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Paul Samuelson's Amazing Intergenerational Transfer¹

I'm deeply honored to participate in this forum celebrating Paul Samuelson's ongoing contributions to economics. Paul's work has profoundly influenced, irrevocably altered, and dramatically improved economic analysis in virtually all areas of economics. A prime example is the field of generational policy, which focuses on the extent and means by which governments redistribute across generations. Paul's masterpiece -- "An Exact Consumption-Loan Model of Interest with and without the Social Contrivance of Money" -- is *the* seminal article in this field and permeates virtually all postwar research on the issue. The paper's insights and messages have particular salience today given what many view as the grave demographic/fiscal threat facing the developed world.

Like all of Paul's writings, this paper is a literary gem with copious references to our intellectual forefathers. Bentham, Mills, Engels, Myrdal, Kant, Robertson, Böhm-Bawerk, Harrod, Fisher, Landry, Hobbes, Rousseau and others make an appearance. But the paper's real appeal is the theoretical vistas it provides. Here in one fell swoop we learn that competitive economies can be Pareto inefficient (dynamically inefficient), that altruism can promote survival, that constitutions and social norms can have economic determinants, that dynamic economies can have an infinite number of equilibria, that biology can determine interest and inflation rates, that financial markets can be highly volatile, if not unstable, that monetary and fiscal policy can be isomorphic, that fiscal policy can be endogenous, that the same economic policy can be labeled a zillion different ways, and that there is an economic limit to expropriating the young.

The paper and Paul's JPE subsequent exchange (Samuelson, 1960) with W. H. Meckling (1960) provide a winding road through this splendid garden of issues and ideas. Just when you think you've come to the end, there's another twist presenting an even more striking view. Paul clearly delights in story telling, knows how to keep his reader in suspense, and waits until the last minute to pull his paper oblongs out of the hat.

A simple rendition of Paul's story takes place on a very hot island with very tall cocoa trees, which only the young can climb. At the top of the trees grow the only source of sustenance -- Hershey chocolate bars.² The young climb the trees, harvest the bars, and eat them immediately for they won't keep in the heat. The earth-bound elderly grovel for chocolate, but to no avail. Their elevated kids see no quo for their quid, have no *Up* in their *Uc*, and experience no qualm in watching their parents starve.

¹ I thank Herakles Polemarcharkis and Karl Shell for extensive and extremely valuable comments.

² Karl Shell appears to be the first to use the chocolate metaphor for the good in Paul's model.

This unfortunate state of competition continues year after year until some enterprising generation of oldsters offers to swap chocolate for pink sea shells that have washed up on shore. The young could not care less about sea shells, but they make the swap in order to have shells with which to swap when old.

Voila! The economy moves from brutish to blissful. People no longer starve when old, everyone is better off, and the young and old celebrate the economy's Pareto improvement by washing each other with melted chocolate.

Now what determines the price level – the rate at which shells swap for chocolate? The answer is expectations. The price today depends on what people think it will be tomorrow. But what it will be tomorrow depends on what tomorrow's people think it will be the day after, and so on. Nothing in the economic environment pins down these expectations, so nothing limits the number of paths the price level and, thus, the economy can take. In particular, nothing says the price level will change in line with population growth or, if this does occur, when such a steady state will arise. Moreover, since the public's expectations of future prices determine the course of chocolate transfers, such transfers, which might be termed fiscal policy, are endogenous.

Since the change in the price level determines the rate at which one can swap consumption today for consumption tomorrow, the dynamics of the price level are also those of the implicit interest rate. When prices move in line with population growth, the interest rate equals the population growth rate (the biological rate). But when they don't, the interest rate can go its merry way, including fluctuating wildly. So our little chocolate paradise can have lots of what some would describe as "financial instability" along any given equilibrium path or, indeed, across paths, if the economy jumps equilibria.³

Regardless of what path the economy takes – what equilibrium prevails -- birds (non-edible ones) perched in the cocoa trees will take notice. They'll no longer hear the moans of starving geezers or watch the young pelt the old with candy wrappers. But while the birds will all agree about the amount of chocolate being passed from the young to the old each period, they'll vehemently disagree as to the policy in place. Some birds will claim that monetary policy is at work and that the shells are money. Others will see a pay-as-you-go social security system in which the chocolate handed over when young constitutes a tax and the shells simply represent bookkeeping for one's future claim to chocolate social security benefits. Yet others will claim the shells are bonds that are purchased when young and sold when old. And there will even be some birds who'll claim that the shells are irrelevant – just a shell game, if you will – and that the chocolate eaters must have drawn up a constitution forcing each generation of young to make transfers to the old.

