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Stopping Agents from "Cheating"

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When a principal hires two (or more) agents to work in a correlated environment, each agent's reward will depend on the other's performance. Unfortunately, with just the usual optimal (incentive-constrained) contracts being offered, the agents strictly prefer to play equilibrium strategies other than those desired by the principal; they "cheat." However, the principal can use a more subtle contractual mechanism to stop them from cheating at no extra cost. This mechanism uses one agent to police the other, and exploits certain properties of the optimal contracts. *Journal of Economic Literature* Classification Numbers: 022, 026. © 1988 Academic Press, Inc.

1. INTRODUCTION

The method of maximizing subject to "incentive constraints" is an indispensable tool in the design and analysis of optimal mechanisms for use in environments of incomplete information. But it is not always adequately taken into account that incentive compatibility is only a *necessary* requirement, and may not be sufficient. The difficulty is that in many models, the optimal incentive compatible mechanism may have, in addition

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to the desired equilibrium, several undesired equilibria. In this paper we look at such a model, in the context of agency theory.[1]

Principal/multi-agent models in general appear to be susceptible to the problem of multiple equilibria in the game played by the agents. A number of recent papers have made this point: Demski and Sappington [2], their paper with Spiller [3], and Mookherjee [7]. Our particular concern is with the model of Demski and Sappington [2], although we anticipate that some variant of the method we propose here will prove to be useful in dealing with unwanted equilibria in other models too.[2] Demski and Sappington examine a situation in which two agents' production functions are correlated: specifically, agents observe correlated state variables before they take actions. A pair of optimal incentive contracts—one for each agent—exploits this correlation by making payments contingent not only on an agent's own output but also on the other agent's output. The incentive constraints ensure that it is a Bayes–Nash equilibrium for each agent to accept his contract and to produce the output which the principal wants from him (as a function of what he has observed).[3] However, there is a difficulty. Demski and Sappington found that given a pair of optimal contracts, there exists another pair of *equilibrium* strategies whose outcome, from the agents' point of view, Pareto dominates the equilibrium outcome which the principal wants to implement. Loosely put, the agents can "cheat" the principal.

In [2], Demski and Sappington propose to avoid the problem of multiple equilibria by *strengthening* the principal's constraint set so that one agent's output choice (as a function of his individual observation) is a dominant strategy for him. (This agent has the "dual role of productive

[1] For a class of models where the direct mechanism exhibits multiple Bayes–Nash equilibria, see Laffont and Maskin [4].

[2] See Ma [5].

[3] Clearly, such a contract represents rather a narrowly specified mechanism. From the Revelation Principle, we know that the most efficient contract would require each agent in effect to report his private observation—so that if necessary, his output, as well as his payment, could depend on *both* agents' observations. But notice that there is still likely to be the problem of multiple equilibria even in these more general "revelation" mechanisms. The point is that there is a distinction between a revelation mechanism that "truthfully implements" (i.e., in which truth-telling is *one* equilibrium) and an abstract mechanism that "fully implements" (i.e., in which the desired strategies constitute the unique equilibrium). On this point, see, for example, Dasgupta et al. [1] and Repullo [11].

Demski and Sappington in [2] restricted their attention to the simpler kind of contracts in which agents choose their outputs solely on the basis of their individual observations, and did not consider revelation mechanisms. We shall follow them in this regard in order to facilitate a direct comparison with their results. However, in Ma and Moore [6], we suggest some reasons why general contracts might be infeasible. Also, it appears that a method similar to the one we develop in this paper to knock out unwanted equilibria would work for more general contracts.

agent and information supplier" [2, p. 168].) Of course it costs the principal to do so.

In this paper we solve the problem of multiple equilibria raised by Demski and Sappington, without incurring any additional cost to the principal. We show how the principal can use one agent to police the other by offering him a set of additional output levels from which to choose—although these will not be used in equilibrium. It therefore turns out that there is no need for the principal to adopt the (costly) solution of acquiring information from an agent whose actions have been artificially constrained to be a dominant strategy. Instead, the principal can proceed as if the only constraints were those required to ensure that the agents' output choices constitute a Bayes–Nash equilibrium.

We also characterize the optimal contracts—i.e., when only the (Bayesian) incentive constraints have been imposed. It is important to characterize these optimal contracts since our mechanism relies on their properties. Parts of the characterization theorem are not standard, owing to the interrelated nature of the agents' contracts. For example, unlike the single-agent model, agents may not receive rents in their high productivity states. Also, optimal contracts may not exhibit the usual monotonicity: an agent may produce more output even though he is less productive.

Before starting our analysis, we should mention some recent related work by Postlewaite and Schmeidler [10] and Palfrey and Srivastava [8] (also see their paper [9]). The results of these papers may be applied to the general question of what can be uniquely implemented as a Bayes–Nash equilibrium in exchange environments with incomplete information. They show that if the desired outcomes satisfy certain conditions (the key one being "Bayesian monotonicity"), and if there are three or more agents, then there exists a mechanism which generates those outcomes uniquely. Clearly this work is germane to the present paper, in that a similar issue is being addressed. But there are two respects in which our work is distinct. First, the abstract mechanisms in [10, 8] only deal with exchange economies. Moreover they do not deal with the case of two agents (it is well known from the implementation literature that in general two-agent models pose particular difficulties). Second, we are concerned here to find a mechanism that is reasonably simple and which can be interpreted, whereas the mechanisms in [10, 8] are rather abstruse. That is not surprising, though, because those mechanisms are designed for a broad context. We are likely to be able to find a mechanism that is "reasonably simple," and then judge whether it can be "interpreted," only if we work with a specific model.

The plan of the paper is as follows. Section 2 introduces the model. Optimal (second-best) contracts are characterized in Section 3. In Section 4, we describe a mechanism that implements the second best as a unique Bayes–Nash equilibrium. Open questions are in Section 5.

