Paying For Joint Costs in Health Care

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The paper analyzes a regulatory game between a public and a private payer to finance hospital joint costs (mainly capital and technology expenses). The public payer (inspired by the federal Medicare program) may both directly reimburse for joint costs ("pass-through" payments) and add a margin over variable costs paid per discharge, while the private payer can only use a margin policy. The hospital chooses joint costs in response to payers' overall payment incentives. Without pass-through payments, under provision of joint costs results from free-riding behavior of payers and the first-mover advantage of the public payer. Using pass-through policy in its self-interest, the public payer actually may moderate the under provision of joint costs; under some conditions, the equilibrium allocation may be socially efficient. Our results bear directly on current Medicare policy, which is phasing out pass-through payments.

1. Introduction

Joint (or common) costs are those costs of a multiproduct firm that cannot be attributed to specific outputs. The major components of joint costs in hospitals—capital and technology-related expenses—are at the center of the health policy debate. Although capital costs are only about 10% of total hospital costs, most health analysts agree with Evans (1986), Newhouse (1988), and Weisbrod (1991), who hold the accommodating regulatory and financing environment for new technology responsible for the inexorable increases in health care costs occurring in the United States since 1960. How joint costs are paid for

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will determine the incentives for capital accumulation and technology adoption.

The regulation literature has considered investment many times, and joint costs in our model, as well as in many others in the literature, are a form of investment. However, previous papers that are concerned with joint costs typically assume that the level of joint costs is predetermined. In the regulation model of a multiproduct monopoly, the fixed cost of production has to be borne by consumers in different markets (see Braeutigam, 1989, for a survey). Furthermore, Braeutigam and Panzar (1989), Brennan (1990), and Rogerson (1990) consider the effect of mechanisms for financing joint costs when prices of the firm's outputs are regulated and unregulated, or more generally, regulated in different manners. Although these papers analyze the efficiency effects of regulation in terms of pricing policies in different markets and the firm's input or output choices, the level of joint costs is given. The normative Ramsey-pricing literature is concerned with finding the set of prices that minimizes the efficiency costs of divergence from marginal cost pricing while financing a set level of joint costs (Baumol and Bradford, 1970). Finally, in the cost-based pricing literature, concern has been about financing a specified level of joint costs as a question of fairness or as an outcome of a cooperative game; see Faulhaber (1975), and Spulber (1989) for a unified review.¹

A cost can be independent of the level of output (a fixed cost) but still be chosen by the firm. In a departure from earlier approaches, we model joint costs as a decision made by the firm. Responding to payment mechanisms set by regulators, the firm chooses the level of joint costs to enhance demand and to maximize profit.² Thus, the joint cost decision is subject to efficiency concerns. Furthermore, we explicitly model strategic interactions between regulators. Our methodology is in the spirit of models of the provision of a public good (see Varian, 1990). But in our model, instead of contributing a share of a public good directly, each regulator devises a component of an overall incentive mechanism for a public good provider.³ Initially, we

1. The multiproduct cost functions in Baumol et al. (1982) contain a joint input that is variable. The theory of contestable markets studies the level of this and other inputs in an industry equilibrium with free entry.

2. Joint costs enhance demand by raising the quality of the good. Because of our concern with the health market and because of the presence of insurance in this market, we will assume that demands are price inelastic. Hence, we do not address the usual problem of the inefficient market provision of quality due to differences in consumers’ marginal and average valuations (Spence, 1975).

3. Baumol et al. (1982) refer to an input equally available to production of a firm’s multiple outputs as a “public input,” regarding this public property of an input as the source of economies of scope. They also make the analogy between any fixed cost and public goods.
assume the two regulators, whom we call "public and private payers," set payment policies simultaneously. Later, we modify this to allow the public payer to commit to a policy prior to the private payer. Our model was developed to address questions of public policy in the hospital sector. Public payment programs are beginning to pay for capital and technology in new ways. Furthermore, health care reform in the United States is likely to enhance the pressure on private payers (traditional third-party payers or their replacements) to contract with hospitals to limit costs. Payments for joint costs will be a key part of a private contracting strategy.

The largest payer for hospital services (accounting for about 40% of hospital revenues) is the $150 billion Medicare program, which pays on behalf of the elderly and disabled. Medicare’s Prospective Payment System (PPS) pays for hospital operating costs on the basis of the patient’s classification at discharge into a diagnosis-related group (DRG). Prior to October 1991, Medicare paid 85% of its share of capital and direct medical education costs by direct payments known as “pass-through” payments. Medicare direct payments for capital costs are being phased out over a 10-year period; eventually, payments for capital will be included in the per discharge payment. For purposes of discussion, we will consider capital costs to be joint. Some of the

4. For example, if a hospital spends $100 on a piece of machinery, and 50% of the hospital stays are Medicare, Medicare would have paid the hospital $42.50 (= $100(.50)(.85)) in pass-through payments.

5. During the 10-year transition period, Medicare will pay hospitals with above average capital costs differently than those with below average cost. Hospitals with costs above the average will be paid under the old pass-through formula for "old" assets, those acquired or obligated before January 1, 1991. "New" assets will be paid prospectively at the national average rate. For hospitals with costs below the average, the rate will be a weighted average ("blend") of the hospital’s historic capital cost per discharge and the national average rate, with the blend beginning at 90/10 hospital specific/national, and moving to 100% national after 10 years. When the transition is complete, all hospitals will be paid at the same, national rate, although there will be adjustments for certain characteristics of the hospital, such as urban/rural, teaching hospitals, local wage costs, and percent of poor patients served. See Burge, Cromwell, and Adamache (1991) for a discussion of these rules and their implications. A detailed description of the rules are also contained in the Federal Register, Vol. 56, No. 169, August, 1991, pp. 43358–43524.

6. Empirical studies of hospital costs do not permit a precise estimate of the magnitude of joint costs, although the available evidence is consistent with the intuition that lumpy specialized services contain a large element of joint costs. A series of papers estimate the ratio of marginal to average cost of hospital discharges at between .85 to 1.0, leaving room for substantial costs not varying directly with output (Friedman and Pauly, 1981, 1983; Gaynor and Anderson, 1991; Pauly and Wilson, 1986). "Quality" is controlled for imperfectly in these studies, and as our analysis suggests, it is endogenous to output and cost determination in any case. Dranove, Shanley, and Simon (1992) study the supply of specialized services in hospital markets in California. Although they do not estimate cost functions, they conclude that the pattern of service suggests "substantial scale economies for many services."
state-administered Medicaid programs (paying for health care for some of the poor) follow Medicare and pay prospectively for operating costs but pay directly for capital. Other Medicaid programs include any contribution to joint or capital costs in the per discharge payment or in a per diem rate. Private payers make no direct joint costs payments and include any contribution to joint costs in the rates they pay for operating costs. Thus, when a hospital makes a capital investment in joint costs, part of the costs are passed on directly to Medicare (and in some states, Medicaid), and part of the costs are paid for through the margins over operating costs paid by other payers.

