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IMPLEMENTATION IN DYNAMIC JOB TRANSFERS

Ching-to Albert MA *

Boston University, Boston, MA 02215, USA

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The implementation problem in a dynamic incentive contract with job transfers is studied. We find that in the job transfers contract, there are multiple equilibria, in some of which the agents can lie. We construct a mechanism to uniquely implement the principal's desired equilibrium under job transfers.

1. Introduction

It is well known in the implementation literature that non-trivial multiple equilibria cannot arise when one principal contracts with one agent [see e.g., Postlewaite and Schmeidler (1986)]. This is also true if a principal contracts with many agents who possess independent private information, since the (optimal) reward scheme for an agent will be independent of actions being taken by others. It is natural to ask whether this is true in dynamic contracts with many agents who have independent private information. In multi-period contracts, one aspect is very important, and that is the revelation of information over time. Even though agents' information is independent, this may be effective for more than one period. The principal may make use of the information revealed by an agent in one period to alleviate the incentive problem in the next period, during which another agent may be operating. An agent's payoff may then be based upon actions of other agents in previous periods. Because of this dependency, multiple equilibria may arise. Using a model by Ickes and Samuelson (1987), we examine and solve the implementation problem in such an environment.

In Ickes and Samuelson (1987), a job transfers scheme is used to minimize the cost of the ratchet effect in a multi-period contract. In the model, a job carries a certain characteristic that is efffective for more than one period. An agent responsible for the job is able to observe this characteristic privately, and characteristics of jobs are independent random variables. Job transfers can avoid the extra incentives (the ratchet effect) required to elicit the private information from an agent. The intuition is that when agents exchange jobs, an agent will only be attached to the job for one period and his honesty need merely be guaranteed by an incentive scheme that corresponds to that of a static model. However, we find multiple equilibria in the job transfers scheme: besides truthful revelation, there is another equilibrium in which both agents lie. In this equilibrium, they will both be better off in a later period (and the principal worse off) since the principal mistakenly believes that the information obtained earlier is genuine. We solve the multiple equilibria problem by proposing an alternative mechanism and show how the second-best identified by Ickes and Samuelson can be uniquely implemented.

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2. The model and the result

We first outline the job transfers model and then we present our result. There are two agents, A and B, who are risk-neutral. There are two jobs available and each job lasts for two periods. Each job carries a certain characteristic θ, which is a binary random variable with support $\{\underline{\theta}, \bar{\theta}\}$, and characteristics in different jobs are independent. Let prob $(\bar{\theta}) = p$. The property of the job lasts for the whole contracting horizon and is known to an agent before a contract is signed. Agent A (resp. B) can choose between high effort $\bar{e}^{A}$ (resp. $\bar{e}^{B}$) and low effort $\underline{e}^{A}$ (resp. $\underline{e}^{B}$), $\bar{e}^{i} \geq \underline{e}^{i}$, $i =$ A,B. The production technology $f^{i}(\theta, e^{i})$ satisfies

$$f_1^i \equiv f^i(\bar{\theta}, \bar{e}^i) > f^i(\bar{\theta}, \underline{e}^i) \equiv f_2^i \equiv f^i(\underline{\theta}, \bar{e}^i) > f^i(\underline{\theta}, \underline{e}^i) \equiv f_3^i, \qquad i = \text{A, B}.$$

That is, when f_2^i occurs, the principal (who never observes θs) cannot infer whether it is due to high effort ($\bar{e}^i$) or due to a 'good' state of nature ($\bar{\theta}$). An agent's preference is given by $R - e^i$, where R is remuneration from the principal and $e^i = \bar{e}^i$, $\underline{e}^i$, $i =$ A, B. Each agent has a reservation utility that is normalized to zero. We suppose that the principal is only able to offer short term contracts, i.e., contracts that are effective for the current period.

