Good rankings are bad - Why reliable rankings can hurt consumers*

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Abstract

Ranking have become increasingly popular on markets for study programs, restaurants, wines, cars, etc. This paper analyses the welfare implication of such rankings. Consumers have to make a choice between two goods of unknown quality with exogenous presence or absence of an informative ranking. We show that existence of the ranking might make all consumers worse off. The existence of a ranking changes the demand structure of consumers. With rigid prices and rationing, the change can be detrimental to consumers due to its effect on rationing. Furthermore, this change in demand can also be detrimental due to consumption externalities. Finally, with perfectly flexible prices the ranking might increase the market power of firms and hence lead to losses for all consumers.

1 Introduction

During the last 20 years, rankings have become increasingly popular. While restaurants and hotels have been ranked for a long time (the first red Michelin guide appeared more than 100 years ago), relatively new rankings cover study programs (see e.g. US News and World Report's rankings of colleges and graduate schools, and the FT's ranking of MBA programs), wine (e.g. the famous Parker guide), health insurances and health care providers (e.g. The

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National Committee for Quality Assurance's plan ratings). Furthermore, the internet allows rankings based on consumers evaluation, e.g. in the fields of gastronomy, medical services (see e.g. Yelp.com), and cars (see e.g. cars.com).

Most of the scientific research on rankings investigates the quality of rankings. In the context of scientific journals in economics, ranking methods have been discussed e.g. by Kalaitzidakis et al. (2010), and Palacios-Huerta and Volij (2004). There has been quite some concern that firms or other interested parties might manipulate the outcome of rankings (see Dranove et al. 2003 for a case with consumer-input technology; Glazer and McGuire (2006), and Glazer et al., 2008 for multitasking issues and Sorensen (2006) for effects on products diversity) and that this might be harmful for (some) consumers. But very little research has been done on the welfare impact of (correct) rankings (see below for few exceptions). The literature investigating the quality of rankings seem to be based on the implicit assumption that better rankings are better - more information should not hurt. If the ranking would only be used in individual decision problems, better information could indeed never hurt. However, in many "real-world" markets where rankings play a role agents' choices are not adequately described as individual decision problems. Consider study programs: students learn from their peers, and the network generated at school is crucial for future professional success. Or consider restaurants: for many customers the value of a dinner is influenced by the identity of the other customers of the restaurant. Hence, these markets are characterized by consumption externalities. Furthermore, in some market prices are not perfectly flexible, leading to rationing. A prototypical example are again study programs, where good programs are typically oversubscribed and hence schools choose among applicants. Finally, even in markets with fully flexible prices and without externalities rankings might have negative welfare impacts, if the existence of a ranking increases the market power of the firms.

To analyze the impact of rankings generated by rationing, externalities, and market power we use a framework with two types of goods that differ in price and quality. Ex-ante it is not known whether the expensive good is the good one. We compare the situations with and without rankings. Since we do not deal with the question of the credibility of rankings, we assume that the ranking (if it exists) is always informative. Furthermore, there are two types of consumers, who differ in terms of appreciation for the goods, and in terms of externalities they inflict on other consumers of the same good. For given prices, we show that without consumption externalities and capacity constraints, the existence of a ranking makes all types of consumers weakly better off, and that it can lead to a pareto improvement. But with externalities or with capacity constraints, the situation is very different. The ranking might lead to an ex-ante pareto deterioration - before the results of the ranking are known, the expected utility of both types of consumers is strictly lower with than without the ranking. If the market exhibits capacity constraints and consumer externalities at the same time, the ranking might even be ex-post detrimental: for all possible ranking outcomes both types of consumers are worse off with than without the ranking. These harmful effects are more likely the more random the rationing procedure is and the more important consumption externalities are.

If one allows for flexible prices and some idiosyncratic preference of each consumer for one of the goods, the existence of a ranking might hurt the consumers even in markets without capacity constraints and consumption externalities. Flexible prices do not necessarily overcome the problem of welfare decreasing rankings. To the contrary, they allow for an additional reason why rankings can hurt consumers. This negative effect is more likely the more important is the good's intrinsic quality - about which the ranking informs the consumers - relative to the consumers' idiosyncratic preferences for a particular good.

It has been argued that (correct) rankings can hurt some consumers. But our paper is one of the first showing that rankings can hurt all consumers. Gavazza and Lizzeri (2007) analyses the impact of rationing on the overall surplus of consumers. In particular, they show that more information about school quality reduces overall surplus when slots for oversubscribed schools are distributed randomly. Unlike our paper no form of pareto-efficiency with respect to the different types of students is considered. Furthermore, this model does not analyze the impact of consumer's externalities and flexible prices.[1] In the context of flexible prices, Anderson and Renault (2009) show that better information might decrease consumer's surplus. But this results is derived within a matching framework, where the information refers to horizontal aspects of the good and not to its quality. Morris and Shin (2002) analyze the impact of public information in a setting in which (i) there is strategic complementary in agents' actions, and (ii) agents hold private information. They show that increased precision of public information may be harmful. The reason is that, because of coordination motivations, agents disregard their private (and more precise) information. More precise public information might thus lead to more "mistakes" by the agents. Though driven by the coordination effect of public information, our results are fundamentally different. First, agents do not have private information to disregard. Second, public information has a coordination effect because it reveals that one product is intrinsically better than the other, not because it indirectly reveals what other agents are going to do. Both in the rationing and in the consumption externality cases, the coordination effect of public infor-

[1] It actually suggests that such a negative effect does not hold when prices are flexible.

mation leads to collateral damages that exists because of the strategic complementaries in consumers' actions.

The rest of the paper is organized as follows. Section 2 lays out the setup. Section 3 analyzes the effects of rankings under the assumption that prices are given and equal to the marginal costs. It identifies two features of market that may make rankings undesirable: capacity constraints and consumption externalities. Section 4 relaxes the assumption of given prices and analyzes the effects of rankings under price flexibility. It shows that rankings may affect firms market power through an increase in products differentiation. Ratings thus affect prices and may ultimately diminish consumers welfare. Section 5 concludes.

2 Setup

We consider a market where each consumer acquires one unit of a good. There are two goods available, and the quality of each good is unknown. If the ranking exists, it provides additional information about the qualities. Consumption of a particular good may lead to externalities for the other consumers of the same brand. Capacity constraints may prevent one firm to fully satisfy the demand.

Goods. Each consumer acquires one unit of one good. There are two goods[2], $m \in M = \{e, c\}$. Both goods are produced by a constant returns to scale technology, with g_m denoting the unit costs of production of good m. We assume that good e has higher production costs, i.e. $g_e > g_c$. The price for good m is denoted by f^m. In Section 3 we will analyze the case were prices are fixed with $f^e > f^c$, whereas in section in Section 4 we study the effects of rankings when firms compete in prices.

One of the two goods is of high intrinsic quality, the other one of low quality. With a commonly known ex-ante probability $\lambda^0 > 0.5$ good e has the higher quality. The quality is measured such that it is proportional to the direct utility gained from consuming the respective good, without taking the (potential) externality into account.

Ranking. If a ranking exists, it is published before the consumers decide which good to request. It (partially) resolves the uncertainty about the intrinsic quality of the good.[3] More precisely, with the commonly known probability $\alpha \in (0.5, 1]$ the ranking reflects the true qualities. In case that good e is rated higher than good c, Bayesian updating implies

[2] The two goods should be viewed as close substitutes, e.g. dinner in two different restaurants, study programs in the same field at two different universities, etc.

[3] In a previous version of the paper we allowed the ranking to also reflects the externalities. All of the main results could be derived with such a formulation, but this analysis is considerably more complicated.

that the probability of good e being the good one is

$$\lambda^1 = \frac{\alpha\lambda^0}{\alpha\lambda^0 + (1-\alpha)(1-\lambda^0)} > \lambda^0. \tag{1}$$

In case the ranking ranks good c higher, the probability of good e being the good one is

$$\lambda^2 = \frac{(1-\alpha)\lambda^0}{(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)} < \lambda^0. \tag{2}$$

We will not consider the exceptional case where α is such that $\lambda^2 = \frac{1}{2}$. We use the superscript $\rho \in \{0, 1, 2\}$ to identify the three different situations with respect to the ranking. A value of $\rho = 0$ refers to a situation without ranking, a value of $\rho = 1$ to a situation in which the ranking ranks good e higher (called *confirmative ranking*), and a value of $\rho = 2$ to a situation in which the ranking ranks good c higher (called *surprise ranking*).

Consumers. There is a continuum $[0, 2]$ of consumers. There are two types of consumers, $t \in T = \{1, 2\}$ with type 1 providing the "better" externalities if any (the exact meaning of the types will be defined below). The consumers' population is composed of equal fractions of these two types.

