



## Managed care and shadow price

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### Summary

A managed-care company must decide on allocating resources of many services to many groups of enrollees. The profit-maximizing allocation rule is characterized. For each group, the marginal utilities across all services are equalized. The equilibrium has an enrollee group shadow price interpretation. The equilibrium spending allocation can be implemented by letting utilitarian physicians decide on service spending on an enrollee group subject to a budget for the group. Copyright © 2003 John Wiley & Sons, Ltd.

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### Introduction

Managed care refers to the set of instruments for controlling the delivery of health services when minimal financial incentives are imposed upon consumers. Managed care increasingly has replaced traditional financial control such as patient deductibles, and copayments. The formal modeling of the way managed care delivers services to consumers continues to be an important research topic.

An early attempt by Baumgardner [1] simply postulates that a fixed quantity of service will be supplied under managed care (see also [2]). More recently, Keeler *et al.* [3] put forward a shadow price approach to model the way managed-care companies allocate resources to patients. Frank *et al.* [4] further develop this methodology for managed care into a model with many patient groups and many services; they use it to study selection and empirically estimate the extent of

distortion. More recently, Glazer and McGuire [5] extend it to study policy implications.

The shadow price approach posits that a managed-care company will allocate resources of a service to its consumer groups until each group's marginal benefit is equal to the service shadow price.<sup>a</sup> The shadow price approach has been a significant theoretical development. First, it is a simple theory, and under some assumptions, can be used for empirical analysis. Second, given the service shadow prices, resources are allocated efficiently. Third, it also has been claimed that managed-care firms setting shadow prices may be consistent with profit maximization and capture health plans' actions in practice.

In this note, I examine the theoretical foundation of service shadow price. Contrary to the earlier analysis, I do not require that a managed-care company must impose a single shadow price on each service. Here, setting service shadow price is a feasible strategy, but a managed-care plan is free to allocate resources of many services to many

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groups to maximize profit. I show that it is never profit-maximizing to use service shadow prices. The optimal strategy is for the managed-care firm to allocate resources to a group of individuals to equalize marginal utilities of all services. In other words, the equilibrium can be described by a set of group shadow prices, one for each group of individuals covered by the managed-care company.

Given a fixed revenue rate for each group of consumers, profit maximization by the managed-care company can be given the following interpretation. First, the managed-care company chooses a total budget for each group. Given the budget for a group, the final allocation will maximize the utility of consumers in that group. Relative to the total amount of resources set aside to a group of consumers, these consumers receive an efficient allocation. Maximizing consumer utility is profit maximizing because that will attract more consumers, each of whom yields a fixed revenue for the firm. To maximize the utility of consumers in a group, each dollar spent on each service for the group must yield the same marginal utility; were this condition not satisfied, the managed-care company could have reallocated the same spending amount to increase utility, and hence profit.

## Equilibrium shadow price

A set of consumers consider joining and receiving services from a managed-care company. There are  $I$  different groups of consumers. For my purpose, it is unnecessary to consider aggregation issues. So I regard each group as consisting of a single, representative individual. The representative preferences of each group are denoted by a strictly increasing and concave function  $U_i$  defined on a managed-care organization's spending on  $S$  services. The index  $i = 1, \dots, I$  is for the consumer groups; the index  $s = 1, \dots, S$ , for the services. If the managed-care company allocates a monetary amount  $m_{is}$  for service  $s$  on group  $i$ , group  $i$  consumers have a utility  $U_i(m_{i1}, \dots, m_{iS}) + \mu_i$ , where  $\mu_i$  follows a distribution  $F_i$  with density  $f_i$ . Group  $i$  consumers have a reservation utility  $\bar{U}_i$ . The managed-care organization receives a capitation rate  $r_i$  for providing services to group  $i$  consumers. These capitation rates are assumed to be exogenous; consumers do not directly pay for services.

