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Living with Risk

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Living with risk can lead to anticipatory feelings such as anxiety or hopefulness. Such feelings can affect the choice between lotteries that will be played out in the future—choice may be motivated not only by the (static) risks involved but also by the desire to reduce anxiety or to promote savouring. This paper provides a model of preference in a three-period setting that is axiomatic and includes a role for anticipatory feelings. It is shown that the model of preference can accommodate intuitive patterns of demand for information such as information seeking when a favourable outcome is very likely and information aversion when it is more likely that the outcome will be unfavourable. Behavioural meaning is given to statements such as "individual 1 is anxious" and "2 is more anxious than 1". Finally, the model is differentiated sharply from the classic model due to Kreps and Porteus.

1. INTRODUCTION

1.1. *Objectives and outline*

Living with risk can lead to anticipatory feelings such as anxiety that the eventual outcome will be bad, or hopefulness that it will be good.[1] Such feelings can affect the choice between lotteries that will be played out in the future—choice may be motivated not only by the (static) risks involved but also by the desire to reduce anxiety or to promote savouring. This paper provides a model of preference in a three-period setting that is axiomatic and includes a role for anticipatory feelings. It is intuitive that the latter affect the demand for information. It is shown that the model of preference can accommodate intuitive patterns of demand for information such as information seeking when a favourable outcome is very likely and information aversion when it is more likely that the outcome will be unfavourable. Behavioural meaning is given to statements such as "individual 1 is anxious" and "2 is more anxious than 1". Finally, the model is differentiated sharply from the classic model due to Kreps and Porteus (1978).

Consider risky prospects that pay off at a fixed time in the future. Standard expected utility maximizers care about the riskiness of prospects, but they are indifferent to when risk is resolved. Thus, it is not possible within the standard framework to distinguish between individuals who prefer early resolution, perhaps because they are anxious and cannot bear to live with risk, and those who prefer to delay resolution, perhaps because they wish to savour the prospect (or illusion) of a favourable outcome. Kreps and Porteus, henceforth KP, permit such a distinction. A key to their model is expansion of the domain of objects of choice from the set of lotteries (or lottery streams) to the domain of dynamic choice problems (p. 187), or suitably defined menus. These include, in particular, the set of multi-stage or temporal lotteries that distinguish between risks according to their temporal resolution.

To illustrate our model and its difference from KP's, consider a concrete example—we focus on savouring, though examples highlighting anxiety can also be constructed (Section 5.1). There are three time periods: 0, 1, and 2. Let p and q be two lotteries, where p represents a lotto ticket

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[1] See Caplin and Leahy (2001) for references to psychological research on anticipatory emotions.

that is resolved at time 2 and q is riskless. In choosing between them at time 0, the agent is influenced not only by the riskiness of p but also by the long time interval during which she can savour the possibility of winning a large prize. The latter may dominate and lead to the choice of p over q. (In the formal model, the choice of p amounts to the commitment to receiving p at time 1. Thus, while she may not physically hold the ticket at 0, she is certain then that the ticket will be hers at time 1, and thus, it is intuitive that she may already begin to savour the possibility of a good outcome.) The influence of savouring on choice is demonstrated by also considering the choice, still at time 0, between q and the hypothetical lottery $\widehat{p}$, where $\widehat{p}$ has the same probability distribution over outcomes as does p but differs from p in that it is completely resolved at time 1. Since savouring with regard to $\widehat{p}$ is limited to a shorter period, risk aversion may dominate and lead to the time 0 strict preference for q over $\widehat{p}$.

The preceding is also permitted in the KP model. The difference between the two models arises when considering the agent's choice at 0 between $\{p\}$ and the menu $\{p, q\}$; here, $\{p\}$ represents commitment to p as mentioned above, and $\{p, q\}$ represents the option of deferring to period 1 the decision between p and q. The ranking of $\{p\}$ vs. $\{p, q\}$ depends on what choice the agent expects to make out of $\{p, q\}$ at time 1 should she choose that menu at 0. But the comparison at 1 between p and q is completely analogous to that between $\widehat{p}$ and q at time 0—in particular, in both cases, the agent compares the deterministic prize q with a lotto ticket that is realized in the next period. Thus, if there is no reason for the difference in calendar dates to matter,[2] the agent should expect her time 1 ranking to agree with her time 0 preference for q over $\widehat{p}$. Therefore, she should expect the menu $\{p, q\}$ to lead ultimately to the choice of q. But, as described above, p is the better choice from the time 0 perspective, which attaches a large weight to savouring. Foreseeing all this, she would prefer to commit herself and choose $\{p\}$ over $\{p, q\}$. In contrast, commitment is not valuable in the KP model because they implicitly assume that the agent expects her time 1 ranking of p vs. q to coincide with her ranking at time 0, even though the importance of savouring is presumably different in those two situations.

Speaking more formally, we adopt the KP domain, specialized to our setting of three periods and terminal payoffs.[3] The common domain permits a sharp comparison of the two models. The key difference is that the KP model violates our central axiom, illustrated above and called Expected Stationarity. Its essence is the assumption that for lotteries that resolve within one period, the agent expects her future preference over such lotteries to be the same as her current one, that is the ranking of such "one-step-ahead lotteries" is expected to be independent of the calendar date. Given such an expectation of stationarity, we show that if the agent cares about the temporal resolution of risk, then she will be led to value commitment. Thus, anticipatory feelings imply a demand for commitment. Commitment is not valuable in the KP model only because of their implicit assumption, made explicit here, that risk preferences are expected to be *non-stationary* in the above sense. Since such non-stationarity is arguably unnatural in many settings, our analysis raises questions about the suitability of the KP model as a model of living with risk.[4]

Though simple from a technical point of view, we feel that our results are useful for three (related) reasons. First, they demonstrate that anticipatory feelings can be accommodated in an axiomatic choice-theoretic framework. Second, they cast new light on the seminal model of KP. Finally, the model suggests a positive answer to the question posed by Eliaz and Spiegler (2006):

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[2] Calendar dates may matter if savouring also applies to deterministic outcomes. Then, the choice between p and q at time 1 is different from the hypothetical choice problem at 0 because in the former, consumption occurs in the next period, while in the latter, consumption lies two periods into the future. We focus on modelling "living with risk" and thus assume that savouring and anxiety are limited to risky prospects.

[3] Henceforth, when referring to the KP model, the intention is to this specialization of their model.

[4] There are other reasons for caring about the temporal resolution of risk—one is that early resolution may facilitate planning—and KP are non-commital about which story they have in mind. Our objection is only to the anxiety, or living-with-risk, story.

"Can anticipatory feelings explain anomalous choices of information sources?" In particular, the intuitive pattern of information demand described in the paragraph above can be accommodated (Section 5).

1.2. *Sophistication*

The assumption of a forward-looking individual who anticipates her future tastes is key and may merit some justification. Such sophistication is plausibly associated with a rational, cool-headed individual. Thus, it may seem non-intuitive as an assumption about our agent at time 0, who, it might be argued, is driven by her "emotional side", the one concerned with anxiety or savouring. Does it make sense that this emotional self anticipate and try to constrain her future "narrowly rational" self, the one who is concerned primarily with the "actual" risk? Possibly not if we accept the preceding characterization. However, we do not accept it as the only way to think about anxiety and savouring.

Our view is that the length of time that you have to live with a risk is as much a part of the risk as is its mean payoff or its riskiness, say variance for concreteness.[5] Thus, the relevant choice is between bundles of the form (*mean*, *variance*, *time*). Caring about the time component is perfectly rational—time is "consumed" just as the mean and variance are. Moreover, its magnitude plausibly affects the trade-off between mean and variance—such non-separability is a way to think about the change in the ranking of risks induced by anticipatory feelings.

To reinforce the parallel with consumer theory, consider the following setup that mirrors ours but where the "commodities" involved are more standard—coffee, orange juice (OJ), and sugar. The individual must choose a beverage. She prefers coffee if there is sugar, and OJ otherwise. The supply of sugar is exogenous—a copious supply is available at time 0, but it disappears by time 1. Therefore, if the choice is made at time 0, it would be (suitably sweetened) coffee and would be consumed immediately. Suppose, however, that the individual can postpone the choice until time 1—formally, she could choose the menu {coffee, OJ} at time 0 rather than committing to, and actually drinking, the coffee. How would we model her evaluation of the menu? We suspect that most modellers would assume that she understands that sugar will disappear and anticipates (or knows) her dislike of unsweetened coffee. Thus, she would anticipate at time 0 that the non-singleton menu would lead to OJ rather than to her currently most preferred option.[6] Accordingly, we would predict a preference to commit by drinking the coffee at time 0. Such commitment arises not because the time 0 ranking of beverages is "correct" and the individual feels that her future ranking is "irrational". Each ranking is correct for its situation (sugar or no sugar) and wrong for the other situation. Commitment is desired simply because of the expectation of a change.

The translation into our setting is clear. Time is part of the consumption bundle just as is sugar, and a given lottery unresolved for a long period is different from one that resolves quickly, just as sweetened and unsweetened coffee are different commodities. Finally, and more pertinent to the issue of sophistication, the effects on tastes of the passage of time might be anticipated just as are the effects of a reduction in the availability of sugar. Admittedly, one can imagine situations where anxiety could be so intense as to preclude clear thinking—perhaps surrounding potentially life-relevant medical diagnoses; and even where there are no catastrophic outcomes possible, some individuals may suffer anxiety attacks that prevent them from thinking clearly. Nevertheless, our presumption is that for many risks and individuals, a "rational" model of

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[5] Skewness is also an important characteristic of a lottery when considering the demand for lotto tickets, as in the example above, or insurance, as in the example in Section 5.1.

[6] The juice would be consumed at time 1 rather than at time 0. However, assume zero time preference in this regard.

anxiety and savouring makes intuitive sense.[7] Besides, it seems sensible also on methodological grounds to explore how far the model of full rationality can take us.