Strategies. All consumers decide simultaneously which good they request to buy. The set of pure strategies of consumer i is given by:

$$s_i \in \{e, c\}$$

where $s_i = m$ means that consumer i wants to buy good m. We denote by x_1 (x_2) the fraction of type 1 (2) consumers wanting to buy good e.

Capacity constraints and selection process. For simplicity we assume that the supply of each good is the same and denoted by z.[4] To make sure that the whole market can be served, z is larger than 1. If $z \geq 2$ then the whole market can be served just by one good - there are no capacity constraints. If $z < 2$, the provider of a particular good has to select among their potential consumers whenever all the consumers want the same good. We assume that those consumers who do not get their preferred good are willing to buy the other good - it is always better to consume any of the two goods than none of them. Formally, we denote by $n_t(x_1, x_2)$ the mass of consumers of type t who get good e when x_1 type 1 and x_2 type 2 consumer want good e. This function is assumed to be continuous. For simplification, we assume that both types of producers use the same selection process. This implies that $n_t(x_1, x_2) = 1 - n_t(1 - x_1, 1 - x_2)$.

[4] Allowing for different supplies would considerably complicate the notation without changing any of our results.

The probability of an agent to get a particular good depends on his type, on his strategy, and on the strategy distribution of the whole population. Denote by $p_t^m(s_i, x_1, x_2)$ the probability of an agent of type t to get good m when his strategy is s_i and (x_1, x_2) is the strategy distribution of the whole population. Since the whole market can be served by both goods together and each agent gets one good, $p_t^m(s_i, x_1, x_2) = 1 - p_t^{m'}(s_i, x_1, x_2)$ for $m \neq m'$. Hence, we simplify notation by $p_t^e(s_i, x_1, x_2) = p_t(s_i, x_1, x_2)$. In order to further characterize $n_t(x_1, x_2)$ and $p_t(s_i, x_1, x_2)$ we have to distinguish between three cases: $x_1 + x_2 > z$, $z \geq x_1 + x_2 \geq 2 - x_1 - x_2$, and $x_1 + x_2 < 2 - x_1 - x_2$.

If $x_1 + x_2 > z$, there is excess demand for good e. Since the good is sold up to the capacity constraint, it holds that $n_1(x_1, x_2) = z - n_2(x_1, x_2)$. The providers may want to select for one type of consumers, but we assume that this selection process is not perfect. So even if the providers would prefer only consumers of one type, they cannot detect all of them. This implies that $x_1 > n_1 > 0$, $x_2 > n_2 > 0$. If x_1 strictly increases (for given x_2), n_1 strictly increases and n_2 strictly decreases. If x_2 strictly increases (for given x_1), n_1 strictly decreases and n_2 strictly increases. The probabilities of getting the goods are given by:

$$\begin{aligned} p_1(e, x_1, x_2) &= \frac{n_1(x_1, x_2)}{x_1};\ p_2(e, x_1, x_2) = \frac{z - n_1(x_1, x_2)}{x_2}; \\ p_1(c, x_1, x_2) &= p_2(c, x_1, x_2) = 0. \end{aligned}$$

If $z \geq x_1 + x_2 \geq 2 - x_1 - x_2$, there is no excess demand for any of the goods, and each consumer gets the good he wants. This implies that $n_1(x_1, x_2) = x_1$ and $n_2(x_1, x_2) = x_2$. Furthermore

$$\begin{aligned} p_1(e, x_1, x_2) &= p_2(e, x_1, x_2) = 1, \\ p_1(c, x_1, x_2) &= p_2(c, x_1, x_2) = 0. \end{aligned}$$

If $x_1 + x_2 < 2 - x_1 - x_2$, there is excess demand for good c, and $1 - n_1(x_1, x_2) = z - (1 - n_2(x_1, x_2))$. Since the selection process is the same for both types of producers, $1 - n_t(x_1, x_2) = n_t(1 - x_1, 1 - x_2)$. The mass of consumers of type 1 who consume good c, $1 - n_1(x_1, x_2)$, strictly increases in the number of type-1 applications for c, $1 - x_1$, and strictly decreases in the number of type 2 applications for c, $1 - x_2$. The acceptance probabilities are given by

$$\begin{aligned} p_1(e, x_1, x_2) &= p_2(e, x_1, x_2) = 1, \\ p_1(c, x_1, x_2) &= 1 - \frac{n_1(1 - x_1, 1 - x_2)}{1 - x_1};\ p_2(c, x_1, x_2) = 1 - \frac{n_2(1 - x_1, 1 - x_2)}{1 - x_1}. \end{aligned}$$

Note that for all three cases, $n_1(x_1, x_2)$ strictly increases in x_1 and weakly decreases in x_2. Conversely, $n_2(x_1, x_2)$ strictly increases in x_2 and weakly decreases in x_1. Furthermore,

$p_t(e, x_1, x_2) > p_t(c, x_1, x_2)$ - the probability of getting a good is higher when one applies for it than when one does not.

Externalities. Consumption may lead to (positive or negative) externalities. Type 1 consumers create better (i.e. either more positive or less negative) externalities than type 2 consumers. Denote by $q^m(n_1, n_2)$ the externality experienced by a consumers of good m when n_1 (n_2) type 1 (2) consumers consume good e. The externalities are assumed to increase in the number of type 1 and to decrease in the number of type 2 consumers of the good. To simplify we assume that the externalities depend only on the composition of actual consumers and not on the good of the good, i.e. $q^e(n_1, n_2) = q^c(1 - n_1, 1 - n_2)$. Let $q(x_1, x_2) = q^e(n_1(x_1, x_2), n_2(x_1, x_2)) - q^c(n_1(x_1, x_2), n_2(x_1, x_2))$. Since the number of type 1 (2) consumers of good e strictly increases in x_1 (x_2) and weakly decreases in x_2 (x_1), it holds that $q(x_1, x_2)$ strictly increases in x_1 and strictly decreases in x_2. For the case without externalities, we have $q^e(n_1, n_2) = 0$.

Utility. Denote by b_t^h (b_t^l) the evaluation by a type t consumer of the intrinsic quality of a good when this quality is high (low), with $b_t^h > b_t^l$. The expected utility of applying for good s_i depends on whether the consumer actually gets the good e or c, whether this good is of high/low intrinsic quality, on how the consumer values this intrinsic quality, and on the externalities (i.e. the behavior of other consumers). Formally

$$\begin{aligned} u_t^\rho(s_i, x_1^\rho, x_2^\rho) = \\ & p_t(s_i, x_1^\rho, x_2^\rho)\left(-f^e + \lambda^\rho b_t^h + (1-\lambda^\rho) b_t^l + q^e(n_1(x_1, x_2), n_2(x_1, x_2))\right) + \\ & (1 - p_t(s_i, x_1^\rho, x_2^\rho))\left(-f^c + (1-\lambda^\rho) b_t^h + \lambda^\rho b_t^l + q^c(n_1(x_1, x_2), n_2(x_1, x_2))\right). \end{aligned} \tag{3}$$

3 Capacity Constraints and Externalities

In this section, we study the case where prices are fixed, with good e being more expensive than good c (i.e. $f^e > f^c$). With fixed prices, we show that rankings are always welfare improving provided that there are no capacity constraints or consumption externalities. But the presence of capacity constraints or consumption externalities rankings can lead to an *ex-ante* Pareto deterioration. And with a combination of both capacity constraints and consumption externalities the rankings might be Pareto-dominated even from an *ex-post* viewpoint.

3.1 Equilibrium Analysis

We characterize the set of equilibria for generic parameter values (i.e. we exclude the case in which $b_1^h - b_1^l = b_2^h - b_2^l$) when prices are given.

Since $p_t(e, x_1, x_2) > p_t(c, x_1, x_2)$ it holds that $u_t(e, x_1, x_2) \geq u_t(c, x_1, x_2)$ whenever

$$\begin{aligned} & -f^e + \lambda b_t^h + (1-\lambda) b_t^l + q^e(n_1(x_1, x_2), n_2(x_1, x_2)) \\ \geq \quad & -f^c + (1-\lambda) b_t^h + \lambda b_t^l + q^c(n_1(x_1, x_2), n_2(x_1, x_2)). \end{aligned} \tag{4}$$

Letting $f = f^e - f^c > 0$ and $b_t = b_t^h - b_t^l > 0$, and $q(x_1, x_2) = q^e(n_1(x_1, x_2), n_2(x_1, x_2)) - q^c(n_1(x_1, x_2), n_2(x_1, x_2))$, the equilibrium condition becomes:

$$b_t(2\lambda - 1) + q(x_1, x_2) - f \geq 0, \tag{5}$$

The following Lemma follows directly from condition (5).