In this article, we are concerned with the implementation of $\bar{e}^i$ from both agents. Consider for the moment the case in which an agent is associated to a job for two periods. Implementation of $\bar{e}^i$ from both types of agents requires driving a large wedge in the agents's period 1 remunerations in outputs f_1^i and f_2^i. Otherwise, an agent knowing $\bar{\theta}$ may shirk to mimic a $\underline{\theta}$ agent in period 1 in order to earn a bonus in period 2 – the ratchet effect. Ickes and Samuelson propose an interesting mechanism to avoid the ratchet effect. Their idea is: ask agents to *exchange jobs* after period 1. In this job transfers arrangement, there is no need for an excessive wedge in payments to motivate agents to perform $\bar{e}^i$ since an agent is only responsible for the job in one period. But the characteristic of a job lies solely with the job; the principal then uses the information revealed by the outgoing agent to structure a reward schedule for the incoming agent. He can achieve the first-best in period 2! [1] Formally, in Ickes and Samuelson, the payments proposed by the principal are given in table 1.

In period 1, an agent's reward depends on the output, e.g., if the output is f_2^i, he is paid $\bar{e}^i$. The period 2 output from agent j depends on the output of agent i in period 1; if in period 1 agent i

Table 1

Output	Payment
Period 1	
f_1^i	$2\bar{e}^i - \underline{e}^i$
f_2^i	$\bar{e}^i$
f_3^i	$\underline{e}^i$
Period 2	
f_1^j	$\bar{e}^j$
f_2^j	$\bar{e}^j$
f_3^j	$\underline{e}^j$
$i, j =$ A, B, $\quad i \neq j$.	

[1] Ickes and Samuelson derive conditions under which a job transfers scheme is optimal, when there are beneficial human capital externalities in period 2. The interested reader is referred to the paper. This note is only concerned with the game form that implements the job transfers scheme.

Table 2
Agent A's payments.

	f_1^{B}	f_2^{B}	f_3^{B}
f_1^{A}	$2\bar{e}^{\mathrm{A}} - \underline{e}^{\mathrm{A}}$	$2\bar{e}^{\mathrm{A}} - \underline{e}^{\mathrm{A}}$	$2\bar{e}^{\mathrm{A}} - \underline{e}^{\mathrm{A}} + \alpha$
f_2^{A}	$\bar{e}^{\mathrm{A}} - \delta(t^{\mathrm{A}})$	$\bar{e}^{\mathrm{A}} + \epsilon(t^{\mathrm{A}})$	$\bar{e}^{\mathrm{A}}$
f_3^{A}	$\underline{e}^{\mathrm{A}} - \alpha$	$\underline{e}^{\mathrm{A}} - \alpha$	$\underline{e}^{\mathrm{A}} - \alpha$

produces f_1^i (resp. f_2^i), then agent j is also required to produce f_1^j (resp. f_2^j) in period 2. In this game, it is easy to prove that agent i taking $\bar{e}^i$, $i = \mathrm{A}$, B, in both periods form a Bayes–Nash equilibrium and this is the equilibrium the principal seeks to implement under job transfers. Notice that in this equilibrium an agent in period 2 has a zero utility. However, note that in period 1, agent A is indifferent between $\bar{e}^{\mathrm{A}}$ and $\underline{e}^{\mathrm{A}}$ when he is type $\bar{\theta}$. If he shirks in that state and produces f_2^{A}, then the principal will mistakenly infer that he is type $\underline{\theta}$. The incoming agent (who observes $\bar{\theta}$) in period 2 can then produce f_2^{B} by choosing effort $\underline{e}^{\mathrm{B}}$, earning a utility $\bar{e}^{\mathrm{B}} - \underline{e}^{\mathrm{B}} > 0$. This argument applies to the other agent as well. In sum, there is another equilibrium in which each agent produces f_2^i (irrespective of this type) in both periods, and they are both better off in this equilibrium than in the 'honest' equilibrium. [2]