The rationale for including joint cost payments in Medicare’s per discharge payment is “to provide hospitals with incentives to control the increase in capital costs” (ProPAC, June 1991, p. 69). This argument seems to ignore the fact that folding capital payments into the discharge payment necessarily increases the price above marginal cost, a source of economic inefficiency. Moreover, with price above marginal cost, hospitals will expand volume by (among other ways) increasing the attractiveness of their facility through capital expenditures.8

In the context of a single payer attempting to align a hospital’s profit-maximizing joint cost decision with social welfare, a pass-through policy together with a margin policy must perform better than a margin policy alone. Indeed, our analysis in Section 4 demonstrates this formally. What is more striking is our result that with multiple payers, each of whom receives only the fraction of social benefit according to its market share, the use of pass-through policy by a payer will not only enhance its strategic advantage over the other payer but also enhance the aggregate social welfare. Thus, although there is no single benevolent regulator in our model attempting to maximize social welfare, a payer’s ability to commit to pass-through and margin policies may lead to social efficiency.

Our results bear on fairness as well as efficiency. We cast our fairness results in terms of “cost shifting,” adopting the natural definition that a payer cost shifts if it pays a share of joint costs less than its share of total discharges. Cost shifting in hospitals is essentially concerned with payment for joint costs (Hadley and Feder, 1985). A payer’s choice of strategy reflects a balance of two competing objectives. On the one hand, each payer values joint costs because they

7. State Medicaid programs operate within Federal guidelines. See Waid (1990) for an explanation of Medicaid eligibility and program operations.

8. In one of the early studies of quality competition, Feldstein (1977) found that the elasticity of hospital admissions at the market level with respect to (all) costs was .44.
enhance the value of output. On the other hand, each payer would like to shift the joint costs to the other. In the joint cost game without pass through (payers setting margins only), the smaller payer is better able to free ride. The public payer’s first-mover advantage figures into the sequential-move equilibrium. The joint cost game with pass-through is, however, no contest: The public payer can always cost shift to the private payer.

While the extent of free riding (or cost shifting in our context) and inefficiency tends to be positively related in many models with public goods, their relationship exhibits more variety in ours. When payers can only use margin policies, free riding must lead to a suboptimal level of joint costs; when the public payer exploits its first-mover advantage, under provision may be exacerbated. Surprisingly, with the pass-through policy available, free riding and efficiency need not be inconsistent. In our most interesting efficiency result, the public payer’s use of pass-through payments always allows it to free ride on the private payer, but the under provision is counteracted. In fact, when extreme free riding is possible so that the public payer appropriates the entire social surplus (despite that it has only a fraction of the market), equilibrium joint costs become socially efficient.

Our model of a hospital selling discharges to a public and a private payer is in Section 2. Patients covered by each of the two payers demand discharges based on the level of joint costs, which are chosen by a profit-maximizing hospital. Section 3 considers equilibria when the payers set per discharge payments only. Section 4 analyzes equilibria when the public payer can make a direct payment for a share of the hospital’s joint costs. Section 5 discusses our findings with an emphasis on the pass-through strategy.

Throughout the paper, it is assumed that payments to hospitals can be based upon the level of joint costs ex post, but that hospitals cannot be directed to adopt a level of joint costs ex ante. Reimbursing a percentage of a hospital’s joint costs is a feasible policy, but specifying a particular level is not. This assumption reflects important practical problems in health care regulation. Payers typically do not have the expertise to specify technology and capital in detail. Moreover, because of the large number of hospitals in the country, it would be unrealistic for any payer to specify every piece of technology and capital equipment in each of them.9

9. Some states do regulate new hospital construction and large capital purchases, although state health planning activity has decreased in the last decade. The federal government primarily uses payment system incentives to affect hospital decisions about capital.
2. The Model

We study the regulation of a single hospital by two regulators. In this section we lay out the basic analytical framework and establish an efficiency benchmark for comparison. First, we describe the demand side of the market. Generally, the demand for health care will be a function of price and the quality of care. For simplicity, however, we assume that consumers in the health care market are fully insured, and, hence, market demand is assumed to be price inelastic. In practice, a consumer may have to incur some uninsurable or out-of-pocket expenses (such as time and travel costs) when seeking treatments from the hospital; this cost, $e$, will only be used in Remark 1 in Section 4. Thus, for brevity we will suppress the price or uninsurable expense argument in the demand function. Turning to the quality of care, because joint costs are typically capital and technology-related expenses, it is natural for one to assume that a higher level of joint costs represents a better quality of care, which consumers desire. Hence, demand for health care depends positively on the level of joint costs $T$, and the strictly increasing function $X(T)$ specifies the total number of hospital discharges demanded by patients. We also assume that $X$ is strictly concave; thus, the second-order derivative is strictly negative, $X''(T) < 0$. We further assume that $X(T)X''(T)/[X'(T)^2]$, or equivalently, that the ratio $X''/X'$ to $X'/X$, has a strictly negative derivative; a sufficient condition for this is that the third derivative of $X$ is nonpositive. This regularity assumption allows us to make a sharper prediction on the distortion in the equilibrium allocations. Finally, a fraction $\theta$ of patients are insured by the public payer, and the remaining $(1 - \theta)$ fraction of patients are covered by the private payer.