Lemma 1 *The Nash equilibria of the games with and without ranking, $x^{\rho*} = (x_1^{1*}, x_2^{1*}, x_1^{2*}, x_2^{2*})$ and $x^{0*} = (x_1^{0*}, x_2^{0*})$, fulfill the following conditions:*

(i) $x_1^{\rho*} = 0 \Rightarrow -f + b_1(2\lambda^\rho - 1) + q(x_1^{\rho*}, x_2^{\rho*}) \leq 0$,

(ii) $x_1^{\rho*} \in (0,1) \Rightarrow -f + b_1(2\lambda^\rho - 1) + q(x_1^{\rho*}, x_2^{\rho*}) = 0$,

(iii) $x_1^{\rho*} = 1 \Rightarrow -f + b_1(2\lambda^\rho - 1) + q(x_1^{\rho*}, x_2^{\rho*}) \geq 0$,

(iv) $x_2^{\rho*} = 0 \Rightarrow -f + b_2(2\lambda^\rho - 1) + q(x_1^{\rho*}, x_2^{\rho*}) \leq 0$,

(v) $x_2^{\rho*} \in (0,1) \Rightarrow -f + b_2(2\lambda^\rho - 1) + q(x_1^{\rho*}, x_2^{\rho*}) = 0$,

(vi) $x_2^{\rho*} = 1 \Rightarrow -f + b_2(2\lambda^\rho - 1) + q(x_1^{\rho*}, x_2^{\rho*}) \geq 0$,

for any $\rho \in \{0, 1, 2\}$.

Combining this result with condition (5), we obtain the following Lemma:

Lemma 2 *For any $\rho \in \{0, 1, 2\}$, we have that:*

i) If $b_1(2\lambda^\rho - 1) > b_2(2\lambda^\rho - 1)$, then $x_2^{\rho*} > 0$ implies $x_1^{\rho*} = 1$.

ii) If $b_1(2\lambda^\rho - 1) < b_2(2\lambda^\rho - 1)$, then $x_2^{\rho*} < 1$ implies $x_1^{\rho*} = 0$.

Using this Lemma, we can characterize the set of equilibria for generic parameter values:

Proposition 1 *The equilibria of the game with and without rankings, i.e. $x^{\rho*}$ and x^{0*}, are fully characterized by the following conditions for generic parameter values:*

1. *Whenever $b_1(2\lambda^\rho - 1) > b_2(2\lambda^\rho - 1)$:*

   (a) *If $b_2(2\lambda^\rho - 1) + q(1,1) > f$, then $x_1^{\rho*} = x_2^{\rho*} = 1$;*

(b) *If* $b_2(2\lambda^\rho - 1) + q(1,0) > f$ *and* $b_2(2\lambda^\rho - 1) + q(1,1) < f$*, then* $x_1^{\rho*} = 1$ *and* $x_2^* \in (0,1)$ *solves* $b_2(2\lambda^\rho - 1) + q(1, x_2^{\rho*}) = f$*;*

(c) *If* $b_1(2\lambda^\rho - 1) + q(1,0) > f$ *and* $b_2(2\lambda^\rho - 1) + q(1,0) < f$*, then* $x_1^{\rho*} = 1$ *and* $x_2^{\rho*} = 0$*;*

(d) *If* $b_1(2\lambda^\rho - 1) + q(1,0) > f$ *and* $b_1(2\lambda^\rho - 1) + q(0,0) < f$*, then* $x_2^{\rho*} = 0$ *and* $x_1^* \in (0,1)$ *solves* $b_1(2\lambda^\rho - 1) + q(x_1^{\rho*}, 0) = f$*;*

(e) *If* $b_1(2\lambda^\rho - 1) + q(0,0) < f$*, then* $x_1^{\rho*} = x_2^{\rho*} = 0$*.*

2. *Whenever* $b_1(2\lambda^\rho - 1) < b_2(2\lambda^\rho - 1)$*:*

(a) *If* $b_1(2\lambda^\rho - 1) + q(1,1) > f, x_1^{\rho*} = x_2^{\rho*} = 1$*;*

(b) *If* $b_1(2\lambda^\rho - 1) + q(0,1) < f$ *and* $b_1(2\lambda^\rho - 1) + q(1,1) > f$*, then* $x_2^{\rho*} = 1$ *and* $x_1^* \in (0,1)$ *solves* $b_1(2\lambda^\rho - 1) + q(x_1^{\rho*}, 1) = f$*;*

(c) *If* $b_1(2\lambda^\rho - 1) + q(0,1) < f$ *and* $b_2(2\lambda^\rho - 1) + q(0,1) > f$*, then* $x_1^{\rho*} = 0$ *and* $x_2^{\rho*} = 1$*;*

(d) *If* $b_2(2\lambda^\rho - 1) + q(0,1) > f$ *and* $b_2(2\lambda^\rho - 1) + q(0,0) < f$*, then* $x_1^{\rho*} = 0$ *and* $x_2^* \in (0,1)$ *solves* $b_2(2\lambda^\rho - 1) + q(0, x_2^{\rho*}) = f$*;*

(e) *If* $b_1(2\lambda^\rho - 1) + q(0,0) < f$*, then* $x_1^{\rho*} = x_2^{\rho*} = 0$*.*

Proof: see Appendix

It can be easily checked that Proposition 1 guarantees the existence of pure strategy equilibria. Furthermore, without externalities, i.e. if q is a constant, for generic parameter values the equilibrium of each of the games is unique and symmetric, i.e. all consumers of the same type request the same good.

3.2 Welfare analysis

For the welfare analysis, we will not take into account which of the individual agents consumes which good, since this is generally also determined by the random selection process. Rather, the welfare analysis will investigate the allocation of goods between the two types, i.e. n_1 and n_2, and the resulting average payoff of an agent of a particular type. Denote by $n^0 = (n_1^0, n_2^0)$, $n^1 = (n_1^1, n_2^1)$, $n^2 = (n_1^2, n_2^2)$ the allocations without ranking, with confirmative ranking (e is ranked better than c), and with surprise ranking (c ranked higher than e) respectively. Since the population of each type is 1, n_t^ρ denotes also the probability that a consumer of type t consumes good e for $\rho = 0, 1, 2$. The expected payoffs without, with confirmative, and with surprise ranking are given by:

$$\begin{aligned} U_t^0(n^0) &= \lambda^0[n_t^0\left(-f^e+b_t^h+q^e(n_1^0,n_2^0)\right)+(1-n_t^0)\left(-f^c+b_t^l+q^c(n_1^0,n_2^0)\right)]+ \\ & \left(1-\lambda^0\right)[n_t^0\left(-f^e+b_t^l+q^e(n_1^0,n_2^0)\right)+(1-n_t^0)\left(-f^c+b_t^h+q^c(n_1^0,n_2^0)\right)] \quad (6) \\ U_t^1(n^1) &= \lambda^1\left[n_t^1\left(-f^e+b_t^h+q^e(n_1^1,n_2^1)\right)+(1-n_t^1)\left(-f^c+b_t^l+q^c(n_1^1,n_2^1)\right)\right]+ \\ & +(1-\lambda^1)\left[n_t^1\left(-f^e+b_t^l+q^e(n_1^1,n_2^1)\right)+(1-n_t^1)\left(-f^c+b_t^h+q^c(n_1^1,n_2^1)\right)\right] (7) \\ U_t^2(n^2) &= \lambda^2\left[n_t^2\left(-f^e+b_t^h+q^e(n_1^2,n_2^2)\right)+(1-n_t^2)\left(-f^c+b_t^l+q^c(n_1^2,n_2^2)\right)\right]+ \\ & +(1-\lambda^2)\left[n_t^2\left(-f^e+b_t^l+q^e(n_1^2,n_2^2)\right)+(1-n_t^2)\left(-f^c+b_t^h+q^c(n_1^2,n_2^2)\right)\right] (8) \end{aligned}$$

Using these expected payoffs and taking into account the possible multiplicity of equilibria we introduce the following efficiency notion (remember that α is the probability that the ranking reflects the true qualities):

Definition 1 *The presence of the ranking ex-ante dominates its absence if for all equilibrium values of n^0, n^1, and n^2 and for both t the following holds:*

$$\left[\lambda^0\alpha+(1-\lambda^0)(1-\alpha)\right]U_t^1(n^1,n^2)+\left[(1-\alpha)\lambda^0+\alpha(1-\lambda^0)\right]U_t^2(n^1,n^2)\geq U_t^0(n^0), \quad (9)$$

with strict inequality for one of these inequalities.
The absence of the ranking ex-ante dominates its presence when the opposite holds.

Using this efficiency notion, we get the following proposition:

Proposition 2 *Without consumption externalities and without capacity constraint, for any equilibrium $x_t^*=(x_t^{0*},x_t^{1*},x_t^{2*})$ for generic values of b_t^l, b_t^h, f^e, f^c,λ^0, and α and for $t\in\{1,2\}$:*
(i) Condition (9) is satisfied (with strict inequality whenever $x_t^{0*}\neq x_t^{1*}$ or $x_t^{0*}\neq x_t^{2*}$).
(ii) There is an open set of parameter values such that the ranking ex-ante dominates its absence.