2. The Model

The principal owns two production processes, $x^l = X^l(a^l, \theta^l)$, $l = A, B$, to be operated by two agents, A and B. θ^l is a random variable with binary support $\{\theta_1^l, \theta_2^l\}$. For $l = A, B$, let

$$p_i^l = \mathrm{Prob}\{\theta^l = \theta_i^l\} > 0 \qquad (i = 1, 2)$$
$$p_1^l + p_2^l = 1.$$

The level of effort, a^l, exerted by agent l is not observable, and hence compensation cannot be made contingent on it. Assume that for each l there are decreasing returns to effort: $X_a^l > 0$ and $X_{aa}^l < 0$. Also

$$X^l(a^l, \theta_1^l) < X^l(a^l, \theta_2^l) \qquad \text{for all } a^l,$$

so θ_2^l represents a "good" state and θ_1^l a "bad" state.

The state variables are positively but imperfectly correlated. That is, putting

$$q_i^A = \mathrm{Prob}\{\theta^B = \theta_1^B \mid \theta^A = \theta_i^A\}$$

and

$$q_i^B = \mathrm{Prob}\{\theta^A = \theta_1^A \mid \theta^B = \theta_i^B\},$$

we assume for $l = A, B$ that

$$1 > q_1^l > q_2^l > 0.$$

(If ϕ_{jk} is the joint probability that $\theta^A = \theta_j^A$ and $\theta^B = \theta_k^B$, where $j, k = 1, 2$, then for $i = 1, 2$: $p_i^A = (\phi_{i1} + \phi_{i2})$; $p_i^B = (\phi_{1i} + \phi_{2i})$; $q_i^A = \phi_{i1}/(\phi_{i1} + \phi_{i2})$; and $q_i^B = \phi_{1i}/(\phi_{1i} + \phi_{2i})$.)

Agent l $(= A, B)$ *privately* observes θ^l before he signs a contract with the principal. Since neither θ^l nor a^l is publicly observed, an enforceable incentive scheme, or *contract*, can only be based on the level(s) of output(s) of one or both agents.

We assume that the principal is risk neutral, and he maximizes expected profit. Prices of the x's are normalized so that x^A and x^B represent both outputs and revenues. Agents are risk averse and dislike effort increasingly. They have additively separable utility functions $u^l(R^l, a^l) \equiv U^l(R^l) - V^l(a^l)$, for $l = A, B$, where R^l is payment from principal; U^l is increasing and strictly concave in R^l, and V^l is increasing and strictly convex in a^l.

Knowing the utility function of an agent l (A or B), one can compute the level of effort, a^l, required to produce a certain level of output, x^l, in state θ^l. Hence, one can define the agent's disutility, D^l, say, of effort in terms of his output level and the state variable: $D^l(x^l, \theta^l)$. From the assumptions on $X^l(\cdot, \cdot)$ and $V^l(\cdot)$ given earlier, it follows that $D_x^l > 0$, $D_{xx}^l > 0$, and $D^l(x^l, \theta_1^l) > D^l(x^l, \theta_2^l)$ for all x^l. We also suppose that

$$D_x^l(x^l, \theta_1^l) > D_x^l(x^l, \theta_2^l) \qquad \text{for all } x^l.$$

Thus for any given level of output, the marginal (as well as the total) disutility for the agent in a good state is smaller than that in a bad state. Note that this last assumption cannot be derived from the assumptions on $X^l(\cdot, \cdot)$ and $V^l(\cdot)$, but it will simplify the analysis when we handle incentive constraints. This assumption—the "Spence/Mirrlees condition"—is very common in self-selection models.

There exists a reservation utility level, $\bar{U}^l$, for each agent $l = A, B$, that represents the expected utility that he would obtain if he refused to sign a contract with the principal.

The principal offers each agent a contract. These contracts may be interdependent. For example, consider a typical contract offered to agent A; it will read

> "You may choose to produce either x_1^A or x_2^A. Your payment, R^A, will depend not only on your own output, but also on what agent B does; if you choose to produce x_i^A $(i = 1, 2)$, then[4]
>
> if agent B produces x_1^B, you will be paid R_{i1}^A
> if agent B produces x_2^B, you will be paid R_{i2}^A
> if agent B does not sign his contract, you will be paid R_{i0}^A."

3. Characterization

To provide a benchmark, we begin by writing down the optimal contract in a first-best world where the principal can observe or monitor an agent's effort costlessly. It is obvious that this is equivalent to the case where the principal can observe θ^l, $l = A, B$, before he signs a contract with an agent. Also, it is enough to consider the optimal contract with respect to one agent; accordingly, we shall leave out the superscripts $l = A, B$ whenever so doing does not create confusion. A first-best contract will have the following properties:

[4] If it turns out that in agent B's optimal contract x_1^B precisely equals x_2^B, then agent A's payment could not be made contingent on B's output choice. We ignore this possibility throughout our analysis—for if necessary, B's output choices x_1^B and x_2^B could be made distinct but arbitrarily close, with arbitrarily small loss to the principal. The same argument applies if A's output choices x_1^A and x_2^A happen to coincide at the optimum.

(i) $U(R_i^*) - D(x_i^*, \theta_i) = \bar{U}$, for $i = 1, 2$,
(ii) $U'(R_i^*) = D_x(x_i^*, \theta_i)$, for $i = 1, 2$,
(iii) $x_1^* < x_2^*$.