1.3. *Related literature*

Grant, Kajii and Polak (2000, pp. 277–279) and Caplin and Leahy (2001) emphasize that dynamic inconsistency arises naturally in the presence of anticipation. The former suggest that modelling the intrinsic demand for information without assuming dynamic consistency, would be a fruitful avenue for further research. The latter propose a model. In it, preferences are defined not only over temporal lotteries (of the sort considered by KP) but also over "psychological lotteries". Elsewhere, Caplin and Leahy (2004) acknowledge that such an expanded domain poses challenges for gathering evidence and also suggest that surveys, physiological measures, and brain scans might serve as sources of evidence. Here, we follow the more traditional revealed preference approach in which economic choices alone constitute the relevant evidence.

Given "changing tastes", behaviour could be determined as in Strotz (1955), by assuming that the agent chooses the plan that is optimal among those that will be implemented. We adapt instead the alternative approach put forth by Gul and Pesendorfer in a series of papers, whereby the demand for commitment is modelled using a single preference, albeit over menus (or choice problems) rather than over lotteries.[8] A menu, which limits options for actions ex post, is selected *ex ante*. The individual's expectation is that later, when she decides on an action, she may be tempted to deviate from the choice that is optimal *ex ante*, that is if she were able to commit. In the general model of Gul and Pesendorfer (2001), henceforth GP, self-control might be exerted and the temptation resisted. A special case assumes no self-control (NSC) and this model is closest to ours. The GP analysis does not apply directly, however, because they assume the Independence axiom and we do not.[9] Neither is the later paper of Gul and Pesendorfer (2005) directly applicable. Here, the authors deal exclusively with temptation in the absence of self-control, but they restrict themselves to finite-choice problems (and thus Independence is not meaningful). We deal with continuous choice problems without Independence. However, we do not claim any technical novelty. In particular, the axiomatics are simple in our setting because the domain (including *two-stage* lotteries) and our specific story of why temptation arises allow us to express the "temptation ranking" explicitly in terms of the given preference over choice problems.

It may be worthwhile, in concluding this introduction, to elaborate on the connection to the literature on "non-reduction" of multi-stage risks. We have already mentioned the seminal paper by KP; other examples include Segal (1990) and Epstein and Zin (1989). These models differ in details from one another and from the present model—for example, for risks that resolve within one period (one-step-ahead risks), KP assume vNM preferences, Segal sometimes assumes rank-dependent expected utility theory, and Epstein and Zin as well as this paper's model permit general preferences satisfying only some monotonicity and technical conditions. However, these differences are not important here. The crucial and common feature of the other models is that they rule out a demand for commitment.[10] Formally, our contribution is to permit commitment

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[7] Also in the beverage example, one can imagine that some people may be incapable of clear forward-looking thinking before having had their cup of coffee.

[8] Gul and Pesendorfer (2005) describe advantages of their approach, which also apply to our setting.

[9] We do not assume Independence because (i) it is not necessary, the axiomatics are simple even without it, and (ii) it precludes some intuitive patterns for the demand for information (Section 5.1).

[10] This is explicit in the KP model since their agents rank dynamic choice problems, which are menus, and any menu is valued by its best element. It is also explicit in the Epstein–Zin model in the way in which they apply preferences to consumption–savings portfolio choice problems—smaller menus, or budget sets in their setting, can never be strictly preferable according to the induced indirect utility, and it is clear that Segal has in mind that menus are valued by their best alternatives, thus ruling out a value for commitment.

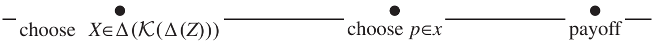
to be valuable without imposing reduction (thus letting temporal resolution matter), and to do so in an axiomatic framework. Our motivation is the intuition, described above, that anticipatory feelings lead to a value for commitment. To express the desire for commitment requires that we consider a domain for preference that includes menus. Indeed, if we observe only the ranking of multi-stage lotteries, as opposed to non-singleton menus, then our model is indistinguishable from those cited (apart from the noted minor differences). Put another way, it is *illegitimate* to interpret a preference over multi-stage lotteries alone as reflecting anticipatory feelings (of a self-aware individual)—one must also observe its extension to the ranking of menus. Naturally, this is not to say that the demand for commitment always justifies an interpretation in terms of anticipatory feelings—there are, of course, many reasons one may wish to commit. Our axioms, specifically Expected Stationarity, describe the pattern of commitment corresponding to anticipation.

2. THE MODEL

2.1. *Random menus*

There are three periods, $t = 0, 1, 2$. Let Z denote a space of outcomes. We assume that Z is compact metric and connected. Elements of $\mathcal{K}(\Delta(Z))$ are called *menus*.[11] Objects of choice at time 0 are lotteries over menus, or *random menus*. Thus, preference $\succeq$ is defined on $\Delta(\mathcal{K}(\Delta(Z)))$.

For interpretation, see the time line below. At $t = 0$, the agent chooses a random menu X. At $t = 1^-$, X delivers a menu $x \in \mathcal{K}(\Delta(Z))$. At $t = 1$, she chooses $p \in x$, and finally, all risk is resolved and an outcome is realized at $t = 2$. Note that the time line is intended as a description of the agent's perception when she evaluates random menus at time 0. Thus, the time 1 choice of p from x is her time 0 expectation of what she will do if facing the menu x.



Think of any random menu X as modelling a physical action undertaken at time 0. Such an action determines, along with stochastic factors described by the probabilities prescribed by X, a set of options for further action at time 1; the latter are modelled by lotteries over Z, the set of outcomes or payoffs. A random menu X that has support on singleton menus leaves room for only trivial choices at time 1 and commits the agent to a two-stage lottery. In fact, the set $\Delta(\Delta(Z))$ of two-stage lotteries can be identified with the set of elements in $\Delta(\mathcal{K}(\Delta(Z)))$ that provide commitment.

The choice of the set of random menus as our domain can be "rationalized" as follows:[12] The proper domain should include both the set $\Delta(\Delta(Z))$ of two-stage lotteries, in order to address the attitude towards the temporal resolution of risk, and also the set of menus $\mathcal{K}(\Delta(Z))$, in order to model the demand for commitment. Indeed, our central axioms and the principal content of our model concern only preference restricted to their union $\mathcal{D}$,

$$\mathcal{D} = \Delta(\Delta(Z)) \cup \mathcal{K}(\Delta(Z)). \tag{2.1}$$

We could have adopted $\mathcal{D}$ as the domain of preference. We chose instead to adopt the larger domain $\Delta(\mathcal{K}(\Delta(Z)))$ because (i) it is unifying and more elegant; (ii) preference can be extended

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[11] For any compact metric space Y, $\Delta(Y)$ denotes the space of Borel probability measures endowed with the weak convergence topology, and $\mathcal{K}(Y)$ denotes the space of compact subsets of Y endowed with the Hausdorff metric. Both $\Delta(Y)$ and $\mathcal{K}(Y)$ are compact metric. Finally, δ_y is the probability measure on Y that assigns probability 1 to y.

[12] A different domain is called for if one wishes to admit subjective beliefs. For that purpose, one could take as domain $\Delta(\mathcal{K}(\mathcal{H}))$, where $\mathcal{H}$ is the set of Anscombe-Aumann acts over a state space S.

uniquely from $\mathcal{D}$ to $\Delta(\mathcal{K}(\Delta(Z)))$ under relatively mild assumptions (as explained below); (iii) KP also use $\Delta(\mathcal{K}(\Delta(Z)))$ as their domain (specializing their model to our simpler setting of three periods and terminal payoffs), and having a common domain facilitates comparison of the two models, which is a principal objective; and (iv) the larger domain broadens the range of applicability of the model to include the (arguably typical) case where the set of options available at time 1 is not entirely within the agent's control but depends also on stochastic factors.

The adoption of a three-period horizon is not innocuous. It is well known that Strotz-like representations may not be well defined, given longer horizons (see, *e.g.* Peleg and Yaari, 1973). To accommodate an arbitrary finite horizon, Gul and Pesendorfer (2005) adopt two alternative strategies. In one, they limit the agent to finite-choice problems. In the second, they show in a setting with infinitely many choices that Strotz-like behaviour can be approximated arbitrarily well by a well-defined representation. We suspect that both strategies could be adapted to our setting, but we have chosen instead to focus on the simplest framework that permits modelling the notion of living with risk.

2.2. *Axioms*

We adopt several axioms for the binary relation $\succeq$ on $\Delta(\mathcal{K}(\Delta(Z)))$.

Axiom 1 (Order). $\succeq$ *is complete and transitive.*

Identify $\mathcal{K}(\Delta(Z))$ with a subset of $\Delta(\mathcal{K}(\Delta(Z)))$, where x is identified with δ_x. Then, $\succeq$ induces a ranking of menus, also denoted by $\succeq$. Thus, we often write $x' \succeq x$ rather than $\delta_{x'} \succeq \delta_x$.

In the standard model, a menu is as good as the best alternative that it contains, a property that is captured by the following axiom.

Strategic Rationality (SR). For all menus x and y, $x \succeq y \Longrightarrow x \sim x \cup y$.

SR is not intuitive in our setting, as illustrated by the example in the Introduction. To see the intuition in slightly more general terms, consider the agent at time 0 evaluating the menu $x \subset \Delta(Z)$ from which a choice p is to be made at time 1. Her *ex ante* view of these lotteries includes not only the risk associated with each p but also the fact that she will have to live with this risk for two periods—anticipatory feelings or anxiety affects her evaluation of each lottery and therefore also of x. But these are less relevant at time 1, and thus, she may view lotteries differently then. Being forward looking, she foresees this consequence of the passage of time when evaluating x, or when choosing between any two menus. As a result, she may value commitment and thus violate SR.

