ADVERSE SELECTION IN DYNAMIC MORAL HAZARD*

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This paper studies a multiperiod moral hazard problem under two assumptions: (i) contracts are subject to renegotiations; (ii) the agent's action has long-term effects. The action is also interpreted as a choice of characteristic or "type." Renegotiation-proof contracts that implement various actions, including random ones, are characterized. Under appropriate conditions, the equilibrium involves the principal implementing a random action. Therefore, the equilibrium has standard properties of "adverse selection" models.

I. INTRODUCTION

In this paper we study a dynamic moral hazard model with renegotiation. Principal-agent relationships are usually associated with imperfect information about actions that an agent performs on behalf of the principal. This typically leads to a conflict between incentive provision and risk sharing. As a result, incentive schemes are used to motivate the agent. If the principal and the agent interact for a long time, it is often advantageous for the principal to commit to a long-term incentive scheme, specifying all the payments for the agent as time and events unfold. However, it is likely that the parties will renegotiate the contract as the relationship progresses, looking for mutually beneficial gains.

In dynamic relationships the agent's action may have long-term consequences. For example, an agent's current decision may affect production possibilities in the future. Under such circumstances, when parties renegotiate the contract, they must realize that the project has already been through the early stages and some characteristics have been decided. Moreover, the principal must recognize that the agent now possesses some private information about the action that has been taken. Even if the parties are initially symmetrically informed, the agent may later (endogenously) acquire some private information. Renegotiation must then trade off the cost of extracting the agent's superior information

*My thanks to Kalyan Chatterjee, Alan Manning, Michael Manove, John Moore, Ailsa Roell, and Ariel Rubinstein for helpful discussions. Also, I received useful comments and suggestions from Eric Maskin, two referees, and seminar participants at Boston University, the Econometric Society North American Summer Meeting at Minneapolis, 1988, and Penn State University. The Committee of Vice Chancellors and Principals of Universities in the United Kingdom, and the LSE 1980s Funds provided research support.

The Quarterly Journal of Economics, February 1991

with the benefit of insuring her. More important, renegotiation may affect the action that the principal would like to implement.

In this paper we investigate the effects of renegotiation. Our model has two crucial assumptions: (i) the agent's choice of action affects production for more than one period, i.e., an action has long-term effects; (ii) the principal offers multiperiod contracts that may later be renegotiated. In the paper the agent's action is regarded as her choice of "type."

Assumption (ii) reflects our view that the principal's ability to commit to long-term contracts is frequently incredible. It is natural to allow the parties to renegotiate an existing contract, when it is in their joint interest to do so. In our model long-term contracts are time-inconsistent, implying that when parties act as the optimal long-term contract prescribes, they both prefer to renegotiate the contract later. Hence explicit modeling of renegotiation is necessary.[1]

Assumption (i) serves as a good description in many interesting situations. In the abstract model the agent chooses an action in period 1 that induces probability distributions on outputs in period 1 *and* period 2. One can imagine that the agent is responsible for a long-term project; her choice of project or the action that she performs on a given project then influence the characteristic of the possible outcomes. Alternatively, the agent may be investing in human capital, the level of which determines productivities in the future.[2] In many situations this "spillover" effect is quite common. For example, the profitability of a firm over a number of years depends on the initial strategy management adopts. As another example, the success of a long-term R&D project may be affected by an investment made when the project begins.

When the principal recognizes (i) and (ii), there are important questions he has to consider. When the agent is risk averse and the principal is risk neutral, inducing a costly action (say, working hard) from the agent requires a departure from optimal risk

1. Fudenberg, Holmstrom, and Milgrom [1987] investigate when a sequence of short-term contracts performs as well as a long-term contract. Also, Rey and Salanie [1990] consider multiperiod contracts with renegotiation and other commitment assumptions. The idea of renegotiation in contracts can also be found in Stiglitz and Weiss [1983].

2. An alternative assumption that will give rise to the same formal model (with suitably chosen parameters) is the following. The agent chooses an action in period 1, and she will be constrained to provide the same action in period 2. One interpretation is that the agent is selecting a long-term project that demands a specific input in all time periods. Once the project is under way, it is too costly to change. The real choice for the agent is therefore a supply of a sequence of actions.

sharing, and her remuneration varies according to stochastic production outcomes. At the renegotiation stage, the principal's contract revision offer depends on his belief about the unobservable chosen action. For example, if the principal believes that the agent has chosen diligence with probability one, he will replace any (costly) contingent scheme by a full insurance wage. In fact, the cost of implementation increases as the principal's belief becomes more accurate. With almost degenerate beliefs, proper incentives can only be provided before the renegotiation stage. Our major result is that the principal may *not* always want the agent to work hard. Indeed, we give a condition for a mixed strategy equilibrium in which the agent is indifferent between diligence and shirking, and she randomizes between these two options. Thus, in equilibrium, at the renegotiation stage, the game proceeds as if there *exists* asymmetric information between the parties, and the (continuation) equilibrium has standard features of adverse selection models. Essentially, a stochastic decision between diligence and shirking prevents the principal from knowing precisely at the renegotiation stage what the agent has done. It will then be optimal for the principal to offer contingent payments in order to screen different types of the agent. From an ex ante point of view, this relaxes the action choice incentive constraint, which sometimes may be the dominating factor.

The paper is organized as follows. Section II presents the model. We work with a simple model of dynamic moral hazard that satisfies (i) and (ii). In Section III, to provide a benchmark, we establish the corresponding static (one-period) optimal scheme and discuss briefly the optimal multiperiod scheme under full commitment. We study in Section IV the dynamic game between the principal and the agent under the assumption that contracts may be renegotiated. Renegotiation-proof contracts that implement various probability distributions on the action set are characterized. Section V proves that for appropriate parameter values, the principal implements a random action. In this equilibrium "adverse selection" is the feature in period 2. The last section concludes.

II. The Model

We study a simple principal-agent model with "hidden action" or "moral hazard." The relationship between the principal and the agent lasts two periods. The principal has a stochastic production process; in each period there are two possible, publicly observable states, x_1 and x_2. We identify them as outputs and assume that $x_1 < x_2$; prices are normalized so that x_1 and x_2 also denote revenues. For simplicity, we assume that parties do not discount future payoffs; all results generalize straightforwardly to the case where both parties have the same discount rate. The principal, who never observes the agent's action, is risk neutral and maximizes total expected profits (revenues minus agent's compensations). The agent, having no access to a capital market, is unable to save or borrow. She is strictly risk averse with respect to incomes from the principal, and has a von Neumann-Morgenstern utility function $V(y_1) + V(y_2) - 2\,G(a)$, where y_t is her income in period t, $t = 1,2$ and $2\,G(a)$ is the disutility of action a.[3] There are two available actions, a_1 and a_2, where $G(a_1) < G(a_2)$. Sometimes, we find it convenient to say that she works hard (respectively, shirks) when she selects a_2 (respectively, a_1), even though we also interpret her action as a choice of characteristic or type. At the beginning of period 1 the agent has a reservation utility $\overline{U}$ that represents her utility payoff if she does not take part in production; $\overline{U}$ is normalized to zero. In the sequel it is useful to imagine that the principal pays the agent in utility units. With the notation $v = V(y)$ and $h = V^{-1}$, the term utility-payment (v) means an equivalent income ($h(v)$). Notice that h is strictly convex since V is strictly concave.

