In this chapter, we present the theory of consumer preferences on risky outcomes. The theory is then applied to study the demand for insurance. Consider the following story. John wants to mail a package to his cousin in another state. This is a birthday present, and John has spent $100 buying this. At the post office, the question comes up: "Would you like to purchase insurance for the mailing?" John would like more about the details of the insurance option, and he gets the following answer: "For $5, if your package is lost in transit, then you can file a claim, and you receive $100. Well, if your package isn’t lost, all is well."

John thinks about it for 2 seconds, and decides that it is a “good deal,” so he pays $5 extra for the insurance. By the way, another customer, Mary, faces the same scenario, but she decides against it, declines the insurance.

1 Uncertainty and Preferences

1.1 Consumption Bundles

In the standard economic model of consumer demand, we begin with a set of preferences on bundles of goods. For example, consider two bundles, each of which consists of two goods such as the number of internet music downloads and the number of cellular phone text messages. For example, a bundle (20,40) denotes 20 music downloads and 40 text messages, say, in a week, and another bundle (25,20) denotes 25 music downloads and 20 text messages. A consumer, John, is assumed to have preferences over these two bundles. For example, John may prefer the bundle (20,40) to the bundle (25,20). The standard consumer demand theory then takes John’s ranking of all sorts of such bundles as given, and studies how John would choose the best bundle given the prices of the goods and an available income.

As an illustration, suppose that a music download costs $1, while a text message costs 25 cents, and John has $30 available for these two goods. Now, John realizes that each of the two bundles, (20,40) and (25,20), costs exactly $30. Because he prefers (20,40) over (25,20), consumer demand theory then says that he will choose the (20,40) bundle. Essentially, John is willing to given up the five music download (reducing from 25 to 20) in order to send twenty more text messages (increasing from 20 to 40).

The standard economic model of consumer demand is about the trade-off between goods. The market
prices give the consumer that opportunity. Adjusting his consumption from (25,20) to (20,40) is feasible for John, since giving up five music downloads saves him $5, which he can use to purchase twenty more text messages.

1.2 Decision under Uncertainty

Let us get back to the post office. How should John and Mary think about the insurance purchase. Here is the economic perspective. John faces some uncertainty as to whether the package will be lost in the mail. With some chance the package will be lost, and he will be $100 short; otherwise, it will be fine, and there is no loss. There are, therefore, two possibilities: the package arrives in tact, and the package is lost. John is uncertain about whether which of these possibilities will ensure. For a price of $5, the insurance, he can make sure that even in the event that the package is lost, he will be compensated. So his decision concerns whether it is worth his while to pay $5 to insure against this loss. John comes down with a purchase decision: he buys the insurance. Mary, on the other hand, decides against it. Apparently, Mary thinks that the insurance is not worthwhile. Each consumer faces a tradeoff.

How do consumers make trade-off under uncertainty? First, we must understand how uncertainty is modelled. Uncertainty is defined to be a set of possible states, one and only one of which will be the actual state. For example, the weather tomorrow can be either sunny or rainy. There are two possible states: sunny and rainy. Today we know that each of these two states is possible; by tomorrow, we will find out exactly what the weather will be. Your health can be either good or bad tomorrow. Today you do not know whether you will be healthy or sick tomorrow. Again, by tomorrow, you will find out. By its definition, uncertainty already embeds a notion of time. There is a difference between today and tomorrow: the weather tomorrow is uncertain, but by tomorrow that uncertainty will be resolved; one’s health tomorrow is uncertain, but by tomorrow that uncertainty will be resolved. We often call the time frame when the uncertainty exists the ex ante, and the time frame when the uncertainty is resolved the ex post.

Second, we have to understand what a bundle of goods means under uncertainty. A good is now defined by its availability contingent upon a state. For example, a sum of $25 will be available to John if the weather is sunny tomorrow, or $20 will be available to John if the weather is rainy. Here, John will have $20 no matter
what the weather is tomorrow, but he will have $5 more if it is sunny. His income \textit{ex ante} is uncertain; his income \textit{ex post} will turn out to be either $20 or $25 depending on whether the weather is sunny. As another example, good health may be interpreted as no loss of income, but bad health may correspond to a loss. If John remains healthy tomorrow, his income stays at $25, but if he is ill, that income drops to $20.