For example, suppose that $\{p\} \succeq \{q\}$, reflecting the fact that at 0, when she must live with risk for two periods, she would prefer to commit to p rather than to q. SR would require that $\{p, q\} \sim \{p\}$. This is possible here if p is preferable also at 1. But suppose that at 1, when savouring and anxiety are less important, that q is more attractive. Then, she will choose q if it is feasible, that is if $\{p, q\}$ is chosen at 0. Thus, both $\{p, q\}$ and $\{q\}$ lead to the *ex post* choice of q. All this is foreseen. Therefore, $\{p, q\} \sim \{q\}$. This is the intuition for the following weakening of SR.

Axiom 2 (NSC). *For all x and y,*

$$x \cup y \sim x \text{ or } x \cup y \sim y. \tag{2.2}$$

The axiom strengthens GP's central axiom Set-Betweenness so as to express a lack of self-control. In our case, if at time 1, when anticipatory feelings are not as important as they were *ex ante*, the agent is tempted to choose a lottery that was not optimal *ex ante* under commitment, there is no reason for her to resist. That is, she does not exert self-control in the face of such temptations.

According to NSC, for every pair of menus with $x \succ y$, either there is no temptation ($x \sim x \cup y$) or *ex post* choice is from the tempting menu ($x \cup y \sim y$). We now go further and specify circumstances when each case obtains. Consider lotteries over Z that are resolved at time 2. The noted circumstances centre on how such lotteries are evaluated from the perspectives of times 0 and 1. Usually, it is assumed that the ranking of lotteries that are resolved and paid at a fixed time T is the same regardless of when this ranking is done. This is decidedly not the case here—anticipatory feelings depend on the temporal distance from T; in the present three-period setting, they are presumably more important at time 0. Thus, we consider both perspectives $t = 0$ and $t = 1$ explicitly.

Consider the ranking of lotteries $\Delta(Z)$ at time 0 when the agent can commit. Such rankings take the form $\delta_{\{p'\}} \succeq \delta_{\{p\}}$, or given the notational convention introduced above, $\{p'\} \succeq \{p\}$. Since the above-mentioned lotteries are not resolved until time 2, they constitute *delayed risks* from the perspective of time 0. To emphasize this, we introduce special notation, and we define the order $\succeq_{\text{del}}$ on $\Delta(Z)$ by:

$$p' \succeq_{\text{del}} p \text{ iff } \{p'\} \succeq \{p\}.$$

Given any lottery over Z, we can imagine it alternatively playing out earlier, at time 1. Thus, for any lottery p in $\Delta(Z)$ having finite support, $p = \Sigma_z p(z)\delta_z$, define the random menu $X_p \in \Delta(\Delta(Z))$ by:

$$X_p = \Sigma_z p(z)\delta_{\{\delta_z\}}.$$

Thus, X_p yields the terminal payoff z with probability $p(z)$, just as does p, but for X_p, the outcome will be known at time 1.[13] Therefore, from the perspective of time 0, any such X_p constitutes an *immediate risk*. More generally, for any $p \in \Delta(Z)$, the immediate risk corresponding to p is the two-stage lottery $X_p \in \Delta(\Delta(Z))$ defined by:

$$X_p(B) = p(e^{-1}(B)),$$

for any measurable $B \subset \Delta(Z)$, where $e : Z \to \Delta(Z)$ is the natural embedding, $e(z) = \delta_z$. We introduce notation also for the ranking of immediate risks: Let $\succeq_{\text{imm}}$ be the ranking on $\Delta(Z)$ be defined by:

$$p' \succeq_{\text{imm}} p \text{ if } X_{p'} \succeq X_p.$$

Then, $p' \succeq_{\text{imm}} p$ indicates the time 0 preference for the risk or lottery p' over p when these are to be resolved next period, while $p' \succeq_{\text{del}} p$ indicates the same ranking when the two lotteries are to be resolved only at time 2.

Next consider the perspective of time 1. Though we are given only the time 0 preference $\succeq$, it suggests a time 1 perspective as we now show. At the intermediate time, objects of choice are lotteries that are resolved one period later, that is risks that are immediate from the time 1 perspective. At time 0, immediate risks are ranked via $\succeq_{\text{imm}}$. It follows that in a "stationary" environment, where the calendar date alone is not important, the time 1 ranking of lotteries should also be given by $\succeq_{\text{imm}}$. Suppose that the agent foresees these time 1 preferences. Then, she foresees choosing lotteries out of menus so as to maximize $\succeq_{\text{imm}}$. But at time 0, those lotteries constitute delayed risks and thus are ranked according to $\succeq_{\text{del}}$. Thus, where these orders disagree, she will prefer to limit options for future choice. Specifically, we adopt the following.

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[13] In the introductory example, X_p was denoted by $\widehat{p}$.

Axiom 3 (Expected Stationarity). *For all lotteries* p' *and* p *in* $\Delta(Z)$, $\{p'\} \succ \{p', p\}$ *if and only if* $p' \succ_{\text{del}} p$ *and* $p' \prec_{\text{imm}} p$.

Above, we gave intuition for the "if" part of the axiom. The converse ensures that the difference between $\succ_{\text{del}}$ and $\succ_{\text{imm}}$, and hence the effects of differing temporal distance from the time of resolution, are the *only* reason for commitment. (There is an implicit tie-breaking rule: When $p' \sim_{\text{imm}} p$, she assumes that she will make the choice that is best according to $\succeq_{\text{del}}$.) In its absence, the remainder of the axiom ("if") could, for example be satisfied vacuously if $\succ_{\text{del}}$ and $\succ_{\text{imm}}$ are identical, yet commitment could be valuable for reasons having nothing to do with savouring or anxiety. (See the example labelled "time-varying risk aversion" in Section 4.)

We offer three more remarks on the axiom. First, though its interpretation refers to expectations about future behaviour, the axiom is exclusively an assumption about the time 0 ranking of random menus. Second, note that the demand for commitment expressed in Expected Stationarity could be either due to anxiety or due to savouring. Finally, if our model is truly about living with risk, then there should not be any demand for commitment when the prospects involved are deterministic; in other words, it should be that $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ agree on Z. But that is easily seen to be true:

$$\delta_{z'} \succeq_{\text{imm}} \delta_z \Longleftrightarrow \delta_{\{\delta_{z'}\}} \succeq \delta_{\{\delta_z\}} \Longleftrightarrow \delta_{z'} \succeq_{\text{del}} \delta_z. \tag{2.3}$$

Some form of continuity is needed. GP observe that continuity of preference (closed weakly better than and weakly worse than sets) is not to be expected in a model of temptation without self-control, and they use a weaker form of continuity (axioms 2a to 2c). The following adapts their axioms to our setting.

Axiom 4 (Limited Continuity).

(a) *Upper semi-continuity: The sets* $\{y \in \mathcal{K}(\Delta(Z)) : y \succeq x\}$ *are closed.*
(b) *Lower singleton continuity: The sets* $\{p' \in \Delta(Z) : p' \preceq_{\text{del}} p\}$ *and* $\{p' \in \Delta(Z) : p' \preceq_{\text{imm}} p\}$ *are closed.*
(c) *For every* $x \in \mathcal{K}(\Delta(Z))$, *there exists* $p \in \Delta(Z)$ *such that* $x \sim \{p\}$.

Parts (a) and (b) imply that both $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ are continuous. We use condition (c) to prove that $\succeq$ has a utility function representing it on $\Delta(\mathcal{K}(\Delta(Z)))$, which is otherwise not guaranteed in the absence of continuity. (GP exploit instead the Independence axiom to prove existence of a representation.)

Let Y be any (compact metric) space and $\gtrapprox$ an order on $\Delta(Y)$. Say that $\gtrapprox$ is *FSD-increasing* if, for all lotteries p' and p in $\Delta(Y)$, $p' \gtrapprox p$ whenever:

$$p'(\{y : \delta_y \gtrapprox \delta_{y^*}\}) \geq p(\{y : \delta_y \gtrapprox \delta_{y^*}\}) \quad \text{for every } y^* \text{ in } Y, \tag{2.4}$$

that is if, for every y^*, the set of outcomes better than y^* according to $\gtrapprox$ has larger probability under p' than under p; refer also to any representing utility function as FSD-increasing. If equation (2.4) is satisfied, write $p' \gtrapprox^{\text{FSD}} p$, which is to be read as "p' first order stochastically dominates p with respect to the order on Y induced by $\gtrapprox$". The preceding applies in particular to $\succeq$, an order on $\Delta(\mathcal{K}(\Delta(Z)))$, and to $\succeq_{\text{imm}}$ and $\succeq_{\text{del}}$, both of which are orders on $\Delta(Z)$. In all these cases, the indicated sets $\{y : \delta_y \gtrapprox \delta_{y^*}\}$ are closed, hence measurable, by Limited Continuity.

The assumption that preference on a space of lotteries is FSD-increasing is common and is not especially problematic for our setting. Therefore, we assume the following.

Axiom 5 (Monotonicity). *Both $\succeq$ and $\succeq_{\text{del}}$ are FSD-increasing.*

The following is an important implication of the assumption that $\succeq$ is FSD-increasing: Two random menus X' and X must be indifferent if they induce the same distribution, that is if there is equality in the appropriate form of (2.4) for every menu y^* in $\mathcal{K}(\Delta(Z))$. If we denote the latter condition by $X' \approx^{\text{FSD}} X$, then this implication can be written in the form:

$$X' \approx^{\text{FSD}} X \implies X' \sim X. \tag{2.5}$$

Finally, note that, given the other axioms, if $\succeq$ is FSD-increasing, then so is $\succeq_{\text{imm}}$.[14] However, that $\succeq_{\text{del}}$ is FSD-increasing is not implied, and the axiom imposes this requirement separately.

