution of valuations can be interpreted as horizontal product differentiation à la Hotelling: consumers are uniformly distributed on a line of length $l$, with firm D being located at the origin and firm R at point $l$. A consumer located at $x$, where $0 \le x \le l$, has to incur either a disutility of $tx$ if he purchases from firm D, or a disutility of $t(l - x)$ if he purchases from firm R, where $t > 0$ can be regarded as the "mismatch" or "transportation cost" parameter.

Consumers' utility functions are linear in benefits from products and money. If $P_D$ and $P_R$ are the respective prices at firm D and firm R, then $K - tx - P_D$ and $-t(l - x) - P_R$ are the respective net utilities for the consumer with index $x$ if he purchases a good from these firms.

Each consumer buys from the firm that offers the higher utility. Hence, firm D's demand function is easily derived:

$$x(K, P_D, P_R) = \begin{cases} l & tl < K - P_D + P_R \\ l/2 + (K - P_D + P_R)/2t \quad \text{if} & 0 \le K - P_D + P_R \le tl \\ 0 & K - P_D + P_R < 0 \end{cases}. \tag{1}$$

Firm R's demand function is simply $l - x(K, P_D, P_R)$. We shall assume that $K_H < tl$, so that, in all the allocations to be derived, both firms will obtain strictly positive market shares.

The unit cost of production at each firm is constant and normalized to zero. Each firm maximizes expected profits. We assume that firm R is an active Stackelberg follower; it reacts to firm D's action through its pricing policy.[3] Under this assumption, the unregulated firm has more flexibility when adjusting its pricing policies than does the regulated firm. This appears to be a realistic assumption and is consistent with the regulatory practices in the telecommunication and other industries. For example, until recently, AT&T's price

[3] In Section 5, we shall discuss the alternative assumptions that the firms move simultaneously and that the regulator can commit to a policy of adjusting firm D's price contingent on firm R's reaction to firm D's initial price.

change applications went through a hearing process at the Federal Communications Commission before their final approval, whereas MCI, Sprint, and other unregulated companies were free to change prices.

The unregulated firm's price response distinguishes our model from those in the literature, in which the usual assumption is that the unregulated firm is a passive fringe with no market power. We believe this assumption severely limits the range of applications of those models. In the context of the regulation of a dominant firm, unregulated rival firms often have significant market power. Thus, our model captures a very important aspect of the market structure that often has been ignored.

Given $P_D$ and $K$, firm R chooses $P_R$ to maximize profit, $P_R(l - x(K, P_D, P_R))$ . When firms have positive market shares, firm D's demand is given by the middle part of (1). Thus, firm R's price best-response function is given by

$$P_R(K, P_D) = \frac{tl - K + P_D}{2}. \tag{2}$$

If we substitute firm R's best-response function (2) into firm D's demand function (1), then we obtain firm D's reduced-form demand function:

$$X(K, P_D) = \frac{3}{4}l + \frac{K - P_D}{4t}. \tag{3}$$

This reduced-form demand function is also firm D's market share.

Although firm R's pricing policy is positively related to firm D's price, its optimal price reaction is only half of firm D's price change. Moreover, firm R's optimal price is negatively related to $K$: it charges a lower price to compensate for its disadvantage due to consumers' higher valuations of firm D's products. Finally, even if the regulator sets firm D's price at marginal cost, firm R has no incentive to do so. Instead, by (2), firm R will price above marginal cost and profit by reducing its market share.

The regulator's policy instruments for the dominant firm are menus of two-part tariffs, each of which consists of a lump-sum transfer and a per-unit product price. Although in principle the dominant firm's product price and transfer can be functions of its sales, we believe that such policies are difficult to implement in practice. As other authors have argued, quantity and accounting information can be hidden or manipulated easily (Lewis and Sappington, 1988, 1989).

In regulatory regimes, the regulator's objective is to maximize a welfare index equal to the weighted sum of aggregate consumer utility[4] less any lump-sum transfer firm D receives, firm D's profit, and firm R's profit, with corresponding weights $1 - \alpha_D - \alpha_R$, $\alpha_D$, and $\alpha_R$, where each $\alpha$ is strictly between zero and one-third. Aggregate consumer utility is

$$\int_0^{X(K,P_D)} [K - P_D - tx]dx + \int_{X(K,P_D)}^{l} [-P_R(K, P_D) - t(l - x)]dx,$$

or, equivalently,

$$[K - P_D]X(K, P_D) - \frac{tX(K, P_D)^2}{2} - P_R(K, P_D)[l - X(K, P_D)] - \frac{t[l - X(K, P_D)]^2}{2}. \tag{4}$$

[4] Because consumers' utility function is separable in benefits and money, aggregate utility is the same as (gross) consumer surplus.

Although the unregulated firm does not necessarily price at marginal cost, welfare always improves by its presence in the market. If firm D is a monopolist, then the optimal regulatory policy sets its price at marginal cost (Lewis and Sappington, 1988). With the unregulated rival in the market, a feasible policy is to price firm D's product at marginal cost. Compared to the monopoly case under optimal regulation, welfare must increase: those consumers who continue to buy from firm D are equally well off, whereas those who switch to buy from firm R must be better off, profits at firm D remain at zero, and profits at firm R must be positive. The argument is equally valid when firms possess private information about demands. Hence, even in the incomplete-information model in Section 4, welfare improves from firm R's presence in the market.

## 3. Efficient allocation and complete-information optimal regulation

■ We now establish two benchmarks. We start by deriving the efficient allocation when firm D's knowledge about demand is public information. In this planning regime, the regulator can directly allocate consumers and control productions at the two firms. Therefore, an allocation is simply defined by $x$, where consumers with indexes in $[0,x]$ obtain the product from firm D and the remaining consumers obtain the product from firm R. The efficient allocation, $X^P$, is one that maximizes aggregate consumer utility, $Kx - tx^2/2 - t(l - x)^2/2$. Straightforward computation yields $X^P = (K + tl)/2t$. Observe that, when the regulator can set prices at both firms, the efficient planning allocation can be implemented by setting prices at both firms equal to their marginal cost of zero and then asking firms to satisfy demands.

Even if firm R is unregulated and sets its price according to (2), the efficient allocation of consumers is still feasible. From firm R's best-response function (2), we know that, if $P_D$ is set at $tl - K$, then firm R will respond by choosing the same price. With equal prices at the two firms, firm D has a market share of $X^P$. Nevertheless, when $P_D = P_R = tl - K$, both firms earn strictly positive profits. Although firm D's profit may be taxed away through a transfer, firm R's profit may be undesirable because the regulator puts a relatively smaller weight on the firm's profits than on consumer surplus ($\alpha_R < 1/3$). Thus, even under complete information, the regulator faces a distribution tradeoff. The implementation of the efficient allocation results in a welfare loss due to the excessive profits at firm R. Likewise, reducing the profit at firm R by lowering firm D's price results in an inefficient allocation of consumers among the two firms.

We characterize the complete-information optimal regulatory policy for firm D when firm R is unregulated. A regulatory policy on firm D is a two-part tariff, $(T_D, P_D)$. Given the policy and price $P_R$ at firm R, the aggregate consumer utility is (4) less $T_D$. Profits at firm D and firm R are $P_D X + T_D$ and $P_R(l - X)$, respectively. The regulator's objective is to choose a policy $(T_D, P_D)$ to maximize a weighted sum of aggregate consumer utility and profits at firm D and firm R subject to (2) and (3). As both $\alpha_D$ and $\alpha_R$ are less than one-third, for any $P_D$, $T_D$ will be chosen to make firm D's profits zero. Hence, we have $T_D = -P_D X$. Substituting this expression for $T_D$ into the objective function, we eliminate the $T_D$ instrument. We can now write out the program as follows: choose $P_D$ to maximize

$$(1 - \alpha_D - \alpha_R)\left\{K X - \frac{tX^2}{2} - P_R(l - X) - \frac{t(l - X)^2}{2}\right\} + \alpha_R(P_R(l - X))$$

subject to (2) and (3). The next proposition characterizes the optimal regulatory policy.