Third, we will assume that a consumer is able to rank bundles. Suppose now the pair (25,20) denotes an income of $25 if it is sunny tomorrow and an income of $20 if it is rainy. The pair (20,40) denotes an income of $20 if it is sunny, and an income of $40 if it is rainy. John is able to tell whether he prefers (25,20) to (20,40), or otherwise, so he may prefer the bundle (25,20) to (20,40).

As one can see, the notation for bundles of goods that are of the same physical characteristics (say, money) available across different states looks identical to the notation for goods that are physically different (say, music downloads and text messages). One, however, intuitively suspects that the preferences for physically identical goods whose availability is contingent upon different states must be quite different from preferences for physically different goods but whose availability is not contingent upon states at all. We feel differently when we know that some future events are uncertain. We do not feel the same as we do with physical goods which do not exhibit uncertain characteristics. There is no uncertainty about music downloads and text messages like there is regarding one’s health.

In fact, a theory about preferences on bundles that are contingent on states turns out to be a theory about attitudes towards risks, and how such attitudes can be characterized. For over half a century, the expected utility hypothesis has been the leading theory for such a characterization. In standard consumer demand theory, preferences for physical goods can be represented by a utility function, which offers a lot of computational convenience. The expected utility hypothesis is the analogue: it says that preferences for goods that are contingent on uncertain states can be represented by the expected value of a utility function.

We now proceed to explain the expected utility hypothesis.

\section*{1.3 The Expected Utility Hypothesis}

We develop this theory using a simple environment. The object of choice for the consumer is now a \textit{lottery}. We begin with a simple lottery: there are two prizes, $0 and $100, and the probability of winning the $100
prize is \( p \), where \( 0 \leq p \leq 1 \). The notation \((0, 100; 1 - p, p)\) denotes such a lottery; in this notation the prizes and their probabilities are listed correspondingly. The lottery \((0, 100; 0.5, 0.5)\) means that the consumer may win $0 or $100 with equal probabilities. The lottery \((0, 100; 0.3, 0.7)\) means that the consumer may win $100 with probability 0.7, and nothing with probability 0.3. One can think of there being two states: winning the low prize and winning the high prize, and these states may happen with probabilities \(1 - p\) and \(p\). Some lotteries are degenerate. A lottery of the form \((50, 50; 0.25, 0.75)\) says that one wins $50 with probability 0.25 or one wins $100 with probability 0.75, which translates to the certain outcome of winning $50. The expected utility hypothesis is about how we may represent consumers’ ranking over all lotteries under some assumptions.

Now consider all those lotteries with the two prizes of $0 and $100. A consumer should prefer those lotteries with higher probabilities of winning the higher prize. Comparing lotteries \((0, 100; 0.5, 0.5)\) and \((0, 100; 0.25, 0.75)\) seems straightforward. The first lottery allows a consumer to win $100 half the time, while the second lottery allows a consumer to win $100 three quarters of the time. Naturally, a consumer prefers the second lottery. One can understand this as simply asserting that more is better than less: a higher chance of winning the larger prize must be better than a lower chance. We call this a **monotonicity** assumption: for simple lotteries such as \((0, 100; 1 - p, p)\), a lottery with a higher value of \(p\) is preferred to a lottery with a lower value.

Next, consider three lotteries with three different probabilities of winning the $100 prize, say \(p_1\), \(p_2\), and \(p_3\). Suppose that \(p_1 > p_2\) and \(p_2 > p_3\), so that the first lottery is preferred to the second, and the second is preferred to the third. We can conclude that the first lottery is preferred to the third, simply because we can infer that \(p_1 > p_3\). Pairwise comparisons between lotteries can be extended. We call this a **transitivity** assumption: pairwise comparisons between lotteries are consistent.