3. UTILITY

The two primitive components of the functional form are utility functions $U, V : \Delta(Z) \rightarrow \mathbb{R}$; they will represent $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ respectively. We assume that they are continuous, FSD-increasing, and are ordinally equivalent on Z. Refer to any pair of utility functions (or corresponding orders) satisfying these properties as *compatible*. Then, it is without loss of generality (wlog), by taking a monotonic transformation of U or V, to assume that:

$$V(\delta_z) = U(\delta_z) \text{ for every } z \text{ in } Z. \tag{3.1}$$

(To see why, ordinal equivalence on Z implies that $V(\delta_z) = \phi(U(\delta_z))$ for some strictly increasing and continuous

$$\phi : U(Z) \equiv \{U(\delta_z) : z \in Z\} \rightarrow \mathbb{R}.$$

Since Z is connected, $U(Z) = U(\Delta(Z))$, that is for every p, there exists z such that $U(p) = U(\delta_z)$. Therefore, ϕ is strictly increasing and continuous on $U(\Delta(Z))$, and $\phi(U)$ is ordinally equivalent to U on $\Delta(Z)$. Then, equation (3.1) is satisfied if we use $\phi(U)$ in place of U.)

To describe how U and V determine a utility function on the entire domain $\Delta(\mathcal{K}(\Delta(Z)))$, we proceed in stages. Since the formulae for finite-support lotteries are more transparent, we define utility first for $\Delta_s(\mathcal{K}(\Delta(Z)))$, the set of *simple* random menus. We do this by describing first how utility is defined on menus ($\mathcal{K}(\Delta(Z))$); then how it is defined on the set $\Delta_s(\Delta(Z))$ of (simple) two-stage lotteries; and finally, how it is extended to all simple random menus. Finally, we extend the definition of utility to all random menus.

Consider the agent at time 0 evaluating a menu $x \subset \Delta(Z)$ from which a choice p is to be made at time 1. Think of U as describing the time 0 valuation of lotteries to be played out beginning at time 1 (delayed risks), and suppose that she expects V to describe risk preferences at time 1. Suppose further that the agent anticipates that she will not exert self-control at time 1. Therefore, she expects the time 1 choice out of any menu x to maximize V; maximization of U enters only when there is indifference according to V. This leads to the Strotz-like utility for any menu given by:

$$\mathcal{U}(x) = \max\left\{U(p) : p \in \arg\max_{p' \in x} V(p')\right\}. \tag{3.2}$$

In particular,

$$\mathcal{U}(\{p\}) = U(p), \tag{3.3}$$

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[14] Given a menu x^*, Limited Continuity and connectedness of Z imply that there exists z^* in Z such that $\{\delta_{z^*}\} \sim x^*$. Then, $\{z : \{\delta_z\} \succeq x^*\} = \{z : \{\delta_z\} \succeq \{\delta_{z^*}\}\}$. It follows, using also equation (2.3), that $p' \succeq^{\text{FSD}}_{\text{imm}} p \implies X_{p'} \succeq^{\text{FSD}} X_p \implies X_{p'} \succeq X_p \implies p' \succeq_{\text{imm}} p$.

so that $U(\cdot)$ ranks delayed risks and hence represents $\succeq_{\text{del}}$. Note also that SR is satisfied if and only if U and V describe the same risk preferences (*i.e.* they are ordinally equivalent on $\Delta(Z)$).

Next, define utility on $\Delta_s(\Delta(Z))$. Let $X = \Sigma_p X(\{p\})\delta_{\{p\}}$ be a two-stage lottery. Since X provides perfect commitment, its evaluation is based on the time 0 perspective alone—there is no conflict with later preferences and thus no reason to violate recursivity. Therefore, utility is computed by backward induction: For each p that is realized at the first stage, replace it by a certainty equivalent $z_p \in Z$. In this way, X is transformed into the single-stage lottery $\widehat{X} = \Sigma_p X(\{p\})\delta_{\{\delta_{z_p}\}}$, which is assigned a suitable utility level. The question is how to compute certainty equivalents at the second stage and utility levels at the first stage. The function U is used to compute certainty equivalents, that is z_p is defined as any outcome in Z satisfying:

$$U(p) = U(\delta_{z_p}). \tag{3.4}$$

(There exists such a z_p because U is continuous and Z is connected.) We use U because each p is a delayed risk (it is resolved only at time 2) and because, as just shown, U gives the utility of delayed risks. On the other hand, the single-stage lottery $\widehat{X}$ constructed above has all risk resolved by time 1—at that point, the agent will receive some δ_{z_p}, and thus, she will be certain that z_p will be forthcoming at time 2. The utility function V is used to evaluate immediate risks. Putting the two steps together yields the following expression for the utility of X:

$$\mathcal{U}(\Sigma_p X(\{p\})\delta_{\{p\}}) = V\left(\Sigma_p X(\{p\})\delta_{z_p}\right), \tag{3.5}$$

where z_p is *any* solution to equation (3.4).[15]

The preceding expression applies in particular to a two-stage lottery $X = \Sigma_p X(\{p\})\delta_{\{p\}}$ that is an immediate risk. Then, each p is degenerate, $X = \Sigma_z X(z)\delta_{\{\delta_z\}}$ and taking z to be a certainty equivalent for δ_z:

$$\mathcal{U}(X) = \mathcal{U}\left(\Sigma_z X(z)\delta_{\{\delta_z\}}\right) = V(\Sigma_z X(z)\delta_z), \tag{3.6}$$

that is V represents $\succeq_{\text{imm}}$.

On the surface, there may appear to be a contradiction between the way we arrived at equation (3.2) vs. the way in which we implemented the recursive calculation (3.5). In the context of the former, we referred to V as describing expected risk preferences at time 1, while in the latter, V was used to evaluate an immediate risk from the perspective of time 0. These dual roles for V are perfectly consistent and reflect our axiom Expected Stationarity—the expectation that one-step-ahead risks will be evaluated in the same way regardless of the calendar date.

Finally, with regard to (simple) two-stage lotteries, there is indifference to the temporal resolution of risk if and only if U and V are ordinally equivalent on $\Delta(Z)$, which, in turn, is equivalent to SR.

Thus far, we have defined utility $\mathcal{U}$ on $\Delta_s(\Delta(Z)) \cup \mathcal{K}(\Delta(Z))$. Next $\mathcal{U}$ can be extended uniquely to $\Delta_s(\mathcal{K}(\Delta(Z)))$ in such a way as to be FSD-increasing. The extension is defined as follows. For any menu x, let $p_x \in \Delta(Z)$ be any lottery such that $\mathcal{U}(x) = \mathcal{U}(\{p_x\}) = U(p_x)$. Then, $\Sigma_x X(x)\delta_{\{p_x\}}$ is a two-stage lottery, whose utility has been defined, and $\Sigma_x X(x)\delta_{\{p_x\}} \approx^{\text{FSD}} X$. Thus, in order for the extension of $\mathcal{U}$ to be FSD-increasing, it must be that it satisfy:

$$\mathcal{U}(\Sigma_x X(x)\delta_x) = \mathcal{U}\left(\Sigma_x X(x)\delta_{\{p_x\}}\right). \tag{3.7}$$

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[15] We show in the theorem below, using equation (3.1), that utility is well defined on $\Delta_s(\Delta(Z))$, that is (i) the right side of equation (3.5) is invariant to the choice of z_ps and (ii) the utility values defined by equations (3.5) and (3.2) agree on the intersection $\Delta_s(\Delta(Z)) \cap \mathcal{K}(\Delta(Z))$.

Utility is well defined—the R.H.S. is independent of the particular choice of p_x's because $\mathcal{U}$, as constructed above, is FSD-increasing on $\Delta_s(\Delta(Z))$ (recall equation (2.5)).

Example (Linear Model). Let U and V be continuous and linear, and denote by u the vNM index of U. To ensure ordinal equivalence on Z (and the normalization (3.1)), let:[16]

$$V(p) = \Theta^{-1}(E_p\Theta(u)),$$

for some $\Theta : U(\Delta(Z)) \to \mathbb{R}$ strictly increasing and continuous. Then, U and V are compatible (any linear utility function is FSD-increasing).

The utility of a (non-random) menu x is:

$$\mathcal{U}^{\text{lin}}(x) = \max\left\{E_p u : p \in \arg\max_{p' \in x} E_{p'}\Theta(u)\right\}.$$

Evidently, preference is strategically rational if and only if Θ is linear (U and V represent the same risk preferences). The utility of any two-stage lottery $X \in \Delta_s(\Delta(Z))$ is:

$$\mathcal{U}^{\text{lin}}(X) = \Theta^{-1}(\Sigma_x X(\{p\})\Theta(E_p u)).$$

There is indifference to the temporal resolution of risk if and only if Θ is linear, thus tying together violations of SR and the non-reduction of two-stage lotteries.

The unifying expression that describes the utility of any random menu $X \in \Delta_s(\mathcal{K}(\Delta(Z)))$ is:

$$\Theta(\mathcal{U}^{\text{lin}}(X)) = \Sigma_x X(x)\Theta\left(\max\left\{E_p u : p \in \arg\max_{p' \in x} E_{p'}\Theta(u)\right\}\right).$$

This completes the example.

The preceding example has an obvious extension to non-simple random menus. The same is true of our general model as we now describe. Specifically, we extend the utility specifications (3.5) and (3.7); the specification (3.2) is unaltered and defines utility $\mathcal{U}$ on $\mathcal{K}(\Delta(Z))$.