Properties (i) and (ii) together say that production levels are efficient and the principal can hold an agent to his reservation utility in both states. To see (iii), note that if $x_1^* \geqslant x_2^*$, then $D_x(x_1^*, \theta_1) > D_x(x_1^*, \theta_2)$ and $D(x_1^*, \theta_1) > D(x_2^*, \theta_2)$. The first inequality implies $R_1^* < R_2^*$ from (ii); but the second inequality implies $R_1^* > R_2^*$ from (i). Hence $x_1^* < x_2^*$. These properties will be useful for the characterization problem later.

Turning now to the second best, Demski and Sappington partially characterized the solution of the program that restricts the agents' output choices to be a Bayes–Nash equilibrium, given that they are guaranteed at least their reservation utilities (conditional on their private knowledge). The following program gives the principal's problem for each agent (again, we drop the superscript $l = A, B$):

$$\text{(P}\cdot\text{BN)} \qquad \underset{x_i,\, R_{ij}}{\text{maximize}}\; p_1[q_1(x_1 - R_{11}) + (1-q_1)(x_1 - R_{12})] + p_2[q_2(x_2 - R_{21}) + (1-q_2)(x_2 - R_{22})]$$

subject to

$$q_i U(R_{i1}) + (1-q_i)\, U(R_{i2}) - D(x_i, \theta_i) \geqslant \bar{U}, \qquad i = 1, 2 \qquad (1, i)$$

$$q_i U(R_{i1}) + (1-q_i)\, U(R_{i2}) - D(x_i, \theta_i) \geqslant q_i U(R_{j1}) + (1-q_i)\, U(R_{j2}) - D(x_j, \theta_i), \qquad i, j = 1, 2;\ i \neq j. \quad (2, i)$$

PROPOSITION 1. *Optimal (second-best) contracts have the following properties (omitting superscripts $l = A, B$):*

(ai) $R_{11} > R_{12}$, (aii) $R_{21} = R_{22} = R_2$ (say).
(bi) $q_1 U(R_{11}) + (1-q_1)\, U(R_{12}) - D(x_1, \theta_1) = \bar{U}$,
(bii) $U(R_2) - D(x_2, \theta_2) \geqslant \bar{U}$.
(ci) $q_1 U'(R_{11}) + (1-q_1)\, U'(R_{12}) > D_x(x_1, \theta_1)$,
(cii) $U'(R_2) = D_x(x_2, \theta_2)$.
(di) $q_1 U(R_{11}) + (1-q_1)\, U(R_{12}) - D(x_1, \theta_1) > U(R_2) - D(x_2, \theta_1)$,
(dii) $U(R_2) - D(x_2, \theta_2) = q_2 U(R_{11}) + (1-q_2)\, U(R_{12}) - D(x_1, \theta_2)$.
(e) $\pi_2 > \pi_1$, *where* $\pi_i \equiv x_i - q_i R_{i1} - (1-q_i)\, R_{i2}$ *for* $i = 1, 2$.
(f) $x_2 \leqslant x_2^*$, *with equality if there is an equality in* (bii).

(g) *If* $U(\cdot)$ *exhibits constant absolute risk aversion, then* (i) $x_1 < x_1^*$, *and* (ii) $x_1 < x_2$. (*These are also true for increasing absolute risk aversion.*)

The nature of many of these results is well known, although there are a couple of surprising possibilities. The key point is that only one of the incentive constraints, (dii), binds at an optimum: the principal has to discourage an agent from choosing output x_1 when he has in fact observed θ_2. Thus the only useful distortions from efficient risk sharing and efficient production (at the margin) are for those parts of the contract designed for an agent who has observed θ_1. In (ai), the lottery (R_{1j}) helps with incentives because if the agent has in fact observed θ_2, not θ_1, then he has a lower probability ($q_2 < q_1$) of getting the higher payment ($R_{11} > R_{12}$).[5] In (ci), the inefficiently low output x_1 (which is offset by a reduction in the payments R_{1j}) helps with incentives because if the agent has in fact observed θ_2, not θ_1, then he has a lower marginal disutility of effort ($D_x(\cdot, \theta_2) < D_x(\cdot, \theta_1)$). Part (f) of the proposition says that the output x_2 for an agent who has observed θ_2 never exceeds the corresponding level in the first best, and will be less only if he is receiving some rent from his private information. Notice that it is indeed possible for him *not* to receive any rent (bii)! This stark contrast to the single-agent model reflects the fact that the principal is exploiting the correlation between the two agents' observations. Stronger results on the levels of outputs are obtained in (g), when the assumption of constant (or increasing) absolute risk aversion is made. With decreasing absolute risk aversion (a usual assumption), it is quite possible that output x_1 in the bad state θ_1 is *higher* not only than output x_2 in the good state θ_2, but also than the first-best outputs (x_1^*, x_2^*) in either state. Again, this surprising possibility arises from the interrelated nature of the two agents' contracts.

The proof of Proposition 1 is also somewhat nonstandard, in that it is difficult to show directly that the incentive constraint (di) is slack. Instead, one proves this indirectly by first showing that the principal's expected profit from an agent in a bad state is less than that in a good state (e). See the Appendix.

4. An Indirect Mechanism with a Unique Equilibrium

It is important to realize what Proposition 1 does and does not tell us. We have characterized contracts that yield the principal the highest *possible* expected profit. If the principal offers each agent $l = A, B$ the choice of

[5] There would be no profit in adding any further, extraneous uncertainty to the payments R. The reason is that the agents are risk averse, and the incentive constraints would not be relaxed because the agents' attitudes to risk are (by assumption) independent of their observation θ.

— produce x_1^l and receive (stochastic) payment $\{R_{11}^l, R_{12}^l\}$, or
— produce x_2^l and receive payment R_2^l

then the principal *will* earn the maximum profit *if the agents both respond as the principal desires*: viz., sign their respective contracts and produce output x_i^l when they observe θ_i^l $(i=1,2)$. (Throughout the previous section, we supposed that agents will sign their contract; that is why it was unnecessary to specify the payments R_{i0}^l made to an agent l $(=A, B)$ who produces output x_i^l when the other agent refuses to sign.) By construction, it must be an equilibrium for the agents to respond in this way. Let this equilibrium be E^*, say. But the proposition does not tell us if there are other equilibria in the game played by the agents.