The contribution of this article is to show how the principal can easily avoid the undesired equilibrium in the job transfers scheme. Certain features of the mechanism we construct here are similar to those in Ma, Moore and Turnbull (1987) in which the implementation problem in another many-agent adverse selection model is discussed. The crucial element we introduce is that, in period 1, the principal asks agents to police each another by obtaining from them signals that indicate whether they are performing 'properly'. In particular, we allow each agent a range of other strategies: agent i can announce a real number t^i, where $p \geq t^i \geq 0$, when he produces f_2^i, $i = \mathrm{A}$, B. As will be seen, a report of $t^i \neq p$ by agent i serves as a signal to the principal that he believes $\bar{e}^j$ is not being taken when agent j is of type $\bar{\theta}$. Also, the principal adopts the period 2 payments in table 1 and, for each job, he uses the period 1 outputs to determine the corresponding period 2 output requirements. The full period 1 payment matrices of our mechanism are in tables 2 and 3, in which an entry in a matrix denotes the payment to an agent if the corresponding outputs are being produced (in period 1) and the corresponding announcements (if any) are being made.

In tables 2 and 3, $\epsilon(t^i)$ and $\delta(t^i)$ are continuous non-negative functions of t^i and satisfy $\epsilon(p) = \delta(p) = 0$,

$$-\delta(t^i)t^i + \epsilon(t^i)(1 - t^i) > 0, \tag{1}$$

$$-\delta(t^i)p^i + \epsilon(t^i)(1 - p^i) < 0, \tag{2}$$

[2] Ickes and Samuelson argue that they adopt a tie-breaking rule that basically says that if an agent is indifferent between two actions, he performs according to the principal's wish. This certainly is a harmless assumption when there is one agent. But in effect, when there are many agents, this amounts to a convention of *equilibrium selection* in agents' game! One possible way to strengthen the truth-telling incentive in the mechanism defined in table 1 is to write $2\bar{e}^i - \underline{e}^i + \epsilon$, instead of $2\bar{e}^i - \underline{e}^i$ at the top left corner, where ϵ is positive and arbitrarily small. Then a type $\bar{\theta}$ agent will strictly prefer to perform $\bar{e}^i$. We do not find this method convincing, for two reasons. First, as we show later, there is no need to pay ϵ more to the agent. Second, the payment ϵ presumably serves as a bribe to a type $\bar{\theta}$ agent to conform to the principal's intention. If the principal can do this, why cannot the other agent? More precisely, let agent B send agent A a message that reads 'Please produce f_2^{A} when you are type $\bar{\theta}$. *Whenever* I see f_2^{A} occur, I will pay you 2ϵ'. This 'side contract' is at least as credible as the principal's bribe, since ϵ is arbitrarily small. [Other papers that discuss the effects of side contracts include Eswaran and Kotwal (1984) and Tirole (1986).] In other words, the slightest possibility of 'collusion' between agents will undo the principal's bribe.

Table 3
Agent B's payments.

	f_1^A	f_2^A $t^A = p$	f_2^A $t^A < p$	f_3^A
f_1^B	$2\bar{e}^B - \underline{e}^B$	$2\bar{e}^B - \underline{e}^B$	$2\bar{e}^B - \underline{e}^B + \alpha$	$2\bar{e}^B - \underline{e}^B + \alpha$
f_2^B	$\bar{e}^B - \delta(t^B)$	$\bar{e}^B + \epsilon(t^B)$	$\bar{e}^B$	$\bar{e}^B$
f_3^B	$\underline{e}^B - \alpha$	$\underline{e}^B - \alpha$	$\underline{e}^B - \alpha$	$\underline{e}^B - \alpha$

for $0 \le t^i < p$, $i = \text{A, B}$;

$$\alpha > \max\big(\delta(t^i), 0 \le t^i \le p, i = A, B\big). \tag{3}$$

We are now in a position to state and prove our result.