³ Paul's concern (Samuelson, 1960), raised in response to Meckling (1960), that "a perpetual competitive system seems to be an indeterminate one" depends, as most recently shown by Rochon and Polemarchakis (2005), on whether the economy in question is dynamically efficient.

After the birds spend several centuries arguing and forming societies call the Monetarists, the Socialists, the Keynesians, and the Strict Constructionists, a young bird named Paul points out that the argument isn't about economics, but about language. This stops the fight for a full nanosecond, after which it proceeds apace.

Like this island, our society contains a lot of bird brains, many located in Washington, who constantly mistake linguistics for economics. They would do well to read Paul's article.

We economists, in contrast, have read and reread Paul's article and incorporated it fully into our teaching and research. In this respect, Paul's paper is the gift that keeps on giving -- not a chocolate bar, but a cocoa tree! This is plain to see in Edmund Phelps's (1961, 1965) work on the golden rule and dynamic efficiency, Peter Diamond's (1965) analysis of debt in an OLG model, Martin Feldstein's (1974) work on pay-as-you-go social security, Robert Barro's (1974) work on Ricardian Equivalence, Karl Shell's (1971, 1977) work on the economics of infinity and sunspots, David Cass and Karl Shell's (1983, 1989) work on sunspots, Costas Azariadis (1981) and Roger Farmer's (1993) work on sunspots, Yves Balasko and Karl Shell's (1980, 1981a, 1981b) detailed investigation of the OG model, Glenn Loury's (1981) work on intra-family human capital transfers, Jean-Michel Grandmont's (1985) work on 'temporary equilibrium,' Richard Benveniste and David Cass' (1986) work on optimal stationary equilibria, Andy Abel, Greg Mankiw, Larry Summers, and Richard Zeckhauser's (1989) work on dynamic efficiency under uncertainty, Michael Woodford's (1990) work on the convergence of rational expectations equilibrium under adaptive learning, the recent work by Rochon and Polemarchakis (2005) distinguishing money and debt in OLG models, Heckman, Lochner, and Taber's (1998) work on human capital accumulation in OLG models, and in the work of literally thousands of others. Indeed, since 1988 alone Paul's paper has been cited 638 times in published articles and books!

For my part, I encountered Paul's paper in several courses in graduate school and was immediately intrigued by the issues it raised, although I wasn't able to clearly sort them out in my mind or fully grasp the lessons being taught. (I'm still doing that.) But Paul's article, its offshoots, and the strong influence of my thesis advisor, Marty Feldstein, got me completely hooked on generational economics.

Let me connect my own research in this field to Paul's paper. I do so not to put my work on the same plane. It's in a much lower dimensional manifold. I do so to suggest the reach of Paul's intellectual transfer.

One of the key issues raised by Paul's paper is the role of intergenerational altruism in society. In considering the possibility that kids might let their parents starve, Paul was inviting economists to find out if that was really the case. And, by extension, he was inviting us to study whether parents would let their kids starve.

Starvation is, of course, an extreme outcome that arises only in unusual circumstances. So we need to test altruism in our everyday world in which parents and kids both have

access to their own chocolate. The simple way to test intergenerational altruism is to see whether parents and kids share resources when it comes to consumption. Consumption sharing means that the consumption levels of those doing the sharing should move together. If parents get more (less) income, both their consumption and that of their kids should rise (fall). If kids get more (less) income, the same thing should happen. Stated differently, the ratio of the kids' consumption to that of the parents should be independent of the ratio of the kids' resources to that of the parents. Moreover, a dollar taken from a child and given to a parent should lead to a dollar increase in the parent's transfer to the child. Finally, when altruism is operational, other family decisions, like living together, should depend on the sum, not the division, of resources.