The challenging observation made by Demski and Sappington [2] is that the desired equilibrium E^* is dominated (*from the agents' perspective*) by another Bayes–Nash equilibrium. Specifically, it is an equilibrium (E°, say) for both agents to *always* choose the output x_1^l corresponding to their bad state θ_1^l $(l=A, B)$, and in all states they will both be strictly better off than in equilibrium E^*. (From now on, let x_1^l, x_2^l, R_{11}^l, R_{12}^l, and R_2^l refer to a solution of (P.BN) for agents $l=A$ and B, respectively.) Of course, the principal will be strictly worse off.

The point is that if agent A (say) is in the good state θ_2^A then he is indifferent between choosing x_2^A and x_1^A, provided that agent B is playing according to E^*—that is, provided agent A assesses the respective probabilities of payments R_{11}^A and R_{12}^A to be q_2^A and $(1-q_2^A)$ should he choose x_1^A. (See (dii) of Proposition 1.) However, suppose that agent B always chooses output x_1^B. Then in state θ_2^A agent A will strictly prefer producing x_1^A to x_2^A, since with the former choice he will now be paid R_{11}^A with certainty and $R_{11}^A > R_{12}^A$ from (ai) of Proposition 1. This argument works symmetrically for agent B. E° is therefore an equilibrium, and it dominates E^* since both agents avoid the low payments R_{12}^l.

Thus Demski and Sappington argued that "explicit attention must be afforded [to] alternative strategies that the agents might adopt. In particular, the equilibrium in which both agents simultaneously claim to always be unproductive must be explicitly avoided" [2, p. 166]. Their approach is to strengthen the incentive constraints of *one* agent to make his output choices a dominant strategy for him. (They do not analyze *which* agent the principal would choose to place in this dominant strategy position.) But although this method does guarantee a unique equilibrium, it is also costly since the principal has to strengthen the incentive constraints in (P.BN).

We propose here an alternative, costless method of stopping the agents from "cheating": the principal enriches their strategy sets. We show that the principal can guarantee the second-best outcome from (P.BN) as a *unique* —and hence undominated (from the agents' perspective)—equilibrium of an appropriately designed mechanism.

The key to the indirect mechanism is that the principal offers one agent, say agent A, a range of extra output options $x_1^A(\varepsilon)$—indexed by ε, where $0<\varepsilon\leqslant 1-q_1^A$. If agent A chooses one of these options $x_1^A(\varepsilon)$ then he essentially produces x_1^A—except that $x_1^A(\varepsilon)$ has some inconsequential modification "ε" which is costless for agent A to effect.[6] The importance of ε is that it acts as a signal that agent A sends to the principal:

> "Agent B is cheating; from my perspective, the probability that he is choosing x_1^B is at least $(q_1^A+\varepsilon)$."

In the light of such a signal from agent A, if agent B chooses x_1^B, the principal pays him an amount $\bar{R}_1^B$, where

$$U^B(\bar{R}_1^B) = q_2^B\, U^B(R_{11}^B) + (1-q_2^B)\, U^B(R_{12}^B). \tag{3}$$

That is, the principal pays the certainty equivalent of the lottery agent B would face if he had observed θ_2^B. However, if agent B actually chooses x_2^B—and agent A signals some $\varepsilon>0$ by choosing $x_1^A(\varepsilon)$—then agent B is, so to speak, "compensated for the slur on his character" by receiving a higher payment $(R_2^B+\gamma)$. (The increase $\gamma>0$ must not be too great, though. It turns out that too high a compensation $(R_2^B+\gamma)$ might admit unwanted equilibria. More on this below.)

It must be the case that agent A has an incentive to exercise one of the options $x_1^A(\varepsilon)$ if agent B is choosing x_1^B more often than he would in equilibrium E^*. With this in mind, agent A is "rewarded" for signalling ε as follows: in return for producing $x_1^A(\varepsilon)$, he is paid

$$\begin{array}{ll} R_{11}^A + s(\varepsilon) & \text{if agent } B \text{ chooses } x_1^B \\ R_{12}^A - t(\varepsilon) & \text{if agent } B \text{ chooses } x_2^B, \end{array}$$

where the (continuous) functions $s(\varepsilon)$ and $t(\varepsilon)$ are both strictly positive for $0<\varepsilon\leqslant 1-q_1^A$, and satisfy

$$\begin{aligned}(q_1^A+\varepsilon)\, U^A(R_{11}^A+s(\varepsilon)) + (1-q_1^A-\varepsilon)\, U^A(R_{12}^A-t(\varepsilon))\\ = (q_1^A+\varepsilon)\, U^A(R_{11}^A) + (1-q_1^A-\varepsilon)\, U^A(R_{12}^A).\end{aligned} \tag{4}$$

[6] In case the idea of $x_1^A(\varepsilon)$ being a qualitative modification "ε" of x_1^A seems unappealing, there is a purely quantitative way of achieving the same ends. Namely, let the quantity $x_1^A(\varepsilon)$ vary slightly, and strictly monotonically, with ε, so that $x_1^A(\varepsilon)\to x_1^A$ as $\varepsilon\to 0$. We have chosen not to model the mechanism this way because, although it works, the analysis would be somewhat more complicated and less transparent.