Consider the lottery \((0, 100; 0.5, 0.5)\). If we take each prize, multiply it by its probability, then add these products together, we have the average prize, or the **expected prize**. In this case, it is \(0 \cdot 0.5 + 100 \cdot 0.5 = 50\). For the lottery \((0, 100; 1 - p, p)\), the expected prize is \(100p\). How would a consumer compare an arbitrary lottery such as \((0, 100; 1 - p, p)\) with a sure prize of $50 (the degenerate lottery \((50, 50; p, 1 - p)\)?)
Let us begin with some extreme cases. Suppose that $\pi = 0.99$. This is a lottery that almost surely will win $100 for the consumer, and it seems likely that a consumer will prefer $(0, 100; 0.01, 0.99)$ to the sure prize of $50$. Next, suppose that $p = 0.01$. This is a lottery that almost surely will win nothing, and it seems likely that a consumer will prefer $50 to the lottery $(0, 100; 0.99, 0.01)$. At some extreme, when $p$ is nearly 1, a consumer will prefer the lottery over $50; at the other extreme, when $p$ is nearly 0, a consumer will prefer the sure prize $50 over the lottery. Now, at some intermediate value of the winning probability, a consumer will be indifferent between the lottery $(0, 100; 1 - p, p)$ and the sure prize $50$. This value of the winning probability may be 0.3 for one consumer, 0.6 for another. For example, John will be equally happy between playing the lottery $(0, 100; 0.7, 0.3)$ and getting the sure prize $50, while Mary will be equally happy between $(0, 100, 0.4, 0.6)$ and $50$.

We assume that there is an intermediate probability $p$ such that a consumer is indifferent between the lottery $(0, 100; 1 - p, p)$ and a sure prize, such as $50$. This is called the continuity assumption. There is something important about this probability that makes a consumer indifferent between the prize $50 and the lottery $(0, 100; 1 - p, p)$. It signifies what the sure prize $50 is worth to a consumer, relative to the lottery with prizes 0 and $100. In fact, suppose that prizes give rise to utilities, much in the same way that bundles of goods give rise to utilities. A lottery of the form $(0, 100; 1 - p, p)$ gives rise to some utility. Suppose we use the following benchmarks. Let the utility of the lottery $(0, 100; 0, 1)$, the sure prize of $100, be set at 1, and the utility of the lottery $(0, 100; 1, 0)$, the sure prize of $0, be set at 0. As a convention, we have chosen a scale in which the utility of $100 is 1 while the utility of $0 is 0. We call this a utility function $U$, which maps a lottery or an income level to a number, the utility of the lottery or the income. We can write $U(100) = 1, and $U(0) = 0. There is actually nothing special about this convention. We can alternatively set $U(100)$ at 100, and $U(0)$ at 75. Any arbitrary numbers for $U(100)$ and $U(0)$ work fine as long as $U(100) > U(0)$.

Ultimately, we would like to associate a utility level to each wealth level. As a first step, we can establish our association of utility levels to lotteries. Extending these benchmarks, we can set the utility of the general lottery $(0, 100; 1 - p, p) at p. Certainly, this is consistent with our benchmarks of $U(100) = 1, and $U(0) = 0, because the lotteries $(0, 100; 0, 1)$ and $(0, 100; 1, 0)$ are respectively the sure prizes $100 and $0. As a matter of notation, we can write $U(0, 100; 1 - p, p) = p$, where it is understood that the more complicated argument
or input in the utility function denotes a lottery.

Now John is indifferent between \((0,100;0.7,0.3)\) and the sure prize $50. We have assigned the utility 0.3 to the lottery \((0,100;0.7,0.3)\). If we are interested in associating a utility value to a prize level, we can then set the utility of $50 at 0.3, so \(U(50) = 0.3\). The utility level 0.3, again, simply means that John is indifferent between the lottery \((0,100;0.7,0.3)\) and the sure prize $50.

We can consider other sure prizes, such as $40 and $60. Now suppose that John likes equally the lottery \((0,100;0.75,0.25)\) and the sure prize $40. We associate the utility of 0.25 to the prize $40, so \(U(40) = 0.25\). Similarly, if John likes equally the lottery \((0,100;0.5,0.5)\) and the sure prize $60, we set \(U(60) = 0.5\). By asking John repeatedly how much of the winning probability will be equivalent to any sure prize level between $0 and $100, we can map out John’s utility function, \(U(W)\) for all \(W\) between 0 and 100. The value of \(U(W)\) simply states the probability of winning $100 which yields the same satisfaction level as getting \(W\) for sure: John is indifferent between getting \(W\) for sure and playing the lottery \((0,100;1−p,p)\) with \(U(W) = p\).