For a given action each period, there is a probability distribution on (x_1,x_2). Thus, our model differs from the standard principal-agent model in that the agent's action has long-term effects on productions. It is further assumed that for each action the distributions are identical and independent across periods. In the paper we use the following simple technology:

Prob	x_1	x_2
a_1	1	0
a_2	$1-\gamma$	γ

That is, in each period the high output level occurs with positive probability γ if and only if the agent is diligent. For future use we let $\epsilon = 1/\gamma$ and $G_j \equiv G(a_j)$, $j = 1,2$.

3. The reason why we write $2\,G(a)$ rather than $G(a)$ as the utility cost of action will become obvious when we define the corresponding one-period model in the next section.

It is useful to discuss briefly the first-best (full information solution) when the agent's choice of action can be monitored. Because the agent is risk averse, it is optimal for the principal to give her a constant payment independent of the (random) outputs in each period. Also, due to risk aversion and her inability to borrow and save, the principal tries to smooth the agent's income across periods. Clearly, it is enough to pay her an amount such that she obtains $\overline{U}$ $(\equiv 0)$. Therefore, the first-best utility-payments in each period that induce the agent to perform a_1 and a_2 are, respectively, G_1 and G_2.

III. One-Period and Full Commitment Contracts

In this section we first define a corresponding one-period model and then discuss the full commitment two-period optimal contracts. To define the corresponding one-period model, we take the model introduced in the last section but let the relationship terminate at the end of period 1. The agent's choice is between a_1 and a_2 for one period with respective utility costs $G(a_1)$ and $G(a_2)$. (An action now has half the utility cost compared with that in the two-period model.) A contract promises utility-payment u_i conditional on output level x_i, $i = 1,2$. We say that a contract (u_1,u_2) implements action a_j if the agent optimally selects a_j given the contract, and this action gives her at least the reservation utility.

The form of these utility-payments depends on the objective of the principal. First, since a_1 minimizes the agent's utility cost among her choices, the principal can use the first-best utility-payment G_1 to implement a_1. Second, the implementation of action a_2 requires an incentive contract. Formally, an optimal contract (v_1,v_2) for action a_2 is a solution to the following program: Choose (v_1,v_2) to minimize

$$(1-\gamma)h(v_1)+\gamma h(v_2),$$

subject to

$$(1-\gamma)v_1+\gamma v_2-G(a_2)\geq 0 \tag{1}$$

$$(1-\gamma)v_1+\gamma v_2-G(a_2)\geq v_1-G(a_1). \tag{2}$$

Inequality (1) guarantees the agent at least zero when she works hard, while (2) says diligence is preferred to shirking. The objective function represents the principal's expected cost. At a solution to the above program, the two constraints (1) and (2) must be binding,[4] and we can use them to solve for the solution (v_1^*,v_2^*) explicitly:

$$v_1^* = G_1 \tag{3}$$

$$v_2^* = G_1 + \epsilon(G_2 - G_1). \tag{4}$$

(Recall that $\epsilon = 1/\gamma > 1$.) Notice that due to (2), $v_2^* > v_1^*$. To summarize, in the corresponding one-period model, if the principal motivates the agent to work hard, the optimal contract is given by (v_1^*,v_2^*) in (3) and (4); the minimum cost is $(1 - \gamma)h(v_1^*) + \gamma h(v_2^*)$.

In this paper we are interested in a situation where diligence is desirable for the principal:

ASSUMPTION (A.1).

$$\Pi(a_2) - (1 - \gamma)\,h(v_1^*) - \gamma\,h(v_2^*) > \Pi(a_1) - h(G_1), \tag{5}$$

where $\quad \Pi(a_2) = (1 - \gamma)x_1 + \gamma x_2 \quad$ and $\Pi(a_1) = x_1$.

Assumption (A.1) simply says that for the principal, the benefit of action a_2 net of its minimum cost of implementation is greater than that of a_1. (A.1) is retained throughout this paper, i.e., for the two-period model as well. v_1^* and v_2^* (i.e., (3) and (4)) will be used later.

In the dynamic model the agent's action has effects on outputs in periods 1 and 2, and she may be paid in both periods. We are not directly concerned with the principal's problem when fully committed (long-term) contracts are feasible, but the properties of these contracts are worth mentioning. For a detailed study, we refer the reader to Rogerson [1985]. To implement a_1, it is clear that the optimal contract offers G_1 in each period, because a_1 is the least cost action. The implementation of action a_2 involves a subtle arrangement of utility-payments according to the history of outcomes, i.e., period 2 rewards depend on both period 1 and period 2 outputs. We would like to emphasize one important point. Because the agent is risk averse, it minimizes cost by spreading incentives over the two periods. As a result, a fully committed optimal contract for action a_2 will have period 2 payments dependent on period 2 outcomes (a

4. Suppose at a solution that (1) did not bind. Then v_1 and v_2 could be reduced without affecting (2). This is a contradiction. Next, suppose that (2) did not bind, then $v_1 = v_2 = G_2$. But then (2) is violated given this contract. This is again a contradiction.

penalty if x_1 is observed, a higher payment if x_2 is observed) when x_1 has occurred in period 1. Thus, an efficient long-term contract ignores that the agent takes an action only in period 1. Now suppose that the contract may be revised at period 2. Then if x_1 has occurred in period 1, the parties can reach a Pareto improvement by rescinding the original contract and agreeing on a noncontingent payment.

IV. Renegotiation-Proof Contracts

We now analyze the dynamic game under the assumption that the principal's contract offer may be renegotiated. We begin by describing the sequence of moves and events that take place in the two periods. At the beginning of period 1, the principal proposes a contract $C = [(u_1,u_2);M]$, where u_i is a utility-payment to the agent if x_i occurs in period 1, and M is a mechanism to determine the agent's reward in period 2 for each possible outcome; the details of M will be explained shortly. Then the agent has to choose between signing the contract C or not. If C is accepted, an unobservable action is taken by the agent; we allow mixed strategies so that the agent may choose an action according to the realization of a probability distribution on $\{a_1,a_2\}$. At the end of period 1, one of x_1 or x_2 is realized, and the agent is paid accordingly. At the beginning of period 2, the principal may propose an alternative mechanism N to replace M. Then the agent chooses between M and N. Finally, an output realizes at the end of period 2, and the agent is paid according to the mechanism that is in force.