The idea of this "reward scheme" is as follows. Consider agent A who has observed θ_1^A. Suppose he assesses that agent B is choosing x_1^B more than he would in equilibrium E^*—say with a probability of $q > q_1^A$. Then the construction (4), together with the fact that $s(\varepsilon)$ and $t(\varepsilon)$ are both positive, means that for all $0 < \varepsilon < (q - q_1^A)$, agent A prefers to choose $x_1^A(\varepsilon)$ rather than x_1^A. To see why, define the difference in payoff between choosing $x_1^A(\varepsilon)$ and x_1^A as

$$\Delta^\varepsilon(q) \equiv \{[qU^A(R_{11}^A + s(\varepsilon)) + (1-q)\, U^A(R_{12}^A - t(\varepsilon)) - D^A(x_1^A, \theta_1^A)] - [qU^A(R_{11}^A) + (1-q)\, U^A(R_{12}^A) - D^A(x_1^A, \theta_1^A)]\}.$$

Then $\partial \Delta^\varepsilon(q)/\partial q = [U^A(R_{11}^A + s(\varepsilon)) - U^A(R_{12}^A - t(\varepsilon))] - [U^A(R_{11}^A) - U^A(R_{12}^A)] > 0$, and—since (4) implies $\Delta^\varepsilon(q_1^A + \varepsilon) = 0$—it follows that $\Delta^\varepsilon(q) > 0$ for $q > q_1^A + \varepsilon$.

Two other aspects of this construction are important. First, if agent B is in fact choosing output as in equilibrium E^*, then agent A must not have an incentive to signal some $\varepsilon > 0$. This has been taken into account: (4) implies $\Delta^\varepsilon(q_1^A + \varepsilon) = 0$, which (since $\Delta^\varepsilon(\cdot)$ is strictly increasing) in turn implies $\Delta^\varepsilon(q_1^A) < 0$ and $\Delta^\varepsilon(q_2^A) < 0$ for all $\varepsilon > 0$.

Second, we do not want our uniqueness result to be contrived in the sense that it hinges on either agent (in particular, agent A) maximizing over a noncompact strategy set. In other words, we would be unhappy proposing a resolution of the multiple equilibria problem which exploited the nonattainability of some supremum. Therefore, we have taken care to ensure that each agent maximizes a continuous payoff over a compact strategy set. For agent A, the limit point $\varepsilon = 0$ is a potential problem. But by choosing $s(\varepsilon)$ and $t(\varepsilon)$ so that

$$\operatorname*{Lim}_{\varepsilon \to 0} s(\varepsilon) = \operatorname*{Lim}_{\varepsilon \to 0} t(\varepsilon) = 0,$$

the limit point $\varepsilon = 0$ simply corresponds to choosing x_1^A—since in the limit agent A produces $x_1^A(0)$ ($\equiv x_1^A$) for random payment (R_{11}^A, R_{12}^A).

An issue we must consider is how much should an agent be paid for producing output x_i^I $(i = 1, 2)$ if the other agent refuses to sign his contract—i.e., what values should R_{i0}^I take? The principal wants to implement the second best *uniquely*. So he must avoid any equilibrium in which one or both agents refuse to sign their contracts. Rather than give the details here of how the principal can do this, we leave them to the proof of Proposition 2.

Before stating the proposition, it may be helpful to display the full "payment matrices" of the mechanism which uniquely implements the second best. These are given in Table I. The various aspects of the

TABLE I

The Mechanism

Agent A's Payments

A's choice \ B's choice	x_1^B	x_2^B	Refuse
x_1^A	R_{11}^A	R_{12}^A	R_{11}^A
$x_1^A(\varepsilon)$	$R_{11}^A + s(\varepsilon)$	$R_{12}^A - t(\varepsilon)$	$R_{11}^A + s(\varepsilon)$
x_2^A	R_2^A	R_2^A	$R_2^A - \gamma$
Refuse	—	—	—

Agent B's Payments

A's choice \ B's choice	x_1^B	x_2^B	Refuse
x_1^A	R_{11}^B	R_2^B	—
$x_1^A(\varepsilon)$	$\bar{R}_1^B$	$R_2^B + \gamma$	—
x_2^A	R_{12}^B	R_2^B	—
Refuse	R_{11}^B	$R_2^B - \gamma$	—

mechanism which have been highlighted in the preceding discussion should be apparent.

One last point to which we alluded earlier. Agent B's "compensation" γ—see the payment $R_2^B + \gamma$ in the second row/second column box of his payment matrix—cannot be too great, otherwise unwanted equilibria may be admitted. Specifically, we assume that $\gamma > 0$ is chosen sufficiently small so that

$$q_2^B\, U^B(R_2^B + \gamma) + (1 - q_2^B)\, U^B(R_2^B) - D^B(x_2^B, \theta_2^B) < q_2^B\, U^B(\bar{R}_1^B) + (1 - q_2^B)\, U^B(R_{11}^B) - D^B(x_1^B, \theta_2^B) \tag{5}$$

and

$$U^B(R_2^B + \gamma) - D^B(x_2^B, \theta_1^B) < \bar{U}^B. \tag{6}$$

These two upper bounds on γ will be made use of in the proof of Proposition 2. (Note that (5) *is* feasible for sufficiently small $\gamma > 0$: this follows from (dii) and (ai) in Proposition 1, together with the definition of

$\bar{R}_1^B$ in (3) and the fact that $q_2^B < 1$. Also (6) *is* feasible for sufficiently small $\gamma > 0$, directly because of (bi) and (di) in Proposition 1.)

(Incidentally, we have used γ in two other boxes of the payoff matrices—the bottom right-hand boxes. This is not significant; it would suffice to subtract any positive amount from these payments R_{20}^l $(= R_2^l - \gamma)$ for $l = A, B$. The critical upper bounds (5) and (6) pertain only to agent B's payment $R_2^B + \gamma$ when he chooses x_2^B, but agent A signals some $\varepsilon > 0$ by choosing $x_1^A(\varepsilon)$.)