Over the years I and a variety of co-authors have conducted a number of studies to test these propositions using cross section, cohort, and time series data.⁴ We've even tested altruism within extended families in which parents are actively making transfers to their children. The tests have all strongly rejected intergenerational altruism. Thus, Paul's assumption of selfish behavior is, regrettably, on the mark. In particular, when it comes to consumption, there is no evidence that cohorts share resources very much, that extended family members share resources very much, or that nuclear families share resources very much. And there is no evidence that parents whose incomes rise substantially compensate their kids whose incomes fall. This last finding holds even for parents who are actively making transfers to their children!

Paul's paper also raises questions about the range of social compacts that can be sustained and the manner of their enforcement. In Kotlikoff, Persson, and Swensson (1988) my coauthors and I pointed out that the old can sell the young more than simply covenants to support the elderly. They can also sell the young other economic laws, including a law prohibiting capital levies. Such a law can prevent second best taxation from degenerating into third best taxation due to the time inconsistency problem noted by Fischer (1980). The young can make their payment in the form of tax contributions to finance public goods for the old or transfer payments to the old.

Kotlikoff, Persson, and Swensson (1988) also explore ways of enforcing the sale of social contracts. In particular, we showed that if setting up new social contracts involves transactions costs, it will be easier to sustain existing social contracts. The first generation that sets up such a contract is forced to pay the setup costs, but has an offsetting advantage in not having to pay for the law. Subsequent generations find it cheaper to buy the law and then resell it (effectively rent the law) than incurring the set up costs from scratch. Our paper also showed that laws could be sold (laws are assets) even in the absence of transactions costs if generations adopt strategies under which they only purchase laws that have never been broken.

A third research area that intersects with Paul's paper and Diamond's (1965) own diamond involves trying to understand how interest and wage rates evolve in dynamic general equilibrium. My work with Alan Auerbach (Auerbach and Kotlikoff, 1987) and others in this area adds capital accumulation, variable labor supply, demographics,

⁴ See Kotlikoff (2001).

multiple periods, and a variety of fiscal policies to Paul's framework.⁵ These additions plus the assumption of CES production and CES intertemporal preferences admit unique and dynamically efficient equilibria.

My graduate student, Javier Hamann (1992), showed that adding money to this model as well as nominal government liabilities permits one to calculate a unique path for the price level. Here we see the price level endogenously determining real fiscal policy, just as in Paul's paper, Sargent and Wallace's examination of monetarist arithmetic, and Woodford's (1994) fiscal theory of the price level. The fact that nicely behaved, dynamically efficient, neoclassical models can have unique equilibria is reassuring given Paul's concern, raised in response to Meckling (Meckling, 1960; Samuelson, 1960), that "a perpetual competitive system seems to be an indeterminate one." But, as Nakajima and Polemarchakis' (2005) work suggests, indeterminacy may yet rear its ugly head if the monetary-fiscal authorities adjust real fiscal policy in response to changes in prices and interest rates, where such changes are governed by rational, but otherwise freely determined expectations.

A fourth connection between my work and Paul's paper involves generational accounting. The impetus for generational accounting derives from Paul's insight that the same policy can be "run" in different ways. But since the math makes no distinction between one way and the other, any one way can be called the other. So Paul's point is really that a given policy can be labeled different ways; i.e., whether we call a policy one thing or another is a matter of language, not economics.

This labeling problem is not specific to Paul's model, a point I belabor in Kotlikoff (2003). The problem is generic to any neoclassical model with rational agents. This fact calls into question essentially all conventional analysis of fiscal affairs given that such analysis is predicated on deficit accounting. Paul's point, writ large, is that governments are free to choose fiscal labels so as to report any time-path of deficits or surpluses independent of the actual policy they are running. We could well call this Samuelson's Relativity Theory. It shows that each observer's reference point (her choice of labels) alters the perception of economic policy, as conventionally measured, but not the reality of what the policy actually is or what it is doing to the economy.⁶

Generational accounting, when properly conducted, doesn't suffer from this labeling problem. Its assessment of the fiscal burden facing future generations is the same regardless of the government's nomenclature. So are its measures of changes in the fiscal burdens facing current and generations arising from policy changes. This is not surprising since generational accounting is trying to answer an economic question rather than engage in mindless measurement.