*Proposition 1*. Suppose firms' knowledge about demand is public information. The optimal regulatory policy sets firm D's product price at

$$P_D^\dagger = (K - tl)\frac{1 - \alpha_D - 3\alpha_R}{3 - 3\alpha_D - 5\alpha_R}. \tag{5}$$

The optimal transfer $T^{\dagger}$ is chosen to make firm D's profit zero when its product price is $P_D^{\dagger}$. Moreover, firm D's market share is always greater than its efficient share $X^P$.

Proposition 1 says that the optimally regulated price for firm D depends crucially on $A$, where

$$A \equiv \frac{1 - \alpha_D - 3\alpha_R}{3 - 3\alpha_D - 5\alpha_R}.$$

Because $K < tl$, firm D's regulated price $P_D^{\dagger}$ has the opposite sign of $A$. For $\alpha_D$ and $\alpha_R$ close to one-third, $A$ is negative; however, as they decrease, $A$ becomes positive. So when the regulator has a large valuation for profits ($\alpha_D$ and $\alpha_R$ close to one-third), the dominant firm's price is above marginal cost, and vice versa for a low valuation. It is easy to verify that, when $\alpha_D$ and $\alpha_R$ are both decreased from one-third to zero, the solution $P_D^{\dagger}$ is decreased from $tl - K$ to $(K - tl)/3 < 0$ and firm D's market share is increased from $X^P$ to $(5tl + K)/6$.

The intuition for Proposition 1 is as follows. First, consider the limiting case $\alpha_D = \alpha_R = 1/3$. In this case, the objective function puts equal weights on consumer utility and profits. Thus, there is no welfare loss due to excess profits. From (5), $P_D^{\dagger}$ then becomes $tl - K$ and firm D's market share becomes $X^P$. This confirms our earlier intuition that the efficient allocation $X^P$ is feasible and implemented when excess profits do not decrease welfare.

For $\alpha_D$ and $\alpha_R$ less than one-third, profits at the firms become a welfare loss. It is straightforward to verify that the firms' equilibrium profits are increasing in $P_D$, so to reduce firms' profits, $P_D^{\dagger}$ must be decreased from $tl - K$. Firm D's market share increases from $X^P$ as a result of a decrease in price from $P_D^{\dagger}$. Depressing $P_D$ thus reduces the welfare loss due to excessive profits at the expense of allocating too many consumers to firm D. As the welfare weights $\alpha_D$ and $\alpha_R$ continue to decrease from one-third, the regulated price at firm D must also continue to decrease from $tl - K$. In the extreme, when $\alpha_D$ and $\alpha_R$ are sufficiently close to zero, $P_D^{\dagger}$ will be less than marginal cost.

Observe that the larger the value of $K$, the smaller is the deviation of $P_D$ from marginal cost. In equilibrium, a larger value of $K$ increases firm D's market share (and reduces firm R's) and lowers firm R's price. It can be shown that the effect of a marginal change in $P_D$ on welfare is proportional to both firm R's market share and price. Thus, as the value of $K$ increases, the change in welfare due to a change in $P_D$ becomes smaller, and there is less gain in welfare from deviating from marginal cost pricing.[5]

From the discussion of the effects of changing $\alpha_D$ and $\alpha_R$ on the optimal policy, it is clear that $P_D$ is set so as to reduce firm R's profits at the expense of distorting the efficient allocation of consumers. Because $K$ measures consumers' higher valuation of firm D's products over firm R's, a higher value of $K$ means that firm D has larger market power and that firm R will earn less profits. Thus, setting firm D's price different from marginal cost to reduce firm R's profit becomes less important for larger values of $K$.

Finally, observe that optimally regulated prices under complete information may be nonmonotonic with respect to the demand parameter. Suppose $P_L^{\dagger}$ and $P_H^{\dagger}$ are, respectively,

[5] Suppose $P_D$ is raised from 0, while a transfer $T$ compensates firm D's profit. This has two first-order effects in the regulator's objective function: $P_R$ constant, and $P_R$ with $P_D$ constant. The first is the change in firm R's profit, $X_R(\partial X_R/\partial P_D)dP_D$, where $X_R = 1 - X$; because of $T$ and the marginal consumer's indifference, consumer welfare is constant. The second concerns only consumers buying from $R$, $X_R(\partial P_R/\partial P_D)dP_D$—by the envelope theorem, firm R's profit is unchanged. With firm R's optimal price for $P_D = 0$ and $X_R = P_R/2t$, we have $\partial P_R/\partial P_D = t\partial X_R/\partial P_D = 1/2$. So the objective function varies according to $X_R$, which is proportional to $K - tl$. With $P_D$ at other prices, there are other terms proportional to $P_D$. We thank Michael Whinston for this interpretation.

the prices from Proposition 1 for $K = K_L$ and $K = K_H$, with $K_L < K_H$. Then, if $A > 0$, we have $P_H^\dagger > P_L^\dagger$. But if $A < 0$, then we have $P_H^\dagger < P_L^\dagger$. So for $A > 0$, the higher the demand parameter, the higher is the optimally regulated price for firm D. But for $A < 0$, the higher the demand parameter, the smaller is the optimally regulated price (compare Figures 1 and 2 below). As we shall see, this nonmonotonicity property turns out to be important for characterizing the incomplete-information optimal regulatory policy.

In summary, optimal regulation under complete information already incorporates second-best distribution considerations. When firms possess private information, the regulator must consider the distortion due to information rents. Hence, regulatory mechanisms must consider three kinds of tradeoffs: efficiently allocating consumers between firms, limiting firm R's profits, and limiting firm D's information rents.

## 4. Optimal regulation under incomplete information

■ We now study the regulation model when firm D's relative demand is unknown to the regulator. A regulatory policy on firm D is a menu of two-part tariffs (items in the menu are allowed to be identical). Because there are two possible values for the demand parameter, $K_L$ and $K_H$, we assume that a menu contains two schemes: $\{(T_L, P_L), (T_H, P_H)\}$, where $P_i$ denotes a product price and $T_i$ denotes a lump-sum transfer, $i = L, H$. The dominant firm will select a scheme from the menu.