Discharges are produced by the hospital at constant operating cost $c$ per discharge, independent of the level of joint costs in the hospital. Notice that this cost structure exhibits increasing returns. Each payer pays $c$ per discharge plus a margin equal to $\alpha$ for the public program and $\beta$ for the private program. The margins are the payers’ contribution to joint costs and may be positive, zero, or negative. In addition, the public payer may directly pay for some of the joint costs. This reimbursement policy is represented by a pass-through parameter $\rho$, between zero and one, which entitles the hospital to reimbursement $\rho T$ if its level of joint costs is $T$. Notice that a policy cannot specify the exact value of $T$, but after the hospital has chosen $T$, it may request that $\rho T$ be reimbursed; in other words, joint costs are not ex ante contractible, but its level is ex post verifiable. We study cases in which the public payer can set $\rho$ and $\alpha$ as well as cases in which pass-throughs are forbidden ($\rho = 0$), and payers choose only $\alpha$ and $\beta$. 
We assume that the hospital must satisfy all demands (provided that it makes nonnegative profits). Given the payers’ margins, the pass-through, and the joint costs level, hospital’s profits are total revenues less total costs: \( \Pi = (\theta \alpha + (1 - \theta)\beta)X(T) - (1 - \rho)T \). The hospital is assumed to maximize profits by choosing the level of joint costs \( T \). By the concavity of \( X(T) \), the profit maximizing joint costs level is fully characterized by the first-order condition:

\[
\left[ \theta \alpha + (1 - \theta)\beta \right] \frac{\partial X(T)}{\partial T} = (1 - \rho).
\] (1)

In general, the gross social benefit depends on the number of discharges and the quality of care consumers receive. However, because in our model both demand and quality of care are functions of the level of joint costs, we will simply write the gross social benefit as \( U(T) \). In many applications in the literature, \( U(T) \) is taken to be consumer surplus. Here, we leave \( U \) as a general function, including consumer surplus as one possibility.\(^{10}\) Because the public and the private payers are respectively responsible for \( \theta \) and \( (1 - \theta) \) of hospital discharges, we assume that they also split the social benefit proportionally.\(^{11}\) Therefore, for given margins \( \alpha \) and \( \beta \), and public payer’s pass-through, public and private payers’ net utilities are respectively:

\[
V = \theta[U(T) - (c + \alpha)X(T)] - \rho T
\] (2)

\[
W = (1 - \theta)[U(T) - (c + \beta)X(T)].
\] (3)

Before equilibria of models of joint costs provision are characterized, we write down the efficiency benchmark. Social welfare is given by \( U(T) - cX(T) - T \); we assume that this is a strictly concave function. The efficient level of joint costs is \( T^E \), where

\[
U'(T^E) - cX'(T^E) = 1.
\] (4)

Because \( X \) is a concave function, the concavity of \( U - cX \) requires

\(^{10}\) Note that patient demand for discharges ignores the marginal cost of a discharge \( c \), because patients are insured. Thus \( U(\cdot) \) can be interpreted as second-best welfare. Social welfare maximizing \( T \) will take patient behavior into account. Baumgardner (1991) employs a different depiction of technology than the one considered here but makes a similar point about the value of quality improvements in a second-best world. In Baumgardner, an “improvement” in technology takes the form of an increase in the amount of health stock that can be recovered after illness. Because in equilibrium some patients would restore “too much” health stock after an illness because of moral hazard but do not do so because of the limits of technology, technological change can have ambiguous welfare properties, even if the insurer chooses the extent of coverage (the coinsurance) to attain the second-best equilibrium.

\(^{11}\) We assume the public payer maximizes public patients’ welfare because of benevolence, and the private payer maximizes private patients’ welfare because of competitive pressures. Thus, the most natural interpretation of \( U(\cdot) \) is consumer surplus.
that $U$ be "more" concave than $X$. On occasions, we will strengthen the concavity of $U - cX$ to

**Assumption A:**

$$\frac{U''(T)}{U'(T)} \leq \frac{X''(T)}{X'(T)}, \text{ for all } T.$$ 

Under Assumption A the curvature of $U$ is always higher than that of $X$. Although Assumption A allows us to make sharper comparison between equilibrium and efficient joint costs levels, it is unnecessary for the derivation of equilibrium joint costs levels.

In addition to an efficiency standard, we will evaluate equilibria in terms of fairness. A fair equilibrium is one with no cost shifting, defined as follows. For any given level of joint costs $T$, the hospital's total revenue net of operating costs is $[\theta \alpha + (1 - \theta) \beta]X(T) + \rho T$. The public and the private payers contribute $\theta \alpha X(T) + \rho T$ and $(1 - \theta) \beta X(T)$, respectively. We will say that the public payer cost shifts if its share of the total contribution is less than $\theta$, that is,

$$\frac{\theta \alpha X(T) + \rho T}{[\theta \alpha + (1 - \theta) \beta]X(T) + \rho T} < \theta.$$  \hspace{1cm} (5)

An equilibrium can be fair, without cost shifting, or a payer can cost shift.

### 3. Equilibrium Joint Costs Provision Without Pass-Through

In this section we study two models of joint costs provision when the pass-through instrument is unavailable; hence, $\rho$ is set at zero in this section. In the first model, the public and private payers simultaneously choose their respective margins, $\alpha$ and $\beta$. The second model describes the process as a multistage game: The public payer first chooses $\alpha$, and then the private payer chooses $\beta$. In the latter model, the private payer's margin is made conditional on the public payer's move, that is, the private payer's strategy is a function of the public payer's margin. The second, sequential move game is probably more representative of the U.S. health policy context in which private payers can make policy changes much more quickly than can government. In both models, after the payers have decided on the margins, the hospital then chooses a joint costs level, and finally the public and private patients consume health services (in terms of discharges). Notice that for any given pair of $\alpha$ and $\beta$, the hospital's optimal choice
of joint costs is given by eq. (1). Given the hospital’s joint costs level, demand and discharges will be given by $X(T)$.

In the simultaneous move game, a (Nash) equilibrium is a pair of strategies $(\alpha^+, \beta^+)$ that are mutual best responses. That is, given that the private payer chooses $\beta = \beta^+$, $\alpha^+$ maximizes the public payer’s objective function (2), with $T$ determined by (1); similarly for $\beta^+$.

In the sequential move game, a (subgame-perfect) equilibrium is a pair of strategies $(\alpha^-, \beta^-)$ for which, given that $T$ is determined by eq. (1), (a) $\beta^-$ (a function of the public payer’s margin) maximizes the private payer’s welfare (3) for any margin set by the public payer, and (b) $\alpha^-$ maximizes the public payer’s objective (2) given $\beta^-$. 