We are now in position to state and prove our central result.

PROPOSITION 2. *Suppose the payments are as given in Table I. Then the second best is implemented as the unique Bayes–Nash equilibrium* E^*.

Proof. We adopt the following shorthand. Let $A1$ $(A2)$ denote agent A if he has observed θ_1^A (θ_2^A), and let $B1$ $(B2)$ denote agent B if he has observed θ_1^B (θ_2^B).

The proof is divided into nine steps:

Step 1: $A1$ never chooses x_2^A.

This is because refusing the contract strictly dominates: use (bi) and (di) of Proposition 1.

Step 2: $B1$ never chooses x_2^B.

Again, this is because refusing the contract strictly dominates: use (bi) and (di) of Proposition 1, together with (6).

Step 3: For each ε $(0 < \varepsilon \leqslant 1 - q_1^A)$, A never randomizes between $x_1^A(\varepsilon)$ and x_1^A, or between $x_1^A(\varepsilon)$ and x_2^A, or between $x_1^A(\varepsilon)$ and refusing the contract.

Suppose that this were not true for Ai, where $i = 1$ or 2. If Ai chooses some $x_1^A(\varepsilon)$, where $0 < \varepsilon \leqslant 1 - q_1^A$, then

$$\text{prob}_{Ai}\{B \text{ choosing } x_2^B\} < 1 - q_1^A, \tag{†}$$

otherwise Ai would strictly prefer choosing x_1^A to $x_1^A(\varepsilon)$, since $\Delta^\varepsilon(q_1^A) < 0$. Inequality (†) means that Ai strictly prefers x_1^A to both x_2^A and refusing the contract (using (ai), (bi), (bii), and (dii) of Proposition 1). This means that Ai must be randomizing between $x_1^A(\varepsilon)$ and x_1^A, and hence he must be indifferent between them. But (4) implies that he would strictly prefer $x_1^A(\varepsilon/2)$, a contradiction.

Step 4: $A1$ never chooses $x_1^A(\varepsilon)$, for any $\varepsilon > 0$.

Suppose that this were not true. Then by Step 3, $A1$ must always choose from the set $\{x_1^A(\varepsilon) \mid 0 < \varepsilon \leqslant 1 - q_1^A\}$. This means that

$$\text{prob}_{A1}\{B \text{ chooses } x_2^B\} < 1 - q_1^A,$$

which, from Step 2, implies

$$\text{prob}_{A2}\{B \text{ chooses } x_2^B\} < 1 - q_2^A. \tag{††}$$

Now (††), together with (bii) and (dii) of Proposition 1, means that $A2$ strictly prefers x_1^A to both x_2^A and refusing the contract. From Step 3, we know that there are only two possibilities: *either* (a) $A2$ always chooses x_1^A *or* (b) $A2$ always chooses from the set $\{x_1^A(\varepsilon) \mid 0 < \varepsilon \leqslant 1 - q_1^A\}$.

Suppose (a): $A2$ always chooses x_1^A. Then, in the light of (5), $B2$ would choose x_1^B. But then B is never choosing x_2^B, and so $A2$ strictly prefers some $x_1^A(\varepsilon)$ to x_1^A, a contradiction.

Now suppose (b): $A2$ always chooses from the set $\{x_1^A(\varepsilon) \mid 0 < \varepsilon \leqslant 1 - q_1^A\}$. Then by construction (3) and the fact that $\gamma > 0$, $B2$ strictly prefers x_2^B to both x_1^B and refusing the contract. But this contradicts (††).

Step 5: $B2$ chooses x_2^B with certainty.

Suppose not. Then

$$\text{prob}_{A1}\{B \text{ chooses } x_2^B\} < 1 - q_1^A,$$

which, since $\Delta^\varepsilon(q) > 0$ for some $\varepsilon > 0$ if $q > q_1^A$, implies that $A1$ would choose some $x_1^A(\varepsilon)$. This contradicts Step 4.

Step 6: $A2$ never chooses $x_1^A(\varepsilon)$, for any $\varepsilon > 0$.

From Steps 2 and 5,

$$\text{prob}_{A2}\{B \text{ chooses } x_2^B\} = 1 - q_2^A. \tag{†††}$$

Hence $A2$ strictly prefers x_1^A to any $x_1^A(\varepsilon)$, since $\Delta^\varepsilon(q) < 0$ for $q \leqslant q_1^A$.

Step 7: $A2$ never chooses x_1^A.

If he did, then from Step 1,

$$\text{prob}_{B2}\{A \text{ chooses } x_2^A\} < 1 - q_2^B.$$

Hence, from (ai) and (dii) of Proposition 1, $B2$ strictly prefers x_1^B to x_2^B, which contradicts Step 5.

Step 8: A never refuses to sign the contract.

Suppose A refuses to sign with some positive probability. Notice from Steps 4 and 6 that A never chooses any $x_1^A(\varepsilon)$ where $\varepsilon > 0$. Therefore, using (dii) of Proposition 1, $B2$ strictly prefers x_1^B to x_2^B (since $(-\gamma) < 0$). This contradicts Step 5.

Step 9: $B1$ never refuses to sign the contract.

Suppose $B1$ refuses to sign with some positive probability. Then, using

(†††) and (dii) of Proposition 1, $A2$ strictly prefers x_1^A to x_2^A (again, since $(-\gamma)<0$). This contradicts Step 7.

In sum, the only possible equilibrium is one in which A chooses output according to E^* (by Steps 1, 4, 6, 7, and 8), and B does likewise (by Steps 2, 5, and 9). It is straightforward to confirm that this *is* an equilibrium. Q.E.D.