Recall that a firm D with demand parameter $K_i$ is called firm $D_i$. Without loss of generality, suppose that, in an equilibrium, firm $D_L$ selects $(T_L, P_L)$ and firm $D_H$ selects $(T_H, P_H)$. Using firm D's reduced-form demand function (3), we can compute firm $D_L$'s profits when $(T_L, P_L)$ and $(T_H, P_H)$ are selected. These are, respectively,

$$P_L\left[\frac{3}{4}l + \frac{K_L - P_L}{4t}\right] + T_L \quad \text{and} \quad P_H\left[\frac{3}{4}l + \frac{K_L - P_H}{4t}\right] + T_H.$$

Firm $D_L$ selects $(T_L, P_L)$ when the incentive-compatibility constraint (IC–L) holds:

$$P_L\left[\frac{3}{4}l + \frac{K_L - P_L}{4t}\right] + T_L \geq P_H\left[\frac{3}{4}l + \frac{K_L - P_H}{4t}\right] + T_H. \tag{IC–L}$$

Similarly, the incentive-compatibility constraint for firm $D_H$, (IC–H), is

$$P_H\left[\frac{3}{4}l + \frac{K_H - P_H}{4t}\right] + T_H \geq P_L\left[\frac{3}{4}l + \frac{K_H - P_L}{4t}\right] + T_L. \tag{IC–H}$$

Moreover, each firm must earn nonnegative profits to participate so that the respective participatory constraints, (PC–L) and (PC–H), are

$$P_L\left[\frac{3}{4}l + \frac{K_L - P_L}{4t}\right] + T_L \equiv \Pi(K_L) \geq 0, \tag{PC–L}$$

$$P_H\left[\frac{3}{4}l + \frac{K_H - P_H}{4t}\right] + T_H \equiv \Pi(K_H) \geq 0. \tag{PC–H}$$

Consider an equilibrium in which firm $D_i$ selects $(T_i, P_i)$, $i = L, H$. Suppose firm D's relative demand is $K_i$, then aggregate consumers' utility less transfer becomes

$$(K_i - P_i)X_i - \frac{tX_i^2}{2} - P_{Ri}(l - X_i) - \frac{t(l - X_i)^2}{2} - T_i \equiv U_i,$$

where

$$P_{Ri} = \frac{tl - K_i + P_i}{2} \quad \text{and} \quad X_i = \frac{3}{4}l + \frac{K_i - P_i}{4t}. \tag{6}$$

The regulator's objective function is the expectation of his payoffs over states of firm D's relative demands:

$$\mu\{(1-\alpha_D-\alpha_R)U_L+\alpha_D\,\Pi(K_L)+\alpha_R[P_{RL}(l-X_L)]\}$$
$$+(1-\mu)\{(1-\alpha_D-\alpha_R)U_H+\alpha_D\,\Pi(K_H)+\alpha_R[P_{RH}(l-X_H)]\}. \quad (7)$$

The regulator chooses $P_L$, $P_H$, $T_L$, and $T_H$ to solve the following program: maximize (7) subject to the incentive and participatory constraints (IC–L), (IC–H), (PC–L), and (PC–H).

The constraints in the above program are identical to those that would arise if firm D were a monopolist facing a demand function given by (3).[6] If either firm D is a monopolist or firm R is a passive firm always charging price at marginal cost, then the optimal regulation is for firm D's price to be marginal cost. This implements the efficient allocation even if firms possess private information—a simple extension of the Lewis and Sappington (1988) efficiency result when marginal cost is nondecreasing.

Our model is different from the usual monopoly model, because the dominant firm's rival sets its price strategically. Whereas firm D cannot benefit from its private information if the regulator sets its price at marginal cost and transfer at zero, a discrepancy between price and marginal cost can turn into a source of information rent when firms possess private information about demands. Suppose that the price at firm D is set above marginal cost, a regime in Proposition 1 in which firms' profits are given large valuations, or $A < 0$. Then, from (IC–H) and (PC–L), we get $\Pi(K_H) \geq P_L(K_H - K_L)/4t > 0$. Thus, firm $D_H$ will earn strictly positive profits, or information rent. Likewise, suppose that the price at firm D is set below marginal cost, a regime in Proposition 1 in which firms' profits are given small valuations, or $A > 0$. From (IC–L) and (PC–H), we obtain $\Pi(K_L) \geq P_H(K_L - K_H)/4t > 0$. Here, firm $D_L$ will earn information rent. Thus, optimal policies must suppress information rent, and which of the two types of firm D may earn information rent is directly dependent on the value of $A$.

Before presenting the next result, we define

$$B \equiv \frac{1-2\alpha_D-\alpha_R}{3-3\alpha_D-5\alpha_R} > 0$$

$$\underline{\mu} \equiv \frac{2(1-2\alpha_D-\alpha_R)(K_H-K_L)}{2(1-2\alpha_D-\alpha_R)(K_H-K_L)-(1-\alpha_D-3\alpha_R)(tl-K_L)} \equiv \frac{1}{\dfrac{A(K_L-tl)}{2B(K_H-K_L)}+1}$$

$$\bar{\mu} \equiv \frac{2(1-2\alpha_D-\alpha_R)}{1-3\alpha_D+\alpha_R}.$$

If $A < 0$ and $K_H < tl$, we have $0 < \underline{\mu} < \bar{\mu} < 1$. Let $(P_i^\dagger, T_i^\dagger)$ denote the complete-information optimal policy for firm $D_i$, $i = L, H$; from Proposition 1, $P_i^\dagger = (K_i - tl)A$. The next proposition completely characterizes the optimal regulatory policies when the welfare weights on profits are relatively large.

*Proposition 2*. Suppose $A < 0$. At the solution to the program for the incomplete-information optimal policy, incentive constraint (IC–H) and participatory constraint (PC–L) always bind. The optimal regulatory policy $\{(T_L^*, P_L^*), (T_H^*, P_H^*)\}$ is given as follows.

---

[6] Observe also that, because demand functions are linear, on any isoprofit curve firm D's marginal rate of substitution between price and transfer is monotonic in its demand $K$; the "single-crossing" property is satisfied. (That is, the ratio between the partial derivatives of profit with respect to price and transfer is monotonic in $K$.)

1. For $0 \leq \mu \leq \underline{\mu}$, where $\mu$ is the prior probability of $K = K_L$, incentive constraint (IC–L) does not bind and participatory constraint (PC–H) binds. The optimal prices are

$$P_L^* = 0 \qquad P_H^* = P_H^\dagger,$$

and the optimal transfers are

$$T_L^* = 0 \qquad T_H^* = -P_H^*\left[\frac{3}{4}l + \frac{K_H - P_H^*}{4t}\right] = T_H^\dagger.$$

2. For $\underline{\mu} < \mu \leq \bar{\mu}$, neither incentive constraint (IC–L) nor participatory constraint (PC–H) binds; firm $D_H$ earns strictly positive profit. The optimal prices are

$$P_L^* = P_L^\dagger - 2B(K_H - K_L)\frac{1-\mu}{\mu} > 0 \qquad P_H^* = P_H^\dagger,$$

and the optimal transfers are

$$T_L^* = -P_L^*\left[\frac{3}{4}l + \frac{K_L - P_L^*}{4t}\right] \qquad T_H^* = P_L^*\frac{K_H - K_L}{4t} - P_H^*\left[\frac{3}{4}l + \frac{K_H - P_H^*}{4t}\right].$$

3. For $\bar{\mu} < \mu \leq 1$, incentive constraint (IC–L) binds and participatory constraint (PC–H) does not bind. The optimal prices are

$$P_L^* = P_H^* = (\bar{K} - tl)A - 2B(K_H - K_L)(1-\mu) > P_H^\dagger,$$

and the optimal transfers are

$$T_L^* = T_H^* = -P_L^*\left[\frac{3}{4}l + \frac{K_L - P_L^*}{4t}\right].$$

Figure 1 illustrates the optimal prices in this proposition. We provide the intuition for our results. Under the hypothesis in Proposition 2, the complete-information optimal policies for firm D satisfy $0 < P_H^\dagger < P_L^\dagger$ and ensure that each firm D earns zero profit. To begin, observe that the properties of the complete-information policies imply that firm $D_H$ will prefer the policy intended for firm $D_L$ if $\{(P_L^\dagger, T_L^\dagger), (P_H^\dagger, T_H^\dagger)\}$ is offered:

$$\begin{aligned} 0 &= P_H^\dagger\left[\frac{3}{4}l + \frac{K_H - P_H^\dagger}{4t}\right] + T_H^\dagger \\ &= P_L^\dagger\left[\frac{3}{4}l + \frac{K_L - P_L^\dagger}{4t}\right] + T_L^\dagger < P_L^\dagger\left[\frac{3}{4}l + \frac{K_H - P_L^\dagger}{4t}\right] + T_L^\dagger. \end{aligned}$$

