Optimal Public Rationing and Price Response

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Motivation

- Crowd out: more public supply means less private supply
- Rationing design
- Which kind of information for rationing?
- Why nonprice rationing?
- Why noncompetitive private market?
Optimal rationing: Public sector has limited budget. How to allocate?

Private market: Rationed consumers may purchase privately

Information regimes: Rationing policies based on different consumer information

1. Public supplier sets rationing policies
2. Private firm sets prices
Information

- Public sector observes *wealth* OR *wealth and cost*: rationing based on available information

- Private sector observes *cost*: prices based on cost

Two-Dimensional Model of Public-Private Strategic Interaction
Cutler and Gruber (QJE 1996) on crowd out

P. Barros and P. Olivella (JEMS, 2005) on waiting lists and patients selection

M. Hoel (JHE, 2007) on cost effectiveness when a competitive market is available

many (older) papers on mixed oligopoly

Grassi and Ma (2008)
Results: Equilibrium Rationing and Prices

Public sector observes WEALTH

- Optimal scheme MUST ration some poor consumers
- In order to force a price reduction from private market

Public sector observes WEALTH and COST

- Rationing based on cost-effectiveness
- Optimal scheme rations consumers if and only if their costs are high
- Wealth information is not used
- Cost information much more important than wealth
The Model

- Each consumer gets 0 or 1 unit

- Cost of one unit of the good: $c > 0$

- A consumer’s wealth $w_1$ or $w_2$: $w_1 < w_2$; poor and rich consumers

- $m_1$ and $m_2$ are masses of poor and rich consumers

- Cost $c$ has distribution $G(c)$ and density $g(c)$ on $[c, \bar{c}]$

- Let $\gamma = \int c \, dG(c)$ be expected cost

- Cost and Wealth are independently distributed
• Consumer gets 1 unit of utility from the good—whether it is from public or private provider

• Utility from purchase at price $p$: $U(w_i - p) + 1$

• Utility from no purchase: $U(w_i)$

• Utility from public supply $U(w_i) + 1$

• $U$ strictly increasing and strictly concave: $U' > 0$, $U'' < 0$
• Willingness-to-pay $\tau_i$:

\[ U(w_i - \tau_i) + 1 = U(w_i) \]

\[ \tau_1 < \tau_2 \]

The rich is more willing to pay

**Trade Surplus: at price $p$, rich consumers benefit more**

\[ U(w_1 - p) + 1 - U(w_1) < U(w_2 - p) + 1 - U(w_2) \]
The Game

Stage 0: Nature determines \( m_i \) consumers have \( w_i \), draws a cost \( c \) for each consumer.

Stage 1: Public supplier observes \( w \), or \( w \) and \( c \); private firm observes \( c \).

Stage 2: Public supplier chooses rationing function.

Stage 3: Private firm chooses pricing function; consumers without free public supply may purchase from private firm.
Rationing policies:

Based on wealth

\[(\theta_1, \theta_2), \ 0 \leq \theta_i \leq 1, \ \text{supplying} \ (1 - \theta_i) m_i \ \text{of consumers with wealth} \ w_i, \ i = 1, 2\]

Based on wealth and cost

\[(\phi_1, \phi_2), \ \phi_i : [c, \bar{c}] \rightarrow [0, 1], \ \text{supplying} \ [1 - \phi_i(c)] m_i \ \text{of consumers with wealth} \ w_i \ \text{and cost} \ c\]

Motivations for these rationing rules
Equilibrium: rationing based on wealth

Firm’s continuation equilibrium prices given policy \((\theta_1, \theta_2)\)

- Optimal price is either \(\tau_1\) or \(\tau_2\)
- Cost above \(\tau_1\) \implies \(\tau_2\)
- Cost below \(\tau_1\) \implies either \(\tau_1\) or \(\tau_2\)
  - Many poor consumers around: price at \(\tau_1\)
  - Many rich consumers around: price at \(\tau_2\)
  - Very low cost: price at \(\tau_1\)
  - Cost just below \(\tau_1\): price at \(\tau_2\)
Cost threshold for price reduction:

\[ c_1 \equiv \tau_1 - \frac{m_2 \theta_2}{m_1 \theta_1} (\tau_2 - \tau_1) \]

Figure 1: Comparison of profits between setting high and low prices
Public supplier preferences: total consumer utility

- Poor consumers never get any surplus private market
- Rich consumers get surplus if and only if price is low $\tau_1$:
  \[ U(w_2 - \tau_1) + 1 - U(w_2) \]
- In fact, total consumer utility is
  \[
  \text{Constant} + m_2 \theta_2 \int_{c}^{c_1} [U(w_2 - \tau_1) + 1 - U(w_2)] \, g(c) \, dc
  \]
  \[= \text{Constant} + \text{constant} \times m_2 \theta_2 G(c_1)\]
- Price reduction $\Leftrightarrow$ surplus for consumers
- Choices of $(\theta_1, \theta_2)$ implements price reduction threshold $c_1$
Proposition 1: In equilibrium, $\theta_1 > 0$ and $\theta_2 > 0$. There must be some price reduction: $c < c_1 < \bar{c}$

In private market

- Only rich consumers potentially gain
- So ration some rich
- But rich consumers gain if and only if there is price reduction
- And price reduction means some poor consumers in private market
- So ration some poor
What is the equilibrium?