5. Open Questions

It is, of course, a limitation that the model considers only binary-state variables, since presumably very complicated equilibrium strategies can arise in a many-state world. We do not yet know whether there is some indirect mechanism which would uniquely implement the second best if the stochastic structure were richer.

Another interesting area of research is to see whether an approach similar to ours can knock out unwanted equilibria in other principal/multi-agent models (e.g., Mookherjee [7]). Work by one of us, Ma [5], shows that these indirect mechanisms do indeed help substantially.

Appendix: Proof of Proposition 1

Consider the relaxed program (RP) of (P.BN) in which inequality (2, 1) is not included, i.e., we omit the "upward" incentive constraint

$$\begin{aligned} &q_1 U(R_{11})+(1-q_1)\,U(R_{12})-D(x_1,\theta_1)\\ &\quad\geqslant q_1 U(R_{21})+(1-q_1)\,U(R_{22})-D(x_2,\theta_1). \end{aligned} \tag{A3}$$

We shall show that at a solution to (RP), (A3) holds as a strict inequality.

Consider the first-order conditions of (RP) with respect to R_{21} and R_{22}:

$$-p_2q_2+\hat\lambda_2 q_2 U'(R_{21})+\hat\lambda_4 q_2 U'(R_{21})=0$$

$$-p_2(1-q_2)+\hat\lambda_2(1-q_2)\,U'(R_{22})+\hat\lambda_4(1-q_2)\,U'(R_{22})=0,$$

where $\hat\lambda_2$ and $\hat\lambda_4$ are multipliers of (1, 2) and (2, 2). It cannot be the case that both $\hat\lambda_2$ and $\hat\lambda_4$ are zero, otherwise the principal could lower R_{21} and R_{22} to increase his payoff without violating any constraint. These first-order conditions therefore give $R_{21}=R_{22}\equiv R_2$, say. That is, agents do not face any lottery when they produce output x_2. We can rewrite (RP) as

$$\underset{x_i,\,R_{1j},\,R_2}{\text{maximize}}\; p_1[q_1(x_1-R_{11})+(1-q_1)(x_1-R_{12})]+p_2[x_2-R_2]$$

subject to

$$q_1 U(R_{11})+(1-q_1)\,U(R_{12})-D(x_1,\theta_1)\geqslant \bar U \tag{A1}$$

$$U(R_2)-D(x_2,\theta_2)\geqslant \bar U \tag{A2}$$

$$U(R_2)-D(x_2,\theta_2)\geqslant q_2 U(R_{11})+(1-q_2)\,U(R_{12})-D(x_1,\theta_2). \tag{A4}$$

Let λ_1, λ_2, and λ_4 be the multipliers of (A1), (A2), and (A4), respectively. First note that $\lambda_1>0$. For if not, then by complementary slackness (A1) holds as a strict inequality, and the principal can lower R_{11} and R_{12} without violating (A4)—which would be a contradiction.

The first-order conditions are

$$-p_1q_1+\lambda_1 q_1 U'(R_{11})-\lambda_4 q_2 U'(R_{11})=0 \tag{A5}$$

$$-p_1(1-q_1)+\lambda_1(1-q_1)\,U'(R_{12})-\lambda_4(1-q_2)\,U'(R_{12})=0 \tag{A6}$$

$$-p_2+\lambda_2 U'(R_2)+\lambda_4 U'(R_2)=0 \tag{A7}$$

$$p_1-\lambda_1 D_x(x_1,\theta_1)+\lambda_4 D_x(x_1,\theta_2)=0 \tag{A8}$$

$$p_2-\lambda_2 D_x(x_2,\theta_2)-\lambda_4 D_x(x_2,\theta_2)=0. \tag{A9}$$

From (A7) and (A9), $U'(R_2)=D_x(x_2,\theta_2)$ so that production is efficient in a good state. Also $x_2\leqslant x_2^*$, where x_2^* is the first-best level of output in a good state. To see this, suppose to the contrary that $x_2>x_2^*$. Now x_2^* is characterized by $U(R_2^*)-D(x_2^*,\theta_2)=\bar U$ and $U'(R_2^*)=D_x(x_2^*,\theta_2)$, where R_2^* is first-best payment. We thus have $U'(R_2)=D_x(x_2,\theta_2)>D_x(x_2^*,\theta_2)=U'(R_2^*)$, which implies $R_2<R_2^*$. But we also have $D(x_2,\theta_2)>D(x_2^*,\theta_2)$, which with (A2) gives $U(R_2)>U(R_2^*)$, and hence $R_2>R_2^*$. This contradiction proves $x_2\leqslant x_2^*$.

The proposition is proved in the following seven steps.

Step 1: Incentive constraint (A4) binds (with a positive multiplier).

If $\lambda_4=0$, then we have only the two individual rationality constraints to worry about and the principal can attain the first best, which has $R_{11}=R_{12}\equiv R_1^*$, say, and also $U(R_1^*)-D(x_1^*,\theta_1)=U(R_2^*)-D(x_2^*,\theta_2)=\bar U$.

But $U(R_1^*)-D(x_1^*,\theta_2)>U(R_1^*)-D(x_1^*,\theta_1)=\bar U=U(R_2^*)-D(x_2^*,\theta_2)$ so that the incentive constraint (A4) is violated. Thus it must be that $\lambda_4>0$.

Step 2: $R_{11}>R_{12}$.

Dividing (A5) into (A6),

$$\frac{\lambda_1-\lambda_4 q_2/q_1}{\lambda_1-\lambda_4(1-q_2)/(1-q_1)}=\frac{U'(R_{12})}{U'(R_{11})}. \tag{A10}$$

By assumption $q_2 < q_1$ and $(1-q_2) > (1-q_1)$ so that the LHS of (A10) exceeds 1. Hence $U'(R_{11}) < U'(R_{12})$, and $R_{11} > R_{12}$.