Proof. See Appendix ■

Without capacity constraints and without externalities, the "game" is actually an individual decision problem without strategic interaction. Therefore, better information can never hurt.

This result does not hold any longer in the presence of the capacity constraints, as the following proposition shows:

Proposition 3 *In the presence of capacity constraints there is an open set of parameter values b_t^l, b_t^h, f^e, f^c, λ^0, α, z, $n_1(1,1)$, and $n_1(0,0)$ such that the absence of a ranking ex-ante dominates its presence.*

Proof. Take the parameter values $f^c = 0, f^e = 2$, $b_1^l = b_2^l = 1$, $b_1^h = 5$, $b_2^h = 3.2$, $\alpha = 1$, $\lambda^0 = 0.85$, and $z = 1.2$. Furthermore, assume that there are no externalities, and that the selection is characterized by $n_1(1,1) = 0.6$ and $n_1(0,0) = 0.4$. Because of (5), these parameter values imply that in equilibrium $x_1^0 = x_1^1 = 1$, $x_1^2 = 0$ and $x_2^0 = x_2^2 = 0$, $x_2^1 = 1$. Because of the selection this implies that $n_1^0 = 1$, $n_1^1 = 0.6$, $n_1^2 = 0.4$ and $n_2^0 = 0$, $n_2^1 = 0.6$, and $n_2^2 = 0.4$. Inserting into (6), (7), and (8) gives:

$$
\begin{aligned}
U_1^0(n^0) &= 2.4; \quad \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_1^1(n^1,n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_1^2(n^1,n^2) = 2.26 \\
U_2^0(n^0) &= 1.33; \quad \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_2^1(n^1,n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_2^2(n^1,n^2) = 1.18.
\end{aligned}
$$

So both types of consumers are better off without the ranking. Since both inequalities are strict, this holds for an open set of parameters. ■

The intuition for this result is as follows. Without the ranking, consumers of the two types request different goods. They therefore all get the good they request. With ranking, all consumers request good e (in case of the confirmative ranking) or good c (in case of the surprise ranking). Type 1 consumers cannot lose in case of a surprise ranking, because those type 1 who get good c in case of a ranking benefit whereas the situation of the other type 1 does not change due to the surprise ranking. But type 1 consumers lose in case of a confirmative ranking, since now some of them do no longer get good e due to rationing, and the situation of the other type 1 consumers is unchanged. If the negative effect of the confirmative ranking outweights the positive effect of the surprise ranking, type 1 consumers lose in expectations. The situation of type 2 consumers is the mirror image of that of type 1 consumers. For a surprise ranking they lose, because now not all of them get the cheap and high quality good any more. They win in case of a confirmative ranking, since in this case good e is actually desirable, and some of them get it. But the loss due to a surprise ranking might outweight the win due to a confirmative ranking. The example of the proof shows that for some parameter constellations the negative effect is stronger for both types of consumers than the positive ones.

The example in the proof of the previous proposition assumes that producers do not discriminate against any type of consumers. Due to the continuity of all functions the results would not change if the producers select to some extent in favor of one type of consumers. But if there is perfect discrimination in favor of one type of consumer, the ranking cannot be dominated by its absence.

Proposition 4 *For generic values of* b_t^l, b_t^h, f^e, f^c, λ^0 *and* α *the absence of the ranking does not ex-ante dominate its presence when all the producers do discriminate perfectly in favor of one type of consumers.*

Proof. Let t be the type of consumer for which it is discriminated for. This implies every consumer of type t gets the good he wants, i.e. $n_t^\rho = x_t^\rho$. Therefore, the proof of proposition 2 holds for type t. ■

We have seen an example where both types of consumers are strictly better off without the ranking in the presence of capacity constraints and without perfect discrimination. Without capacity constraints, but with externalities one can get the same result.

Proposition 5 *With externalities there is an open set of parameter values* b_t^l, b_t^h, f^e, f^c, λ^0, α,$q^e(1,0)$, $q^e(1,1)$,$q^e(0,0)$, $q^c(1,0)$, $q^c(1,1)$, $q^c(0,0)$ *such that the absence of a ranking ex-ante dominates its presence.*

Proof. Assume that there are no capacity constraints, and take the parameter values $f^c = 0$, $f^e = 0.35$, $b_1^l = b_2^l = 1$, $b_1^h = 1.54$, $b_2^h = 1.3$, $\alpha = 1$, $\lambda^0 = 0.9$, $q^e(1,0) = q^c(0,1) = 0.8$, $q^e(1,1) = q^c(0,0) = 0.71$, $q^e(0,0) = q^c(1,1) = 0.65$, and $q^e(0,1) = q^c(1,0) = 0.7$. Because of (5), these parameter values imply that the unique equilibrium is such that: $x_1^0 = x_1^1 = 1$, $x_1^2 = 0$ and $x_2^0 = x_2^2 = 0$, $x_2^1 = 1$.

Because there is no capacity constraint, $n_t^\rho = x_t^\rho\ \forall t, \rho$. Inserting into (6), (7) , and (8) gives:

$$U_1^0(n^0) = 1.936; \quad \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_t^1(n^1,n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_t^2(n^1,n^2) = 1.935$$
$$U_2^0(n^0) = 1.73; \quad \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_t^1(n^1,n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_t^2(n^1,n^2) = 1.695.$$

So both types of consumers are better off without the ranking. Since both inequalities are strict, this holds for an open set of parameters. ■

As for the case with capacity constraints, the ex-ante dominance of the absence of ranking occurs for parameter values such that (i) without ranking the type 1 consumers go for good e and type 2 consumers go for good c, (ii) with confirmative ranking both types go for good e, and (iii) with surprise ranking both types go for good c.

Compared to the absence of the ranking, type 1 consumers lose in case of a confirmative ranking, since they experience worse externalities. With a surprise ranking the effect is ambiguous on type 1 consumers - they avoid the expensive and bad good, but they suffer from worse externalities. Still, they do not want to switch to good e because having no peer is worse than having all consumers as peers. The overall effect is negative for type 1 consumers. Type 2 consumers gain for sure in case a surprise ranking, because they still buy the cheap high quality good, but now they have better peers. In case of the an confirmative ranking, the effect on type 2 consumers is ambiguous. They now get the better good and better peers, but have to pay the high price. For some parameter constellations it would have been better for them if all of them stayed with good c than all of them buying good e, but because of the revealed high quality of good e, it is individually rational for them to deviate to good e, making all of them worse off. This effect might dominate all the positive effects of the ranking for type 2 consumers. Furthermore, recall that without externalities the ranking can never hurt. Hence, the negative effect of the ranking is more likely the larger the externalities are.

Till now, we have only looked at the ex-ante dominance relation. We found that in presence of the externalities or capacity constraints the absence of ranking may be better for both types of consumers. As suggested by the intuition of Propositions 3 and 5, one could suspect that this is true because the ranking hurts the consumers for one type of ranking outcome (e.g. for the confirmative ranking), and is beneficial for the other outcome (e.g. for the surprise ranking), with the loss in the first case dominating the gain of the second one. One might suspect that that each type of consumer always benefits from a ranking for one type of ranking outcome. But this intuition is wrong whenever there are externalities and rationing at the same time. To see this, take the following efficiency notion:

Definition 2 *The presence of the ranking ex-post dominates its absence if for all equilibrium values of n^0, n^1, n^2 and for both t the following two conditions hold:*

$$\begin{aligned} \textit{i)}\ U_t^1(n^1, n^2) &\geq U_t^0(n^0) \\ \textit{ii)}\ U_t^2(n^1, n^2) &\geq U_t^0(n^0) \end{aligned}$$

with strict inequality for one of these inequalities.
The absence of the ranking ex-post dominates its presence when the opposite holds.

Note that ex-post dominance implies ex-ante dominance, but not vice versa.

Using this ex-post efficiency notion, we get the following proposition:

Proposition 6 *In the presence of capacity constraints and externalities, there is an open set of parameter values b_t^l, b_t^h, f^e, f^c, λ^0, α, z, $n_1(1,1)$, $n_1(1,0)$, $q^e(1,0)$, $q^e(1,1)$,$q^e(0,0)$, $q^c(1,0)$, $q^c(1,1)$, and $q^c(0,0)$ such that, at the ex-post stage, the absence of a ranking dominates its presence.*

Proof. Take the parameter values $f^c = 0$, $f^e = 1.6$, $b_1^l = b_2^l = 1$, $b_1^h = 3.4$, $b_2^h = 2.37$, $\alpha = 0.79$, $\lambda^0 = 0.9$, $z = 1.35$, $n_1(1,1) = 0.96$, $n_1(1,0) = 1$, $n_1(0,0) = 0.04$, $n_2(1,1) = 0.39$, $n_2(0,0) = 0.61$,

$q^e(1,0) = q^c(0,1) = 1.78$, $q^e(1,1) = q^c(0,0) = 1.6413$, $q^e(0,0) = q^c(1,1) = 1.3295$, and $q^e(0,1) = q^c(1,0) = 1.3$.[5]

Because of (5), these parameter values imply that the unique equilibrium is such that: $x_1^0 = x_1^1 = 1$, $x_1^2 = 0$ and $x_2^0 = x_2^2 = 0$, $x_2^1 = 1$.