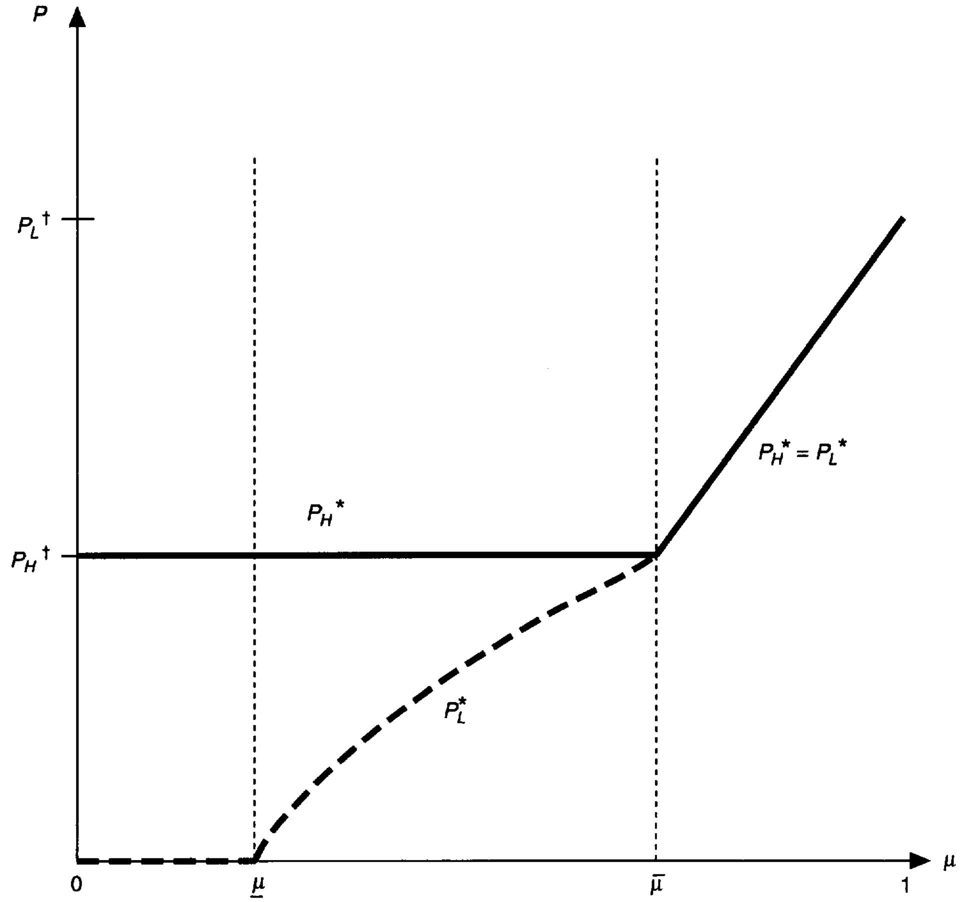
Because its demand is higher, firm $D_H$ will benefit from the higher price–lower transfer policy for firm $D_L$. So constraint (IC–H) should be binding: the incomplete-information optimal policy must deter firm $D_H$ from mimicking firm $D_L$.

Next, from incentive constraints (IC–L) and (IC–H), we have $(K_H - K_L)(P_H - P_L) \geq 0$, so that $P_L \leq P_H$. These two conditions—binding (IC–H) and prices nondecreasing in firm D's demand—are common in asymmetric information models; see Cooper (1984). In fact, they will also imply that the remaining incentive constraint (IC–L) is satisfied. Indeed, if (IC–H) binds, it can be rewritten as

$$P_H\left[\frac{3}{4}l + \frac{K_L - P_H}{4t}\right] + T_H + (P_H - P_L)(K_H - K_L)\frac{1}{4t} = P_L\left[\frac{3}{4}l + \frac{K_L - P_L}{4t}\right] + T_L.$$

Clearly, with $P_L \leq P_H$, this equation implies (IC–L). Thus, it is sufficient to maximize

FIGURE 1
OPTIMAL REGULATORY PRICES IN PROPOSITION 2 ($A < 0$).




the objective function (7) subject to (IC–H), (PC–L), (PC–H), and the monotonicity constraint $P_L \leq P_H$. Obviously, at a solution, participatory constraint (PC–L) must bind so that $\Pi(K_L) = 0$.

As we discussed before Proposition 2, we can combine constraints (IC–H) and (PC–L) to obtain $\Pi(K_H) \geq \Pi(K_L) + P_L(K_H - K_L)/4t$. Hence, the term $P_L(K_H - K_L)/4t$ is firm $D_H$'s information rent. Notice that this information rent is positively related to the price for firm $D_L$. So, to reduce information rent, $P_L$ should be lowered. Moreover, because (PC–L) binds and $\Pi(K_L) = 0$, the participatory constraint (PC–H) is satisfied if and only if $P_L \geq 0$. Thus, (PC–H) can be replaced by $P_L \geq 0$.

The optimal policies in our model are different from those of the standard model in two ways. First, in the solution to the standard model, the monotonicity constraint $P_L \leq P_H$ does not ordinarily bind, from which complete separation follows readily. Surprisingly, in the solution to our model, this monotonicity constraint may bind and identical policies may be offered to both types of firm D. Second, in the standard model, the participatory constraint on the type for whom the incentive constraint binds, namely, (PC–H) for firm $D_H$ in our model, can be ignored. Again, surprisingly, both (PC–L) and (PC–H) may bind in our model; firm $D_H$ need not earn information rent, and the optimal policy for firm $D_H$ will become identical to its corresponding complete-information optimal policy.

The first possibility comes from the following property of the complete-information optimal policies: $0 < P_H^\dagger < P_L^\dagger$; that is, the ranking of complete-information optimal prices is opposite to that of the monotonicity requirement. When the prior probability for firm $D_L$, $\mu$, is close to one, it becomes important to set $P_L$ close to $P_L^\dagger$. To satisfy the monotonicity constraint, however, $P_H$ must be raised accordingly. Because $P_H^\dagger < P_L^\dagger$, when $P_L$ is close to $P_L^\dagger$, the monotonicity condition $P_L \leq P_H$ must imply that $P_H > P_H^\dagger$; that is, monotonicity already pushes $P_H$ higher than $P_H^\dagger$. Setting $P_H$ strictly above $P_L$ therefore exacerbates the distortion on firm $D_H$. Hence, when $\mu$ is sufficiently high, the monotonicity constraint binds: $P_H = P_L$.

Next, observe that the (identical) price for both types of firm D will never be above $P_L^\dagger$. Lowering any price above $P_L^\dagger$ to $P_L^\dagger$ will eliminate distortion for firm $D_L$ and decrease information rent for and distortion on firm $D_H$. Furthermore, the closer $\mu$ is to one, the closer $P_L$ should be to $P_L^\dagger$. These observations together explain the last part of Proposition 2: whenever $\mu$ is sufficiently close to one, $P_L = P_H < P_L^\dagger$, and $P_L$ will increase toward $P_L^\dagger$ as $\mu$ tends to one.