What about more complicated lotteries? Suppose now we consider a richer lottery, with three possible prizes, $0, $50 and $100. We call this a compound lottery. Suppose that the probabilities of winning these three prizes are respectively 0.2, 0.3, and 0.5. We denote this compound lottery by \((0,50,100;0.2,0.3,0.5)\).

What are we to make of the intermediate prize $50? Previously, when there are only two prizes, the comparison is straightforward: the probability of winning the higher prize is what a consumer should consider. Now there is a medium prize to consider. How should a consumer think about the medium prize, versus the low and high prizes?

We can proceed in the following way. Instead of thinking about three prizes, we can instead think about only two prizes! Imagine that John actually wins the $50 prize. After he wins, of course the $0 and $100 prizes are no longer relevant. There is a catch though. The $50 prize is actually going to be replaced by another lottery. In other words, even though John thought that he could win this medium prize with probability 0.3, he was tricked. If he did get the medium prize, the lottery organizer would turn around and instead offer him another lottery. The lottery that substitutes for $50 turns out to be \((0,100;0.7,0.3)\), which is winning $100 with probability 0.3 and nothing with probability 0.7. Should John complain about
it? Perhaps he should not! Why? Recall that John is indifferent between $(0, 100; 0.7, 0.3)$ and the sure prize $50$, so why should he complain? He should get the same satisfaction from the lottery $(0, 100; 0.7, 0.3)$ and the medium prize $50$ after all!

Now if John anticipates this substitution, before he plays, he should really be thinking about the medium prize $50$ as the lottery $(0, 100; 0.7, 0.3)$. Winning the medium prize is like being given the chance of playing a new lottery $(0, 100; 0.7, 0.3)$. In other words, we have reduced John’s compound lottery which has three prizes ($0$, $50$, and $100$) into a simple lottery with only two prizes. This procedure we have just described is called the Reduction of Compound Lotteries.

John’s assessment of actually winning the $100$ prize should be $0.5 + 0.3 \cdot 0.3 = 0.59$. He has a $0.5$ probability of winning the $100$ outright. He has a $0.3$ probability of winning the $50$ prize, but then this translates to a $0.3$ probability of winning the $100$ prize. There is therefore an additional $0.3 \cdot 0.3 = 0.09$ probability of winning $100$. Figure 1 illustrates this reasoning. By the reduction of compound lottery, the compound lottery $(0, 50; 100; 0.2, 0.3, 0.5)$ is equivalent to a simple lottery $(0, 100; 0.41, 0.59)$.

The Expected Utility Hypothesis states that any compound lottery can be evaluated by a suitably defined expectation of the utility of possible prizes. In the compound lottery $(0, 50; 100; 0.2, 0.3, 0.5)$, the three utility levels are $U(0)$, $U(50)$ and $U(100)$. For John, we have $U(0) = 0$, $U(50) = 0.3$ and $U(100) = 1$. The expected utility is the sum of the corresponding products between the utility level and the probabilities:

\[
0.2 \cdot U(0) + 0.3 \cdot U(50) + 0.5 \cdot U(100).
\]

\[
= 0.2 \cdot 0 + 0.3 \cdot 0.3 + 0.5 \cdot 1 = 0.59
\]

This expected utility $0.59$ is how John should use for evaluating this compound lottery. By our construction, the expected utility of the compound lottery is the same as the utility of the simple lottery $(0, 100; 0.41, 0.59)$.

1.4 Some Examples

We work out some examples with utility functions. For ease of notation, we drop the dollar sign from now on. Let John’s utility function be $U(W) = \frac{1}{10} \sqrt{W}$. First, we observe that $U(0) = 0$, and $U(100) = 1$. Now, consider $W = 49$, so its utility is $U(49) = \frac{1}{10} \sqrt{49} = 0.7$. What does this utility level mean? To
understand this, consider a lottery \((0, 100; 0.3, 0.7)\). In this lottery, a prize of 100 is won with probability 0.7, and 0 otherwise. Applying the Expected Utility Hypothesis, we obtain the expected utility of the lottery:

\[
0.3 \times U(0) + 0.7 \times U(100) = 0.7 = U(49).
\]

The utility level \(U(49)\) in fact indicates that John is indifferent between the sure prize $49 and the lottery \((0, 100; 0.3, 0.7)\). It follows that John will prefer the lottery \((0, 100; 0.2, 0.8)\) to the sure prize 49, but the sure prize 49 to the lottery \((0, 100; 0.4, 0.6)\). This is the essence of the Expected Utility Hypothesis. A utility function in fact captures a consumer’s preferences on lotteries, and hence his attitude towards risks.