### 3.1 Simultaneous Moves

We now characterize Nash equilibria in the simultaneous move game between the public and private payers. For any given $\alpha$ and $\beta$, the allocation will be completely defined by the hospital’s profit maximizing joint costs level in eq. (1). Given the public payer’s margin $\alpha$, the private payer chooses $T$ and $\beta$ to maximize its utility $(1 - \theta)[U(T) - (c + \beta)X(T)]$ subject to eq. (1):

$$[\theta \alpha + (1 - \theta)\beta] \frac{\partial X(T)}{\partial T} = 1.$$ 

Differentiating with respect to $T$ and $\beta$, we get the first-order necessary conditions for this constrained maximization program:

$$(1 - \theta)[U'(T) - (c + \beta)X'(T)] + \lambda [\theta \alpha + (1 - \theta)\beta]X''(T) = 0 \quad (6)$$

$$-(1 - \theta)X(T) + \lambda(1 - \theta)X'(T) = 0, \quad (7)$$

where $\lambda$ is the multiplier to the constraint. Using eq. (7), we can express $\lambda$ in terms of $T$; then we substitute for $\lambda$ in eq. (6) to obtain:

$$U'(T) - (c + \beta)X'(T) + \frac{X(T)X''(T)}{X'(T)} \left[\frac{\theta \alpha + (1 - \theta)\beta}{1 - \theta}\right] = 0.$$ 

Using the constraint, we can simplify the previous to

$$U'(T^+) - (c + \beta^+)X'(T^+) + \frac{X(T^+)X''(T^+)}{(1 - \theta)X'(T^+)^2} = 0, \quad (8)$$

where the superscripts on $T$ and $\beta$ emphasize that eq. (8) refers to equilibrium choices.
We can study a similar program for the public payer, who chooses the margin $\alpha$ and $T$ to maximize its utility (2) subject to eq. (1). After almost identical calculation, we obtain the following necessary condition for the public payer's equilibrium choice of $T$ and $\alpha$:

$$U'(T^+) - (c + \alpha^+)X'(T^+) + \frac{X(T^+)X''(T^+)}{\theta X'(T^+)^2} = 0. \quad (9)$$

We can now compare the equilibrium choice of joint costs level in the simultaneous move model with the efficient benchmark. Multiplying eqs. (8) and (9), respectively, by $(1 - \theta)$ and $\theta$, adding and then using the constraint (1), we have

$$U'(T^+) - cX'(T^+) - 1 + \frac{2X(T^+)X''(T^+)}{X'(T^+)^2} = 0. \quad (10)$$

Because $X(T)$ and $X'(T)$ are positive and $X''(T)$ negative, condition (10) implies

$$U'(T^+) - cX'(T^+) > 1.$$ 

Because $U(T) - cX(T)$ is concave, this inequality means that $T^+ < T^E$. Hence, in a result familiar from the public-good context, noncooperative behavior by the two payers leads to underinvestment in joint costs. From eqs. (8) and (9), we can also solve for equilibrium margins $\alpha^+$ and $\beta^+$ in terms of $T^+$. To summarize, we have

**Proposition 1:** In the simultaneous move game, the equilibrium joint costs level, $T^+$, is independent of the market shares, is given by eq. (10), and is lower than the socially efficient level $T^E$. The equilibrium margins chosen by the public and private payers are, respectively

$$\alpha^+ = \frac{U'(T^+) - cX'(T^+)}{X'(T^+)} + \frac{X(T^+)X''(T^+)}{\theta X'(T^+)^3} \quad (11)$$

$$\beta^+ = \frac{U'(T^+) - cX'(T^+)}{X'(T^+)} + \frac{X(T^+)X''(T^+)}{(1 - \theta) X'(T^+)^3}. \quad (12)$$

Perhaps somewhat surprising, the equilibrium joint costs level $T^+$ is independent of the payers' market shares (provided they are strictly positive). Having one payer with almost the entire market does not eliminate the public good problem in this model because the very small payer can set a large and negative margin that has little effect
on $T$ (because the payer is so small) but a large effect on the small payer's net welfare.\textsuperscript{12}

Furthermore, even if there is only one payer, the resulting joint costs level will be inefficient. To see this, suppose for the moment we set both $\theta$ and $\alpha$ exactly equal to zero (a special case of the model with only the private payer). Then eqs. (1) and (8) will characterize equilibrium joint costs:

$$U'(T) - cX'(T) - 1 + \frac{X(T)X''(T)}{X'(T)^2} = 0.$$ Absent free riding, the joint costs level becomes higher than $T^+$ but is still lower than $T^E$. The source of this inefficiency is that the use of the margin for paying for joint costs gives excessive profit to the hospital. The marginal and average revenues to a hospital with respect to $T$ are, respectively, $\alpha X'(T)$ and $\alpha X(T)/T$; by the concavity of $X(T)$, average revenue exceeds marginal revenue. As a second best, a payer limits the hospital's profit by implementing a lower level of joint costs.

Let us now examine the extent of cost shifting in this model. From Proposition 1, eqs. (1) and (10), the public payer's equilibrium share of joint costs contribution is

$$\frac{\theta \alpha^+}{\theta \alpha^+ + (1 - \theta) \beta^+} = \frac{\theta U'(T^+) - cX'(T^+)}{X'(T^+)^2} + \frac{X(T^+)X''(T^+)}{X'(T^+)^2}\left(\theta + (1 - 2\theta)\frac{X(T^+)X''(T^+)}{X'(T^+)^2}\right).$$

Because $X(T)$ is concave, the previous expression is more than $\theta$ if and only if $\theta$ is bigger than a half. In this model, the payer with a smaller market share is able to free ride on the payer with a larger share.

**Proposition 2:** In the simultaneous move game, the public payer will pay more than its share of joint costs if and only if its share of the market is more than that of the private payer.

\textsuperscript{12} On the basis of the hospital's ability to reject a payer with a negative margin, we could restrict the margins to nonnegative values and thereby limit the free riding of the small payer. A more complicated expression would replace eq. (10) in which market shares entered. As an alternative, for reasons of tractability, we proceed with eq. (10) making the additional assumption that both payers are large enough so they wish to set positive margins.
3.2 Sequential Moves

We now study the sequential move game of joint costs provision. In this game, the public payer acts as a Stackelberg leader and chooses a margin first. Let us begin by deriving the private payer’s optimal response against any arbitrary margin selected by the public payer. For any given $\alpha$, the private payer’s optimal response is given by eq. (8) and the hospital profit maximization constraint (1). To characterize the subgame-perfect equilibrium in the sequential move game, we choose $T$, $\alpha$, and $\beta$ to maximize the public payer’s utility, $\theta[U(T) - (c + \alpha)X(T)]$, subject to these two constraints:

$$U'(T) - (c + \beta)X'(T) + \frac{X(T)X''(T)}{(1 - \theta)X'(T)^2} = 0,$$

$$[\theta\alpha + (1 - \theta)\beta]X'(T) = 1.$$

Suppose that $T^-$ is the equilibrium joint costs level in the sequential move game. Given that $T = T^-$, the previous two constraints are two linear equations in $\alpha$ and $\beta$; we can therefore solve for the equilibrium margins easily. In the Appendix we derive the next proposition.