Problem is: choose $\theta_1$, $\theta_2$, and cost threshold $c_1$ to maximize objective function

$$\text{Constant} + \text{constant} \times m_2 \theta_2 G(c_1)$$

subject to

1. Budget constraint
2. Definition of cost threshold
3. Boundary conditions on $\theta_1$ and $\theta_2$
Figure 2: Budget and cost threshold constraints; boundary conditions
Proposition 2: For big budget, equilibrium has $0 < \theta_i < 1$, 
\[
\frac{G(c_1^*)}{g(c_1^*)} = \frac{(\tau_1 - c_1^*)(\tau_2 - c_1^*)}{\tau_2 - \tau_1}
\]
otherwise, either $\theta_1 = 1$ or $\theta_2 = 1$

Remarks:

• For big budget, equilibrium cost threshold and rationing ratio $\frac{\theta_1}{\theta_2}$ constant functions of budget

• For small budget, may ration an entire wealth class

• So there is some crowd out; can be limited by rationing the poor

• What if there is equity concern?

• What if the private market is competitive (price = cost)?
Equilibrium: rationing based on wealth, cost

- Given rationing policies \((\phi_1, \phi_2)\), \(\phi_i : [c, \bar{c}] \rightarrow [0, 1]\)

- Profit selling to poor and rich \(>\) profit selling only to rich \(\iff\) price reduction

- Define price reduction function: \(p : [c, \bar{c}] \rightarrow [0, 1]\)

- At \(c\), \(p(c) = 1\) means low price \(\tau_1\)

Given \((\phi_1, \phi_2)\), continuation equilibrium is a price reduction function \(p\) where

\[
p(c)\{\text{profit from } \tau_1 - \text{profit from } \tau_2\} \geq 0
\]

\[
[1 - p(c)]\{\text{profit from } \tau_1 - \text{profit from } \tau_2\} \leq 0
\]
What is the equilibrium?

Problem is: choose $\phi_1$, $\phi_2$, and price reduction function $p$ to maximize objective function

$$\text{Constant} + \int_{\bar{c}} \left\{ m_1[1 - \phi_1(c)] + m_2[1 - \phi_2(c)] \right\} dG(c)$$

$$+ \int_{\bar{c}} m_2\phi_2(c)p(c)[U(w_2 - \tau_1) + 1 - U(w_2)] dG(c)$$

subject to

1. Budget constraint

2. Price reduction function $p$ consistent with $\phi_1$ and $\phi_2$

3. Boundary conditions on $\phi_1$ and $\phi_2$
Proposition 3: In equilibrium, spend all budget on low cost consumers:

\[
\phi_1(c) = \phi_2(c) = 0 \quad \text{for} \quad c < c^B \\
\phi_1(c) = \phi_2(c) = 1 \quad \text{for} \quad c > c^B,
\]

Consumers with costs below \( c^B \) exhausts the budget:

\[
\int_c^{c^B} (m_1 + m_2)c \, dG(c) = B.
\]

- Cost effectiveness principle
- Wealth irrelevant
- Implementing price reduction inferior
- In fact, optimal policy is the same as if private market didn’t exist!!
Figure 3: Equilibrium rationing and price reduction
Three factors:

1. Cost effectiveness

2. Rationing rich ⇒ price rises; rationing poor ⇒ price falls

3. Rich consumers may gain surplus from market

\[
\frac{\partial L}{\partial \phi_1} = -m_1(1 - \lambda c) + \mu(c)p(c)m_1[\tau_1 - c]
\]

\[
\frac{\partial L}{\partial \phi_2} = -m_2(1 - \lambda c) - \mu(c)p(c)m_2[\tau_2 - \tau_1] + m_2p(c)\Delta,
\]

where \(\Delta\) is surplus for rich consumers
Equilibrium consumer utility higher under rationing based on cost than wealth.

Between cost and wealth information for rationing, cost information better.

- What if there is equity concern?
- What if the private market is competitive (price = cost)?
Conclusions

- A model of rationing
- Optimal design; noncompetitive private market
- Compare wealth and cost
Robustness

- If private firm does not observe cost...
- If private firm observes both...
- If benefit varies, if cost and benefit correlated...
- If public and private move simultaneously...
- What if there are many wealth classes?
- What if public uses monetary subsidies?