The game proceeds in the following way. The managed-care company decides on the spending,  $m_{is}$ ,  $i = 1, \dots, I$ ,  $s = 1, \dots, S$ . Group  $i$  consumers observe the realization of  $\mu_i$ ,  $i = 1, \dots, I$ , and decide whether to join the managed-care plan to enjoy the services. Given the spending allocation, the probability that group  $i$  joins the managed-care firm is  $\Pr(U_i + \mu_i \geq \bar{U}_i)$ ; that is, the demand from group  $i$  is  $1 - F_i(\bar{U}_i - U_i(m_{i1}, \dots, m_{iS}))$ . The managed-care firm makes an expected profit:

$$\sum_{i=1}^I [1 - F_i(\bar{U}_i - U_i(m_{i1}, \dots, m_{iS}))] \left[ r_i - \sum_{s=1}^S m_{is} \right] \quad (1)$$

In an equilibrium, the managed-care firm picks  $m_{is}$ ,  $i = 1, \dots, I$ ,  $s = 1, \dots, S$  to maximize its profits.<sup>b</sup>

To understand how the equilibrium spendings  $m_{is}$  are chosen, break up the maximization problem into two steps. First, for group  $i$ , let the managed-care firm commit to a level of total spending,  $\sum_{s=1}^S m_{is}$ . Then find the allocation of spending to maximize profit given this preset total. Second, adjust the total spending to achieve the global maximum profit.

From (1), once  $\sum_{s=1}^S m_{is}$  is fixed in the first step, the managed-care firm chooses the spending to maximize its demand  $1 - F_i$ . This is equivalent to maximizing the consumer utility function  $U_i$  subject to the spending level. For those services for which the managed-care firm chooses positive spendings, marginal utilities of these services are equalized:

$$\frac{\partial U_i(m_{i1}, \dots, m_{iS})}{\partial m_{is}} = \frac{\partial U_i(m_{i1}, \dots, m_{iS})}{\partial m_{it}},$$

$$s, t = 1, \dots, S.$$

For any given consumer group, each service generates the same marginal utility. So I call this value of marginal utility the *shadow price for the consumer group*. From the second step, given the equal-service-marginal-utility property, the profit-maximizing total spending  $\sum_{s=1}^S m_{is}$  is chosen to satisfy the usual condition: the price-cost margin  $(r_i - \sum_{s=1}^S m_{is}) / \sum_{s=1}^S m_{is}$  will be inversely related to the elasticity of demand.

The following derivation confirms the intuition. The first-order derivative of (1) with respect

to  $m_{is}$  is

$$- [1 - F_i(\bar{U}_i - U_i(m_{i1}, \dots, m_{iS}))] + \left[ r_i - \sum_{t=1}^S m_{it} \right] f_i(\bar{U}_i - U_i(m_{i1}, \dots, m_{iS})) \frac{\partial U_i}{\partial m_{is}} \quad (2)$$

If there is a corner solution, then the above expression will be negative and the value of  $m_{is}$  is set at 0. For an interior solution, I set the first-order derivative to zero. After rearranging, I obtain the necessary condition for a profit-maximizing choice of an interior  $m_{is}$ :

$$\frac{\partial U_i}{\partial m_{is}} = \left[ r_i - \sum_{t=1}^S m_{it} \right]^{-1} \frac{1 - F_i(\bar{U}_i - U_i(m_{i1}, \dots, m_{iS}))}{f_i(\bar{U}_i - U_i(m_{i1}, \dots, m_{iS}))} \quad i = 1, \dots, I; \quad s = 1, \dots, S \quad (3)$$

I consider now those spendings that are strictly positive. The right-hand side expression in (3) holds for all such spendings for service  $s = 1, \dots, S$  for group  $i = 1, \dots, I$ . For group  $i$ , the profit-maximizing spendings will equalize group  $i$ 's marginal utilities for these services. The equilibrium reveals a shadow price,  $P_i$ , for each consumer group  $i$ : for any two services (with strictly positive spendings)  $s$  and  $t$

$$\frac{\partial U_i(m_{i1}, \dots, m_{iS})}{\partial m_{is}} = \frac{\partial U_i(m_{i1}, \dots, m_{iS})}{\partial m_{it}} \equiv P_i$$

Rearranging (3), I also obtain

$$\frac{r_i - \sum_{t=1}^S m_{it}}{\sum_{t=1}^S m_{it}} = \left[ \frac{f_i}{1 - F_i} \frac{\partial U_i}{\partial m_{is}} m_{is} \right]^{-1} \frac{m_{is}}{\sum_{t=1}^S m_{it}}$$

The term inside the square brackets on the right-hand side is simply the elasticity of group  $i$  demand with respect to  $m_{is}$ . The price-cost margin for group  $i$  is inversely related to its demand elasticity.