What is the structure of M and N? To emphasize that M and N may depend on the period 1 outcome, we write $M(x_i)$ and $N(x_i)$, $i = 1,2$. We suppose that M and N consist of sets of messages to be announced by the agent. Notice that in period 2, the principal knows that there are two possibilities: the agent has chosen either action a_1 or a_2. (We shall say that the agent is type a_j if she has taken action a_j in period 1.) From the Revelation Principle we can assume that M and N each consists of two possible messages. Hence, we put $M = \{m^1,m^2\}$ and $N = \{n^1,n^2\}$. A report of m^k (or n^l) determines the agent's reward in period 2. By a slight abuse of notation, we let $m^1(x_1) = (v_1^1,v_2^1)$, $m^2(x_1) = (v_1^2,v_2^2)$, $m^1(x_2) = (w_1^1,w_2^1)$, and $m^2(x_2) = (w_1^2,w_2^2)$. We mean that if $m^k(x_1)$, $k = 1,2$, is the agent's report (after x_1 has occurred in period 1), then she is paid v_i^k contingent on x_i being the output in period 2, $i = 1,2$.

Similarly for $m^k(x_2)$. N is defined analogously.[5] Figure I provides a schematic sketch.

The solution concept for this game is perfect Bayesian equilibrium or sequential equilibrium (see Kreps and Wilson [1982]). For brevity, we do not present a formal definition. In a perfect Bayesian equilibrium whenever a player moves, he or she must maximize his or her payoff, given the strategy of the other player and his or her belief about earlier moves made by the other player; updating of beliefs is according to Bayes rule whenever this is permitted by the strategy. Note that in our model the only information missing to the principal is the agent's choice of action.

At this point, two remarks are in order. First, we know from the Revelation Principal that there is no loss of generality in requiring the payment schemes induced by the reports (m^1,m^2) to be incentive compatible, i.e., for each x_i a "type" a_j agent will optimally report $m^j(x_i)$ or $n^j(x_i), j = 1,2$. Second, there is no loss of generality in focusing only on renegotiation-proof contract offers, i.e., those the principal cannot improve on when he has the opportunity to revise them in period 2. Indeed, any result the principal can achieve with a contract $C = [(u_1,u_2); M]$ in which M is later replaced by M^* can be achieved by a renegotiation-proof contract $C^* = [(u_1,u_2); M^*]$. Clearly, C^* and C give the principal the same outcome. If C^* were not renegotiation-proof, then let M^* be revised to M'; M' must bring about a Pareto improvement in period 2. But then the principal would have proposed M' when renegotiating M, contrary to the assumption that M was revised to M^*. (See also Hart and Tirole [1988] and Fudenberg and Tirole [1990].)

Which contracts are renegotiation-proof? In other words, which contracts leave no room for Pareto improvement in period 2? Notice that the initial contract M may have specified that the agent's compensation be based on outputs in period 2, even though her action has already been taken. Therefore, the principal may want to insure the agent further. However, M may have intended different status quo expected utility levels to an agent according to her type (or chosen action). The renegotiated contract must prevent an agent from lying about her characteristic. Too much insurance encourages lying, but too little is not optimal for risk sharing.

5. An equivalent way of presenting the period 2 message mechanism is to suppose that the M consists of a menu of reward schemes from which the agent may choose.

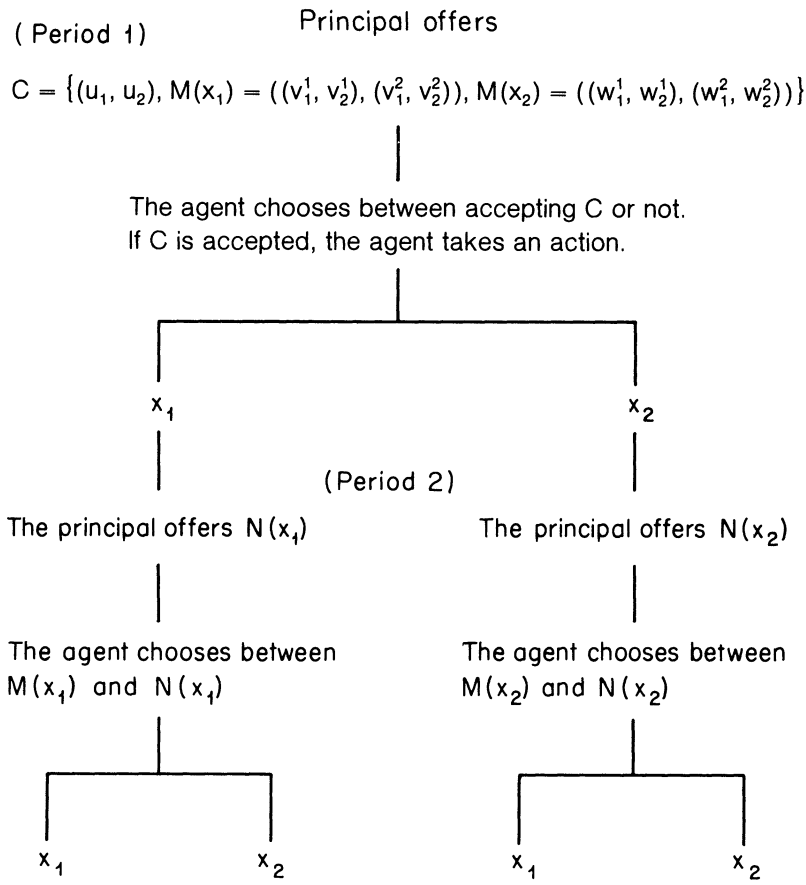

FIGURE I

Before we define a renegotiation-proof contract, let us suppose in period 1 that the agent chooses a_2 with probability p. From the stochastic structure, if x_2 has occurred in period 1, the principal must believe with probability 1 that action a_2 has been chosen. On the other hand, conditional on x_1 in period 1, Bayes rule says that his posterior belief that the agent is type a_2 is

$$
\begin{aligned}
\phi(p) &\equiv \frac{\text{prob}\,(a_2,x_1)}{\text{prob}\,(a_2,x_1) + \text{prob}\,(a_1,x_1)} \\
&= \frac{p(1-\gamma)}{p(1-\gamma) + (1-p)} = \frac{p(1-\gamma)}{1-p\gamma}.
\end{aligned} \tag{6}
$$

Now suppose conditional on x_i in period 1, that the initial contract C specifies $M(x_i) = \{(\bar{v}_1^1,(\bar{v}_2^1), (\bar{v}_1^2,\bar{v}_2^2)\}$ in period 2. Recall that $M(x_i)$ is incentive compatible. Then let $\overline{U}^1$ and $\overline{U}^2$ be, respectively, the period 2 expected utility-payments for type a_1 and a_2 agents under M, i.e., $\overline{U}^1 = \bar{v}_1^1$ and $\overline{U}^2 = (1 - \gamma)\bar{v}_1^2 + \gamma\, \bar{v}_2^2$. $\overline{U}^1$ and $\overline{U}^2$ are "status quo" expected utilities for each type of agent; by rejecting any new contract offer, a type a_j agent receives $\overline{U}^j$ in period 2.