Let $\theta : \Delta(Z) \to Z$ be any measurable map such that $U(p) = U(\delta_{\theta(p)})$ for all p.[17] Then, any $X \in \Delta(\Delta(Z))$ induces the measure $X \circ \theta^{-1}$ on Z defined in the usual way by:[18]

$$(X \circ \theta^{-1})(B) = X(\theta^{-1}(B)), \;\; B \subset Z \text{ measurable.}$$

Define $\mathcal{U}$ on $\Delta(\Delta(Z))$ by:

$$\mathcal{U}(X) = V(X \circ \theta^{-1}). \tag{3.8}$$

For the generalization of equation (3.7), let $\phi : \mathcal{K}(\Delta(Z)) \to \Delta(Z)$ be any measurable map such that $\mathcal{U}(x) = U(\delta_{\phi(x)})$. Then, $X \in \Delta(\mathcal{K}(\Delta(Z)))$ implies that $X \circ \phi^{-1} \in \Delta(\Delta(Z))$, where utility is defined above. Define $\mathcal{U}$ on $\Delta(\mathcal{K}(\Delta(Z)))$ by:

$$\mathcal{U}(X) = \mathcal{U}(X \circ \phi^{-1}). \tag{3.9}$$

This completes the utility specification.

We can now state our main result.

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[16] E_p denotes expectation with respect to p.

[17] Existence of such a measurable map follows from Filipov's Implicit Function Lemma (Aliprantis and Border, 1994, p. 507). Similarly, for ϕ below.

[18] More generally, adopt the following notation: Let (S_i, Σ_i) be measurable spaces for $i = 1, 2$, m_1 a measure on Σ_1, and $\phi : (S_1, \Sigma_1) \to (S_2, \Sigma_2)$ a measurable map (S_2-valued random variable). Then, $m_1 \circ \phi^{-1}$ denotes the measure on Σ_2 induced by m_1 and the random variable ϕ, that is $(m_1 \circ \phi^{-1})(B_2) = m_1(\phi^{-1}(B_2))$ for all B_2 in Σ_2.

Theorem 3.1.

(a) *Utility is well-defined on* $\Delta(\mathcal{K}(\Delta(Z)))$ *by equations* (3.2), (3.8)*, and* (3.9). *The corresponding preference* $\succeq$ *satisfies Order, NSC, Expected Stationarity, Limited Continuity, and Monotonicity. Finally,* U *and* V *represent* $\succeq_{\text{del}}$ *and* $\succeq_{\text{imm}}$ *respectively.*

(b) *Let* $\succeq$ *be a binary relation on* $\Delta(\mathcal{K}(\Delta(Z)))$ *satisfying the axioms in part (a). Then, there exists a compatible pair of utility functions* $U, V : \Delta(Z) \longrightarrow \mathbb{R}$ *such that* $\succeq$ *admits a representation of the form* (3.2), (3.8)*, and* (3.9). *Moreover, preference is represented in this way also by* $U', V' : \Delta(Z) \longrightarrow \mathbb{R}$ *if and only if they are ordinally equivalent to* U *and* V *respectively.*

The bulk of the proof is provided in the Appendix, but the uniqueness property asserted in part (b) is easily understood. First, if U' and V' also represent the preference $\succeq$ via equations (3.2), (3.8), and (3.9), then, by part (a), they represent $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$, as do U and $V$and hence asserted ordinal equivalences. Conversely, it is easily seen that the definition of (ordinal) utility $\mathcal{U}$ on $\Delta(\mathcal{K}(\Delta(Z)))$ uses only the ordinal properties of U and V.

It is noteworthy, at both conceptual and practical levels, that the model described in the theorem is completely specified by a compatible pair of (ordinal) utility functions on $\Delta(Z)$, or equivalently, by the corresponding orders. Modelling anticipatory feelings does not require consideration of "psychological lotteries"—it is sufficient to specify two preferences over ordinary lotteries, interpreted as the rankings of delayed and immediate risks. Moreover, the model is rich in that any compatible pair of orders can be taken as primitives. The large literature, both theoretical and empirical, concerning the ranking of lotteries, makes this starting point convenient. In particular, any specific model of risk preference (satisfying suitable FSD-monotonicity) can be integrated into our model of anxiety axiomatically—one need only assume, in addition to compatibility, that each order satisfies the axioms that characterize the specific model of risk preference that is of interest. Here, we presume that both induced orders conform to the same model of risk preference, which we view as the natural specification. In the example above (linear model), the relevant model of risk preference is expected utility; below, we provide an example where risk preferences are non-linear.

4. EXAMPLES

Example (KP). This is the classic model of preference where the temporal resolution of risk matters. As we see, it violates our axioms.

Preference on $\Delta_s(\mathcal{K}(\Delta(Z)))$ is represented by:[19]

$$\Theta(\mathcal{U}^{KP}(X)) = \Sigma_x X(x)\Theta\left(\max_{p\in x} E_p u\right),$$

where u and Θ are as in the previous example ($u : Z \to \mathbb{R}$ is continuous and $\Theta : u(Z) \to \mathbb{R}$ is strictly increasing and continuous). Kreps and Porteus (1978) formulate utility not only for two-stage lotteries, which is how their model is often described, but also for all random menus. They model agents who not only care about how risk resolves over time but are also dynamically consistent in the usual sense that commitment is never valuable. In particular, and in contrast with our general model,

$$\mathcal{U}^{\text{KP}}(x) \leq \mathcal{U}^{\text{KP}}(x \cup y), \text{ for all menus } x \text{ and } y.$$

More formally, the order $\succeq_{\text{del}}$ is represented by U, where U is linear with vNM utility index u, and $\succeq_{\text{imm}}$ is represented by V, $V(p) = \Theta^{-1}(E_p\Theta(u))$. Apart from the extreme case where Θ

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[19] Throughout this section, we describe utility only for finite-support lotteries.

is linear, $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ are distinct and thus Expected Stationarity is violated: Even though the agent's time 0 ranking of delayed risks differs from her ranking of immediate risks, she does not value commitment. The reason is that when evaluating a menu at time 0, the agent expects her choice out of the menu at time 1 to be guided by $\succeq_{\text{del}}$, which also describes her time 0 ranking of delayed risks. She holds this expectation even though (i) the time 1 choice is between immediate risks and (ii) her current ranking of immediate risks is given by by $\succeq_{\text{imm}}$.

It is readily verified that all other axioms are satisfied; indeed, KP preference satisfies SR, which is stronger than NSC.

Note finally that $\mathcal{U}^{\text{KP}}$ is distinguishable from $\mathcal{U}^{\text{lin}}$ *only if* we can observe rankings of menus. In particular, in both cases, the utility of two-stage lotteries is given by:

$$\mathcal{U}(X) = \Theta^{-1}(\Sigma_p X(\{p\}\Theta(E_p u))).$$

This merits emphasis: *a recursive structure for utility on the domain of two-stage lotteries does not imply that commitment has no value.*

Example (Rank-Dependent-Expected Utility). Here, we describe a special case of our general model where the induced orders $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ both conform to rank-dependent-expected utility (RDEU), a model of risk preferences that has played a large role in attempts to accommodate evidence, such as the Allais Paradox, contradicting the vNM model. See the survey by Starmer (2000), for example.

Let $g, h : [0,1] \to [0,1]$ be increasing and surjective; since they are used to transform (or distort) probabilities, they are sometimes referred to as *distortion functions*. Let $u : Z \longrightarrow \mathbb{R}$ be continuous and define U, for $p = \Sigma_i p_i \delta_{z_i}$, by:

$$U(p) = \Sigma_i [g(\Sigma_{j \geq i} p_j) - g(\Sigma_{j \geq i+1} p_j)] u(z_i), \tag{4.1}$$

where outcomes are ordered so that $u(z_i) \leq u(z_{i+1})$ for all i. Define V similarly using the distortion function h and the vNM index $\upsilon = \Theta(u)$, where $\Theta : u(Z) \to \mathbb{R}$ is strictly increasing and continuous. In order to satisfy the normalization (3.1), let:

$$V(p) = \Theta^{-1}(\Sigma_i [h(\Sigma_{j \geq i} p_j) - h(\Sigma_{j \geq i+1} p_j)] \Theta(u(z_i))).$$

Then, U and V constitute a compatible pair, and thus, they determine a utility function, denoted $\mathcal{U}^{\text{rdeu}}$, consistent with our axioms.

This example generalizes the linear model (our first example), to which it reduces if both g and h are identity functions. We show below that non-linear distortion functions are useful for modelling intuitive patterns of the demand for information.

Axiomatic foundations for RDEU preferences over lotteries can be found in Quiggin (1982), Segal (1989), and Starmer (2000). As described in the last section, axiomatic foundations for $\mathcal{U}^{\text{rdeu}}$ follow by adding these RDEU axioms, applied to $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$, to those in Theorem 3.1. The cited axiomatic studies and the survey by Starmer (2000) can be brought to bear on the plausibility or appeal of $\mathcal{U}^{\text{rdeu}}$. The only difference here from the literature on risk preferences is that the orders $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ deal with lotteries that are resolved only with the delay of at least "one period", and where the length of a period should be significant (on the scale of days or weeks rather than minutes) in order that anticipatory feelings be relevant. However, the axioms characterizing RDEU seem as appealing intuitively (or no more problematic) in our setting, and while we are not aware of any experimental evidence on the descriptive validity of RDEU when resolution is delayed significantly, there is no reason to expect the axioms to perform less well for such risks. Thus, we view $\mathcal{U}^{\text{rdeu}}$, which we use in the next section to model the attitude towards information, as being well founded.

Segal (1990) studies preference over two-stage lotteries where preference at each stage conforms to RDEU. One of his models, extended to the domain of random menus by assuming SR, is related to $\mathcal{U}^{\text{rdeu}}$ in the same way that $\mathcal{U}^{\text{KP}}$ is related to $\mathcal{U}^{\text{lin}}$.