Let Mary’s utility function be \(U(W) = \frac{1}{100} W\). Consider again the lottery \((0, 100; 0.3, 0.7)\). We have seen that John is indifferent between having this lottery and the sure prize \(W = 49\). What about Mary? If Mary’s preferences are represented by the utility function, \(U(W) = \frac{1}{100} W\), her expected utility from the lottery is \(0.3U(0) + 0.7U(100) = 0.3 \times 0 + 0.7 \times \left(\frac{100}{100}\right) = 0.7\). Now the utility level 0.7 is the utility of sure wealth 70, so Mary is indifferent between having a sure prize of 70 and playing the lottery \((0, 100; 0.3, 0.7)\). It follows that Mary will prefer the lottery \((0, 100; 0.2, 0.8)\) to the sure prize 70, but the sure prize 70 to the lottery \((0, 100; 0.4, 0.6)\).

For the same lottery \((0, 100; 0.3, 0.7)\), John views it as equivalent to a sure prize of 49. To Mary, however, the lottery is equivalent to a sure prize of 70! Different utility functions describe how different people express their preferences towards lotteries. This is analogous to standard consumer demand theory, except that the object of choice is a lottery or uncertain incomes.

John’s and Mary’s utility functions are illustrated in Figure ???. There, the concave curve is John’s utility function \(U(W) = \frac{1}{10} \sqrt{W}\), while the straight line is Mary’s utility function \(U(W) = \frac{1}{100} W\). The point 70 on the horizontal axis is the weighted sum of the two prizes: \(70 = 0.3 \times 0 + 0.7 \times 100\); this is the expected value of the lottery. To John, the lottery yields an expected utility of \(0.3 \times \frac{1}{10} \sqrt{0} + 0.7 \times \frac{1}{10} \sqrt{100} = 0.7\). To Mary, the lottery yields the same expected utility, \(0.3 \times \frac{1}{100} \times 0 + 0.7 \times \frac{1}{100} \times 100 = 0.7\). However, for John, the income level that corresponds to a utility level of 0.7 is 49, while for Mary, that income level is 70.
1.5 Risk Aversion, Certainty Equivalent and Risk Premium

1.5.1 Expected Values

For any lottery we can calculate the average prize or its expected value. The expected value of the lottery 
\((0, 100; 0.3, 0.7)\) is \(0.3 \times 0 + 0.7 \times 100 = 70\), for the lottery \((0, 100; 0.2, 0.8)\), it is \(80\), and for the lottery 
\((0, 100; 0.4, 0.6)\), it is \(60\). We take each possible value of the lottery, multiply it with its probability, and then 
add them together. The expected value is simply the average outcome. These lotteries will never pay a prize 
equal to 60, 70, or 80. In the United States, the average number of children in a family is 2.2. Of course, 
there is never going to be any family with 2.2 children! But if we have a large number of families, some will 
have no children, some 1 child, some 2 children, etc. Now we take the total number of children and divide 
by the number of families, then we likely we get very close to 2.2.

Similarly, suppose we play the three lotteries a large number of times. On some plays, the outcome will 
be 0, while on others it will be 100. But if we add all prizes together and divide that by the total number 
of plays, then the average will get very close to each lottery’s expected value. The expected value describes 
a kind of tendency for the average outcome over a large number of plays.

The computation of expected values of compound lotteries is similar. For the lottery \((0, 50, 100; 0.2, 0.3, 0.5)\), 
the expected value is \(0.2 \times 0 + 0.3 \times 50 + 0.5 \times 100 = 65\). The simple lottery \((0, 100; 0.35, 0.65)\) has an ex-
pected value of \(0.35 \times 0 + 0.65 \times 100 = 65\), which is the same as the expected value of the compound lottery 
\((0, 50, 100; 0.2, 0.3, 0.5)\).