An (incentive-compatible) mechanism $M(x_i) = \{\bar{v}_1^1,\bar{v}_2^1), (\bar{v}_1^2,\bar{v}_2^2)\}$ is renegotiation-proof with respect to ϕ (the posterior belief) if it is the solution of the following program:

$$\min\, (1 - \phi)h(u_1^1) + \phi[(1 - \gamma)h(u_1^2) + \gamma h(u_2^2)],$$

subject to

$$u_1^1 \geq \overline{U}^1 \tag{7}$$

$$(1 - \gamma)u_1^2 + \gamma u_2^2 \geq \overline{U}^2 \tag{8}$$

$$u_1^1 \geq u_1^2 \tag{9}$$

$$(1 - \gamma)u_1^2 + \gamma u_2^2 \geq (1 - \gamma)u_1^1 + \gamma u_2^1. \tag{10}$$

Inequalities (7) and (8) say that the new schemes induced by the new mechanism $\{(u_1^1,u_2^1), (u_1^2,u_2^2)\}$ offer at least $\overline{U}^1$ and $\overline{U}^2$, the status quo utility-payments in M for type a_1 and type a_2. Inequalities (9) and (10) are "self-selection" or "truth-telling" constraints. A type a_1 agent prefers (u_1^1,u_2^1) to (u_1^2,u_2^2), while a type a_2 agent prefers (u_1^2,u_2^2). The objective function denotes the cost of the revised scheme given the posterior belief ϕ. Call this program (RNP). Note that the solutions are functions of ϕ, $\overline{U}^1$ and $\overline{U}^2$.

LEMMA 1. If $\phi = 1$, then the solution of the above program has $u_1^2 = u_2^2 = \overline{U}^2$. If $\overline{U}^1 \geq \overline{U}^2$, then the solution of the above program has $u_1^1 = \overline{U}^1$ and $u_1^2 = u_2^2 = \overline{U}^2$. In both cases, (9) and (10) are not binding.

We shall give only an informal proof of Lemma 1. First, consider the case $\phi = 1$, when the principal believes that the agent must be of type a_2. Optimal risk sharing requires the agent to be fully insured. Hence she receives a constant wage. Since the status quo for a type a_2 agent is $\overline{U}^2$, the revised payment must also be $\overline{U}^2$. Next, consider the case $\overline{U}^1 \geq \overline{U}^2$. Here, the status quo gives a type a_1 agent at least an expected utility-payment as that of a type a_2 agent. Observe that the principal can always decrease u_2^1 so that (10) does not bind. This shows that $u_1^2 = u_2^2 = \overline{U}^2$. Since $\overline{U}^1 \geq \overline{U}^2$,

(9) does not bind either. It is then obvious that u_1^1 must be equal to $\overline{U}^1$.

Lemma 1 reports two situations in which the outcome of the contract revision is particularly simple. It is obvious that when $\phi = 1$, the mechanism is degenerate and a fixed payment must be offered. This case is important because whenever x_2 results in period 1, the principal must believe that the agent is type a_2. Thus, any renegotiation-proof $M(x_2)$ may simply be represented by a constant utility-payment w. When $\overline{U}^1 \geq \overline{U}^2$, the truth-telling constraints are slack: a type a_2 can easily be discouraged from mimicking a type a_1 by a penalty (a sufficiently low u_2^1). We now turn to the case $\overline{U}^1 < \overline{U}^2$.

LEMMA 2. Suppose that $\overline{U}^1 < \overline{U}^2$ and $\phi < 1$. The solution of (RNP) satisfies the following:

(i) $u_1^1 = u_1^2 < u_2^2$;
(ii) $(1 - \gamma)u_1^2 + \gamma u_2^2 > u_1^1$;
(iii) $(1 - \gamma)u_1^2 + \gamma u_2^2 = \overline{U}^2$;
(iv) there exists $\hat{\phi}$, $0 < \hat{\phi} < 1$, such that

$$u_1^1 = \overline{U}^1 \text{ for } \phi \text{ in } [0, \hat{\phi}],$$

$$u_1^1 > \overline{U}^1 \text{ for } \phi \text{ in } (\hat{\phi}, 1];$$

(v) if $u_1^1 = \overline{U}^1$, then $u_1^2 = \overline{U}^1$ and $u_2^2 = \overline{U}^1 + \epsilon(\overline{U}^2 - \overline{U}^1)$;
(vi) u_2^2 (respectively, u_1^2) is decreasing (respectively, increasing) in $(\hat{\phi}, 1]$; u_1^1, u_1^2 and u_2^2 all tend to $\overline{U}^2$ as ϕ tends to 1.

Proof of Lemma 2. See the Appendix.

REMARK. Lemma 2 does not give information on u_2^1—this variable does not enter the principal's objective function. Without loss of generality, we may put $u_2^1 = u_1^1$.

The intuition behind Lemma 2 is as follows. In period 2 the principal would prefer to fully insure the risk-averse agent, since the (unobservable) action has already been chosen. However, when $\overline{U}^1 < \overline{U}^2$, the period 2 "message mechanism" must deter a type a_1 agent from mimicking a type a_2, because a type a_2 agent has been promised a higher (expected) utility-payment. Therefore, (9) is binding, $u_1^1 = u_1^2$, and a type a_2 agent still faces a contingent reward scheme. On the other hand, it is never in a type a_2 agent's interest to lie—(10) does not bind. Whether a type a_1 agent will actually be offered more than $\overline{U}^1$ depends on the posterior belief ϕ. Driving down type a_1's utility-payment to $\overline{U}^1$ is costly because the principal has to raise u_2^2 to compensate a type a_2 agent. If the probability of type a_2 is high enough, the optimal way to screen them is to reduce the wedge between u_1^2 and u_2^2, thereby avoiding too high a cost for type a_2. The extra cost of putting u_1^1 above $\overline{U}^1$ is in expected terms not so severe since the chance of facing a type a_1 is relatively small.