Example (Time-Varying Risk Aversion). Time-varying risk aversion is another possible reason for commitment. Consider the utility function:

$$\mathcal{U}^{\text{tvra}}(X) = \Sigma_x X(x) \max\left\{U(p) : p \in \arg\max_{p' \in x} U'(p')\right\}, \quad X \in \Delta(\mathcal{K}(\Delta(Z))),$$

where U and U' are (ordinally distinct) continuous linear functions on $\Delta(Z)$. Then, $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ are *both* represented by U, yet commitment is valuable, thus violating Expected Stationarity.[20] The reason for commitment differs here. In particular, an individual with utility function $\mathcal{U}^{\text{tvra}}$ does not care when risk is resolved: Any two-stage lottery $X = \Sigma_p X(\{p\})\delta_{\{p\}}$ has utility:

$$\mathcal{U}^{\text{tvra}}(\Sigma_p X(\{p\})\delta_{\{p\}}) = \Sigma_p X(\{p\})U(p) = U(\Sigma_p X(\{p\})p),$$

which depends only on the induced distribution over outcomes $\Sigma_p X(\{p\})p$. Yet, she values commitment because she expects her risk preferences to change, and therefore, to choose out of menus at time 1 according to U', while her time 0 utility function over lotteries is U.

Example (Self-Control). Define $\mathcal{U}^{\text{sc}}$ as in the linear example, except that equation (3.2) is replaced by:

$$w(x) = \max_{p \in x}\left[U(p) + \Theta^{-1}(E_p \Theta \circ u)\right] - \max_{p' \in x} \Theta^{-1}(E_p \Theta \circ u).$$

Then, preference violates NSC, though it satisfies GP's weaker axiom Set-Betweenness ($x \succeq y \implies x \succeq x \cup y \succeq y$). Our other axioms are satisfied. In particular, Expected Stationarity is readily verified because $\succeq_{\text{imm}}$ has utility function V, where $V(p) = \Theta^{-1}(E_p \Theta \circ u)$.

5. DEMAND FOR INFORMATION

5.1. *Information, anxiety/savouring, and commitment*

Consider the attitude towards information when it has only psychic, as opposed to planning, consequences. This is reflected in the ranking of random menus that provide commitment, that is in the ranking of two-stage lotteries.

For simplicity, restrict attention to two-stage lotteries with finite support. Given $X = \Sigma_p X(\{p\})\delta_{\{p\}}$, define $EX \in \Delta(Z)$ by:

$$EX = \Sigma_p X(\{p\})p.$$

Then, EX describes the probability distribution over outcomes induced by X, where the temporal resolution of this risk has been removed. Since EX describes the prior risk, it is the counterpart of the Bayesian prior in a model with states of the world and subjective uncertainty. We can modify the temporal resolution prescribed by X and consider two extremes. *No information* (at the first stage) corresponds to $\delta_{\{EX\}}$. The other extreme, all risk being resolved at time 1, corresponds to the two-stage lottery $\Sigma_z (EX)(z)\delta_{\{\delta_z\}}$; we refer to this as *perfect information*. For brevity, we examine these extremes only, though intermediate cases could be considered.

Say that the agent is *information seeking at EX* if she prefers perfect information to no information, that is if:

$$\Sigma_z (EX)(z)\delta_{\{\delta_z\}} \succeq \delta_{\{EX\}};$$

if the reverse ranking holds, refer to her as *information averse at EX*. These notions are weak—an agent can satisfy both, in which case we refer to her as *information neutral* at EX.

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[20] All the other axioms of our model are satisfied.

Theorem 5.1. *The agent with preference $\succeq$ satisfying our axioms is information seeking (averse) at EX if and only if, for every z in Z,*

$$\delta_z \succeq_{\text{imm}} EX \implies (\Longleftarrow)\ \delta_z \succeq_{\text{del}} EX.$$

Proof. By the representation, information seeking at EX is equivalent to:

$$V(EX) = V(\Sigma_z(EX)(z)\delta_z) = \mathcal{U}\left(\Sigma_z(EX)(z)\delta_{\{\delta_z\}}\right) \geq \mathcal{U}(\delta_{\{EX\}}) = U(EX),$$

or

$$V(EX) \geq U(EX). \tag{5.1}$$

Since U and V represent $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ and since they are equal on Z, the asserted condition follows. Similarly for information aversion. ||

The condition in the theorem corresponding to information seeking asserts that whenever $\succeq_{\text{imm}}$ would reject the lottery EX in favour of a certain outcome, then so would $\succeq_{\text{del}}$. In that standard sense, $\succeq_{\text{del}}$ is more risk-averse than $\succeq_{\text{imm}}$ at EX. Thus, the agent is *information seeking at EX if and only if $\succeq_{\text{del}}$ is more risk-averse than $\succeq_{\text{imm}}$ at EX*. Information aversion corresponds to $\succeq_{\text{imm}}$ being more risk-averse.[21]

In the linear model, the condition (5.1) becomes (writing $p = EX$):

$$\Theta^{-1}(E_p\Theta(u)) \geq E_p u, \tag{5.2}$$

which is true if Θ is "convex at $E_p u$". This condition is familiar from the KP model as the condition describing a preference for early resolution of risk. The similarity with what we know from KP is not surprising since we have already noted that their model coincides with our linear model (the example in Section 3) on the subdomain of two-stage lotteries; similarly, our general model coincides there with a non-linear version of KP.

The connection between the demand for information and risk aversion at two points in time has been noted previously by Grant, Kajii and Polak (1998). They assume recursivity, a form of dynamic consistency, but our model is indistinguishable from one with dynamically consistent preferences if only the ranking of two-stage lotteries is observed. Therefore, Theorem 5.1 is a known result—we include it for completeness.[22] The distinctive feature of our model is the connection it implies between the demand for information and the value of commitment and the next result focuses on this connection.

We noted earlier that indifference to the temporal resolution of risk (or a zero demand for information) is equivalent in our model to commitment having no value. Now we go further and relate information seeking (or aversion) to the sort of commitments that are or are not valuable.

Theorem 5.2. *The agent with preference $\succeq$ satisfying our axioms is information seeking at $p \in \Delta(Z)$ if and only if, for every z in Z,*

$$\{p\} \succ \{\delta_z\} \Longrightarrow \{p\} \sim \{p, \delta_z\}, \tag{5.3}$$

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[21] We are not aware of much experimental evidence on how delayed resolution affects risk aversion. Some relevant experiments are reported in Liberman, Sagristano and Trope (2002). But the stakes involved are too small to plausibly generate anxiety or savouring.

[22] In fact, Grant, Kajii and Polak also consider partial changes in information and not just the extremes of perfect vs. no information. Given the focus of this paper, and in the interest of brevity and simplicity, we do not pursue an extension in this direction.

and she is information averse at p if and only if, for every z in Z,

$$\{\delta_z\} \succ \{p\} \Longrightarrow \{\delta_z\} \sim \{p, \delta_z\}. \tag{5.4}$$

Proof. Assume information seeking at p. Then, by equation (5.1), $V(p) \geq U(p)$. If also $\{p\} \succ \{\delta_z\}$, then $U(p) > U(\delta_z) = V(\delta_z) \Longrightarrow V(p) > V(\delta_z)$. Therefore, p is better than δ_z according to both U and V. Conclude that $\{p\} \sim \{p, \delta_z\}$. Conversely, assume equation (5.3), which is equivalent to: for every z,

$$U(p) > U(\delta_z) \Longrightarrow V(p) \geq V(\delta_z).$$

But then $V(p) \geq U(p)$, which implies information seeking at p. If not, then $V(p) < U(p)$, and, because U and V are continuous and Z is connected, there exists z such that $V(p) < V(\delta_z) = U(\delta_z) < U(p)$, a contradiction.

The proof for information aversion is similar. ||

We interpret information seeking (aversion) as the behavioural manifestation of anxiety (savouring). Therefore, both theorems mentioned above describe the revealed preference implications of anxiety and savouring. In fact, to highlight this identification, we use the terms anxiety, savouring, and neutrality (at a lottery p) *interchangeably* with information seeking, aversion, and neutrality (at p) below.

The above-mentioned characterizations are intuitive. Consider the characterization for information seeking at p and, for concreteness, interpret p as representing the risk of a large loss due to a house fire or car accident. Living with this risk entails anxiety and thus leads to a preference for early resolution. Complete insurance is available at a price that would leave the agent with the certain outcome z. Suppose, however, that in spite of the anxiety, she strictly prefers at time 0 to remain uninsured ($\{p\} \succ \{\delta_z\}$). Then having the option to postpone the insurance decision to time 1 is a matter of indifference ($\{p\} \sim \{p, \delta_z\}$)—the agent is certain that insurance will be declined because at time 1, the anxiety argument for insurance is weaker. Conversely, suppose that for any price, if insurance is declined at time 0, then it would also be declined at time 1. Then, the psychic cost of the risk p is smaller at the later time, presumably because it can cause less anxiety at that point. But if p is a source of anxiety, then its early resolution would be preferred. The bottom line is that, given our axioms, *an anxious individual is one who would never strictly prefer to commit to not insuring.*

It might appear surprising that anxiety and savouring are characterized by conditions that express a limited form of SR. However, the qualification "limited" is crucial—other commitments may be strictly valuable. For example, for an anxious individual, the ranking:

$$\{\delta_z\} \succ \{p, \delta_z\} \sim \{p\}, \tag{5.5}$$

is intuitive: Insurance could be chosen at time 0 to provide peace of mind, while if the decision is left for a later time, when anxiety is less important, the individual might decide not to insure. Thus, she may *strictly prefer to commit to insurance*.

The indicated strict preference to commit reflects a strict form of anxiety. More generally, we use the terms information seeking (anxiety) and information aversion (savouring) in the weak sense. Therefore, since every preference is either information seeking or information averse at the given p, the way to express (weak) information aversion (hence savouring) is to exclude strict information seeking, that is to exclude equation (5.5). This explains the characterization (5.4) of information aversion.

