How Much Would You Pay to Resolve Long Run Risk?

Larry G. Epstein     Emmanuel Farhi     Tomasz Strzalecki

July 8, 2014
Bansal (2007): Several key features of asset markets are puzzling. Among others, these include the level of equity premium, asset price volatility, and the large cross-sectional differences in average returns across equity portfolios such as value and growth portfolios. In bond and foreign exchange markets, the violations of the expectations hypothesis and the ensuing return predictability is quantitatively difficult to explain. What risks and investor concerns can provide a unified explanation for these asset market facts? One potential explanation of all these anomalies is provided by the long-run risks (LRR) model

Bansal and Yaron (2004) - **1600+ cites on Google Scholar

We highlight a feature of the LRR model that has been largely overlooked and that some may view as problematic
How do we judge success?

The puzzles are *quantitative*. Calibration is key

Mehra-Prescott lesson: Need not only to match moments, but to do so with parameter values that make sense, in terms of fitting other ‘evidence’ - both market-based and introspection

Attention is paid to elasticity of intertemporal substitution (EIS) and to the degree of relative risk aversion (RRA)

But *there is more to preference than EIS and RRA*

*Utility implies that the temporal resolution of risk matters and a quantitative assessment of how much it matters should be part of the calibration process*
Rules of the Game and Lexicon

Treat the representative agent as real
- there is no sensible aggregation theorem rationalizing a fictional agent

‘Introspection’ = intuitively plausible
You can imagine yourself and/or others behaving in this way
Epstein-Zin (‘CES’) Utility

Continuation utilities $\{U_t\}$ defined recursively at each node in an infinite horizon multistage lottery/tree

$$U_t = \left\{ (1 - \beta) c_t^\rho + \beta \left[ E_t(U_{t+1}^\alpha) \right]^{\rho/\alpha} \right\}^{1/\rho}, \quad \rho \neq 0$$

Interpretation: $RRA = 1 - \alpha \neq 1 - \rho = EIS^{-1}$ “separation”

In calibrations, attention is paid to evidence about $RRA$ & $EIS$
But $\rho$ and $\alpha$ model more than substitution & risk aversion
The temporal resolution of consumption risk matters at a psychic noninstrumental level
Early resolution preferred iff $RRA > EIS^{-1}$, which is the case in BY
Long Run Risk

\[
\begin{align*}
\log \frac{c_{t+1}}{c_t} &= m + x_t + \sigma_t W_{c,t+1} \\
x_{t+1} &= ax_t + \varphi \sigma_t W_{x,t+1} \\
\sigma_{t+1}^2 &= \sigma^2 + \nu (\sigma_t^2 - \sigma^2) + \sigma_w W_{w,t+1}
\end{align*}
\]

W’s are standard iid Gaussian innovations, \(0 < a < 1\)

\(x_t\) is a persistently varying predictable component of the drift in consumption growth. Small innovations to \(x_t\) are important (even if \(\varphi \ll 1\)) because they affect consumption for the indefinite future

Volatility of consumption growth is stochastic
Empirical importance emphasized by Bansal et al (2012), Beeler and Campbell (2012)
Why this consumption (endowment) process?

Arguably difficult to reject empirically, and "it works"

The key mechanism, as described by several of the authors in this literature, is that LR risks are not resolved until much later and are consequently treated differently than are current risks – the preference for early resolution is central

But is the differential treatment required to match asset returns data plausible?

To address this question, we translate the BY assumption into explicit quantitative behavioral terms. This is not difficult or deep, but it clarifies what is being assumed in order to match moments
Evidence?

Not aware of any *other* market evidence or of any evidence from psychology that helps with quantitative assessment. Therefore, we suggest thought experiment & introspection as a guide (treating representative agent as real)

Thought experiments re risk aversion: Kandel and Stambaugh (1990, 1991), Rabin (2000): “how much would you pay to avoid this hypothetical gamble?”

Here: “what fraction of your consumption stream would you give up in order for all risk to resolve next period (month)?”

Subjective, not tied to data, but much better than nothing!
Thought experiment

You are given the LRR consumption process. In particular, the riskiness of consumption resolves only gradually over time ($c_t$ and $x_t$ are realized only at time $t$). You are offered the option of having all risk resolved at time 1. The cost is that you would have to relinquish the fraction $\pi$ of both current consumption and of the consumption that is realized subsequently for every later period. What is the maximum value $\pi^*$ for which you would be willing to accept this offer? Call $\pi^*$ the timing premium

$$((1 - \pi^*) c, \text{early}) \sim (c, \text{late})$$

‘Compensating variation’ of the change to early resolution of the given risk
Timing premia for LRR

We take parameter values from BY (2004)

\[
\begin{array}{cccccc}
\alpha & \nu & \phi & \sigma & \sigma_w^2 & \beta \\
.9790 & .987 & .044 & .0078 & .23 \times 10^{-5} & .998 \\
\end{array}
\]

\[EIS = 1.5\]

Then

\[
\begin{array}{c|c|c}
\text{RRA} = & 7.5 & 10 \\
\pi^* = & 24\% & 31\% \\
\end{array}
\]
DISCUSSION

- Why pay a premium?
- Is introspection possible/useful?
- How related to total welfare cost of risk (Lucas, 1987)?
- What about a different endowment process: iid? disasters?
- What to do now??
Why pay a premium?