From Lemma 1 and Lemma 2, we can now decide whether $M = \{m^1, m^2\}$ in an initial contract is renegotiation-proof, given that the agent chooses action a_2 with probability p in period 1. We first consider $M(x_2)$, the message game in period 2 when x_2 is the output in period 1. According to the stochastic production structure, x_2 can only be due to action a_2. Hence $\phi = 1$ after x_2 has been observed in period 1, and from Lemma 1 we know that any contingent reward scheme will be revised to a noncontingent payment. Any renegotiation-proof $M(x_2)$ can be written as $M(x_2) = w$. Second, consider $M(x_1) = \{m^1(x_1),m^2(x_1)\} = \{(v_1^1,v_2^1), (v_1^2,v_2^2)\}$. Let $\phi(p)$ be the posterior belief that the agent is type a_2 conditional on x_1 (see (6)). We have two cases: (A) $\overline{U}^1 \geq \overline{U}^2$; and (B) $\overline{U}^1 < \overline{U}^2$, where $\overline{U}^1 = v_1^1$ and $\overline{U}^2 = (1 - \gamma)v_1^2 + \gamma v_2^2$. For case (A), Lemma 1 implies that any renegotiation-proof contract must have $v_1^2 = v_2^2$. Therefore, if $M(x_1)$ promises a higher expected utility-payment for a type a_1 agent, then a type a_2 agent must face a constant wage in period 2. Notice that this is independent of ϕ. For case (B), Lemma 2 (i) requires that in any renegotiation-proof $M(x_1)$, $v_1^1 = v_1^2$. In addition, $M(x_1)$ fails to be renegotiation-proof if $\phi(p)$ is sufficiently high since in that situation the principal would raise v_1^1 and reduce the difference between v_2^2 and v_1^2. To summarize, we have

PROPOSITION 1. (I) $M(x_2)$ is renegotiation-proof if and only if $M(x_2) = w$. Also, if $\phi = 1$, $M(x_1)$ is renegotiation-proof if and only if $M(x_1) = v$. (II) Consider $\phi < 1$. Suppose that $\overline{U}^1 \geq \overline{U}^2$. Then $M(x_1)$ is renegotiation-proof if and only if $v_1^2 = v_2^2$ and v_2^1 is sufficiently small. Suppose that $\overline{U}^1 < \overline{U}^2$. $M(x_1)$ is renegotiation-proof with respect to ϕ if and only if $v_1^1 = v_1^2$ and $\phi \leq \hat{\phi}$ where $\hat{\phi}$ is given by Lemma 2 (iv).

The analysis thus far has assumed that the agent chooses action a_2 with probability $0 \leq p \leq 1$. We now want to characterize the optimal contracts that implement various distributions on actions. A contract $C = \{(u_1,u_2);\ M\}$ *implements* action a_2 with probability p if (1) $M(x_1)$ is renegotiation-proof with respect to $\phi(p)$ and $M(x_2) = w$; (2) given C, performing action a_2 with probability p in period 1 is the agent's best choice. (We assume that if the agent is indifferent between actions, she performs according to the principal's wish.) An *optimal* contract for action a_2 with probability p is a least cost contract that implements action a_2 with probability p.

It is obvious that the optimal contract that implements action a_1 can be expressed as a payment G_1 in each period. We now derive the optimal contract that implements action a_2 with probability 1. From Proposition 1, when a_2 is chosen with probability 1, $M(x_1) = v$. C offers contingent utility-payments (u_1,u_2) in period 1; moreover, if x_1 (respectively, x_2) is the output in period 1, the agent is paid v(respectively, w) in period 2. Therefore, the optimal contract for action a_2 (with probability 1) is a solution to the following program: Choose u_1, u_2, v, and w to minimize

$$(1 - \gamma)h(u_1) + \gamma h(u_2) + (1 - \gamma)h(v) + \gamma h(w)$$

subject to

$$(1 - \gamma)u_1 + \gamma u_2 + (1 - \gamma)v + \gamma w - 2G_2 \geq 0 \tag{11}$$

$$(1 - \gamma)u_1 + \gamma u_2 + (1 - \gamma)v + \gamma w - 2G_2 \geq u_1 + v - 2G_1. \tag{12}$$

Inequality (12) is an incentive constraint that guarantees a_2 as the most attractive choice for the agent given C; the first four terms on the left-hand side of (12) constitute the agent's expected utility payment if she picks a_2. Inequality (11) ensures that the agent has at least her reservation utility. The objective function denotes the cost of the contract.

PROPOSITION 2. The optimal contract that implements action a_2 with probability 1 has

$$(u_1,u_2) + (G_1, (G_2 - G_1)/\gamma + G_1),$$

$$M(x_1) = v = G_1, \qquad \text{and} \qquad M(x_2) = w = (G_2 - G_1)/\gamma + G_1.$$

Proof of Proposition 2. At a solution to the above program, (12) and (11) are both binding. It is obvious that (12) binds; otherwise u_1 and u_2 can be decreased. To see that (11) binds, suppose not. It is easy to see that then $u_1 = u_2 = v = w$. But $G_2 > G_1$ implies that (11) would be violated. This is a contradiction. Hence (11) binds. Using (11) and (12) as equalities, we have $u_1 = 2G_1 - v$ and $u_2 = 2(G_2 - G_1)/\gamma + 2G_1 - w$.

After substituting, we obtain the following partial derivatives of the objective function with respect to v and w:

$$(1 - \gamma)h'(v) - (1 - \gamma)h'(2G_1 - v) \tag{13}$$

$$\gamma h'(w) - \gamma h'(2(G_2 - G_1)/\gamma + 2G_1 - w). \tag{14}$$

By putting (13) and (14) equal to zero, we get $v = G_1$ and $w = (G_2 - G_1)/\gamma + G_1$.

Q.E.D.

The optimal contract that implements action a_2 with probability one is quite simple. In fact, $u_1 = v$, and $u_2 = w$. Moreover, $u_1 = v_1^*$, and $u_2 = v_2^*$, where (v_1^*, v_2^*) is the optimal contract for action a_2 in the corresponding one-period model (see (3) and (4) above). Proposition 2 also confirms the intuition that when action a_2 is implemented, all appropriate incentives are provided in the first period; the agent's period 2 compensation is decided once period 1 outcomes are observed. Our result differs from that in the paper by Fudenberg and Tirole [1990], in which implementing a_2 with probability one is never feasible.[6] Furthermore, observe that Proposition 2 and Assumption (A.1) together imply that the principal never implements a_1 in equilibrium.