The characterizations in the theorem are not valid in the KP model. Since commitment is never valuable in their model, both conditions (5.3) and (5.4) are satisfied globally, without implications for the nature of information demand.

5.2. *Comparative anxiety*

Above, we provided behavioural characterizations of anxiety. Here, we go further and give behavioural meaning to statements about comparative anxiety across agents. The obvious modifications corresponding to comparative savouring are left to the reader.

For concreteness, think of the insurance example. We know from Theorem 5.2 and the ensuing discussion that, in the absence of neutrality, anxiety about the possible loss is reflected via the desire to commit to insurance, that is through rankings of the sort:

$$\{\delta_z\} \succ \{p, \delta_z\} \sim \{p\}.$$

Let both $\succeq^1$ and $\succeq^2$ be anxious at p. Say that 2 is *more anxious at* p *than* 1 if whenever 1 strictly prefers to commit to insurance, then so does 2; that is if for every $z \in Z$,

$$\{\delta_z\} \succ^1 \{p, \delta_z\} \sim^1 \{p\} \Longrightarrow \{\delta_z\} \succ^2 \{p, \delta_z\} \sim^2 \{p\}. \tag{5.6}$$

Theorem 5.3. *Let* $\succeq^1$ *and* $\succeq^2$ *satisfy our axioms and let both be anxious at* p. *Suppose in addition that* $\succeq^1$ *and* $\succeq^2$ *agree on* Z. *Then,* 2 *is more anxious at* p *than* 1 *if and only if either* (i) 1 *is neutral at* p *or* (ii) *for every* z,

$$\delta_z \succeq^2_{\text{imm}} p \Longrightarrow \delta_z \succeq^1_{\text{imm}} p \ \left(\succeq^2_{\text{imm}} \textit{ is less risk-averse than } \succeq^1_{\text{imm}} \textit{ at } p \right) \tag{5.7}$$

and

$$\delta_z \succeq^1_{\text{del}} p \Longrightarrow \delta_z \succeq^2_{\text{del}} p \ \left(\succeq^2_{\text{del}} \textit{ is more risk-averse than } \succeq^2_{\text{del}} \textit{ at } p \right). \tag{5.8}$$

The proof is not particularly revealing and is relegated to the Appendix.

To interpret the theorem, consider first the role of neutrality (condition (i)). If 1 is neutral at p, then 1 is both anxious and savouring (and is neither in the strict sense). Therefore, the antecedent in equation (5.6) is not satisfied for any z, and the defining condition is satisfied vacuously. Since 2 is anxious by assumption, it makes sense to refer to her as being weakly more anxious than 1.

Theorem 5.1 characterizes anxiety in terms of $\succeq_{\text{imm}}$ being more risk-averse than $\succeq_{\text{del}}$. This makes conditions (5.7) and (5.8) intuitive: An agent becomes more anxious if she becomes less averse to immediate risks and more averse to delayed risks. Conversely, this is necessary for increased anxiety at p unless equation (5.6) is satisfied vacuously, that is 1 is neutral at p.

These characterizing conditions are readily expressed in terms of the representations. Let (U_i, V_i) represent $\succeq^i$, $i = 1, 2$, as in Theorem 3.1. By construction, $U_i = V_i$ on Z. By the hypothesis that $\succeq^1$ and $\succeq^2$ agree on Z, it follows that U_1 and U_2 are ordinally equivalent on Z. Therefore, by applying a common monotonic transformation to U_1 and V_1, we can assume wlog that:

$$U_1 = U_2 = V_1 = V_2 \text{ on } Z. \tag{5.9}$$

Using these utility functions, it is easily shown (see the proof) that 2 is more anxious at p than 1 if and only if

$$V_2(p) \geq V_1(p) \text{ and } U_2(p) \leq U_1(p).$$

Finally, consider the assumption that the two agents have the same ranking on Z. This restriction seems natural; for example, for preferences over lotteries with vector outcomes, Kihlstrom and Mirman (1974) have pointed out that "more risk-averse than" can be meaningfully defined only when the two agents agree on the ranking of outcomes. However, we have not succeeded in finding a similar conceptual justification for that restriction here—indeed, our definition of "more anxious than" does not require it.

5.3. *“Anomalous” demand for information*

In the linear special case of our model, condition (5.2) shows that the attitude towards information depends on properties of Θ at $E_p u$ rather than on the prior p separately. Thus, the linear model cannot accommodate information attitudes that vary with the prior. Eliaz and Spiegler (2006) emphasize that such dependence on the prior is anomalous in an “expected-utility-based” model.[23]

As an example of such an anomalous information attitude, they consider the intuitive hypothesis that the agent is *information seeking (averse) when the favourable (unfavourable) event is very likely*. Though inconsistent with the linear model, the hypothesis is not anomalous relative to our general model. For example, consider binary lotteries with outcomes z_1 and z_2, where $u(z_1) < u(z_2)$, and take the RDEU model (with Θ linear and hence dispensable). Then, by equation (5.1), the hypothesis is satisfied if and only if:

$$(1-h(p_2))u(z_1)+h(p_2)u(z_2) > (1-g(p_2))u(z_1)+g(p_2)u(z_2), \text{ if } p_2 \text{ is near } 1,$$

$$(1-h(p_2))u(z_1)+h(p_2)u(z_2) < (1-g(p_2))u(z_1)+g(p_2)u(z_2), \text{ if } p_2 \text{ is near } 0.$$

These conditions are satisfied if (and only if) h lies above g for probabilities near 1 and below g for probabilities near 0. Refer to this pattern as “h is s-shaped relative to g”.

As emphasized in the last section, the RDEU special case of our model is axiomatically well founded. Thus, the preceding reflects on the skepticism of Eliaz and Spiegler (2006, p. 16) about the usefulness of non-expected utility theories for addressing anomalous attitudes towards information. Admittedly, they describe other anomalies in addition to the one we have been considering. However, these seem intuitively to be due to something other than anticipation or anxiety (*e.g.* cognitive dissonance or confirmatory bias) and are thus most naturally addressed by other models.

One might also wonder whether the hypothesis that h is s-shaped relative to g is consistent with evidence. Note first that it is consistent with risk aversion for both $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$, for which it suffices that u and $\Theta(u)$ be concave and that both g and h be convex (Chew, Karni and Safra, 1987).[24] Second, there exists evidence about the shapes of distortion functions needed in order for the RDEU risk preference model to accommodate Allais-type behaviour; subject to the qualification described in the discussion of the RDEU example, this evidence is relevant here.[25] However, even if both g and h have these shapes, their *relative* shapes are not pinned down by available evidence. What is needed to determine relative shapes is evidence on how individuals rank *both* immediate risks (risks that resolve within one period) *and* delayed risks (those that resolve within two periods), where payment in both cases is received in period 2.

Other patterns of information attitudes can also be accommodated within the RDEU model. For example, suppose that $h(\cdot)-g(\cdot)$ is single peaked with peak at $p_1 = p_2 = \frac{1}{2}$. This specification models an agent who is information seeking when facing any risk, but “particularly so” when she is less certain about the outcome. Alternatively, if $h-g$ is positive except near $p_1 = p_2 = \frac{1}{2}$, then there is information seeking only when the agent is nearly certain *ex ante* about which outcome will be realized (resembling Eliaz and Spiegler’s Example 3).

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[23] Their model differs from ours in details, but their point is still valid here.

[24] Assume $Z \subset \mathbb{R}^n$ so that risk aversion can be defined.

[25] Starmer (2000, p. 348) describes some support for an *inverted* s-shape (relative to the 45° line) for distortions—concave and lying above the 45° line for small probabilities—those smaller than some p^*—and convex and above the 45° line for probabilities greater than p^*; Quiggin (1982) proposed this form with $p^* = 1/2$. See Tversky and Wakker (1995) for a discussion of the connection between the shape of the distortion function and the theoretical properties of preference.

We emphasize that the RDEU model is but one example of the general framework characterized in Theorem 3.1 and that adopting other models of risk preference will lead to alternative implications for the attitude to information. As one illustration, consider the generalization of RDEU called cumulative prospect theory (Tversky and Kahneman, 1992), in which (*e.g.* real valued) outcomes are measured relative to a reference point and there is risk aversion in gains and risk-loving in losses. By assuming that both $\succeq_{\text{del}}$ and $\succeq_{\text{imm}}$ conform to cumulative prospect theory, one can model anxiety for lotteries that involve only losses and savouring for those that involve only gains.

A final remark is intended for those readers who are still wondering, in spite of our many earlier explanations, "Why do we need a new model for all of this? For example, hasn't Segal (1990) proposed modelling preference over two-stage lotteries using RDEU in a similar way?" The contribution here is to show that such an RDEU-based model is consistent with a demand for commitment, and specifically with Expected Stationarity, which we find intuitive when anxiety or savouring is present. Though agents do not reduce compound lotteries in Segal's model, or in KP's, this cannot, in our view, be interpreted as reflecting anticipatory feelings. In our model, it can be interpreted so.