⁵ See, for example, Auerbach and Kotlikoff (1987).

⁶ The term "relativity theory" seems apt since this situation is akin to Einstein's revelation that different observers of the same physical reality will describe it differently depending on their relative speed.

Given the nature of the fiscal/demographic problems facing the developed world, we no longer have the luxury of relying on inherently uninformative indicators of nations' fiscal conditions. Generational accounting, while far from perfect, is, at this point, a necessity, not an option. In the case of the U.S., generational accounting indicates that if current adults don't step up to the plate, young and future generations will face lifetime net tax rates that are twice those of current adults. Attempting to foist such a burden on the next generation is not only immoral; it's also economically infeasible. There is a limit to fiscal child abuse, and the U.S. and other developed countries are, in my view, rapidly approaching that limit.

Corroborating evidence on this score comes from Gokhale and Smetters's (2005) measure of the fiscal gap, which is closely related to the measure of the collective fiscal burden facing future generations discussed in Auerbach, Kotlikoff, and Gokhale (1991).⁷ The fiscal gap, which is also a label-free measure, compares the present value of all projected future U.S. government expenditures, including official debt service, with the present value of all projected future government receipts. Gokhale and Smetter's estimate of the fiscal gap for 2005 is \$65.9 trillion or 8.5 percent of the present value of GDP. To put this figure in perspective, note that 2004 federal personal and corporate income taxes totaled 8.6 percent of 2004 GDP.

What alternative policies could be taken to eliminate the U.S. fiscal gap? One is to immediately and permanently double person and corporate income taxes.⁸ A second option is to immediately and permanently cut all Social Security and Medicare benefits by two thirds. A third alternative is to cut federal discretionary spending immediately and permanently; but even eliminating all such spending would leave us significantly short of the needed \$65.9 trillion.

The Gokhale and Smetters' estimates are updates of the fiscal gap accounting they did while working at the U.S. Treasury under former Treasury Secretary Paul O'Neill.⁹ Their analysis relies exclusively on government projections or extensions of such projections. These projections are quite optimistic with respect to future demographics and growth in Medicare and Medicaid spending per beneficiary.

Notwithstanding the magnitude of the fiscal gap and the downward bias in its measurement, some prominent economists¹⁰ see the short- and medium term projected deficits as manageable and assume the long run will take care of itself. This, to be kind, is misguided. Paul's relativity theory tells us that we can choose labels to arrive at whatever deficit, tax, and transfer projections we want. Hence, there is no legitimate way to consider the short-term apart from the long-term. Only the infinite horizon measures

⁷ The fiscal gap is the present value net tax burden on future generations that is calculated in Auerbach, Kotlikoff, and Gokhale (1991) less the present value of net taxes future generations would pay were they to face the same lifetime net tax rates as current generations.

⁸ This abstracts from tax evasion and tax distortions as well as "Laffer curve" effects.

⁹ Gokhale and Smetters (2003) is, in fact, the Treasury study commissioned by Treasury Secretary Paul O'Neill. It was published through the American Enterprise Institute rather than the Treasury because the Treasury censured the study within a few days of O'Neill's being fired.

¹⁰ See, for example, Porter (2005).

calculated in the generational accounting and fiscal gap analyses are label-free and, thus, well defined.¹¹

The fiscal/demographic optimists might well respond that the government's real borrowing rate is low relative to the economy's growth rate and that, given this fact, taking from the young and giving to the old could continue to work well for decades, if not centuries. These modern-day Ponzis should re-read the Abel, et. al. (1989) study showing that comparing the government's borrowing rate and the economy's growth rate is not appropriate in a setting in which both growth rates and rates of return to capital are uncertain. They should also think about how Paul's model works if the government finds itself taking ever more chocolate from the young in order to satisfy promises made to the old. At some point, the young run out of chocolate to hand over. At that point, it's the young, not the old, who are starving.

Kotlikoff and Burns (2004) raise the alarm about excessive transfers from the young to the old. In particular, we suggest that the U.S. economy could go critical once financial markets recognize the implications of the magnitude of the fiscal gap for U.S. money creation and inflation. But what if there is no financial meltdown to awaken the country to the limits to fiscal child abuse? How will these limits be reached?