It remains to check that (i) $U_t^1(n^1, n^2) < U_t^0(n^0)$, and (ii) $U_t^2(n^1, n^2) < U_t^0(n^0)$ for all consumers:

$$U_1^0(n^0) = 3.34; \qquad U_2^0(n^0) = 2.437;$$
$$U_1^1(n^1, n^2) = 3.3335; \qquad U_2^1(n^1, n^2) = 2.3701;$$
$$U_1^2(n^1, n^2) = 3.3117; \qquad U_2^2(n^1, n^2) = 2.222$$

So, at the ex-post stage, both types of consumers are better off without the ranking. Since all inequalities are strict, this holds for an open set of parameters. ■

Why are type 1 consumers made worse off? This is easier to understand if we split the discussion in two parts: we compare (i) the situations without ranking and with a confirmative ranking, and (ii) the situations without ranking and with a surprise ranking. In case (i), type 1 consumers who obtain good e (the one they request) are made better off but this effect is attenuated by the externalities. Type 1 consumers who do not get good e are made worse off both because of the revealed low quality of good c and because of the externalities (there are more type 2 consuming good c than good e). Even if this is relatively unlikely for type 1 consumers not to obtain good e, the negative effect is so large that it dominates the positive one. The net effect is thus negative. In case (ii), type 1 consumers who obtain good c (the one they request) are made better off but this effect is attenuated by the externalities. Type 1 consumers who do not get good c are made worse off both because they pay a high price for a good of revealed low quality and because of the externalities. Even if this is relatively unlikely for type 1 consumers not to obtain good e, the negative effect is so large that it dominates the positive one. The net effect is thus negative.

Why are type 2 consumers made worse off? Again, we split the discussion in two parts: we compare (i) the situations without ranking and with a confirmative ranking, and (ii) the situations without ranking and with a surprise ranking. In case (i), type 2 consumers who obtain good e (the one they request) are made worse off. This is so because without the ranking, type 2 consumers were paying a low price for a good who was perceived as of not too low quality. With the confirmative ranking, they consume good e. Even if this good is revealed of higher quality (and the externalities are also better), this is not enough to

[5] These qs are not arbitrarily chosen. They are the result of the following assumptions about the selection of consumers in case there is excess demand. First, consumers submit their requests. Second, firms screen the consumers and accept those identified as type 1 and reject those identified as type 2. If after this step the firm has the capacity to serve more consumers, it select randomly among the consumers with unidentified type until its capacity constraint is binding.

compensate for the price increase. Those who do not get good e are also made worse off even if they gain a bit in term of externalities (some type 1 consume good c). This is so because they loose a lot in term of quality. Bottom line: type 2 consumers are made worse off by the confirmative ranking because it prevent them to benefit from decent quality at a low price. In case (ii), type 2 consumers who obtain good c (the one they request) are made better off: they gain both in quality and in the externalities. Those who do not obtain good c are made worse off: they pay a high price for a low quality-low externality good. There are two factors which make the net effect negative. First, there is a larger fraction of type 2 consumers who do not obtain the good they request. Second, because of the price effect, the worse off effect on consumers who do not obtain good c is larger than the better off effect on consumers who do obtain good c.

Overall, we see that for fixed price capacity constraints and/or externalities might induce rankings to hurt consumers. One might suspect that this is due to fixed prices. But the following section will show that flexible prices might allow for an additional negative effect of rankings.

4 Price Competition

In this section, we analyze the effects of rankings on consumers welfare when two firms compete in prices. We consider markets where there are neither externalities, nor capacity constraints. We show that rankings may affect consumers welfare either positively or negatively (from an ex ante and an ex-post viewpoint). The reason for the negative effect is that more accurate information changes consumers' perception of products differentiation. This increases the market power of the firms and affects consumers welfare negatively. Interestingly, our results suggests some general conditions under which this effects is sufficiently strong to dominate the positive effects of information, i.e. reduction in the risks of choosing the wrong product.

Firms are setting prices simultaneously. If there is a ranking, they fix prices after the ranking has been published but before consumers choose which good to request. In order to concentrate on the impact of rankings, we assume that that there is no asymmetric information about the quality of the product. When making their choices, firms have the same information as consumers. Firms know only the ex-ante probability of providing that there good is highly appreciated by the consumers, and they know the outcome of the ranking if it exists. Therefore, prices do not signal any asymmetric information about the quality of the good. They depend only on the identity of the firm and on the ranking situation. Denote by f_m^ρ the price of good m in ranking situation ρ. Recall that the unit production costs of good e, g_e, are higher that those of good c, g_c.

To ensure the existence of an equilibrium, we need to slightly modify our model by introducing some heterogeneity in the evaluation by consumer of the intrinsic quality of a good. Importantly, all our previous results can be reproduced with such heterogeneity. The intuition is straightforward: if the heterogeneity is very small, for most values of the parameters all consumers of a given type request the same good. The model with heterogeneity then produces similar results to those of the model described in Section 2.

As before, we consider a continuum $[0, 2]$ of consumers, two types of consumers, $t \in T = \{1, 2\}$, and a consumers' population which is composed of equal fractions of these two types. The utility of a consumer i of type-t who buy good c is still:

$$L_t^\rho - f_c^\rho,$$

where $L_t^\rho = \lambda^\rho b_t^l + (1 - \lambda^\rho) b_t^h$. The novelty is that the utility of a consumer i of type-t who buy good e is:

$$H_t^\rho + y_{t,i} - f_e^\rho,$$

where, $H_t^\rho = \lambda^\rho b_t^h + (1 - \lambda^\rho) b_t^l$, and $y_{t,i}$ is distributed uniformly between $-\delta$ and δ. Therefore, a consumer i of type-t request good e if:

$$y_{t,i} > f^\rho - b_t(2\lambda^\rho - 1),$$

where $b_t = b_t^h - b_t^l$, and $f^\rho = f_e^\rho - f_c^\rho$.

4.1 Equilibrium Analysis

Let $\psi^\rho = (2\lambda^\rho - 1)(b_1 + b_2)$. The equilibrium analysis is divided in two parts: if $\frac{g}{3} + \frac{\psi^\rho}{3} - b_t(2\lambda^\rho - 1) \in [-\delta, \delta]$ $\forall t, \rho$ and if $\exists t, \rho$ s.t. $\frac{g}{3} + \frac{\psi^\rho}{3} - b_t(2\lambda^\rho - 1) \notin [-\delta, \delta]$.

4.1.1 If $\frac{g}{3} + \frac{\psi^\rho}{3} - b_t(2\lambda^\rho - 1) \in [-\delta, \delta]$ $\forall t, \rho$

We solve the game backward. Ignoring for the moment the fact that the demand cannot be below zero and above 2, the demand for good e is:

$$\begin{aligned} d_e\left(f_e^\rho, f_c^\rho\right) &= \underbrace{1 - \frac{f^\rho - b_1(2\lambda^\rho - 1) + \delta}{2\delta}}_{\text{Fraction of type-1 with } y_1 \geq f^\rho - b_1(2\lambda^\rho - 1)} + \underbrace{1 - \frac{f^\rho - b_2(2\lambda^\rho - 1) + \delta}{2\delta}}_{\text{Fraction of type-2 with } y_2 \geq f^\rho - b_2(2\lambda^\rho - 1)} \\ &= 1 - \frac{f^\rho}{\delta} + \frac{\psi^\rho}{2\delta}. \end{aligned}$$

where f^ρ denotes the price differentials for $\rho \in \{0, 1, 2\}$.

The demand for good c is:

$$d_c\left(f_e^\rho, f_c^\rho\right) = 1 + \frac{f^\rho}{\delta} - \frac{\psi^\rho}{2\delta}.$$

There are of course parameter values such that $d_e\left(f_e^\rho, f_c^\rho\right)$ and/or $d_c\left(f_e^\rho, f_c^\rho\right) \notin [0,2]$. But we will show that in equilibrium $d_e\left(f_e^\rho, f_c^\rho\right)$ and $d_c\left(f_e^\rho, f_c^\rho\right) \in [0,2]$ whenever $\frac{g}{3}+\frac{\psi^\rho}{3}-b_t(2\lambda^\rho-1) \in [-\delta,\delta]$ $\forall t,\rho$ (which is the case we are discussing in this subsection).