The smaller the prior probability $\mu$ of firm $D_L$, the lower is the cost of setting $P_L$ different from $P_L^\dagger$. For medium values of $\mu$, the tradeoff between the benefit from limiting firm $D_H$'s information rent (by lowering $P_L$) against the cost of maintaining a small distortion on firm $D_L$ (by increasing $P_L$ toward $P_L^\dagger$) will favor the former. Hence, when $\mu$ is sufficiently smaller than one, it becomes optimal to set $P_L$ strictly less than $P_H$: the monotonicity constraint does not bind, and different policies will be offered to the two types of firm D. In this regime, because (IC–H) binds, the price for firm $D_L$ will be distorted below $P_H$, whereas firm $D_H$'s price will be set equal to $P_H^\dagger$. This summarizes the result in the second part of Proposition 2.

Finally, for $\mu$ sufficiently close to zero, the tradeoff is further in favor of limiting firm $D_H$'s information rent; it becomes optimal to depress $P_L$ all the way to marginal cost (zero), because the effect of the distortion on firm $D_L$ is less significant. Observe that setting price below marginal cost is both infeasible and suboptimal even if it were feasible. First, it is infeasible because when (IC–H) binds, (PC–H) is satisfied if and only if $P_L \geq 0$. Second, even if setting $P_L$ below marginal cost is feasible, it will not be used. This is because the information rent for firm $D_H$ already vanishes when $P_L = 0$ and firm $D_H$ is offered its complete-information optimal policy. Moreover, as $P_L^\dagger > 0$, reducing $P_L$ below marginal cost further exacerbates the distortion on firm $D_L$. These reasons explain the first part of Proposition 2.

We now consider the optimal policies when the welfare weights on profits are relatively low. We define

$$\mu^* \equiv \frac{(1 - 3\alpha_D - 3\alpha_R)(tl - K_H)}{2(1 - 2\alpha_D - \alpha_R)(K_H - K_L) + (1 - 3\alpha_D - 3\alpha_R)(tl - K_H)} \equiv \frac{1}{1 + \dfrac{2B(K_H - K_L)}{A(tl - K_H)}},$$

which satisfies $0 < \mu^* < 1$ for $A > 0$.

*Proposition 3*. Suppose $A > 0$. At the solution to the program for the incomplete-information optimal policy, incentive constraint (IC–L) and participatory constraint (PC–H) always bind, whereas incentive constraint (IC–H) never binds. The optimal regulatory policy $\{(T_L^*, P_L^*), (T_H^*, P_H^*)\}$ is given as follows.

1. For $0 \leq \mu \leq \mu^*$, the participatory constraint (PC–L) does not bind. The optimal prices are

$$P_L^* = P_L^\dagger \qquad P_H^* = P_H^\dagger + 2B(K_H - K_L)\frac{\mu}{1-\mu},$$

and the optimal transfers are

$$T_L^* = P_H^* \frac{K_L - K_H}{4t} - P_L^*\left[\frac{3}{4}l + \frac{K_L - P_L^*}{4t}\right] \qquad T_H^* = -P_H^*\left[\frac{3}{4}l + \frac{K_H - P_H^*}{4t}\right].$$

2. For $\mu^* < \mu \leq 1$, the participatory constraint (PC–L) binds. The optimal prices are

$$P_L^* = P_L^\dagger \qquad P_H^* = 0,$$

and the optimal transfers are

$$T_L^* = -P_L^*\left[\frac{3}{4}l + \frac{K_L - P_L^*}{4t}\right] = T_L^\dagger \qquad T_H^* = 0.$$

Figure 2 illustrates the optimal prices in the proposition. Under the hypothesis in Proposition 3, the complete-information optimal regulatory policy sets firm D's prices below marginal cost, and the corresponding optimal transfers compensate firm D for its operating losses. In this regime, if the complete-information optimal policies are offered, firm $D_L$ will prefer the policy intended for firm $D_H$:

$$\begin{aligned} 0 &= P_L^\dagger\left[\frac{3}{4}l + \frac{K_L - P_L^\dagger}{4t}\right] + T_L^\dagger \\ &= P_H^\dagger\left[\frac{3}{4}l + \frac{K_H - P_H^\dagger}{4t}\right] + T_H^\dagger < P_H^\dagger\left[\frac{3}{4}l + \frac{K_L - P_H^\dagger}{4t}\right] + T_H^\dagger \end{aligned}$$

because $P_L^\dagger < P_H^\dagger < 0$ and $K_H > K_L$. Due to its smaller demand, firm $D_L$ will reduce its operating loss by picking a higher price. We conclude that constraint (IC–L) should be binding: the incomplete-information optimal policy must deter firm $D_L$ from mimicking firm $D_H$.

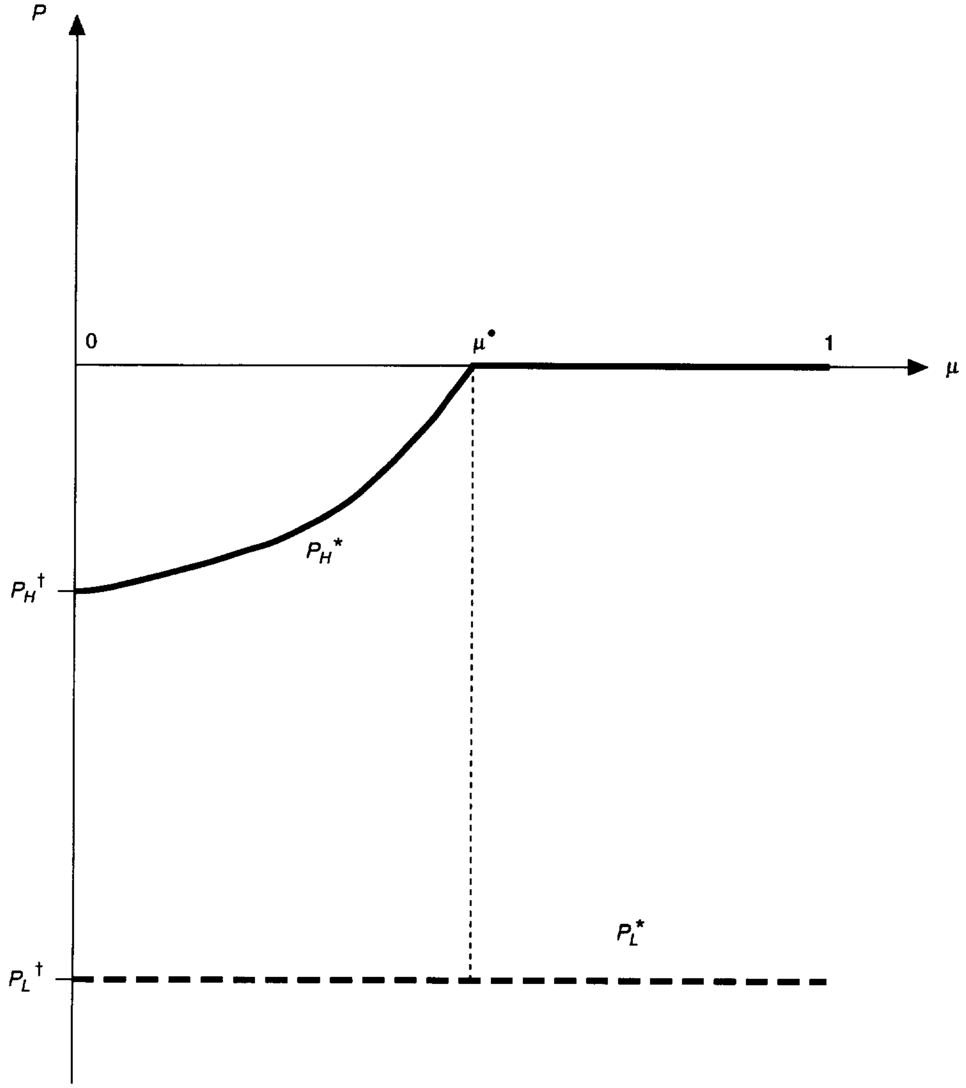
Again, the incentive constraints imply the monotonicity condition $P_L \leq P_H$. In addition, from a similar argument in the discussion of Proposition 2, any policy that satisfies the binding constraint (IC–L) and monotonicity also implies that (IC–H) is fulfilled. Thus, it is sufficient to maximize the objective function (7) subject to (IC–L), (PC–L), (PC–H), and the monotonicity constraint $P_L \leq P_H$. Clearly, at a solution, participatory constraint (PC–H) must bind so that $\Pi(K_H) = 0$. Combining the binding constraints (IC–L) and (PC–H), we obtain $\Pi(K_L) = P_H(K_L - K_H)/4t$, firm $D_L$'s information rent. Notice that (PC–L) is satisfied if and only if $P_H \leq 0$.