Theorem. Suppose the period 1 payments are defined in tables 2 and 3. Then, in both periods, each agent choosing $\bar{e}^i$ $(i = A, B)$ independent of his type (θ) is the unique Bayes–Nash equilibrium. In equilibrium, agent i's period 1 payment is $\bar{e}^i$ when he is type $\underline{\theta}$ and $2\bar{e}^i - \underline{e}^i$ when he is type $\bar{\theta}$, $i = A, B$.

Proof. We shall adopt the following short-hand. Let $\overline{\text{A}}$ (resp. $\underline{\text{A}}$) denote agent A when he is type $\bar{\theta}$ (resp. $\underline{\theta}$), and similarly for agent B. We first note that given job transfers and the principal's insistence that, for each job, an incoming agent must in period 2 produce the corresponding period 1 output of the outgoing agent, it is sufficient to prove that the one-stage game defined by tables 2 and 3 has a unique Bayes–Nash equilibrium with the period 1 properties in the statement of the theorem. We prove the theorem in five steps.

Step 1. Agent $\underline{\text{A}}$ always chooses $\bar{e}^A$. Agent $\underline{\text{B}}$ always chooses $\bar{e}^B$. This is obvious from tables 2 and 3 and eq. (3).

Step 2. Agent $\overline{\text{B}}$ always chooses $\bar{e}^B$.
Suppose not. Suppose $\overline{\text{B}}$ chooses $\underline{e}^B$ with positive probability. Then agent A assesses that f_1^B occurs with probability less than p. From table 2 and eq. (1), the unique best choice of $\overline{\text{A}}$ is $\underline{e}^A$ with some announcement $t^A < p$. Also $\underline{\text{A}}$ will announce some $t^A < p$. However, given that $t^A < p$ is always announced by A, from table 3, $\overline{\text{B}}$'s unique best response is $\bar{e}^B$. Contradiction. Hence $\overline{\text{B}}$ chooses $\bar{e}^B$.

Step 3. Agent $\overline{\text{A}}$ always reports $t^A = p$ if he ever chooses $\underline{e}^A$. Agent $\underline{\text{A}}$ always reports $t^A = p$ when he chooses $\bar{e}^A$.

This is because B always chooses $\bar{e}^B$; eq.(2) then implies that none of the t^As that are strictly less than p may give extra positive expected utility to A.

Step 4. Agent $\overline{\text{A}}$ always chooses $\bar{e}^A$.
Suppose not. Suppose $\overline{\text{A}}$ chooses $\underline{e}^A$ with positive probability. From Steps 1 and 3, A also says $t^A = p$. Now $\overline{\text{B}}$'s unique best response is $\underline{e}^B$ with some $t^B < p$, from table 3 and eq. (1). This contradicts Step 2. Hence $\overline{\text{A}}$ always chooses $\bar{e}^A$.

Step 5. Agent $\underline{\text{B}}$ always announces $t^B = p$.
This follows directly from Steps 1, 3 and 4, and eq. (2).

In sum, we have shown that the only possible equilibrium is A always chooses $\bar{e}^A$ and B always chooses $\bar{e}^B$. Furthermore, $\underline{\text{A}}$ announces $t^A = p$ and $\underline{\text{B}}$ announces $t^B = p$. It is easy to verify that this is an equilibrium. Q.E.D.

References

Eswaran, M. and A. Kotwal, 1984, The moral hazard of budget-breaking, Rand Journal of Economics 15.

Ickes, B.W. and L. Samuelson, 1987, Job transfers and incentives in complex oragnisations: Thwarting the Ratchet effect, Rand Journal of Economics 18.

Ma, C., J. Moore and S. Turnbull, 1987, Stopping agents from 'cheating', Mimeo., Journal of Economic Theory, forthcoming.

Postlewaite, A. and D. Schmeidler, 1986, Implementation in differential information economies, Journal of Economic Theory, 39.

Tirole, J., 1986, Hierarchies and bureaucracies: On the role of collusion in organisations, Journal of Law, Economics and Organisation 2.