Given these demand functions, we can derive the best responses of both firms to the strategy of the other by maximizing profit, denoted by $\pi_m\left(f_e^\rho, f_c^\rho\right)$. For good e, we have the following best response function:

$$f_e^\rho\left(f_c^\rho\right) = \frac{\delta}{2} + \frac{f_c^\rho}{2} + \frac{\psi^\rho}{4} + \frac{g_e}{2}. \tag{10}$$

For good c, we have:

$$f_c^\rho\left(f_e^\rho\right) = \frac{\delta}{2} + \frac{f_e^\rho}{2} - \frac{\psi^\rho}{4} + \frac{g_c}{2}. \tag{11}$$

Using (10) and (11), we find the equilibrium prices (given that the aforementioned assumption is satisfied):

$$f_e^{\rho,*} = \delta + \frac{\psi^\rho}{6} + \frac{2g_e+g_c}{3}, \tag{12}$$

$$f_c^{\rho,*} = \delta - \frac{\psi^\rho}{6} + \frac{2g_c+g_e}{3}. \tag{13}$$

For the ranking situation ρ, we can compute the expected utility of a randomly selected type-t consumer given the equilibrium prices $\left(f_e^{\rho,*}, f_c^{\rho,*}\right)$:

$$U_t^\rho\left(f_e^{\rho,*}, f_c^{\rho,*}\right) = \int_0^{f^{\rho,*}-b_t(2\lambda^\rho-1)} \left(L_t^\rho - f_c^{\rho,*}\right)\frac{1}{2\delta}dy_{t,i} + \int_{f^\rho-b_t(2\lambda^\rho-1)}^{\delta} \left(H_t^\rho + y_{t,i} - f_e^{\rho,*}\right)\frac{1}{2\delta}dy_{t,i}.$$

This boils down to.

$$\begin{aligned} U_t^\rho\left(f_e^{\rho,*}, f_c^{\rho,*}\right) &= \frac{L_t^\rho - f_c^{\rho,*}}{2\delta}\left(f^{\rho,*} - b_t(2\lambda^\rho-1)+\delta\right) + \frac{H_t^{\rho,*} - f_e^{\rho,*}}{2\delta}\left(\delta - f^{\rho,*} + b_t(2\lambda^\rho-1)\right) \\ &\quad + \frac{1}{2\delta}\left(\frac{\delta^2}{2} - \frac{\left(f^{\rho,*}-b_t(2\lambda^\rho-1)\right)^2}{2}\right). \end{aligned}$$

4.1.2 If $\exists t,\rho$ s.t. $\frac{g}{3}+\frac{\psi^\rho}{3}-b_t(2\lambda^\rho-1) \notin [-\delta,\delta]$

The main difference to the previous case concerns the equilibrium prices. There are many potential combinations of t and ρ for which $\frac{g}{3}+\frac{\psi^\rho}{3}-b_t(2\lambda^\rho-1) \notin [-\delta,\delta]$. The analysis of the different cases feature great similitude. To avoid redundancies, we detail only two of these cases (those that will be used in Propositions 7 and 8): Case (I) $\frac{g}{3}+\frac{\psi^1}{3}-b_1(2\lambda^1-1) < -\delta$, and $\frac{g}{3}+\frac{\psi^\rho}{3}-b_t(2\lambda^\rho-1) \in [-\delta,\delta]$ for all other ρ and t, and Case (II) $\frac{g}{3}+\frac{\psi^1}{3}-b_1(2\lambda^1-1) < -\delta$, $\frac{g}{3}+\frac{\psi 2}{3}-b_1(2\lambda^2-1) > \delta$, and $\frac{g}{3}+\frac{\psi^\rho}{3}-b_t(2\lambda^\rho-1) \in [-\delta,\delta]$ for all other ρ and t.

Case (I) For the confirmative ranking $\rho = 1$, the demand for good e is

$$d_e\left(f_e^1, f_c^1\right) = \frac{3}{2} - \frac{f^1}{2\delta} + \frac{b_2\left(2\lambda^1 - 1\right)}{2\delta}, \tag{14}$$

whereas the demand for good c is

$$d_c\left(f_e^1, f_c^1\right) = \frac{1}{2} + \frac{f^1}{2\delta} - \frac{b_2\left(2\lambda^1 - 1\right)}{2\delta}, \tag{15}$$

because in equilibrium (as we will later show) $\frac{g}{3} + \frac{\psi^1}{3} - b_1(2\lambda^1 - 1) < -\delta$, i.e. all type-1 consumers (no matter the $y_{1,i}$) apply to good e when $\rho = 1$. This implies that $1 - F\left(f^1 - b_1\left(2\lambda^1 - 1\right)\right) = 1$.

Using (14) and (15), we find the best responses when $\rho = 1$: for good e

$$f_e^1\left(f_c^1\right) = \frac{3\delta}{2} + \frac{g_e}{2} + \frac{f_c^1}{2} + \frac{b_2\left(2\lambda^1 - 1\right)}{2}, \tag{16}$$

for good c

$$f_c^1\left(f_e^1\right) = \frac{g_c}{2} + \frac{\delta}{2} + \frac{f_e^1}{2} - \frac{b_2\left(2\lambda^1 - 1\right)}{2}. \tag{17}$$

For other ranking situations, i.e. $\rho \neq 1$, the best response functions are given by (10) and (11).

For equilibrium prices, there are two cases: (i) the prices $\left(f_e^{1,*}, f_c^{1,*}\right)$ that we get by using (16) and (17) are such that $f^{1,*} - b_1(2\lambda^1 - 1) < -\delta$, and (ii) the prices $\left(f_e^{1,*}, f_c^{1,*}\right)$ that we get by using (16) and (17) are such that $f^{1,*} - b_1(2\lambda^1 - 1) \in [-\delta, \delta]$. Since our results are based on case (i), we do not investigate case (ii) further.[6]

In case (i), the demand for good e and c are given by (14) and (15) respectively. Therefore, equilibrium prices are:

$$f_e^{1,*} = \frac{7}{3}\delta + \frac{2g_e + g_c}{3} + \frac{b_2\left(2\lambda^1 - 1\right)}{3}, \tag{18}$$

$$f_c^{1,*} = \frac{5}{3}\delta + \frac{g_e + 2g_c}{3} - \frac{b_2\left(2\lambda^1 - 1\right)}{3}. \tag{19}$$

We find those using (16) and (17).

For the ranking situation $\rho = 1$, we can also compute the expected utility of a randomly selected type-t consumer given the equilibrium prices $\left(f_e^{1,*}, f_c^{1,*}\right)$:

$$U_t^1\left(f_e^{1,*}, f_c^{1,*}\right) = \underbrace{\int_{-\delta}^{f^{1,*} - b_t(2\lambda^1 - 1)} \left(L_t^1 - f_c^{1,*}\right)\frac{1}{2\delta}dy_{t,i}}_{=0 \text{ since } f^{1,*} - b_t(2\lambda^1 - 1) < -\delta}$$
$$+ \int_{f^{1,*} - b_t(2\lambda^1 - 1)}^{\delta} \left(H_t^1 + y_{1,i} - f_e^{1,*}\right)\frac{1}{2\delta}dy_{t,i}$$

[6] An analysis of case (ii) is available upon request.

This boils down to

$$U_t^1\left(f_e^{1,*},f_c^{1,*}\right)=\int_{-\delta}^{\delta}\left(H_t^1+y_{t,i}-f_e^{1,*}\right)\frac{1}{2\delta}dy_{t,i}=H_t^1-f_e^{1,*}.$$

For $\rho=0$ and $\rho=2$ the results are the same as those described in section 4.1.1.

Case (II) For the surprise ranking $\rho=2$, the demand for good e is:

$$d_e\left(f_e^2,f_c^2\right)=\frac{1}{2}-\frac{f^2}{2\delta}+\frac{b_2\left(2\lambda^2-1\right)}{2\delta}. \tag{20}$$

The demand for good c is

$$d_c\left(f_e^2,f_c^2\right)=\frac{3}{2}+\frac{f^2}{2\delta}-\frac{b_2\left(2\lambda^2-1\right)}{2\delta}. \tag{21}$$

This comes from the assumption (that will be confirmed in equilibrium) that $\frac{g}{3}+\frac{\psi 2}{3}-b_1(2\lambda^2-1)>\delta$, i.e. all type-1 consumers (no matter the $y_{1,i}$) apply to good c when $\rho=2$. This implies that $F\left(f^2-b_1\left(2\lambda^2-1\right)\right)=1$.