In practice, a managed-care firm has to rely on its physicians and other health-care professionals to deliver services. How can the profit-maximizing allocations be implemented? I now describe this implementation when physicians decide on the services on a utilitarian basis. Suppose the managed-care organization allocates a budget  $B_i$  for consumer group  $i$ ,  $i = 1, \dots, I$ . Under the utilitarian assumption, the physicians will choose service spendings to maximize the group's utility given the resources available to this group. That is, the physicians choose  $m_{is}$  to maximize  $U_i(m_{i1}, \dots, m_{iS})$  subject to  $\sum_{s=1}^S m_{is} = B_i$ .

To maximize utility for consumer group  $i$  subject to budget  $B_i$ , the physicians choose spending  $m_{is}$  characterized by

$$\frac{\partial U_i}{\partial m_{is}} = \frac{\partial U_i}{\partial m_{it}} \quad s, t = 1, \dots, S$$

To implement the profit-maximizing spending, the managed-care company simply picks  $B_i$  such that the above marginal utility equals  $P_i$ . In other words, choose the budget for group  $i$  to ensure that physicians implement spending across services according to the group shadow price  $P_i$ . The implementation of the optimal spending may look similar to that under service shadow price, as Frank, Glazer, and McGuire [4, p. 386] describe: 'Cost-conscious management allocates a budget or a physical capacity for a service. Clinicians working in the service area do the best they can for patients..management is in effect setting a shadow price for a service through its budget allocation.' In fact, here the equilibrium can be implemented by management allocating a budget for a group of enrollees, instead of a service.

## Concluding remarks

I characterize a managed-care company's profit-maximizing spending by a set of shadow prices, one for each of its consumer groups. These group shadow prices depend on the capitation rates and consumer preferences. My results show that service shadow prices are suboptimal. Furthermore, equilibrium properties of managed-care spending are to be recovered from the group shadow prices, not service shadow prices. Therefore, the relevance of empirical estimation of service shadow prices is being questioned. Further empirical work on enrollee group shadow prices may well shed new light on adverse selection and managed care.

Arguments for service shadow price in the literature have centered on the practicality and perhaps fairness of such an approach. Physicians allocating resources to different groups of consumers according to service shadow price will not have to know these consumers' capitated payments. Each group of consumers also obtains the same (marginal) value from a given service. Nevertheless, an enrollee group's capitation rate presumably reflects the group's expected usage

cost and premium. In my model, groups that have higher capitation rates will be allocated a higher budget. This does not seem to be an unfair procedure if higher capitation rates correspond to higher premium rates. In fact, this is the usual way the market allocates resources: consumers who have paid more expect to receive more services.

My model uses the same informational setup as work in the earlier literature (for example, Frank *et al.* [4]) the managed-care company possesses perfect information, and a contractible state for resource allocation is service  $s$  on group  $i$ . In this setup, the managed-care company will not lose profit if it sets a shadow price for each enrollee group, and asks the utilitarian physician to implement the allocation. Conversely, asking utilitarian physicians to implement an allocation via service shadow prices leads to lower profits.

Under perfect information, a managed-care company can simply compute the optimal spending. The usual justification for delegation revolves around asymmetric information and expertise. What is the optimal spending when providers possess private information on patient characteristics and their own costs? What is the second-best allocation, and will service or enrollee group shadow price implement it? There are related issues, too. For example, what are physicians' motives? Is the pure utilitarian assumption a good one? Can physicians dilute or manipulate budgets for different groups? Do physicians' actions fully reflect consumer preferences? For a broader perspective, one must also ask how the capitation rates are set. If these rates do not reflect the group premium rates or costs, adverse selection problems must be addressed. The note here presents a foundation for studying these problems.

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## Notes

- a. Suppose that the utility function of a group  $i$  is  $U_i$ ,  $i = 1, \dots, I$ , and that the shadow price of service  $s$  is  $p_s$ ,  $s = 1, \dots, S$ . According to the service shadow price approach, the resources allocated to group  $i$ ,  $m_{is}$ , satisfy  $\partial U_i(m_{i1}, \dots, m_{is}, \dots, m_{iS}) / \partial m_{is} = p_s$ , for each group  $i$ . For example, a group of individuals who have a low marginal valuation of psychiatric services will be given a smaller number of outpatient visits compared to those who have a high marginal valuation.
- b. The firm must make a nonnegative profit: setting each of  $m_{is}$  to 0 is a feasible allocation.

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