Observe that $P_L^\dagger < P_H^\dagger < 0$: the ranking of complete-information optimal prices with respect to firm D's demand is in line with the monotonicity condition $P_L \leq P_H$. To reduce firm $D_L$'s information rent $(P_H(K_L - K_H)/4t)$, price $P_H$ for firm $D_H$ must be increased from $P_H^\dagger$, resulting in slack in the monotonicity condition. Thus, the optimal policy must always be separating. Also, because (IC–H) does not bind, distorting $P_L$ from $P_L^\dagger$ is unnecessary. We conclude that $P_L = P_L^\dagger$.

When the prior probability for firm $D_H$ is high (a small value of $\mu$), distorting $P_H$ upward from $P_H^\dagger$ is relatively costly. So, for $\mu$ sufficiently small, the increase of $P_H$ from $P_H^\dagger$ will be small. When $\mu$ gradually increases, the cost due to the distortion on firm $D_H$ from raising $P_H$ above $P_H^\dagger$ becomes less severe. Hence, $P_H^*$ gradually increases as well. When $\mu$ becomes sufficiently large, it becomes optimal to raise $P_H$ all the way to marginal cost (zero) to reduce firm $D_L$'s information rent. Last, observe that raising $P_H$ above marginal

FIGURE 2

OPTIMAL REGULATORY PRICES IN PROPOSITION 3 ($A > 0$).




cost both violates (PC–L) and exacerbates the distortion on firm $D_H$. These observations explain the results in Proposition 3.

We can compare the distributions of consumers between the two firms under optimal regulation. First, consider Proposition 2. From Figure 1, we see that $P_L^*$ is always less than $P_L^\dagger$, and therefore more consumers will use firm $D_L$ under incomplete information than under complete information. Also, from Figure 1, for $\mu \leq \bar{\mu}$, $P_H^* = P_H^\dagger$; the distributions of consumers among firm $D_H$ and firm R are identical under complete and incomplete information. But when $\mu > \bar{\mu}$, $P_H^* > P_H^\dagger$; firm $D_H$'s market share becomes smaller under incomplete information. Second, consider Proposition 3. From Figure 2, we know that $P_L^* = P_L^\dagger$ always; hence, the distributions of consumers among firms are identical under both information assumptions. Last, $P_H^* > P_H^\dagger$ so that fewer consumers buy from firm $D_H$ under incomplete information.

Propositions 2 and 3 together illustrate that the optimal policies are particularly sensitive to the value of $A$ at $A = 0$. For example, suppose $\mu$ is sufficiently large (see part 3 of Proposition 2 and part 2 of Proposition 3); then, when $A$ changes from negative to positive, the optimal policy switches from pooling to separating. Moreover, firm $D_H$ may earn information rent only if $A < 0$, whereas firm $D_L$ may earn it only if $A > 0$. Notice, however, that the optimal policies are continuous in $A$. As $A$ converges to zero from below, $P_L^\dagger$, $P_H^\dagger$, $P_L^*$, and $P_H^*$ all converge to zero from above. Symmetrically, as $A$ converges to zero from above, $P_L^\dagger$, $P_H^\dagger$, $P_L^*$, and $P_H^*$ all converge to zero from below.

Proposition 1 points out that a departure from marginal cost pricing is necessary under complete information; moreover, the magnitude of the price-cost distortion depends negatively on the value of $K$. Interestingly, under incomplete information, the optimal policy on some type of firm D may simply be marginal cost pricing, as in part 1 of Proposition 2 and part 2 of Proposition 3. Except in part 3 of Proposition 2, the optimal price-cost margin for a given type is never more than that under complete information. This is because, in our model, marginal cost pricing eliminates information rent altogether: irrespective of its demand, a firm earns zero profit if its products sell at cost. Hence, the predominant feature of the optimal prices in Propositions 2 and 3 is a smaller price-cost margin (in absolute value) as compared to optimal prices under complete information.

Firm D always (weakly) prefers to face an unregulated rival. Whereas a monopolist will not benefit from private information about demand in our model, a dominant firm may, because the regulator may allow it to earn information rent in order to constrain the unregulated firm. The unregulated firm R may or may not be able to earn a higher profit relative to the complete-information regime. Firm R's profit function is increasing in $P_D$. Thus, whenever firm D's price is raised from the corresponding optimally regulated price under complete information, firm R earns a higher profit, as in part 3 of Proposition 2 and in Proposition 3.

For parameter values satisfying the hypothesis of Proposition 3, firm D's price will be set below marginal cost. Although this situation may resemble predatory or limit pricing, the actual interpretations are quite different. First, as we demonstrated at the end of Section 2, social welfare improves when firm R enters the market, regardless of whether firms possess private information on demand. Thus, entry by firm R will be encouraged by the regulator. Second, although the regulator may set firm D's prices below marginal cost, those prices are never set low enough to keep firm R from entering and making positive profits. Indeed, under the optimal regulatory policies, the strategic behavior of firm R is made more in line with the social objective but is not entirely eliminated.

## 5. Conclusion

■ In many situations, regulated firms face competition from unregulated firms. Under demand uncertainty, if the regulated firm competes with passive firms, then optimal regulatory policies will be similar to those obtained by Lewis and Sappington (1988); the regulated firm's private information about demand does not always create regulatory tradeoffs. In particular, when marginal costs are constant, a maintained hypothesis in our article, all distortions can be avoided. In this article, we have shown that when the unregulated firms compete strategically, one must consider the tradeoffs between efficiently allocating consumers across regulated and unregulated firms, lowering the unregulated firm's excess profits, and limiting the regulated firm's information rent. Generally, the existence of strategic rivals imposes new and binding constraints on regulatory problems.

We have used a few strong assumptions: constant marginal costs, linear demands, and a binary information random variable on a demand parameter. It remains an open question whether the specific form of the optimal policies is sensitive to these assumptions. Nevertheless, the insights from our results appear illustrative: in a general model, new regulatory tradeoffs will be created by the existence of the unregulated rival; whether the optimal mechanism is pooling, separating, or constant will depend on the welfare weights and belief parameters; information rent may be completely extracted, whereas the equilibrium allocation remains inefficient.