Using (20) and (21), we find the best responses when $\rho=2$: for firm e

$$f_e^2\left(f_c^2\right)=\frac{\delta}{2}+\frac{g_e}{2}+\frac{f_c^2}{2}+\frac{b_2\left(2\lambda^2-1\right)}{2}, \tag{22}$$

for firm c

$$f_c^2\left(f_e^2\right)=\frac{3\delta}{2}+\frac{g_c}{2}+\frac{f_e^2}{2}-\frac{b_2\left(2\lambda^2-1\right)}{2}. \tag{23}$$

For equilibrium prices, there are two cases to consider: (i) the prices $\left(f_e^{2,*},f_c^{2,*}\right)$ that we get by using (22) and (23) are such that $f^{2,*}-b_1(2\lambda^2-1)>\delta$, and (ii) the prices $\left(f_e^{2,*},f_c^{2,*}\right)$ that we get by using (22) and (23) are such that $f^{2,*}-b_1(2\lambda^2-1)\in[-\delta,\delta]$. We only detail case (i) because it is the one used for the welfare analysis.

In case (i), demand for goods e and c are given by (20) and (21) respectively. Therefore, the equilibrium prices are:

$$f_e^{2,**} = \frac{5}{3}\delta+\frac{2g_e+g_c}{3}+\frac{b_2\left(2\lambda^2-1\right)}{3}, \tag{24}$$

$$f_c^{2,**} = \frac{7}{3}\delta+\frac{g_e+2g_c}{3}-\frac{b_2\left(2\lambda^2-1\right)}{3}. \tag{25}$$

We find those using (22) and (23).

For the ranking situation $\rho=2$, we can also compute the expected utility of a randomly selected type-t consumer given the equilibrium prices $\left(f_e^{2,**},f_c^{2,**}\right)$:

$$\begin{aligned}U_t^2\left(f_e^{2,**},f_c^{2,**}\right) &= \int_{-\delta}^{\delta}\left(L_t^2-f_c^{2,**}\right)\frac{1}{2\delta}dy_{t,i}\\ &= L_t^1-f_c^{1,**}.\end{aligned}$$

We are now in position to analyze the welfare implication of rankings when firms are competing in prices.

4.2 Welfare Analysis

In this section, we show that the ranking may have a positive effect or a negative effect on consumers' welfare, both ex-ante and ex-post. This result will also clarify the conditions under which rankings are beneficial for consumers when prices are flexible.

Without capacity constraints and externalities, the positive effect of the ranking is to reduce uncertainty about the quality of the goods. When firms compete in prices, this is not the only effect of the ranking on consumers welfare. Indeed, the ranking can have a negative effect on consumer welfare through its effect on firms' market power. This is so because, in some sense, the ranking increase the differentiation of the two goods.

We show that the net effect of the ranking is positive when there are consumers of both types that consume both goods in equilibrium (Proposition 7). The net effect of the ranking is negative when the information provided by the ranking induces all consumers of (at least) one type to consume the same good (Proposition 8). In particular, type 1 consumers, who care relatively more about the intrinsic quality of the good, feature intense preferences in favor of the highest ranked good. The firm who produces the highest ranked good can then increase its price at the margin without losing any of those consumers. The ranking thus increases significantly the market power of that firm.

The following proposition shows that the ranking may have a positive effect on consumer welfare both when we evaluate the effect before and after the publication of the ranking.

Proposition 7 *Without consumption externalities and without capacity constraint, when firms compete in prices, there is an open set of parameter values* δ, b_t^l, b_t^h, g^e, g^c, λ^0, *and* α *such that the presence of a ranking ex-ante as well as ex-post dominates its absence.*

Proof. Consider the parameter values $\delta = 0.17, g^c = 0$, $g^e = 0.5$, $b_1^l = b_2^l = 1$, $b_1^h = 2.05$, $b_2^h = 2$, $\alpha = 0.69$, and $\lambda^0 = 0.7$.

For these values of the parameters, we have that $\frac{g}{3} + \frac{\psi^\rho}{3} - b_t(2\lambda^\rho - 1) \in [-\delta, \delta]$ $\forall t, \rho$. Thus, the equilibrium prices are given by (12), (13), (12) and (13):

$$\begin{aligned} f_e^0 &= 0.64,\ f_e^1 = 0.735,\ f_e^2 = 0.511, \\ f_c^0 &= 0.2,\ f_c^1 = 0.105,\ f_c^2 = 0.329. \end{aligned}$$

In equilibrium the demand for good e is:

$$d_e^0 = 0.8235, d_e^1 = 1.3804, d_e^2 = 0.067.$$

The demand for good c is:

$$d_c^0 = 1.1765, d_c^1 = 0.6196, d_c^2 = 1.933.$$

It remains to check that (i) $U_t^1 > U_t^0$, and (ii) $U_t^2 > U_t^0$ for all consumers:

$$U_1^0 = 1.1481; \qquad U_2^0 = 1.1248;$$
$$U_1^1 = 1.1573; \qquad U_2^1 = 1.1258;$$
$$U_1^2 = 1.1842; \qquad U_2^2 = 1.1598.$$

So, ex-post, both types of consumers are better off with the ranking. Since all inequalities are strict, this holds for an open set of parameters. And since ex-post dominance implies ex-ante dominance, the ranking also ex-ante dominates is absence for an open set of parameters ■

The following proposition shows that the ranking may have a negative effect on consumer welfare both when we evaluate the effect before and after the publication of the ranking.

Proposition 8 *Without consumption externalities and without capacity constraint, when firms compete in prices, there is an open set of parameter values δ, b_t^l, b_t^h, g^e, g^c, λ^0, and α such that the absence of a ranking ex-ante as well as ex-post dominates its presence.*

Proof. Consider the parameter values $\delta = 0.2, g^c = 0$, $g^e = 0.5$, $b_1^l = b_2^l = 1$, $b_1^h = 2.4$, $b_2^h = 1.8$, $\alpha = 0.8$, and $\lambda^0 = 0.7$.

For these values of the parameters, we have that $\frac{g}{3} + \frac{\psi^1}{3} - b_1(2\lambda^1 - 1) = -0.371 < -\delta = -0.2$, $\frac{g}{3} + \frac{\psi^2}{3} - b_1(2\lambda^2 - 1) = 0.342 > \delta = 0.2$, and $\frac{g}{3} + \frac{\psi^\rho}{3} - b_t(2\lambda^\rho - 1) \in [-\delta, \delta]$ in all other cases. We thus have that the equilibrium prices are given by (12), (13), (18),(19) , (24) and (25):

$$\begin{aligned} f_e^0 &= 0.68,\ f_e^1 = 1.015,\ f_e^2 = 0.5965, \\ f_c^0 &= 0.22,\ f_c^1 = 0.285,\ f_c^2 = 0.7035. \end{aligned}$$

In equilibrium the demand for good e is:

$$d_e^0 = 0.9, d_e^1 = 1.288, d_e^2 = 0.241.$$

The demand for good c is:

$$d_c^0 = 1.1, d_c^1 = 0.712, d_c^2 = 1.759.$$

It remains to check that (i) $U_t^1 < U_t^0$, and (ii) $U_t^2 < U_t^0$ for all consumers:

$$U_1^0 = 1.308; \qquad U_2^0 = 1.109;$$
$$U_1^1 = 1.196; \qquad U_2^1 = 0.95;$$
$$U_1^2 = 1.163; \qquad U_2^2 = 0.918.$$

So ex-post both types of consumers are better off without the ranking. Since all inequalities are strict, this holds for an open set of parameters. And since ex-post dominance implies ex-ante dominance, the absence of the ranking also ex-ante dominates its presence for an open set of parameters ■

This result shows that the flexibility of prices does not solve the problem of rankings as identified in the previous section. On the contrary, with flexible prices the ranking might create an additional problem for the consumers because it could increase the market power of the individual firms. Furthermore, the two previous propositions indicate that this happens most likely when δ is low. Ironically, when the idiosyncratic preferences of the consumers are relatively unimportant and hence the market is in principle rather competitive, it is most likely that the ranking hurts. The reaons is that it is this case when the outcome of a ranking leads to a big shift in demand. This leads to an increase of the market power of the "winning" firm and finally hurts consumers in the end.

5 Conclusion

In this paper we have demonstrated that the existence of a ranking can be harmful to all consumers when the good is rationed, when there are consumption externalities, and/or when firms have market power. These harmful effects are more likely the more random the rationing procedure is, the more important consumption externalities are, and the more important the good's intrinsic quality is relative to the consumers' idiosyncratic preferences for a particular good. It seems plausible that markets for study programs rationing, externalities, and intrinsic quality play an important role. The same holds e.g. for medical services. Hence, in these markets it is a-priori unclear whether rankings are beneficial for consumers or not. But of course the analysis of the existence and the size of harmful effects of rankings for a particular market is left for future empirical research.

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6 Appendix

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Proof of Proposition 1. Lemma 2 shows that the list of cases of the proposition is exhaustive. We prove the different cases sequentially.