We now proceed to consider implementing action a_2 with probability p, where $0 < p < 1$. Let $C = \{(u_1, u_2); M(x_1) = ((v_1^1, v_2^1), (v_1^2, v_2^2)), M(x_2) = w\}$ be a contract. The form of the renegotiation-proof constraints depends on the relative magnitudes of $\overline{U}^1$ and $\overline{U}^2$ (see Lemmas 1 and 2). An optimal contract C for action a_2 with probability p is a solution to the following program: choose u_1, u_2, v_1^1, v_1^2, v_2^2, and w to minimize

$$(1 - \gamma p)h(u_1) + \gamma p h(u_2) + (1 - \gamma p)[(1 - \phi)h(v_1^1) + \phi\{(1 - \gamma)h(v_1^2) + \gamma h(v_2^2)\}] + \gamma p h(w),$$

subject to

$$u_1 + v_1^1 - 2G_1 \geq 0, \tag{15}$$

$$(1 - \gamma)u_1 + \gamma u_2 + (1 - \gamma)[(1 - \gamma)v_1^2 + \gamma v_2^2] + \gamma w - 2G_2 \geq 0, \tag{16}$$

$$(1 - \gamma)u_1 + \gamma u_2 + (1 - \gamma)[(1 - \gamma)v_1^2 + \gamma v_2^2] + \gamma w - 2G_2 = u_1 + v_1^1 - 2G_1, \tag{17}$$

(18a) if $v_1^1 \geq (1 - \gamma)v_1^2 + \gamma v_2^2$, then $v_1^2 = v_2^2$,

(18b) if $v_1^1 < (1 - \gamma)v_1^2 + \gamma v_2^2$, then $v_1^1 = v_1^2$ and $\phi \leq \hat{\phi}$.

6. In fact, if the period 1 outputs are unobservable, then our model is a special case of Fudenberg and Tirole [1990]. Actually Proposition 2 suggests that if there is some limited commitment, then implementing a costly action with probability one is feasible in their paper. For example, suppose that there is a signal which is correlated with agent's actions. If the signal is observed before the output is, and the contract (which specifies that the agent's award is based on both the signal and output) is not renegotiated until the signal is observed, then implementing a costly action with certainty is feasible.

Inequalities (15) and (16) are reservation utility constraints for the agent; ex ante she obtains at least zero by choosing an action and an appropriate reward scheme. Equation (17) makes the agent indifferent between a_1 and a_2, given C. (18) consists of two parts. The first part deals with the renegotiation-proof constraint corresponding to $\overline{U}^1 \geq \overline{U}^2$; in this case, a type a_2 agent must have full insurance in period 2. The second part concerns $\overline{U}^1 < \overline{U}^2$; here, the period 2 schemes are used to deter a type a_1 agent from mimicking a type a_2 agent ($v_1^1 = v_1^2$), and the values of $\overline{U}^1$ and $\overline{U}^2$ are such that at the renegotiation stage, the principal does not increase the compensation to a type a_1 agent. This means that for any p and hence $\phi(p)$, $\overline{U}^1$ and $\overline{U}^2$ must guarantee $\hat{\phi} \geq \phi$. (See Lemma 2 (iv).) Finally, the objective function is the cost of the contract C when the agent chooses action a_2 with probability p. For a given p, an explicit solution for the above problem depends on the functional form of h and the values of γ, G_1 and G_2. Nevertheless, we are able to show

PROPOSITION 3. In an optimal contract that implements a_2 with probability p, where $0 < p < 1$, ex ante the agent obtains her reservation utility. In other words, constraints (15) and (16) are binding.

Proposition 3, which is proved in the Appendix, is in contrast to a result by Fudenberg and Tirole [1990]. They prove that under certain conditions the principal prefers to give the agent rent in order to relax the renegotiation-proof constraints. This is unnecessary in our model because for any renegotiation game to be played, the principal is able to find suitable u_1 and u_2 so that any rent to the agent is extracted.

V. AN "ADVERSE SELECTION" EQUILIBRIUM

In this section we show that under a suitable form of Assumption (A.1), the principal's utility when he optimally implements action a_2 is less than when he implements a random action (i.e., action a_2 with probability p, $0 < p < 1$). Recall that implementing a_2 (by an optimal contract) is for the principal better than implementing a_1. Therefore, the equilibrium of the game must be one in which the agent chooses an action according to a nondegenerate probability distribution. To do this, we first restate (A.1) (see (5) above) as

$$\Pi(a_2) - \Pi(a_1) = \gamma[h(v_2^*) - h(v_1^*)] + K, \quad K > 0. \tag{19}$$

Using Proposition 2, we can compute the principal's utility if he implements action a_2 with probability one by an optimal contract. Call this $B(a_2)$:

$$B(a_2) = 2\Pi(a_2) - 2(1 - \gamma)h(v_1^*) - 2\gamma h(v_2^*). \tag{20}$$

To obtain the optimal contract that implements a_2 with probability p is difficult because the principal's optimization is nontrivial and simple analytical solutions are unavailable. However, we are able to construct a feasible, renegotiation-proof contract that implements an action with a certain probability $\tilde{p}$. It will be seen that this contract gives the principal a utility higher than $B(a_2)$. The contract we have in mind is

$$\tilde{C} = \{(v_1^*, v_2^*),\quad \tilde{M}(x_1) = ((v_1^*, v_1^*), (v_1^*, v_2^*)),\quad \tilde{M}(x_2) = G_2\},$$

where (v_1^*, v_2^*) is the optimal contract for a_2 in the corresponding one-period model. We shall prove that $\tilde{C}$ gives the agent her reservation utility, makes her indifferent between a_1 and a_2, and that $\tilde{M}(x_1)$ is renegotiation-proof with respect to some probability. Indeed, consider $\tilde{M}(x_1)$. Since $\overline{U}^1 = v_1^* = G_1$ and $\overline{U}^2 = (1 - \gamma)v_1^* + \gamma v_2^* = G_2 > \overline{U}^1$ (see (3) and (4)), by Lemma 2 and Proposition 1, there is some number $\tilde{\phi}$, $0 < \tilde{\phi} < 1$, such that $\tilde{M}(x_1)$ is renegotiation-proof with respect to ϕ, where $0 < \phi \le \tilde{\phi}$. Let $\tilde{p}$ be defined by $\tilde{\phi} = \phi(\tilde{p})$. We shall suppose that the principal uses $\tilde{C}$ to implement a_2 with probability $\tilde{p}$. To check that $\tilde{C}$ does implement such an action, observe that the agent receives expected utility-payment $v_1^* + v_1^* = 2G_1$ by performing a_1, and that she receives $(1 - \gamma)v_1^* + \gamma v_2^* + (1 - \gamma)[(1 - \gamma)v_1^* + \gamma v_2^*] + \gamma G_2 = 2G_2$ by choosing a_2. In other words, she is indifferent between the actions and is guaranteed her reservation utility. By definition of $\tilde{\phi}$ and $\tilde{p}$, $\tilde{M}(x_1)$ is renegotiation-proof if the agent chooses a_2 with probability $\tilde{p}$. Clearly, $\tilde{M}(x_2)$ is renegotiation-proof. In sum, contract $\tilde{C}$ implements a_2 with probability $\tilde{p}$.