**Proposition 3:** In the sequential move game without pass-through, the equilibrium joint costs level, $T^-$, is given by

$$U'(T^-) - cX'(T^-) - 1 + \frac{2X(T^-)X''(T^-)}{X'(T^-)^2}$$

$$+ \frac{X(T^-)}{X'(T^-)} \frac{d}{dT} \left[ \frac{X(T^-)X''(T^-)}{X'(T^-)^2} \right]$$

$$+ \frac{(1 - \theta)X(T^-)}{X'(T^-)} \left\{ U''(T^-) \left[ \frac{U''(T^-)}{U'(T^-)} - \frac{X''(T^-)}{X'(T^-)} \right] 

- \frac{X(T^-)X''(T^-)^2}{(1 - \theta)X'(T^-)^3} \right\} = 0.$$  \hspace{1cm} (13)

Under Assumption A, equilibrium joint costs level $T^-$ is smaller than that in the simultaneous move game and, therefore, smaller than the socially efficient level. The equilibrium margins chosen by the public and private payers are, respectively

$$\alpha^- = \frac{U'(T^-) - cX'(T^-)}{X'(T^-)}$$

$$- \left[ \frac{U'(T^-) - cX'(T^-) - 1}{X'(T^-)} + \frac{X(T^-)X''(T^-)}{X'(T^-)^3} \right] \frac{1}{\theta}.$$  \hspace{1cm} (14)
\[ \beta^- = \frac{U'(T^-) - cX'(T^-)}{X'(T^-)} + \frac{X(T^-)X''(T^-)}{(1 - \theta)X'(T^-)^3}. \]  \hspace{1cm} (15) 

Because \( T^- \) is less than \( T^+ \) under Assumption A, the weighted margin in the sequential move game must be less than that in the simultaneous move game, that is, \( \theta\alpha^- + (1 - \theta)\beta^- < \theta\alpha^+ + (1 - \theta)\beta^+ \). It is also clear that the public payer’s equilibrium utility level in the sequential move game must be higher than in the simultaneous move game, because setting a margin equal to \( \alpha^+ \) is always a feasible policy in the sequential game.

In the sequential move model, the attempt by the public payer to exploit its first-mover advantage may further depress equilibrium joint costs. Notice again that the inefficiency does not disappear even if the public payer’s market share increases toward one (independent of Assumption A). As in the simultaneous move model, the margin method of financing joint costs is the source of this inefficiency.

Because the public payer exercises its first-mover advantage in the sequential game, the public payer can cost shift to the private payer even if its market share is larger than the private payer’s. This contrasts with the cost-shifting result in the simultaneous move model. Indeed, from eq. (14) and the hospital profit maximization constraint, one can verify straightforwardly that the public payer’s share of joint costs contribution is:

\[ \frac{\theta\alpha^-}{\theta\alpha^- + (1 - \theta)\beta^-} = \theta + (1 - 2\theta) \frac{X(T^-)X''(T^-)}{X'(T^-)^2} \]

\[ - (1 - \theta) \left[ U'(T^-) - cX'(T^-) - 1 + \frac{2X(T^-)X''(T^-)}{X'(T^-)^2} \right]. \]

Notice that when \( T^- < T^+ \) and \( T^+ \) is given by eq. (10), the term inside the square brackets must be positive. Even with equal shares (\( \theta = .5 \)), the public payer’s contribution will be less than a half.

**Proposition 4:** Under Assumption A, in the sequential move game without pass-through, the public payer’s share of joint costs contribution is less than \( \theta \) unless \( \theta \) is significantly bigger than a half.

### 4. Direct Reimbursement of Joint Costs

In this section, we are interested in analyzing pass-through payments for joint costs. We give the public payer the option of directly reimbursing the hospital for a share of joint costs in addition to setting a margin over the cost of a discharge. We call the pass-through fraction
If the public payer sets $\rho$ at .425, it means the public payer pays 42.5% of joint costs. We let $\rho$ be any fraction between zero and one. We will mainly consider a game in which the public payer first decides on the pass-through rate and the margin, and then in the second stage, the private payer sets its margin; we believe this is the most “realistic” description of the joint cost payment game.

A strategy for the private payer is a function that maps the public payer’s margin and pass-through to its choice of margin. Given $\alpha$ and $\rho$, the private payer’s best response function is completely characterized by the solution to the following: choose $T$ and $\beta$ to maximize $(1 - \theta)[U(T) - (c + \beta)X(T)]$ subject to $[\theta\alpha + (1 - \theta)\beta]X'(T) = 1 - \rho$. From procedures similar to those in the analysis of the simultaneous move game, we can easily derive the first-order conditions of this program. Thus, the hospital profit maximization constraint (1), together with the following:

$$U'(T) - (c + \beta)X'(T) + \frac{(1 - \rho)X(T)X''(T)}{(1 - \theta)X'(T)^2} = 0,$$

are necessary and sufficient conditions for the private payer’s program.

We can now analyze the public payer’s optimal choice of pass-through and margin given the private payer’s and the hospital’s best reactions. Formally, the public payer chooses $T$, $\alpha$, $\beta$, and $\rho$ to maximize its utility (2), $\theta[U(T) - (c + \alpha)X(T)] - \rho T$, subject to eqs. (1) and (16). We begin by combining the two constraints to eliminate $\beta$. This allows us to consider a program with only $T$, $\alpha$, and $\rho$ as instruments. From eq. (1), we obtain

$$\beta = \frac{1 - \rho - \theta\alpha X'(T)}{(1 - \theta)X'(T)}.$$

Substituting the above for $\beta$ in eq. (16) and after simplifying, we have

$$U'(T) - cX'(T) + \frac{\theta\alpha X'(T)}{1 - \theta} - \frac{1 - \rho}{1 - \theta} \left[ \frac{X'(T)^2 - X(T)X''(T)}{X'(T)^2} \right] = 0.$$

So the public payer’s program becomes choosing $T$, $\alpha$, and $\rho$ to maximize eq. (2) subject to eq. (17).

Let us begin by considering the first-order condition with respect to $\alpha$:

$$\kappa X'(T) \frac{\theta}{1 - \theta} - \theta X(T) = 0,$$
where \( \kappa \) is the multiplier to constraint (17). From eq. (18) we can solve for \( \kappa \). Let us now consider the first-order derivative with respect to \( \rho \):

\[
\frac{\kappa}{1 - \theta} \left[ \frac{X'(T)^2 - X(T)X''(T)}{X'(T)^2} \right] - T
\]

\[= \frac{X(T)}{X'(T)} \left[ \frac{X'(T)^2 - X(T)X''(T)}{X'(T)^2} \right] - T \]

\[= T \left\{ \frac{X(T)/T}{X'(T)} \left[ 1 - \frac{X(T)X''(T)}{X'(T)^2} \right] - 1 \right\}, \quad (19)\]

where the first equality follows from substituting \( \kappa \) by the first-order condition with respect to \( \alpha \). We now show that eq. (19) is positive. Because \( X''(T) \) is negative, \( X(T)/T \) must be greater than \( X'(T) \), and the term inside the square bracket must be greater than one. Thus, the first set of terms inside the curly brackets is a product of two numbers, each of which is greater than one. Hence, eq. (19) must be positive.