We have assumed throughout this article that the unregulated firm is a Stackelberg follower reacting to the dominant firm's strategies. We believe that this is an appealing assumption in regulatory settings. Generally, the filing and hearing processes and the administrative lags associated with them impose significant delays when prices at regulated firms are changed. Thus, it is reasonable to assume that an unregulated firm can change its price more easily. One can, however, imagine an alternative model in which the regulated firm's choice from the policy menu and the unregulated firm's choice of price are performed simultaneously. Although we suspect that intuitions in our model will remain valid, it turns out that the simultaneous-move model is much more awkward to solve,[7] and we have been unable to verify our conjectures.

Our regulatory games implicitly assume that the regulator's pricing policy for firm D cannot be made contingent on firm R's pricing strategy; that is, we rule out a policy that adjusts firm D's price based on firm R's reaction to an initial price for firm D.[8] Such a "reactionary" regulatory policy appears to endow too much commitment power to the regulator in our static model. If such a policy were feasible, then the regulator would set firm D's price at marginal cost, and if firm R reacted by setting a price different from marginal cost, then firm D's price would be decreased sufficiently to drive firm R out of the market. Such a regulatory policy would implement the first-best allocation, because firm R setting its price equal to marginal cost would become a best response. We believe that regulatory responses and commitment issues are best analyzed in a multiperiod model in which the market opens in each period and the regulator can react to the unregulated firm's pricing policy in one period by adjusting the regulated firm's price in the following period. Although commitment and regulatory responses are important research topics, these problems are beyond the scope of this article.

Our model can be extended in various directions. It may be interesting to examine how asymmetry in the firms' cost functions will affect the optimal mechanisms. Generalizing the market structure by letting many unregulated firms enter the market may be worthwhile. Furthermore, firms may have private information about cost, and regulators also may be concerned with cost reductions, quality enhancements, product innovations, and capacity utilizations. Incorporating these elements in a more general model may be fruitful research.

## Appendix

■ Proofs to Propositions 1–3 follow.

*Proof of Proposition 1.* We will only provide a sketch of the proof, because it involves straightforward computation. As we argued earlier, $T$ will be chosen to make firm D's profits zero. Hence, we have $T = -P_D X$. The program for the optimal price at firm D is the following: choose $P_D$ to maximize

$$(1 - \alpha_D - \alpha_R)\left\{KX - \frac{tX^2}{2} - P_R(l - X) - \frac{t(l - X)^2}{2}\right\} + \alpha_R(P_R(l - X))$$

subject to

$$P_R = \frac{tl - K + P_D}{2} \quad \text{and} \quad X = \frac{3}{4}l + \frac{K - P_D}{4t}.$$

[7] First, for a given policy menu, it is no longer true that the continuation equilibrium must be unique. Second, incentive constraints no longer imply that the regulated prices of the dominant firm are monotonic with respect to the demand parameter. Third, the standard techniques of omitting incentive constraints in one direction may fail.

[8] This is because, in the regulatory games, the market opens immediately after firm R's price response, and the regulator is no longer allowed to change firm D's price.

Because firm R can always set its price at marginal cost, a participatory constraint for firm R is unnecessary. After substituting the constraints for $P_R$ and $X$ in the objective functions, we obtain the following first-order condition with respect to $P_D$ after simplification:

$$(1 - \alpha_D - \alpha_R)\left[\frac{K - 3P_D - tl}{8t}\right] + \alpha_R\left[\frac{tl - K + P_D}{4t}\right] = 0.$$

Solving this equation for $P_D$ yields (5). It is straightforward to confirm that firm D's market share is always greater than $X^P$. *Q.E.D.*

*Proof of Proposition 2.* We begin by arguing that incentive constraint (IC–H) must bind. Suppose this is not true; that is, suppose that (IC–H) is not binding. Then, the solution will be given by the solution of a relaxed program in which (7) is maximized subject to (IC–L), (PC–L), and (PC–H). At a solution to this relaxed program, (IC–L) does not bind. Indeed, the complete-information solution, $\{(T_L^\dagger, P_L^\dagger), (T_H^\dagger, P_H^\dagger)\}$, where $P_i^\dagger$ and $T_i^\dagger$ are given by Proposition 1 for $K = K_i$, $i = L, H$, is the solution to the relaxed program. To see this, simply note that $\{(T_L^\dagger, P_L^\dagger), (T_H^\dagger, P_H^\dagger)\}$ satisfies the incentive constraint (IC–L), because, according to the hypothesis of the proposition, $P_L^\dagger > P_H^\dagger > 0$. But routine computation shows that the policy menu $\{(T_L^\dagger, P_L^\dagger), (T_H^\dagger, P_H^\dagger)\}$ violates the omitted incentive constraint (IC–H). Hence, the solution cannot be obtained by omitting incentive constraint (IC–H). We conclude that (IC–H) must be binding.

Second, we show that incentive constraint (IC–L) does not bind if and only if $\mu \le \bar{\mu}$. We do this by considering a relaxed program (*re.P*) in which (IC–L) is omitted, and later verify that the solution to program (*re.P*) satisfies (IC–L) if and only if $\mu \le \bar{\mu}$.

So consider program (*re.P*): the maximization of (7) subject to (IC–H), (PC–L), and (PC–H). Obviously, (IC–H) binds at the solution to (*re.P*). Also, (PC–L) must bind; otherwise, $T_L$ could be reduced, thus increasing the value of (7) without violating any constraint in (*re.P*). Using (PC–L) as an equality, we can solve for $T_L$. Upon substituting $T_L$ to the binding constraint (IC–H), we have

$$P_H\left[\frac{3}{4}l + \frac{K_H - P_H}{4t}\right] + T_H = P_L\frac{K_H - K_L}{4t}.$$

We also can use this equation to solve for $T_H$.

Observe that the last equation implies that (PC–H) is equivalent to $P_L \ge 0$. Furthermore, with (IC–H) holding as an equality, (IC–L) is fulfilled if and only if $P_H \ge P_L \ge 0$.

We substitute for $T_L$ and $T_H$ in the objective function, which then reduces to

$$\mu\left\{(1 - \alpha_D - \alpha_R)\left[K_L X_L - \frac{tX_L^2}{2} - P_{RL}(l - X_L) - \frac{t(l - X_L)^2}{2}\right] + \alpha_R(P_{RL}(l - X_L))\right\}$$
$$+ (1 - \mu)\left\{(1 - \alpha_D - \alpha_R)\left[K_H X_H - \frac{tX_H^2}{2} - P_{RH}(l - X_H) - \frac{t(l - X_H)^2}{2}\right] + \alpha_R(P_{RH}(l - X_H))\right\}$$
$$+ (1 - \mu)\,\alpha_D P_L\frac{K_H - K_L}{4t}, \tag{A1}$$

where $P_{Ri}$ and $X_i$ are defined in (6). Let us choose $P_i$ to maximize (A1) subject to (PC–H), or simply $P_L \ge 0$.

Let $P_L^*$ and $P_H^*$ be the solution. By inspection, $P_H^* = P_H^\dagger$. Next, by using the definitions of $P_{Ri}$ and $X_i$ and putting the first-order partial derivative of (A1) with respect to $P_L$ to zero, we obtain

$$P_L = (K_L - tl)\left[\frac{1 - \alpha_D - 3\alpha_R}{3 - 3\alpha_D - 5\alpha_R}\right] - \frac{1 - 2\alpha_D - \alpha_R}{3 - 3\alpha_D - 5\alpha_R}2(K_H - K_L)\frac{1 - \mu}{\mu}, \tag{A2}$$

the right-hand side of which is the expression for $P_L^*$ in part 2 of Proposition 2.

Define $\underline{\mu}$ and $\bar{\mu}$ by solving for $\mu$ when (A2) is set to zero and $P_H^\dagger$, respectively. It is easily verified that $\underline{\mu}$ and $\bar{\mu}$ are those in Section 4.