The properties of $\tilde{C}$ are noteworthy. Most important, the agent may face a contingent payment scheme (v_1^*, v_2^*) in period 2 when she selects a_2 and x_1 is the output in period 1. As a result, it is no longer necessary to offer the agent such a high period 2 payment in the event x_2 occurs in period 1; $\tilde{C}$ specifies that if the output in period 1 is x_2, she gets only G_2, which is smaller than v_2^*, the corresponding payment in the optimal contract that implements a_2 with probability 1. Thus, incentives are provided more evenly, which saves cost since the agent is risk averse.

When a_2 is implemented with probability $\tilde{p}$ by $\tilde{C}$, the principal's expected revenue is $2(1 - \tilde{p})\Pi(a_1) + 2\tilde{p}\Pi(a_2)$. The period 1 cost associated with $\tilde{C}$ is $(1 - \tilde{p}\gamma)h(v_1^*) + \tilde{p}\gamma h(v_2^*)$. The ex ante period 2 cost is $\tilde{C}$ is

$$(1 - \tilde{p}\gamma)[(1 - \tilde{\phi})h(v_1^*) + \tilde{\phi}[(1 - \gamma)h(v_1^*) + \gamma h(v_2^*)]] + \tilde{p}\gamma h(G_2)$$
$$= (1 - 2\tilde{p}\gamma + \tilde{p}\gamma^2)h(v_1^*) + (\tilde{p}\gamma - \tilde{p}\gamma^2)h(v_2^*) + \tilde{p}\gamma h(G_2),$$

where the last expression is a result of Bayes rule and simplification. When the principal uses $\tilde{C}$ to implement a_2 with probability $\tilde{p}$, his utility is written as

$$B(\tilde{p}; \tilde{C}) = 2(1 - \tilde{p})\Pi(a_1) + 2\tilde{p}\Pi(a_2) - (3\tilde{p}\gamma - \tilde{p}\gamma^2)[h(v_2^*) - h(v_1^*)] + \tilde{p}\gamma[h(v_2^*) - h(G_2)] - 2h(v_1^*). \tag{21}$$

We are now ready to state our main result.

PROPOSITION 4. If the relative superiority of action a_2 over a_1 in the corresponding one-period model is sufficiently small, i.e., $K > 0$ in (19) is small enough, then in equilibrium the agent chooses action a_2 with probability p, where $0 < p < 1$.

Proof of Proposition 4. It is sufficient to prove that for the principal a_1 and a_2 are dominated by a random action. We already know that (A.1) implies that $B(a_2)$ is bigger than the utility from implementing a_1. The proposition is true if $B(a_2)$ is less than the utility from implementing a random action. To show this, let the principal use contract $\tilde{C}$, and consider $B(\tilde{p}; \tilde{C})$, his utility when a_2 with probability $\tilde{p}$ is being implemented by $\tilde{C}$. Using (20) and (21), we compute $B(\tilde{p}; \tilde{C}) - B(a_2)$:

$$\begin{aligned} B(\tilde{p}; \tilde{C}) - B(a_2) &= -2(1 - \tilde{p})[\Pi(a_2) - \Pi(a_1)] \\ &\quad - (3\tilde{p}\gamma - \tilde{p}\gamma^2 - 2\gamma)[h(v_2^*) - h(v_1^*)] \\ &\quad + \tilde{p}\gamma[h(_2^*) - h(G_2)] \\ &= -2(1 - \tilde{p})\gamma[h(v_2^*) - h(v_1^*) + K] \\ &\quad - (3\tilde{p}\gamma - \tilde{p}\gamma^2 - 2\gamma)[h(v_2^*) - h(v_1^*)] \\ &\quad + \tilde{p}\gamma[h(v_2^*) - h(G_2)] \\ &= -2(1 - \tilde{p})\gamma K - (\tilde{p}\gamma - \tilde{p}\gamma^2)[h(v_2^*) - h(v_1^*)] \\ &\quad + \tilde{p}\gamma[h(v_2^*) - h(G_2)] \\ &= -2(1 - \tilde{p})\gamma K + \tilde{p}\gamma[(1 - \gamma)h(v_1^*) \\ &\quad + \gamma h(v_2^*) - h(G_2)], \end{aligned} \tag{22}$$

where the second equality follows from putting (19) into the preceding expression. The term inside the square brackets of (22) is strictly positive since h is convex and $(1 - \gamma)v_1^* + \gamma v_2^* = G_2$. Hence for a given $\tilde{p}$, (22) is positive provided that K is sufficiently small.[7]

Q.E.D.

In conclusion, we have verified that maintaining a degenerate belief may sometimes be too costly for the principal. When the agent always picks a_2, contingent utility-payments in period 1 are the only instruments to motivate her. But if a_2 is being implemented with probability $\tilde{p}$, then (v_1^*,v_2^*) may be offered in both periods. This reduces cost since the agent is risk averse. When $\Pi(a_2)$ is not much greater than $\Pi(a_1)$, indeed, action a_2 is dominated by a random action.

VI. Conclusions

In this paper we have studied a dynamic moral hazard model in which contracts can be renegotiated. The agent's action is interpreted as her choice of types or characteristics. We have proved that under some conditions the principal implements a random action. In period 2 the game possesses standard features of an "adverse selection" model: the principal uses a set of incentive schemes to screen different types of agents. On the empirical side, our results suggest that in long-term relationships, private information may evolve over time, and the agent may have some control over her compensations. If the equilibrium involves screening in period 2, then the riskiness of compensation is decided by the agent: she chooses among a menu of incentive schemes, each of them having a particular risk characteristic. On the other hand, as Proposition 2 illustrates, if in equilibrium, the agent always works hard, then her compensations in a later period are insensitive to her performance, even though production outcomes in period 2 convey important information about the agent's effort level.

Our model is similar to those by Fudenberg and Tirole [1990] and Aron [1987]. In Section IV we have already indicated some

7. The reader may have noticed that the proof of Proposition 4 goes through even if K is zero (or even slightly negative). This might at first sight seem peculiar, for this apparently says that when the principal is indifferent between implementing actions a_1 and a_2 (each with probability 1), he strictly benefits from implementing an action that is a mixture of them. However, notice that (A.1) is derived from the one-period model and has been chosen to ease computation. The correct comparison should be between the renegotiation model and a two-period model with contract commitment. In fact, even when $K = 0$, if the principal can fully commit to a long-term contract, he strictly prefers implementing a_2. Proposition 4 reflects the fact that the principal relies on a random action to enhance his ability to commit, hence reducing the cost of implementation.

similarities and differences with the former paper. In Aron [1987] only short-term contracts are available to the principal, and the agent's utility function is unknown. Thus, the effect of an incentive scheme on the agent is uncertain, and the principal will try to learn about the agent's characteristic. Aron carries out a numerical simulation and shows that in equilibrium the principal may choose to learn about the agent's characteristic gradually. The earlier version of this paper also assumed that only short-term contracts were feasible (and we adopted the interpretation on agent's actions described in footnote 2); under (A.1) the equilibrium of that dynamic game involves the agent choosing a random action.[8]

We have used a very simple model to make possible explicit computations of the principal's utility. However, we believe that the intuition behind our result is quite general. When an agent's action has long-term effects, then without fully committed contracts, one would expect shirking behavior to be more common. For the principal, the difficulty of implementing costly actions compounds over the contracting horizon; this obstacle may outweigh the benefit.