As mentioned, in Paul's model the limit hits when all the chocolate is taken from the young and handed to the old. If we add capital to Paul's model, the limit is reached when the young are using all of their after-tax savings to purchase government bonds and, thus, are unable to accumulate physical capital. At this point, the game is over since production requires capital as well as labor. From a general equilibrium perspective, things would get pretty dicey well before this limit was hit. The reason is that wages depend on capital, so every period the capital stock falls, wages fall as well, which reduces what the young have to save.

This simple model suggests that we need to look at net national saving to understand if we are eating up our capital stock or would be doing so were foreigners to stop investing in the U.S. In this regard, last year's net national saving rate is quite telling. It was only 2.2 percent of national income! With the exception of 2003's 2.1 percent rate, this is the lowest U.S. rate of net national saving in 45 years. By way of comparison, note that the U.S. net national saving rate averaged 13.0 percent in the 1960s, 10.3 percent in the 1970s, 7.6 percent in the 1980s, 5.6 percent in the 1990s, and 3.8 percent since 2000.

¹¹ As a public service, the government should present alternative official deficit time series (past and projected) based on alternative labeling conventions. Some of these series would show the deficit soaring; others would show it heading south at exponential rates; still others would place and keep it at zero. Economists yearning to support their theories of how deficits connect to interest rates, saving rates, or other economic variables would surely find a series to meet their needs. Politicians dying for a balanced budget could die in peace. Others who crave red or black ink would shout for joy. And the public would finally see that the deficit has no clothes.

The decline in the rate of net national saving in the U.S. reflects the ever growing rate of household consumption. Government (federal, state, and local) consumption as a share of national income was 17.6 percent in 2004, which is lower than the average rate observed in each of the last four decades. For example, the government consumption rate was 19.5 percent in the 1970s. In contrast, the ratio of household consumption to the difference between national income and government consumption – what I call the household consumption rate – is now at a postwar high of 97.3 percent. This rate averaged 84.0 percent in the 1960s, 87.2 percent in the 1970s, 90.6 percent in the 1980s, 93.1 percent in the 1990s, and 95.5 since 2000.

As shown in Gokhale, Kotlikoff, and Sabelhaus (1996), the dramatic increase in the household consumption rate can be traced to higher levels of consumption of the elderly. My extrapolation from that study is that the per capita consumption of middle-aged retirees relative to that of middle-aged workers has doubled since 1960. The reason is simple. The government is taking ever more chocolate from young and, implicitly, future generations and giving it to the old, increasingly in the form of medical goods and services.

The huge U.S. current account deficit is, of course, reflective of our country's low saving rate. Last year foreigners invested 3 dollars in the U.S. for every dollar Americans invested here!¹²

While it may be hard to believe, the ever rising transfers to the elderly could shortly lead the U.S. net national saving rate to go negative. At this point, we'll be eating up our national wealth.¹³ Ignoring government assets (could we really sell the White House?) and assets held by non profits, my guesstimate of current national wealth is \$35 trillion. Were the U.S. saving rate to hit, say, - 5 percent, and who's to say it won't given the trend and the pending retirement of the baby boomers, we'd be eating up close to \$2 trillion a year of national wealth. At that rate we'd have only 15 or so years before the country was out of wealth and also out of income from that wealth. At that point we'd have only our wages to finance our consumption. And U.S. labor income is significantly less than U.S. consumption. So we would, indeed, reach a limit to our intergenerational profligacy. Consumption would have to fall. In Herb Stein's words, "Something that can't go on has to stop." The problem with Stein's aphorism is that it fails to clarify that when things that can't go on finally stop, they may stop at a very bad place and stay there forever.

The bottom line here is that Paul's model identifies not just how intergenerational transfers can help the old, but also how they can hurt the young. As we all continue to

¹² In 2004, the U.S. rate of net domestic investment (measured relative to national income) was 8.7 percent. The U.S. net national saving rate was 2.2 percent. The 6.5 percent difference represents the current account deficit as a share of U.S. national income.

¹³ For example, over the past four years Medicare benefits per beneficiary grew 16 times faster than real wages per worker.

study, learn from, and build upon his absolutely brilliant analysis, we, unfortunately, need to keep this concern front and center.

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