For $\mu < \underline{\mu}$, the expression in (A2) yields $P_L < 0$, which implies that (PC–H) is violated. Hence, for $\mu \le \underline{\mu}$, $P_L^* = 0$. For $\underline{\mu} \le \mu \le \bar{\mu}$, the expression in (A2) yields $P_L > 0$ and (PC–H) is satisfied. Now, because $\mu \le \bar{\mu}$, we have $P_L^* \le P_H^*$, implying that (IC–L) holds. Therefore, we conclude that, for $\mu \le \bar{\mu}$, the solution to (*re.P*) solves the original program.

Finally, for $\mu > \bar{\mu}$, the expression in (A2) yields $P_L > P_H^*$. Hence, the solution to program (*re.P*) violates (IC–L). Because (*re.P*) omits only constraint (IC–L), we conclude that, for $\mu > \bar{\mu}$, both (IC–L) and (IC–H) must bind. It follows that, at a solution, $P_L = P_H$ and $T_L = T_H$. The result in the last part of the proposition is obtained simply by maximizing (7) subject to $P_L = P_H$, $T_L = T_H$, and $\Pi(K_L) = 0$. At this solution, it is easy to verify that $\Pi(K_H) > 0$. *Q.E.D.*

*Proof of Proposition 3.* For this proof, we follow along a similar line as that of Proposition 2; hence, we provide only a sketch of the proof. We begin by showing that (IC–L) must bind at a solution to the constrained maximization. Indeed, if (IC–L) were not binding, then the complete-information solution would be feasible: by the hypothesis of the proposition, the complete-information optimal regulatory prices, $P_i^\dagger$, $i = L, H$, satisfy $P_L^\dagger < P_H^\dagger < 0$. It is easy to demonstrate that, at the complete-information solution, (IC–H) is satisfied. But the complete-information optimal regulatory policy violates constraint (IC–L). This contradiction implies that (IC–L) must bind.

Then we consider a relaxed program in which (7) is maximized subject only to (IC–L), (PC–L), and (PC–H); constraint (IC–H) is ignored. Using an almost identical argument as in the proof of Proposition 2, we can show that (IC–L) and (PC–H) must bind. This allows us to solve for $T_L$ and $T_H$ in terms of other variables and parameters and to obtain

$$P_L\left[\frac{3}{4}l + \frac{K_L - P_L}{4t}\right] + T_L = P_H\frac{K_L - K_H}{4t}.$$

Clearly, (PC–L) is fulfilled if and only if $P_H \leq 0$. Also, with (IC–L) holding as an equality, (IC–H) is satisfied if and only if $P_L \leq P_H \leq 0$.

Upon substituting for $T_L$ and $T_H$ in the objective function (7), we simplify it to

$$\mu\left\{(1 - \alpha_D - \alpha_R)\left[K_L X_L - \frac{tX_L^2}{2} - P_{RL}(l - X_L) - \frac{t(l - X_L)^2}{2}\right] + \alpha_R(P_{RL}(l - X_L))\right\}$$

$$+ (1 - \mu)\left\{(1 - \alpha_D - \alpha_R)\left[K_H X_H - \frac{tX_H^2}{2} - P_{RH}(l - X_H) - \frac{t(l - X_H)^2}{2}\right] + \alpha_R(P_{RH}(l - X_H))\right\}$$

$$+ \mu(1 - 2\alpha_D - \alpha_R)P_H\frac{K_H - K_L}{4t}, \tag{A3}$$

where $P_{Ri}$ and $X_i$ are defined in (6). Let us choose $P_i$ to maximize (A3) subject to (PC–L), or simply $P_H \leq 0$.

Denote the solution to this program by $P_L^*$ and $P_H^*$. Clearly, $P_L^* = P_L^\dagger$. Setting the first-order partial derivative of (A3) with respect to $P_H$ to zero yields

$$P_H = (K_H - tl)\left[\frac{1 - \alpha_D - 3\alpha_R}{3 - 3\alpha_D - 5\alpha_R}\right] + \frac{1 - 2\alpha_D - \alpha_R}{3 - 3\alpha_D - 5\alpha_R}(K_H - K_L)\frac{2\mu}{1 - \mu}. \tag{A4}$$

Notice that the right-hand side of (A4) is the same as $P_H^*$ in part 1 of Proposition 3.

Define $\mu^*$ by solving for $\mu$ when (A4) is equated to zero. This expression for $\mu^*$ can be shown to be the one in Section 4. For $0 \leq \mu \leq \mu^*$, $P_H$ in (A4) is strictly negative; (PC–L) is satisfied. Conversely, for $\mu^* < \mu \leq 1$, the value for $P_H$ in (A4) is strictly positive, violating (PC–L). In sum, $P_H^* = 0$ when $\mu^* < \mu \leq 1$ and is given by (A4) otherwise.

Finally, the missing incentive constraint (IC–H) is always satisfied at a solution to the relaxed program. This is a consequence of the fact that, at the solution, (IC–L) holds as an equality and $P_L^* < P_H^* \leq 0$. *Q.E.D.*

## References

AURIOL, E. AND LAFFONT, J.-J. "Regulation by Duopoly." *Journal of Economics & Management Strategy*, Vol. 1 (1992), pp. 507–533.

BARON, D.P. AND MYERSON, R.B. "Regulating a Monopolist with nknown Costs." *Econometrica*, Vol. 50 (1982), pp. 911–930.

BRAEUTIGAM, R.R. "Socially Optimal Pricing with Rivalry and Economies of Scale." *RAND Journal of Economics*, Vol. 15 (1984), pp. 127–134.

CAILLAUD, B. "Regulation, Competition, and Asymmetric Information." *Journal of Economic Theory*, Vol. 52 (1990), pp. 87–110.

COOPER, R. "On Allocative Distortions in Problems of Self-Selection." *RAND Journal of Economics*, Vol. 15 (1984), pp. 568–577.

DANA, J.D. AND SPIER, K.E. "Designing a Private Industry: Government Auctions with Endogenous Market Structure." *Journal of Public Economics*, Vol. 53 (1994), pp. 127–147.

HARRIS, R.G. AND WEINS, E.G. "Government Enterprise: An Instrument for the Internal Regulation of Industry." *Canadian Journal of Economics*, Vol. 13 (1980), pp. 125–132.

LAFFONT, J.-J. AND TIROLE, J. "Using Cost Observation to Regulate Firms." *Journal of Political Economy*, Vol. 94 (1986), pp. 614–641.

——— AND ———. "Repeated Auctions of Incentive Contracts, Investments, and Bidding Parity with an Application to Takeovers." *RAND Journal of Economics*, Vol. 19 (1988), pp. 516–537.

LEWIS, T.R. AND SAPPINGTON, D.E.M. "Regulating a Monopolist with Unknown Demand." *American Economic Review*, Vol. 78 (1988), pp. 986–998.

——— AND ———. "An Informational Effect when Regulated Firms Enter Unregulated Markets." *Journal of Regulatory Economics*, Vol. 1 (1989), pp. 35–45.

MCGUIRE, T.G. AND RIORDAN, M.H. "Incomplete Information and Optimal Market Structure: Public Purchases from Private Providers." *Journal of Public Economics* (forthcoming).

WARE, R. AND WINTER, R.A. "Public Pricing under Imperfect Competition." *International Journal of Industrial Organization*, Vol. 4 (1986), pp. 87–97.

WOLINSKY, A. "Regulation of Duopoly under Asymmetric Information." Discussion Paper, Northwestern University, 1993.