**Proposition 5:** In the equilibrium of the sequential move game with pass-through, the public payer chooses a pass-through equal to one. The hospital’s equilibrium profit is zero.

The solution to the sequential move game with pass-through is both extreme and interesting. It pays for the first-moving public payer to assume responsibility for direct payment for as large a share of joint costs as is feasible. As we demonstrate next, by simultaneously setting a low (and indeed, if feasible, negative) margin, the public payer keeps its total payments low and puts the private payer in a position of having to make a high margin payment. The public payer, in effect, buys out the joint costs, and benefits social welfare in the process.

We proceed to characterize the equilibrium joint costs level and margins. Given any \( \rho < 1 \), one can solve for the solution for \( T \) and \( \alpha \) in the program of maximizing eq. (2) subject to eq. (17). For all \( \rho < 1 \), the hospital’s optimal choice of \( T \) is unique. By the Maximum Theorem, the solutions, \( T(\rho) \) and \( \alpha(\rho) \), are continuous functions of \( \rho \). Proposition 5 establishes that the equilibrium joint costs level and margin are the limits of \( T(\rho) \) and \( \alpha(\rho) \) as \( \rho \) tends to one. Because \( T(\rho) \) and \( \alpha(\rho) \) are continuous in \( \rho \), equivalently one can obtain the equilibrium, \( T(1) \) and \( \alpha(1) \), by choosing \( T \) and \( \alpha \) to maximize eq. (2) subject to eq. (17) and \( \rho = 1 \).

Notice that when \( \rho = 1 \) and \( \theta \alpha + (1 - \theta) \beta = 0 \), the hospital
always earns zero profit: Any choice of \( T \) is profit-maximizing. Nevertheless, in a subgame-perfect equilibrium, the hospital must always choose \( T(1) \). If the hospital chose any other \( T \), the public payer's utility would not be maximized. Then the public payer would strictly prefer setting \( \rho \) slightly less than, but arbitrarily close to, one to induce a unique best response from the hospital, namely \( T(\rho) \), arbitrarily close to \( T(1) \). But by Proposition 5, \( \rho < 1 \) cannot be an equilibrium choice, because any \( \rho \) still closer to one is strictly better for the public payer. Therefore, only \( T(1) \) can be an equilibrium joint costs level.

With \( \rho = 1 \), we can use eq. (17) to solve for \( \alpha \):

\[
\alpha = -\frac{1 - \theta}{\theta} \left[ \frac{U'(T) - cX'(T)}{X'(T)} \right].
\]  

(20)

Substituting this for \( \alpha \) in the public payer's utility function, we have:

\[
\theta[U(T) - cX(T)] + (1 - \theta) \left[ \frac{X(T)}{X'(T)} U'(T) - cX(T) \right] - T.
\]

After simplifying the above expression, we have

\[
U(T) - cX(T) - T + (1 - \theta) \left[ \frac{X(T)}{X'(T)} - \frac{U(T)}{U'(T)} \right] U'(T).
\]  

(21)

In sum, the equilibrium joint costs level is the \( T \) that maximizes the previous function.

The last term in eq. (21) represents the distortion due to strategic behavior between the payers. Equilibrium joint costs level, \( T^\sigma \), will be characterized by the first-order derivative of eq. (21). Comparing this first-order condition (stated in the next proposition) with eq. (13), one can readily verify that \( T^\sigma \) is bigger than \( T^- \), the equilibrium joint costs level in the sequential game without pass-through.

From eq. (20), we can obtain the public payer's equilibrium margin by substituting \( T \) by \( T^\sigma \). With \( \rho = 1 \), the hospital earns zero profits, and its profit maximization constraint becomes \([\theta \alpha + (1 - \theta)\beta] = 0\). Using this and eq. (20), we can compute the private payer's margin.

**Proposition 6:** In the sequential move game with pass-through, the equilibrium joint costs level, \( T^\sigma \), is given by

\[
U'(T^\sigma) - cX'(T^\sigma) - 1 + (1 - \theta)U'(T^\sigma)
\]

\[
\times \frac{X(T^\sigma)}{X'(T^\sigma)} \left[ \frac{U''(T^\sigma)}{U'(T^\sigma)} - \frac{X''(T^\sigma)}{X'(T^\sigma)} \right] = 0.
\]

\( T^\sigma \) is greater than \( T^- \); moreover, under Assumption A, \( T^\sigma \) is at most \( T^E \).
The public and private payer’s equilibrium margins, $\alpha^\sigma$ and $\beta^\sigma$, are

$$
\alpha^\sigma = - \frac{1 - \theta}{\theta} \left[ \frac{U'(T^\sigma) - cX'(T^\sigma)}{X'(T^\sigma)} \right]
$$

(22)

$$
\beta^\sigma = \left[ \frac{U'(T^\sigma) - cX'(T^\sigma)}{X'(T^\sigma)} \right].
$$

(23)

Under Assumption A, strategic behavior by the two payers leads to underinvestment in joint costs compared to the socially efficient level. Nevertheless, the equilibrium level of joint costs is always higher than that without pass-through. Hence, allowing the public payer to reimburse joint costs can improve both the public payer’s and society’s welfare. Indeed, according to Proposition 3, in the sequential move game without pass-through, the inefficiency of $T^-$ remains significant even if the public payer’s market share becomes arbitrarily close to one. By contrast, from Proposition 6, when pass-through is allowed the inefficiency of $T^\sigma$ vanishes as the public regulator’s market share goes to one (independent of Assumption A).