APPENDIX

Proof of Lemma 2. Consider a relaxed program, (RP), of program (RNP). In (RP), (10) is ignored; it will be shown that (10) is satisfied at a solution to (RP). We rewrite (RP) here: given $\overline{U}^1 < \overline{U}^2$ and $\phi < 1$, choose u_1^1, u_1^2, and u_2^2 to minimize

$$(1-\phi)h(u_1^1) + \phi[(1-\gamma)h(u_1^2) + \gamma h(u_2^2)],$$

subject to

$$u_1^1 \geq \overline{U}^1 \tag{7}$$

$$(1-\gamma)u_1^2 + \gamma u_2^2 \geq \overline{U}^2 \tag{8}$$

$$u_1^1 \geq u_1^2. \tag{9}$$

At a solution to (RP), both (8) and (9) bind. First, if (8) did not bind, then u_2^2 could be reduced. This is a contradiction. Second, if (9) did not bind, then because h is convex, $u_1^1 = \overline{U}^1$, $u_1^2 = u_2^2 = \overline{U}^2$. But then $u_1^1 < u_1^2$, violating (9). This is a contradiction. We have therefore proved (iii) and the equality in (i). (v) follows immediately.

8. Other models that deal with adverse selection and the revelation of information over time in situations where only short-term contracts are available include Freixas, Guesnerie, and Tirole [1985] and Laffont and Tirole [1988]. They study the "Ratchet Effect" in the context of a planning–regulation model.

After substituting $u_1^2 = u_1^1$ in (RP), we derive the following first-order conditions:

$$\begin{aligned} (1 - \phi\gamma)h'(u_1^1) + \lambda_1 + (1 - \gamma)\lambda_2 &= 0 \\ \phi\gamma h'(u_2^2) + \gamma\lambda_2 &= 0 \\ \lambda_1[u_1^1 - \overline{U}^1] &= 0, \end{aligned}$$

where $\lambda_1 \leq 0$ and $\lambda_2 < 0$ are multipliers. Now either $\lambda_1 = 0$, or $\lambda_1 < 0$. In the former case, the FOC's imply that

$$[(1 - \phi\gamma)h'(u_1^1)]/[\phi(1 - \gamma)h'(u_2^2)] = 1.$$

Hence $u_2^2 > u_1^1$ because $1 - \phi\gamma > \phi(1 - \gamma)$ and h is convex. In the latter case ($\lambda_1 < 0$), (7) and (8) are equalities and give $u_1^1 = \overline{U}^1$, and $u_2^2 = \overline{U}^1 + \epsilon(\overline{U}^2 - \overline{U}^1) > u_1^1$. This proves the inequality in (i). Now (ii) follows immediately from (i). This also shows that (10) in (RNP) is satisfied at a solution to (RP) if we put $u_2^1 = u_1^1$. It remains to prove (iv) and (vi).

From (i) to (iii) we can solve for u_2^2 in terms of u_1^1:

$$u_2^2 = \epsilon\overline{U}^2 - (\epsilon - 1)u_1^1.$$

The objective function is $C(u_1^1) \equiv (1 - \phi\gamma)\, h(u_1^1) + \phi\gamma h(u_2^2)$. Then

$$(*) \qquad C'(u_1^1) \equiv (1 - \phi\gamma)h'(u_1^1) - \phi(1 - \gamma)h'(u_2^2).$$

Now put $u_1^1 = \overline{U}^1$. Then $u_2^2 = \overline{U}^1 + \epsilon(\overline{U}^2 - \overline{U}^1)$. Note that $u_1^1 < u_2^2$. $(1 - \phi\gamma)$ and $\phi(1 - \gamma)$ have the same limit as ϕ tends to one. Hence for ϕ sufficiently close to one, $C'(\overline{U}^1) < 0$. Therefore, at a solution to (RP), $u_1^1 > \overline{U}^1$ for ϕ sufficiently close to one. Also as ϕ tends to zero, $C'(\overline{U}^1)$ tends to $h'(\overline{U}^1) > 0$. Thus, for ϕ small enough, $u_1^1 = \overline{U}^1$.

To prove (iv), take $\overline{\phi}$ for which $u_1^1(\overline{\phi}) > \overline{U}^1$. We now show that $u_1^1(\overline{\phi})$ has a positive derivative. If $u_1^1(\overline{\phi}) > \overline{U}^1$, then

$$C'(u_1^1) \equiv (1 - \overline{\phi}\gamma)h'(\mathrm{u}_1^1) - \overline{\phi}(1 - \gamma)h'(u_2^2) = 0.$$

Differentiating this FOC with respect to $\overline{\phi}$ and rearrange, we obtain

$$(**) \qquad \frac{du_1^1(\overline{\phi})}{d\overline{\phi}} = \frac{\gamma h'(u_1^1) + (1 - \gamma)h'(u_2^2)}{(1 - \overline{\phi}\gamma)h''(u_1^1) + \overline{\phi}h''(u_2^2)(1 - \gamma)^2/\gamma} > 0.$$

(iv) then follows from (**) and continuity.

Finally, the first part of (vi) is a result of (i), (iii), (iv), and (**). By equating (*) to 0 and take limit as ϕ goes to 1, one has the last part of (vi).

Q.E.D.

Proof of Proposition 3. We consider a program in which (17) is relaxed to

$$(17a)\quad (1-\gamma)u_1 + \gamma u_2 + [(1-\gamma)v_1^2 + \gamma v_2^2] + \gamma w - 2G_2 \geq u_1 + v_1^1 - 2G_1.$$

It will be shown that at a solution to the relaxed program (17a) holds as an equality. We first claim that at least one of (15) and (16) binds. Suppose that it were not true, and neither (15) nor (16) was binding. Then the principal could decrease u_1 and u_2 by the same amount. Obviously (17a) and (18) would not be affected. But the principal's cost was decreased. This is a contradiction.

Next, we show that (15) and (16) *both* bind. First, suppose that (15) does not bind. Then (16) binds. But (15) being a strict inequality and (16) being an equality imply that (17a) is violated. This is a contradiction. Hence (15) is binding. Second, (16) is binding. Notice that if (16) did not bind, then (15) being binding implies that (17a) is a strict inequality. Then the principal could decrease u_2 without violating any constraint in the relaxed program. This is a contradiction. Hence (16) is binding. Now it immediately follows that at a solution (17a) is an equality.

Q.E.D.

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