1.5.2 Certainty Equivalent, Risk Premium

We have seen that for a lottery such as \((0,100; 0.3, 0.7)\), a consumer will be indifferent between playing the 
lottery and winning a sure prize. This sure prize is called the certainty equivalent of the lottery. John, whose 
utility function is \(U(W) = \frac{1}{10} \sqrt{W}\), is indifferent between playing the lottery \((0,100; 0.3, 0.7)\) and the sure 
prize 49. For John, the certainty equivalent of the lottery \((0,100; 0.3, 0.7)\) is 49. Mary, whose utility function 
is \(U(W) = \frac{1}{100} W\), is indifferent between playing the same lottery \((0,100; 0.3, 0.7)\) and the sure prize 70. For 
Mary, the certainty equivalent of the lottery \((0,100; 0.3, 0.7)\) is 70.
Certainty Equivalent:

For a consumer with a given utility function, the certainty equivalent of a lottery is the sure prize at which the consumer is indifferent between playing the lottery and having the sure prize.

The concept of certainty equivalent is a succinct way to summarize how a consumer views a lottery. A lottery is like a sure prize. A lottery, which involves uncertain outcomes (either a prize of 0 or a prize of 100) is the same as a sure prize (the certainty equivalent). The certainty equivalent of a lottery depends on the utility function, which represents a consumer’s preferences towards uncertainty prizes. Certainty equivalent of a lottery is quite different from the expected value of a lottery. For the lottery (0, 100; 0.3, 0.7), John’s certainty equivalent is 49, and Mary’s is 70. However, the expected value of the lottery is always 70; this is simply a property of a lottery, independent of who actually is playing it. See Figure ?? again.

The concept of risk premium relates to how certainty equivalent compares to the expected value of the lottery. The lottery (0, 100; 0.3, 0.7) has an expected value of 70. Yet, John is indifferent between playing this lottery and having his certainty equivalent of 49. John of course is much better off if he can have the expected value of 70 as a sure prize, which gives him a utility of $\frac{1}{10}\sqrt{70} = 0.84$ rather than playing the lottery, which gives him an expected utility 0.7. The difference between the expected value of a lottery and the certainty equivalent of that lottery is called the risk premium. In John’s case, his risk premium is $70 - 49 = 21$.

Risk Premium:

For a consumer with a given utility function, the risk premium is the difference between the expected value of the lottery and the certainty equivalent of the lottery.

The expected value of a lottery can be thought of as a benchmark. Again, If John has the sure prize of 70, which is the expected value of the lottery, his utility is $\frac{1}{10}\sqrt{70} = 0.84$. Yet, if he is asked to play the same lottery, his utility is $\frac{1}{10}\sqrt{49}$ because 49 is his certainty equivalent. John’s income, if he is to play this lottery repeatedly will increase by 70—afterall, that is that meaning of the expected value, the average earning. The only difference is that the lottery yields uncertain outcomes for John. Sometimes, the prize
is 100, but sometimes it is 0. The lower value of the certainty equivalent therefore is a measure how much John dislikes the uncertainty due to the lottery. In order words, John is willing to slash up to 21 from the average earning of 70 to avoid the lottery.

Now if we consider Mary, a different kind of comparison emerges. Recall that for Mary, the certainty equivalent of the same lottery \((0, 100; 0.3, 0.7)\) is 70. The expected value of the lottery is also 70. Mary’s risk premium is just 0. Uncertainty does not reduce Mary’s utility, and Mary is unwilling to give up anything from the average earning of 70 to avoid the lottery.

It should be emphasized that the concepts of expected value of a lottery, the certainty equivalent and risk premium are interrelated. The expected value of a lottery is property of the lottery. It is not referenced with respect to a decision maker or a utility function. On the other hand, certainty equivalent is a notion defined with respect to a decision maker. In the above examples, John and Mary have different utility functions, and for the same lottery, each one has a different certainty equivalent. The risk premium is defined to be the difference between the expected value and the certainty equivalent, so it makes reference to a utility function.

### 1.5.3 Risk Aversion

The comparison between the expected value of a lottery and the certainty equivalent indicates an attitude towards uncertainty. Risk aversion is the idea that a consumer does not like risks. We have seen above that the comparison between the certainty equivalent of a lottery and the expected value tells us something about a consumer’s preferences toward the lottery.