1. $b_1(2\lambda^\rho - 1) > b_2(2\lambda^\rho - 1)$ implies that

$$b_1(2\lambda^\rho - 1) + q(x, y) > b_2(2\lambda^\rho - 1) + q(x, y). \tag{26}$$

We have to distinguish between the following cases:

(a) The condition guarantees that type 2 consumers' optimal choice is e. Due to (26) it is also guaranteed that type 1 consumer's optimal choice is e (see condition (4))

(b) If x_2^* solves

$$b_2(2\lambda^\rho - 1) + q(1, x_2^*) = g,$$

type 2 consumers are indifferent between applying for the good e and c. Due to (26) type 1 consumers' optimal choice is e.

(c) The two conditions guarantee that e is the optimal strategy of consumers of type 1 and c the optimal strategy of consumers of type 2.

(d) Since x_1^* solves

$$b_1(2\lambda^\rho - 1) + q(x_1^*, 0) = g,$$

type 1 consumers are indifferent e and c. Due to (26) type 2 consumers' optimal choice is c.

(e) The condition guarantees that type 1 consumers' optimal choice is c. Due to (26) type 2 consumers' optimal choice is also c.

2. $b_1(2\lambda^\rho - 1) < b_2(2\lambda^\rho - 1)$ implies that

$$b_1(2\lambda^\rho - 1) + q(x, y) < b_2(2\lambda^\rho - 1) + q(x, y). \tag{27}$$

We have to distinguish between the following cases:

(a) The condition guarantees that type 1 consumers' optimal choice is e. Due to (27) it is also guaranteed that type 2 consumer's optimal choice is e (see condition (4))

(b) If x_1^* solves

$$b_1(2\lambda^\rho - 1) + q(x_1^{\rho*}, 1) = g,$$

type 1 consumers are indifferent between applying for the good e and c. Due to (27) type 2 consumers' optimal choice is e.

(c) The two conditions guarantee that e is the optimal strategy of consumers of type 2 and c the optimal strategy of consumers of type 1.

(d) Since x_2^* solves

$$b_2(2\lambda^\rho - 1) + q(0, x_2^{\rho*}) = g,$$

type 2 consumers are indifferent e and c. Due to (27) type 1 consumers' optimal choice is c.

(e) The condition guarantees that type 2 consumers' optimal choice is c. Due to (26) type 1 consumers' optimal choice is also c.

■

Proof of Proposition 2. First, we prove part (i) of the proposition. For generic parameter values, it holds for $\rho = 0, 1, 2$ that

$$b_t(2\lambda^\rho - 1) \neq g.$$

Hence the equilibria with and without ranking are unique, and all consumers of the same type want the same good. Since there is no capacity constraint, each consumer gets his preferred good.

We have to distinguish between two cases. First, it might be that in equilibrium all consumers opt for the same good without ranking, with confirmative ranking, and with a surprising ranking, i.e. $x_t^{0*} = x_t^{1*} = x_t^{2*}$. Since there is no capacity constraint, this implies that $n^0 = n^1 = n^2$. Using this equation and (6), (7), and (8) shows that

$$\left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_t^1(n^1, n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_t^2(n^1, n^2) = U_t^0(n^0) \text{ for all } t.$$

Second, it might be that the ranking induces agents to change their choices. Since $\lambda^1 > \lambda^0 > \lambda^2$ the only possibilities are that a confirmative ranking induces a switch from the good c to good e (i.e. $n_t^0 = n_t^2 = 0$, $n_t^1 = 1$), or a surprise ranking induces a switch from the good e to good c ($n_t^0 = n_t^1 = 1$, $n_t^2 = 0$). Consider the case where type t agents switch from the good c to good e due to a confirmative ranking. Therefore

$$b_t(2\lambda^1 - 1) > g,$$

implying that

$$b_t \frac{\alpha + \lambda^0 - 1}{1 - \alpha - \lambda^0 + 2\alpha\lambda^0} > g. \tag{28}$$

For $n_t^0 = n_t^1 = 1$, $n_t^2 = 0$, we have

$$\begin{aligned} & \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_t^1(n^1, n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_t^2(n^1, n^2) \\ = \;& \lambda^0(\alpha\left(-g^e + b_t^h\right) + (1-\alpha)\left(-g^c + b_t^l\right)) + \left(1-\lambda^0\right)\left(\alpha\left(-g^c + b_t^h\right) + (1-\alpha)(-g^e + b_t^l)\right), \end{aligned}$$

and

$$U_t^0(n^0) = \lambda^0(-g^c + b_t^l) + \left(1 - \lambda^0\right)\left(-g^c + b_t^h\right).$$

Therefore, condition in (9) boils down to

$$\begin{aligned} & \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_t^1(n^1, n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_t^2(n^1, n^2) - U_t^0(n^0) \\ = \;& \lambda^0\alpha\left(-g^e + b_t^h + g^c - b_t^l\right) + (1-\lambda^0)(1-\alpha)\left(g^c - b_t^h - g^e + b_t^l\right) \\ = \;& \lambda^0\alpha(-g + b_t) + (1-\lambda^0)(1-\alpha)(-g - b_t). \end{aligned} \tag{29}$$

Hence

$$\begin{aligned} \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_t^1(n^1, n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_t^2(n^1, n^2) - U_t^0(n^0) &> 0 \Leftrightarrow \\ b\frac{\alpha + \lambda^0 - 1}{1 - \alpha - \lambda^0 + 2\alpha\lambda^0} &> g, \end{aligned}$$

which is guaranteed by (28).

A similar argument can be made for a switch to good c due to a surprise ranking. In this case

$$b_t(2\lambda^2 - 1) < g.$$

Substituting (2) and noting that $\alpha + \lambda^0 - 2\alpha\lambda^0 > 0$ implies

$$b_t \frac{\lambda^0 - \alpha}{\lambda^0 + \alpha - 2\lambda^0\alpha} < g. \tag{30}$$

For $n_t^0 = n_t^1 = 1$, $n_t^2 = 0$, we have

$$\begin{aligned} & \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_t^1(n^1, n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_t^2(n^1, n^2) \\ = \;& \lambda^0(\alpha\left(-g^e + b_t^h\right) + (1-\alpha)\left(-g^c + b_t^l\right)) + \left(1-\lambda^0\right)\left(\alpha\left(-g^c + b_t^h\right) + (1-\alpha)\left(-g^e + b_t^l\right)\right), \end{aligned}$$

and

$$U_t^0(n^0) = \lambda^0(-g^e + b_t^h) + (1 - \lambda^0)(-g^e + b_t^l).$$

Therefore, condition in (9) boils down to

$$\begin{aligned} & \left[\lambda^0\alpha + (1-\lambda^0)(1-\alpha)\right] U_t^1(n^1, n^2) + \left[(1-\alpha)\lambda^0 + \alpha(1-\lambda^0)\right] U_t^2(n^1, n^2) - U_t^0(n^0) \\ = \;& \lambda^0(1-\alpha)\left(g^e - b_t^h - g^c + b_t^l\right) + (1-\lambda^0)\alpha\left(-g^c + b_t^h + g^e - b_t^l\right) \\ = \;& \lambda^0(1-\alpha)(g - b_t) + (1-\lambda^0)\alpha(g + b_t). \end{aligned} \tag{31}$$

Hence

$$\begin{aligned}\left[\lambda^0\alpha+(1-\lambda^0)(1-\alpha)\right]U_t^1(n^1,n^2)+\left[(1-\alpha)\lambda^0+\alpha(1-\lambda^0)\right]U_t^2(n^1,n^2)-U_t^0(n^0) &> 0 \Leftrightarrow \\ b\frac{\alpha-\lambda^0}{2\alpha\lambda^0-\alpha-\lambda^0} &< g\end{aligned}$$

which is guaranteed because of (30).

Second, we prove part (ii) of the proposition. Let $g^c = 0$, $g^e = 2$, $b_1^l = b_2^l = 1$, $b_1^h = 5$, $b_2^h = 3.2$, $\alpha = 1$ and $\lambda^0 = 0.7$. Because of (5) and since there are no capacity constraints, it holds in equilibrium that $n_1^0 = n_1^2 = 0$, $n_1^1 = 1$ and $n_2^0 = n_2^2 = 0$, $n_2^1 = 1$. Inserting into (31) and (29) gives:

$$\begin{aligned}\left[\lambda^0\alpha+(1-\lambda^0)(1-\alpha)\right]U_1^1(n^1,n^2)+\left[(1-\alpha)\lambda^0+\alpha(1-\lambda^0)\right]U_1^2(n^1,n^2)-U_1^0(n^0) &= 1.4 \\ \left[\lambda^0\alpha+(1-\lambda^0)(1-\alpha)\right]U_2^1(n^1,n^2)+\left[(1-\alpha)\lambda^0+\alpha(1-\lambda^0)\right]U_2^2(n^1,n^2)-U_2^0(n^0) &= 0.14\end{aligned}$$

Since both payoff differences are strictly positive, there is an open neighborhood of these parameter values such that the ranking dominates its absence ■