Now write the contract the principal offers the agent in question as

$$\{Z_1, Z_2\} \equiv \{(x_1, R_{11}, R_{12}), (x_2, R_{21}, R_{22})\}$$

and define

$$\pi_i \equiv x_i - q_i R_{i1} - (1-q_i) R_{i2} \qquad \text{for } i = 1, 2.$$

(Note that $R_{21} = R_{22} = R_2$.)

Step 3: $\pi_2 > \pi_1$.

Suppose not, i.e., suppose $\pi_2 \leqslant \pi_1$. First notice that since $R_{11} > R_{12}$ and $q_1 > q_2$,

$$x_1 - q_2 R_{11} - (1-q_2) R_{12} > \pi_1 \geqslant \pi_2.$$

Let the principal offer $\{Z_1, Z_1\}$ instead of $\{Z_1, Z_2\}$. This increases the principal's expected profit. Moreover $\{Z_1, Z_1\}$ is admissible in (RP): First, the agent can attain his reservation utility. This is obvious if he is type θ_1. And if he is type θ_2, then by Step 1 and (A2), he also obtains at least $\bar{U}$. Second, the incentive constraint (A4) is trivially satisfied. But this contradicts the optimality of $\{Z_1, Z_2\}$. Hence $\pi_2 > \pi_1$.

Step 4: Incentive constraint (A3) holds as a strict inequality.

Suppose not, i.e., suppose the agent did not strictly prefer Z_1 to Z_2, when he is type θ_1. The principal could then offer $\{Z_2, Z_2\}$ rather than $\{Z_1, Z_2\}$. $\{Z_2, Z_2\}$ is admissible in (RP): First, the agent can attain his reservation utility. This is obvious if he is type θ_2. And, since he weakly prefers Z_2 to Z_1, he can also have at least $\bar{U}$ if he is type θ_1. Second, the incentive constraint (A4) is trivially satisfied. But by Step 3, $\pi_2 > \pi_1$, and so the contract $\{Z_2, Z_2\}$ will strictly increase the principal's expected profit, which is a contradiction. Hence (A3) holds as a strict inequality.

Step 5: Production is inefficiently low in a "bad" state θ_1; i.e., $q_1 U'(R_{11}) + (1-q_1) U'(R_{12}) > D_x(x_1, \theta_1)$.

Divide (A5), (A6), and (A8) by $D_x(x_1, \theta_1)$ and add them together:

$$\lambda_1 \left\{ q_1 \frac{U'(R_{11})}{D_x(x_1, \theta_1)} + (1-q_1) \frac{U'(R_{12})}{D_x(x_1, \theta_1)} - 1 \right\}$$
$$= \lambda_4 \left\{ q_2 \frac{U'(R_{11})}{D_x(x_1, \theta_1)} + (1-q_2) \frac{U'(R_{12})}{D_x(x_1, \theta_1)} - \frac{D_x(x_1, \theta_2)}{D_x(x_1, \theta_1)} \right\}. \quad \text{(A11)}$$

By (A6) and $(1-q_1) < (1-q_2)$, we have $\lambda_1 > \lambda_4$. Also the terms inside the curly brackets of (A11) must be of the same sign. Since $R_{11} > R_{12}$, $q_1 > q_2$, and $D_x(x_1, \theta_2) < D_x(x_1, \theta_1)$, the term inside the curly brackets on the LHS of (A11) must be strictly less than the term inside the curly brackets on the RHS of (A11). Hence both terms in the curly brackets of (A11) must be positive. Thus

$$q_1 U'(R_{11}) + (1-q_1) U'(R_{12}) > D_x(x_1, \theta_1).$$

Step 6: $x_1 < x_1^*$ if $U(\cdot)$ exhibits constant absolute risk aversion (CARA).

Suppose not, i.e., suppose $x_1 \geqslant x_1^*$. From $\lambda_1 > 0$, complementary slackness, and the properties of the first best, we have

$$q_1 U(R_{11}) + (1-q_1) U(R_{12}) = \bar{U} + D(x_1, \theta_1)$$

and

$$U(R_1^*) = \bar{U} + D(x_1^*, \theta_1).$$

Therefore

$$q_1 U(R_{11}) + (1-q_1) U(R_{12}) \geqslant U(R_1^*)$$

since $D(x_1, \theta_1) \geqslant D(x_1^*, \theta_1)$. CARA implies that

$$q_1 U'(R_{11}) + (1-q_1) U'(R_{12}) \leqslant U'(R_1^*).$$

By Step 5 and a property of the first best,

$$D_x(x_1, \theta_1) < q_1 U'(R_{11}) + (1-q_1) U'(R_{12}) \leqslant U'(R_1^*) = D_x(x_1^*, \theta_1),$$

implying $x_1^* > x_1$, contrary to hypothesis. Hence $x_1 < x_1^*$.

Step 7: $x_1 < x_2$ if $U(\cdot)$ exhibits CARA.

Suppose not, i.e., suppose $x_1 \geqslant x_2$. By Step 5 and (cii) in Proposition 1, we have

$$q_2 U'(R_{11}) + (1-q_2) U'(R_{12}) > D_x(x_1, \theta_2) \geqslant D_x(x_2, \theta_2) = U'(R_2).$$

With CARA, this implies

$$q_2 U(R_{11}) + (1-q_2) U(R_{12}) < U(R_2).$$

Putting this into the (binding) incentive constraint (A4), it follows, from $D(x_1, \theta_2) < D(x_2, \theta_2)$, that $x_1 < x_2$, contrary to hypothesis. Hence $x_1 < x_2$.

Note. The arguments in Steps 6 and 7 are still valid if $U(\cdot)$ exhibits nondecreasing absolute risk aversion (NDARA). Q.E.D.

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