Given a utility function, a consumer is said to be risk averse if his certainty equivalent for the lottery is less than the expected value of the lottery. Conversely, he is said to be risk loving if his certainty equivalent for the lottery is more than the expected value. Finally, he is said to be risk neutral if the certainty equivalent and the expected value are the same.

\[
\text{Risk Averse: Certainty Equivalent} < \text{Expected Value} \\
\text{Risk Loving: Certainty Equivalent} > \text{Expected Value}
\]
Risk Neutral: Certainty Equivalent = Expected Value

For John, the expected value of the lottery \((0, 100; 0.3, 0.7)\) is 70, but his certainty equivalent is only 49, so he falls into the risk averse category. Now for Mary, for the lottery \((0, 100; 0.3, 0.7)\), the expected value is the same as the certainty equivalent, so she falls into the risk neutral category.

Recall that the risk premium is the difference between the certainty equivalent and the expected value, so we can in turn classify risk attitude in the following way:

- Risk Averse: Risk Premium = Expected Value - Certainty Equivalent > 0
- Risk Loving: Risk Premium = Expected Value - Certainty Equivalent < 0
- Risk Neutral: Risk Premium = Expected Value - Certainty Equivalent < 0

1.6 Comparative Statics about Certainty Equivalents and Risk Premiums

We now make a number of observations concerning the concepts in this chapter. Certainty equivalent of a lottery summarizes how a consumer views a lottery. We have seen how differently John and Mary view the same lottery. This view may well depend on how much initial endowment a consumer has. Our discussion so far has assumed that John and Mary have no initial endowment. Let us now consider the effect of endowment.

Again, consider the lottery \((0, 100; 0.3, 0.7)\), but now suppose that John starts with an endowment of 100. Now if he wins the lottery, his income becomes 200, while if he does not, it remains at 100.

Being solely a property of the lottery, the expected value of the lottery remains at 70. What about his risk premium? We can now calculate it in the following way. When he plays the lottery, his income is 100 with probability 0.3 (not winning), or 200 with probability 0.7 (winning). Hence, his expected utility is

\[
0.3U(100) + 0.7U(200) = 0.3 \times \frac{\sqrt{100}}{10} + 0.7 \times \frac{\sqrt{200}}{10} = 1.29.
\]

An endowment of 100 obviously gives John more utility than before. Now the expected utility from the lottery, at an initial wealth of 100, becomes 1.29.
What is his certainty equivalent now? To calculate this, we consider a sure prize, to be added to his initial wealth of 100 so that the utility level becomes 1.29. Let $CE$ be this certainty equivalent, so the equation for it is

\[
U(100 + CE) = 1.29 \\
\frac{\sqrt{100 + CE}}{10} = 1.29 \\
100 + CE = 10 \cdot 1.29^2 \\
CE = 66.
\]

At this initial wealth of 100, John views this lottery differently than when his wealth is 0. Now the certainty equivalent becomes 66, higher than the 49 when he has zero wealth. When John is richer, the difference between the certainty equivalent and the expected value has become smaller.

As John’s wealth level as gone from 0 to 100, the certainty equivalent has increased from 49 to 66. Now the certainty equivalent is still below the expected value of 70, so John remains risk averse. Alternatively, we can say that his risk premium has decreased from 21 to 5. A smaller risk premium indicates that John has become less risk averse.

Certainty equivalent and risk premium generally depend on the consumer’s wealth level. As a consumer becomes richer, he may well tolerate risks better. For John, when he has no endowment, his risk premium is high, but with some income, it becomes lower. In fact, the calculations will show that as John’s wealth increases, the risk premium decreases, although it will never reach 0. An alternative way to describe this is that John’s risk aversion decreases with income.