APPENDIX

Proof of Theorem 3.1. (a) Utility is well defined: Show that utility on $\Delta(\Delta(Z))$ that is defined in equation (3.8) is invariant to the choice of θ. If θ_1 is another such map, then $U(p) = U(\delta_{\theta_1(p)})$:

$$\begin{aligned}
&\Longrightarrow U(\delta_{\theta_1(p)}) = U(\delta_{\theta(p)})\\
&\Longrightarrow V(\delta_{\theta_1(p)}) = V(\delta_{\theta(p)}) \;\; (U \text{ and } V \text{ ordinally equivalent on } Z)\\
&\Longrightarrow X \circ \theta_1^{-1} \approx^{\text{FSD}} X \circ \theta^{-1}, \;\text{ for any } X \in \Delta(\Delta(Z)),\\
&\Longrightarrow V(X \circ \theta_1^{-1}) = V(X \circ \theta^{-1}),
\end{aligned}$$

where: $\approx^{\text{FSD}}$ is defined as in equation (2.5), and the last equality follows from the assumption that V is FSD-increasing. The second step in showing that utility is well defined requires showing that the utility values defined by equations (3.8) and (3.2) agree on the intersection $\Delta(\Delta(Z)) \cap \mathcal{K}(\Delta(Z))$. An element in the intersection must have the form $\{p\}$, for some delayed risk $p \in \Delta(Z)$ that resolves at time 2. Then, p can also be viewed as the two-stage lottery that produces p with certainty at the first stage. According to equation (3.8), the latter has utility $V(\delta_{z_p})$ where z_p solves equation (3.4), and according to equation (3.2) (see also equation (3.3)), the singleton menu $\{p\}$ has utility $U(p) = U(z_p)$. But $V(\delta_{z_p}) = U(\delta_{z_p})$ by equation (3.1).

The extension of utility to $\Delta(\mathcal{K}(\Delta(Z)))$ is well defined by equation (3.7) because $\mathcal{U}$ is FSD-increasing on $\Delta(\Delta(Z))$.

Necessity of axioms: Order is obvious. Monotonicity and Expected Stationarity are readily verified.
NSC: Let $U(x) = \max_{p \in x} U(p)$, $U^{-1}(x) = \arg\max_{p \in x} U(p)$, and similarly for V. In this notation:

$$\mathcal{U}(y) = \max\{U(p) : p \in V^{-1}(y)\}.$$

(i) $V(x') > V(x) \Longrightarrow x' \sim x' \cup x$: Hypothesis implies that $V^{-1}(x' \cup x) = V^{-1}(x')$.
(ii) $V(x') < V(x) \Longrightarrow x \sim x' \cup x$: Hypothesis implies that $V^{-1}(x' \cup x) = V^{-1}(x)$.
(iii) $V(x') = V(x) \Longrightarrow x' \sim x' \cup x$ if $x' \succeq x$: Hypothesis implies that $V^{-1}(x' \cup x) = V^{-1}(x') \cup V^{-1}(x)$, and hence, $\mathcal{U}(x' \cup x) = \max\{\mathcal{U}(x'), \mathcal{U}(x)\}$.

Limited Continuity: (a) follows because $\mathcal{U}$ defined in equation (3.2) is usc on $\mathcal{K}(\Delta(Z))$ by a form of the Maximum Theorem. Parts (b) and (c) are obvious.

(b) Sufficiency of the axioms:

Step 1. There exists a representation w of $\succeq$ on $\mathcal{K}(\Delta(Z))$: $\Delta(Z)$ is separable (because it is compact metric) and connected. Hence, $\succeq_{\text{del}}$ has a continuous (and FSD-increasing) utility function $U : \Delta(Z) \longrightarrow \mathbb{R}$. For any menu x, define:

$$w(x) = U(p),$$

for any p such that $x \sim \{p\}$. Existence of p is ensured by Limited Continuity. If p and p' are two such measures, then $p' \sim_{\text{del}} p$ and thus $U(p') = U(p)$; hence, w is well defined. Moreover,

$$w(\{p\}) = U(p) \text{ for every } p.$$

Step 2. Let V be a continuous utility function for $\succeq_{\text{imm}}$. It exists since $\succeq_{\text{imm}}$ is continuous by Limited Continuity and it is FSD-increasing by Monotonicity.

As observed in equation (2.3), $\succeq_{\text{imm}}$ agrees with $\succeq_{\text{del}}$ on Z. Therefore, it is wlog to assume (3.1)—U and V are identical on Z.

Step 3. $\{p'\} \sim \{p', p\}$ if $p' \succeq_{\text{imm}} p$ and $p' \succeq_{\text{del}} p$: Let $p' \succeq_{\text{imm}} p$ and $p' \succeq_{\text{del}} p$. By definition, the latter implies $\{p'\} \succeq \{p\}$. By NSC, there are two cases. Case 1: $\{p'\} \sim \{p', p\} \succeq \{p\}$. This is consistent with the desired conclusion. Case 2: $\{p'\} \succ \{p', p\} \sim \{p\}$. Then, Expected Stationarity implies $p' \prec_{\text{imm}} p$, contradicting our hypothesis.

Step 4. $\{p'\} \sim \{p', p\}$ if $p' \succ_{\text{imm}} p$: Suppose $p' \succ_{\text{imm}} p$ and $\{p'\} \not\sim \{p', p\}$. Then, Step 3 implies that $p' \not\succeq_{\text{del}} p$, that is $p' \prec_{\text{del}} p$. Thus, $\{p\} \succ \{p', p\}$ by Expected Stationarity. But this contradicts NSC.

Step 5. Prove the representation for finite non-random menus: Argue as in GP (p. 1429). Let x be finite and let $p^* \in x$ satisfy:

$$w(\{p^*\}) = \max\{w(\{p\}) : p \in \arg\max_{p' \in x} V(p')\}.$$

Note that $x = \cup_{p' \in x} \{p^*, p'\}$. Since w represents $\succeq$ and the latter satisfies NSC, then:

$$w(x) = w(\{p^*, p'\}) \text{ for some } p' \in x.$$

Since V represents $\succeq_{\text{imm}}$, Steps 3 and 4 imply that:

$$w(\{p^*, p'\}) = w(\{p^*\}) = U(p^*).$$

This yields the desired result:

$$w(x) = w(\{p^*\}) = \max\{U(p) : p \in \arg\max_{p' \in x} V(p')\}.$$

Step 6. Extend the representation to $\mathcal{K}(\Delta(Z))$: GP's lemma 8 and the ensuing paragraph (p. 1430) deliver the extension. Limited Continuity provides the continuity properties needed by their argument. It follows from the Maximum Theorem that w is usc.

Step 7. Prove the desired representation on $\Delta(\Delta(Z))$: Define utility via equation (3.8), that is for any $X \in \Delta(\Delta(Z))$,

$$\mathcal{U}(X) = V(X \circ \theta^{-1}),$$

where $\theta : \Delta(Z) \to Z$ satisfies $U(p) = U(\delta_{\theta(p)})$. Argue as in the proof of part (a) to show that utility is well defined.

It follows from Monotonicity, specifically from $\succeq$ being FSD-increasing, that[26]

$$X \sim ((X \circ \theta^{-1}) \circ e^{-1}).$$

Informally: If X has finite support, then it assigns probability $X(p)$ to each p in its support, while $(X \circ \theta^{-1}) \circ e^{-1}$ assigns probability $X(p)$ to $\delta_{\theta(p)}$, the certainty equivalent of p according to U. “Therefore, they are indifferent by backward induction”.

Now, for any $X', X \in \Delta(\Delta(Z))$, $X' \succeq X$

$$\begin{aligned}
&\iff ((X' \circ \theta^{-1}) \circ e^{-1}) \succeq ((X \circ \theta^{-1}) \circ e^{-1}) \\
&\iff (X' \circ \theta^{-1}) \succeq_{\text{imm}} (X \circ \theta^{-1}) \\
&\iff V(X' \circ \theta^{-1}) \geq V(X \circ \theta^{-1}) \text{ (by Step 2)} \\
&\iff \mathcal{U}(X') \geq \mathcal{U}(X).
\end{aligned}$$

Step 8. Extend the representation to all random menus: Define utility on $\Delta(\mathcal{K}(\Delta(Z)))$ by equation (3.9), that is for any $X \in \Delta(\mathcal{K}(\Delta(Z)))$,

$$\mathcal{U}(X) = \mathcal{U}(X \circ \phi^{-1}),$$

where $\phi : \mathcal{K}(\Delta(Z)) \to \Delta(Z)$ satisfies $U(x) = U(\delta_{\phi(x)})$; note that $X \circ \phi^{-1}$ is a two-stage lottery, and thus, its utility was defined in the previous step. Argue as in the proof of part (a) to show that utility is well defined.

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[26] Recall the definition of $\succeq_{\text{imm}}$ and that $e : Z \longrightarrow \Delta(Z)$ is the natural embedding.

It follows from $\succeq$ being FSD-increasing, that:

$$X \sim X \circ \phi^{-1}.$$

Since $\mathcal{U}$ represents preference on $\Delta(\Delta(Z))$, the extension defined here represents preference on $\Delta(\mathcal{K}(\Delta(Z)))$. ‖

Proof of Theorem 5.3. Let (U_i, V_i) represent $\succeq^i$, $i = 1, 2$, and satisfy equation (5.9).

If 1 is neutral at p, then equation (5.6) is satisfied vacuously. Assume equations (5.7) and (5.8). Then, by the representations,

$$V_2(\delta_z) \geq V_2(p) \Longrightarrow V_1(\delta_z) \geq V_1(p) \text{ and} \tag{A.1}$$

$$U_1(\delta_z) \geq U_1(p) \Longrightarrow U_2(\delta_z) \geq U_2(p). \tag{A.2}$$

It follows from continuity of the representing functions, connectedness of Z and from equation (5.9) that:

$$V_2(p) \geq V_1(p) \text{ and } U_2(p) \leq U_1(p). \tag{A.3}$$

This, in turn, implies, given equation (5.9), that:

$$[U_1(\delta_z) > U_1(p), V_1(\delta_z) < V_1(p)] \Longrightarrow [U_2(\delta_z) > U_2(p), V_2(\delta_z) < V_2(p)], \tag{A.4}$$

which is equivalent to equation (5.6).

Conversely, suppose that 1 is not neutral at p. Since 1 is anxious, $U_1(p) < V_1(p)$. Then, (5.6) $\Longrightarrow$ (A.4) $\Longrightarrow$ (A.1) and (A.2) $\Longrightarrow$ (5.7) and (5.8). ‖

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