What explains the power of the pass-through policy? Consider the case of a single payer, a special case of the two-payer model with $\theta$ set at one. Given pass-through $\rho$ and margin $\alpha$, the hospital’s profit is $(\alpha + c)X(T) - (1 - \rho)T - cX(T)$ and the profit maximizing choice of joint costs, $T^*$, satisfies

$$
\alpha X'(T^*) - (1 - \rho) = 0.
$$

(24)

The payer’s total expenditure (equal to the hospital’s revenue) is

$$
\rho T^* + (\alpha + c)X(T^*).
$$

(25)

Now, imagine that the payer attempts to implement the same level of joint costs $T^*$ at minimum cost. That is, it chooses $\alpha$ and $\rho$ to minimize eq. (25) subject to eq. (24). By solving for $\alpha$ from eq. (24) and substituting it into eq. (25), we express the payer’s total expenditure as

$$
\rho T^* + (1 - \rho) \frac{X(T^*)}{X'(T^*)} + cX(T^*),
$$

whose first-order derivative with respect to $\rho$ becomes

$$
\left[ 1 - \frac{X(T^*)}{T^*} \right] T^* < 0,
$$
\[(1 - \theta) \left\{ U(T) - cX(T) - \frac{U'(T^\sigma) - cX'(T^\sigma)}{X'(T^\sigma)} X(T) \right\}. \quad (26)\]

The equilibrium joint costs level, \( T^\sigma \), is also given by the \( T \) that maximizes eq. (26). Hence, in equilibrium, the private payer behaves as if it can demand any level of joint costs at a constant price of \( c + \beta^\sigma \). The private payer can earn inframarginal value of joint costs \( T^\sigma \). To see this, consider the first-order derivative of (26) with respect to \( T \):

\[(1 - \theta)X'(T) \left\{ \frac{U'(T) - cX'(T)}{X'(T)} - \frac{U'(T^\sigma) - cX'(T^\sigma)}{X'(T^\sigma)} \right\}.\]

Whenever the first term in the curly brackets is higher than the second, the private payer earns positive inframarginal utility. But if the two terms are exactly equal, then in equilibrium all social surplus will be extracted by the public payer. The following result is immediate.

**Corollary 1:** If the weak inequality in Assumption A holds as an equality, then equilibrium joint costs level is socially efficient; that is, \( T^\sigma \) is equal to \( T^E \), the public payer’s equilibrium utility is equal to social surplus at \( T^E \), and the private payer’s equilibrium utility is zero.

**Remark 1:** When \( X(T) \) is either additively or multiplicatively separable in \( T \) and price, (that is, when \( X(T,P) = f(T) + g(P) \) or \( X(T,P) = f(T)g(P) \), where \( P \) denotes price, and \( f \) and \( g \) are two functions), and when \( U(T) \) is defined to be consumer surplus, (that is, \( U = \int_0^e X(T,z)dz \), where \( e \) denotes consumer’s out-of-pocket expense), the weak inequality in Assumption A holds as an equality.

Although there is no social planner in our model to align the hospital’s profit-maximizing incentive with the social objective, the efficient allocation can be an equilibrium outcome if and only if the pass-through policy is available. Moreover, efficiency obtains when free riding between payers is at the most extreme; when the condition in Assumption A holds as an equality, the public payer appropriates the entire social surplus and achieves social efficiency. Observe also that the hypothesis in Corollary 1 is independent of the market share parameter \( \theta \), and is satisfied by a broad and important class of demand functions as Remark 1 states.

Finally, we examine cost shifting in the sequential model with pass-through. Because in equilibrium \( \theta \alpha^\sigma + (1 - \theta)\beta^\sigma = 0 \), and \( \rho = 1 \), the public payer’s share of contribution becomes
\[
\frac{\theta \alpha^\sigma X(T^\sigma) + \rho T^\sigma}{[\theta \alpha^\sigma + (1 - \theta) \beta^\sigma]X(T^\sigma) + \rho T^\sigma} = \theta \alpha^\sigma \frac{X(T^\sigma)}{T^\sigma} + 1
\]

\[
= \theta - (1 - \theta) \left\{ \frac{[U'(T^\sigma) - cX'(T^\sigma)] X(T^\sigma)/T^\sigma}{X'(T^\sigma)} - 1 \right\}.
\]

Because \( T^\sigma \) is at most \( T^E \) when Assumption A holds, \( U'(T^\sigma) - cX'(T^\sigma) \) must be at least one. Hence, by the strict concavity of \( X \), the term inside the curly brackets must be strictly positive. Therefore, we conclude that:

**Proposition 7:** Under Assumption A, in the sequential move game with pass-through, the public payer’s share of joint costs contribution is always less than its share of the market.

Results in this section may also shed light on current health care payment policy debate. We have shown that the public payer (and society as well) benefits by putting the pass-through payment as high as possible. If the margin must be nonnegative, perhaps due to institutional and political constraints, then the “constrained optimal” pass-through will be set at a value between zero and one. To see this, notice that if eq. (17) is interpreted as a constraint to define a functional relationship between \( \alpha \) and \( \rho \) for any given \( T \), then these two variables are negatively related. According to Proposition 6, when unconstrained, \( \rho \) will be chosen to be one and \( \alpha \) will be negative. If \( \alpha = 0 \), then eq. (17) implies that \( \rho \) must be between zero and one. Ironically, this policy, setting positive pass-through payments and a zero margin, describes the policy presently being abandoned by Medicare in favor of a policy of eliminating pass-through payments and increasing the margin.

In our analysis, we have only allowed the public payer to use pass-through payments to reimburse the hospital directly. What would happen if both regulators are allowed to use a pass-through? The mathematics of the situation would seem to imply that each payer would try to substitute pass-through payments for margin payments, leading to a total pass-through exceeding one, and margins that sum to a negative number. We do not pursue this question in detail, because we believe it is implausible to assume that private payers can be expected to make a direct contribution to joint costs.\(^{13}\)

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13. We believe that if the hospital’s joint costs cannot be more than fully covered by pass-through payments, the equilibrium in Section 4 still obtains in the sequential move game. The reason is obvious, because in that equilibrium the public payer adopts a pass-through equal to one, the private payer’s power to compensate joint costs directly
5. Conclusion

Baumol et al. (1982) label joint costs as a "public input." We show that when large buyers behave strategically in setting payment for the public input, an under provision result emerges. The degree of under provision in equilibrium and the fairness of the equilibrium depend on each payer's ability to free ride on other payers, on the payer's ability to commit to a strategy, and on the payer's set of available strategies.

The application to the hospital sector contained in this paper is an important one. Hospital capital costs, teaching costs, capacity, and administrative costs are key to health policy. The "quality" of hospital care—from both value and cost perspectives—is by nature shared concern of all patients and the public and private plans that pay for care.

We do not claim that our model is descriptive of how payment for joint cost has taken place in the past. Therefore, we do not draw the inference that there is now "too little" hospital joint costs for the simple reason that payers are unlikely to have been following the dictates of our model. Medicare's prospective payment system is itself relatively recent, and indeed, in the early years of PPS, Medicare paid both its full share of joint costs in a pass-through, and paid a margin over variable costs more favorable than that set by private payers! (Cromwell and Burge, 1991). Furthermore, we use a single-hospital framework and, thus, fail to consider what has been labeled the "medical arms race" allegedly run by hospitals in which expensive technologies are bought in a form of "wasteful competition" (Robinson and Luft, 1985).