In most circumstances, one expects that risk aversion does decrease with income. This is driven by the fact that as income becomes high, the fluctuation due to the lottery accounts for a smaller amount of the total income. For example, if John has a wealth level of 10,000, then a 100 win will be like a 1% increase in his income. However, for some utility functions risk aversion does not decrease with income, so it is not an implication of the Expected Utility Hypothesis. More generally, risk aversion may sometimes decrease and sometimes increase with wealth.
1.7 Demand for Insurance

A risk averse consumer is willing to pay a positive amount of money to get rid of the risk he is facing. This simply is another way of saying that a risk averse consumer has a positive risk premium. This demand for insurance is best illustrated by a very simple example. Take John’s utility function \( U(W) = \frac{1}{10} \sqrt{W} \). We already have seen that John is risk averse. Suppose now \( W = 200 \), so that John has an income or wealth at 200. Consider a lottery: \((-100, 100; 0.5, 0.5)\). Unlike lotteries that we have considered earlier, this lottery can actually mean a loss! John can win or lose 100 with equal probabilities!

Suppose that John has to play this lottery. What is his expected utility? We can calculate this as

\[
0.5U(200 - 100) + 0.5U(200 + 100),
\]

where the first term comes from losing 100, and the second term from winning 100. We can substitute for the utility function, and compute

\[
0.5 \left( \frac{1}{10} \sqrt{200 - 100} \right) + 0.5 \left( \frac{1}{10} \sqrt{200 + 100} \right)
\]

\[
= 0.5 \left( \frac{1}{10} \sqrt{100} \right) + 0.5 \left( \frac{1}{10} \sqrt{300} \right)
\]

\[
= 0.5[1] + 0.5[1.73]
\]

\[
= 1.37.
\]

If John does not play this lottery, and gets only his own endowment of 200, his utility is

\[
\frac{1}{10} \sqrt{200}
\]

\[
= 1.41.
\]

We can see that

\[
1.37 = 0.5U(200 - 100) + 0.5U(200 + 100) < U(200) = 1.41.
\]

John is worse off playing this lottery. The expected utility is 1.37 whereas the utility from his original income of 200 is 1.41.

We can understand how this deterioration comes about. Consider the increase of John’s income from 200 to 300, which corresponds to winning 100. The utility level of \( W = 200 \) is 1.41; the utility level of
\( W = 300 \) is 1.73. Therefore, \( U(300) - U(200) = 0.32 \). The utility level of \( W = 100 \) is 1. Therefore \( U(200) - U(100) = 0.41 \). By playing the lottery, John has the opportunity of increasing his wealth from 200 to 300, and this means a gain of utility from \( U(200) \) to \( U(300) \) or 0.32. However, John also may lose. In that case, his wealth decreases from 200 to 100, and this means a loss of utility from \( U(200) \) to \( U(100) \) or 0.41. Clearly, the loss is higher than the gain, and since both happen with equal probabilities, John has become worse off playing the lottery.

The lottery is fair: winning or losing 100 with equal probabilities, its expected value being zero. What the lottery does is that it introduces fluctuations in John’s income, which now can be either 100 or 300 but never 200 anymore. The increment in utility (0.32, from income increasing from 200 to 300) is smaller than the decrement in utility (0.41, from income decreasing from 200 to 100). Even though the level of income goes up or down by an equal amount (100), the utility level does NOT go up or down by the same amount. Figure ?? illustrates this asymmetry: for the either winning 100 or losing 100, the utility decreases more than it increases, and when the gain and the loss occur with equal probabilities, the expected utility has decreased.

An improvement can easily be implemented. This in fact illustrates how an insurance policy can improve John’s welfare. Suppose that at no cost to John, the lottery is taken away from him. So his income is always 200, and there is no more fluctuation of his income between 100 and 300. We have already seen that for John this is better than playing the lottery. In insurance policy is just like that. Suppose that an insurance company comes along, and offers to take the lottery from John, at absolutely no cost to him. John will gladly accept this policy.

Now imagine that a less favorable policy comes along. Suppose that an insurance company offers John a policy that guarantees his income to be 190. In other words, he pays a premium of 10. If he happens to win the lottery, the insurance company receives the 100, but if he loses, the insurance company assumes all the loss. For a premium of 10, John is insured against any income fluctuation. Will John take this offer? Well,
we can calculate John’s utility under this policy. It is

\[
\frac{1}{10}\sqrt{190} \quad = \quad 1.38.
\]

This expected utility is still higher than his expected utility of playing the lottery, which is 1.37, so John will accept this policy. Putting this in the language we have developed, we have shown that the 10 premium is lower than the risk premium, so John is better off avoiding the risk by buying this policy.

In the next chapter, we will fully develop the demand for insurance under different market structures.