At issue is consumption risk, not income or return risk

Anxiety? But arguably would lead to violation of stationarity and thus is not a valid interpretation of recursive utility
To extent that introspection is derived in part from considerations of anxiety, stated timing premia exceed premia that are consistent with LRR

Hidden unmodeled planning problem (Kreps & Porteus, 1978): How to make this concrete/quantitative? Can it make plausible a timing premium of 25%?

Plausibility in theory vs in quantitative/empirical applications
Applied papers have accepted timing nonindifference uncritically
Is introspection possible/useful?

*Artificial/unfamiliar situation:* But starkness arguably helps introspection. One might feel “why should I give up 25% ... just to know earlier, when I can’t use that information?”

Arguably easier to introspect than if allowed to use the information, in which case self assessment of premium would involve introspection about all of substitutability, risk aversion and early resolution, as well as about all consumption processes in an expected budget set.

*Introspection is at best a matter of opinion:* inherently subjective. We are not arguing that a consensus is possible. But exercise may help *some* people understand the LRR model more fully.

*Alternative is to leave completely undisciplined*
How related to total cost of risk?

Are the timing premia large only because the cost of risk is large? Or are they large also in relative terms?

Lucas-style calculation: Consider the deterministic consumption process $\bar{c} = (\bar{c}_t)$ where $\bar{c}_t = E_0 c_t$, starting with $x_0 = 0, \sigma_0 = \sigma$

Compute $\bar{U}_0$ and risk premium $\bar{\pi}$

\[
(1 - \bar{\pi}) \bar{U}_0 = U_0
\]

Then

<table>
<thead>
<tr>
<th>RRA</th>
<th>7.5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^*$</td>
<td>24%</td>
<td>31%</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>48%</td>
<td>57%</td>
</tr>
</tbody>
</table>
IID endowment process

$$\beta = .998, \, \log\left(\frac{c_{t+1}}{c_t}\right) \sim N(.0015, .00007),$$ matching annual moments

<table>
<thead>
<tr>
<th>$RRA \setminus EIS$</th>
<th>1.5</th>
<th>1</th>
<th>.2</th>
<th>.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>9.6%</td>
<td>7.9%</td>
<td>1.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>7.5</td>
<td>6.9%</td>
<td>5.6%</td>
<td>0.4%</td>
<td>−0.5%</td>
</tr>
<tr>
<td>5</td>
<td>4.3%</td>
<td>3.4%</td>
<td>0.0%</td>
<td>−0.8%</td>
</tr>
<tr>
<td>2</td>
<td>1.2%</td>
<td>0.8%</td>
<td>−0.9%</td>
<td>−1.1%</td>
</tr>
<tr>
<td>1</td>
<td>1.0%</td>
<td>0.0%</td>
<td>−1.0%</td>
<td>−1.2%</td>
</tr>
</tbody>
</table>

Premia smaller than for LRR process: 9.6% vs 31%
Premium small for $(10, 0.2)$, which is still "far" from expected utility
Disasters

EZ utility but change in endowment process – small probability of large drop in consumption growth

Premia computed for annual frequency

Prob = probability of disaster

<table>
<thead>
<tr>
<th></th>
<th>EIS</th>
<th>RRA</th>
<th>Prob</th>
<th>$\pi^*$</th>
<th>$\bar{\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barro 2009</td>
<td>2</td>
<td>4</td>
<td>iid</td>
<td>18%</td>
<td>29%</td>
</tr>
<tr>
<td>Wachter 2013</td>
<td>1</td>
<td>3</td>
<td>persistent</td>
<td>42%</td>
<td>65%</td>
</tr>
</tbody>
</table>

As in LRR, persistence leads to large timing premia
CONCLUDE

“Don’t Bansal and Yaron show that the timing premium implicit in asset markets is large? Why bother with introspection?”

The same could be said about risk aversion in the equity premium context. Want a model that is consistent with a broad range of ‘facts.’ Our objective is to remind readers that this principle has not been applied as consistently as it should be.

Not saying that LRR should be abandoned—timing premium is only 1 dimension

*Our objective is to inject the subject into the discussion of the quantitative properties of LRR & related models*
How might we improve models?

(i) Different specification of preference parameters and endowment process?
(ii) A more general model of preference that yields a three-way separation?

Another competing model: Campbell-Cochrane (1999) – external habits

Plausibility of the specified habit process has been judged solely by how it
matches asset market data–an exception is Uhlig and Ljungqvist (2014)

A final word:

It may be difficult to find market evidence about the timing premium and about
the habit formation process. But these should not become free parameters