We do believe our model has some prescriptive value. While the problem of undercapitalization of hospitals may appear presently

is moot. When both payers can use a pass-through and move simultaneously, there are two symmetric equilibria. In each case one payer adopts the strategy of setting the pass-through equal to one. These equilibria lead to the outcome in Section 4; they simply interchange the roles of the public and the private payers in that equilibrium. We do not know whether there are any other equilibria.

14. A pair of recent papers using data from California call into question the empirical importance of the tendency of hospitals to wastefully mimic one another's technologies. Dranove et al. (1992) find that the effect of hospital competition (the "medical arms race effect") is small in relation to the "extent of the market" effect. Starr-McCluer (1992) notes the important distinction between a hospital producing a service (which may be evidence for wasteful competition) and offering a service through a contract with an outside supplier (which does not duplicate fixed costs). She finds that a rival hospital's offering a radiology service has a significant effect on a hospital's own offering but a negligible effect on production decisions.
remote, the ills of strategic free riding may become more serious as all payers including Medicare move to margin payments, and all payers come under increasing pressure to minimize costs. In a related point, the public-good nature of joint costs constitutes an argument for unified regulation of hospital payment rates, to eliminate free riding.\(^\text{15}\)

We present arguments against margins and in favor of the pass-through policy. If one takes as a point of reference capacity pricing in public utilities (Panzar, 1976), it might appear appropriate to include a joint-cost charge in the price of a hospital discharge. The analogy to hospitals is, however, wrong. The discharge price is not paid to the hospital by consumers, so a margin payment is incapable of properly equilibrating demand and supply. The margin policy is an unnecessarily expensive way to induce joint-cost purchase by hospitals because of its unfavorable average/marginal properties. Covering joint costs by pass-through payments ameliorates the problem.

Before transporting this result to the real world, limitations of the model and institutional factors must be considered. Joint costs could be "public inputs" as well as "public goods," affecting operating cost in addition to the value of output.\(^\text{16}\) Recognition of substitution in production may lead to an interior solution for the pass-through amount. Nevertheless, the first-order savings at zero pass-through will still be present, implying that the pass-through amount should in general be positive. The poorly understood competitive behavior of hospitals will also affect, but not reverse, our result about the pass-through. Given any joint cost desired to be implemented, \(X(T)\) will remain concave at the hospital level, and the preferred policy for implementation will include positive pass-through payments.

Our approach to hospital behavior in this paper has been very simple. The hospital is a price (margin)-taking organization interested in maximizing profits. Most acute-care hospitals in the United States are nonprofit corporations so their motives are probably more complex than simple profit maximizing, although we think it is unlikely that the profit incentives created by margin payments will be ignored entirely. While it is realistic to regard Medicare, Medicaid, and the major Blue Cross plans as margin setters, it is plausible that hospitals have some price-setting power in relation to small private insurers. A high

\(^{15}\) Usually, the argument for a single rate structure is made in terms of fairness (avoiding cost shifting) rather than efficiency. See Ginsburg and Thorpe (1992) for a recent discussion.

\(^{16}\) Pope (1989) constructs a model of hospital quality competition in which a hospital chooses the level of quality inputs on a per discharge level. Folding in capital payments in this constant return to scale technology would do no harm. There is no margin as such because the discharge payment just covers average and marginal costs.
monopoly markup would tend to encourage investment in joint costs. Incorporating a third set of payers with hospital market power would moderate our findings, but the market power of hospitals is probably diminishing as more and more payers adopt price-setting policies and selective hospital contracts.

**APPENDIX**

*Proof of Proposition 3.* We begin by studying the public payer’s equilibrium choices of $T$, $\alpha$, and $\beta$ to maximize $\theta[U(T) - (c + \alpha)X(T)]$

subject to

$$U'(T) - (c + \beta)X'(T) + \frac{X(T)X''(T)}{(1 - \theta)X'(T)^2} = 0 \quad (27)$$

$$[\theta \alpha + (1 - \theta)\beta]X'(T) = 1. \quad (28)$$

Recall that the two constraints are the first-order conditions of the private payer’s constrained optimization program. It can be shown that these are necessary and sufficient for that program. From eqs. (27) and (28), we can straightforwardly solve for $\alpha$ and $\beta$. Therefore, we obtain eqs. (14) and (15).

The first-order conditions with respect to $\alpha$, $\beta$, and $T$ are, respectively:

$$-\theta X(T) + \mu \theta X'(T) = 0$$

$$-\eta X'(T) + \mu (1 - \theta) X'(T) = 0$$

$$\theta[U'(T) - (c + \alpha)X'(T)] + \mu X''(T)[\theta \alpha + (1 - \theta)\beta]$$

$$+ \eta \left\{ U''(T) - (c + \beta)X''(T) + \frac{1}{1 - \theta} \frac{d}{dT} \left[ \frac{X(T)X''(T)}{X'(T)^2} \right] \right\} = 0,$$

where $\eta$ and $\mu$ are the multipliers corresponding to eqs. (27) and (28), respectively. With the first two equations, we solve for $\mu$ and $\eta$, and then substitute them into the third. After simplification, the third equation reduces to

$$\theta[U'(T) - (c + \alpha)X'(T)] + [\theta \alpha + (1 - \theta)\beta] \frac{X(T)X''(T)}{X'(T)}$$

$$+ \frac{(1 - \theta)X(T)}{X'(T)} \left\{ U''(T) - (c + \beta)X''(T) \right\}$$

$$+ \frac{1}{1 - \theta} \frac{d}{dT} \left[ \frac{X(T)X''(T)}{X'(T)^2} \right] = 0.$$
Then we substitute constraint (28) into the above, multiply constraint (27) by \( (1 - \theta) \), and add the two resulting equations together to obtain

\[
U'(T) - cX'(T) - 1 + \frac{2X(T)X''(T)}{X'(T)^2} + \frac{(1 - \theta)X(T)}{X'(T)}
\]

\[
\times \left\{ U''(T) - (c + \beta)X''(T) + \frac{1}{1 - \theta} \frac{d}{dT} \left[ \frac{X(T^-)X''(T^-)}{X'(T^-)^2} \right] \right\} = 0.
\]

If we substitute eq. (15) for \( \beta \) in the previous equation and simplify, we get eq. (13). Finally, notice that in eq. (13), under Assumption A, each term inside the curly brackets is negative. Hence, by the second-order necessary condition, we conclude that \( T^- \) must be less than \( T^+ \). This completes the proof of Proposition 3.

\[ \square \]

**References**


