

The Effect of Annuity Insurance on Savings and Inequality

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This paper examines the amount of precautionary savings and wealth inequality arising from life-span uncertainty by comparing saving behavior under perfect insurance arrangements with that arising under imperfect arrangements, namely, when longevity risk can be pooled only with members of one's own family. The central findings of the paper are that (1) perfecting insurance arrangements can sharply lower savings in both intergenerationally altruistic and life-cycle economies and that (2) in altruistic economies perfecting annuity insurance can greatly influence inequality; indeed, in the long run in our model, switching from imperfect family insurance to perfect insurance can mean the difference between absolute inequality and absolute equality.

I. Introduction

While death and taxes may be the two most certain elements in one's life, the date of one's death is surely one of the least certain. Uncertainty concerning the length of life is obviously of great importance to saving

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decisions; “taking it with you” is not an option, but immediately consuming all resources (less an intended bequest) is no alternative either since with high probability one will still be alive but will be very poor in the immediate future. To avoid the prospect of remaining alive in a state of relative poverty, risk-averse individuals will be cautious about their rate of consumption. The extent of precautionary saving motivated by life-span uncertainty depends on the availability of market and nonmarket annuity insurance. Annuity insurance permits those who live longer than average to share the economic risks of this outcome with those who live shorter than average. This paper examines the amount of precautionary savings arising from life-span uncertainty by comparing saving behavior under perfect insurance arrangements with that arising under imperfect arrangements, namely, when longevity risk can be pooled only with members of one’s own family. We consider both intergenerationally altruistic preferences and selfish (zero bequest motive) life-cycle preferences, determining stochastic steady-state wealth levels and wealth distributions in both models.¹

The central findings of the paper are that (1) perfecting insurance arrangements can sharply lower savings in both intergenerationally altruistic and life-cycle economies and (2) in intergenerationally altruistic economies perfecting annuity insurance can greatly influence the degree of inequality; indeed, in the long run, switching from imperfect family insurance to perfect insurance can mean the difference between absolute inequality and absolute equality.

In addition to several important analyses of the relation of intergenerational transfers to inequality (Stiglitz 1978, pp. 271–303; Stiglitz and Bevan 1979), there are now several studies that examine the effect of annuity insurance on saving. Sheshinski and Weiss (1981) were the first to point out that a fully funded social security program could alter household saving through its provision of annuity insurance, assuming such insurance is not otherwise available. Hubbard (1983) reaches a similar conclusion in a partial equilibrium life-cycle analysis. Davies (1981) and Kotlikoff and Spivak (1981) discuss the size of involuntary bequests in life-cycle models in the complete absence of insurance arrangements. Fuller descriptions of life-cycle economies in the absence of annuity insurance are presented by Abel (1983) and Eckstein, Eichenbaum, and Peled (1983). Both papers consider the stochastic steady-state properties of economies in which agents involuntarily leave bequests to their children.

While each of these papers indicates that improvements in annuity insurance reduces precautionary saving, their specification of bequest behavior in the absence of annuity insurance is either incomplete or

¹ Our discussion of behavior in the life-cycle model draws heavily on Kotlikoff, Shoven, and Spivak (1983).

rather arbitrary. For example, Eckstein et al. and Abel assume that while parents are selfish and have no interest in their children, they nonetheless involuntarily bequeath all their wealth to precisely their own children, extracting no quid pro quo in exchange. An alternative assumption explored by Kotlikoff and Spivak (1981) is that selfish parents and selfish children jointly pool longevity risk in a manner that is mutually beneficial. Kotlikoff and Spivak point out that longevity risks are substantial as measured by the amount of resources that even mildly risk-averse selfish individuals would sacrifice to gain access to fair annuity insurance. In addition they demonstrate that risk sharing among even as few as two relatives can provide a large fraction of the gains available from perfect insurance. Hence, in selfish life-cycle models, longevity risk pooling between parents and children appears well “worth the trouble,” with the gains far exceeding any reasonable transactions costs. In contrast to the selfish life-cycle model, in the altruistic model the pooling of otherwise uninsurable longevity risk between parents and children is an immediate implication of utility maximization since the utility of children is an argument in the utility function of parents.

Our model of risk-sharing arrangements between parents and children assumes the smallest possible family risk-sharing pool, namely, at most one parent sharing risk with one child at any point in time. Since family insurance approaches perfect insurance as the number of family members increases, this assumption leads to the largest precautionary motive for saving and provides an upper bound for calculating the potential decline in savings resulting from perfecting annuity insurance arrangements.²

While at most two relatives are simultaneously alive, risk sharing in both the altruistic and the selfish models also involves all future generations. In the altruistic model current relatives consciously take into account the effect of their behavior on their descendants' inheritances and, therefore, on their levels of expected welfare. As a consequence, altruistic families automatically share any current adverse shocks with future generations by reducing the bequests they leave to future generations. In the life-cycle model the risk-sharing arrangements between current parents and children is chosen, taking into account that today's adverse shocks will affect the future bargaining position of current children when they strike selfish deals with their children. This future bargaining with the next generation is, in turn, contingent on the nature of subsequent future bargaining with the succeeding generation, and on and on. Hence the solution to the bargaining problem between living family members takes account of the infinite sequence of bargains struck by family descendants.

² See Kotlikoff and Spivak (1981). This statement assumes independent morality probabilities across family members.

The next section considers the effect of annuity insurance in an intertemporal altruistic economy in which families can borrow the present value of their certain future labor earnings. Asymptotically, the distribution of resources and consumption in this economy approaches complete inequality, with successively fewer families holding all the economy's resources and engaging in all the economy's consumption. Section III considers a presumably more realistic financial setting in which altruistic families cannot borrow against the present value of their infinite stream of future earnings. In contrast to the unconstrained borrowing case in which the altruistic economy's savings is infinitely elastic at a particular interest rate, with borrowing constraints aggregate savings is a continuous increasing function of the interest rate. In addition, the reduction in steady-state wealth from introducing perfect annuity insurance is considerably larger if there are borrowing constraints in the altruistic economy. The borrowing constraints also lead to a stationary wealth distribution in contrast to the increasingly unequal, asymptotically degenerate wealth distribution arising in the altruistic economy without borrowing constraints. The fourth section contains our life-cycle model of bargaining in which nonaltruistic agents consider the infinite sequence of selfish bargaining agreements. The calculations in this section suggest a very sizable effect of perfecting annuity insurance on long-run savings.

The models in Sections II and IV have 4 potential periods of life, while Section III's model has at most 2 periods of life. A minimum of 4 periods is necessary to examine selfish life-cycle bargaining. While Section III's analysis of liquidity-constrained altruistic behavior could be formulated in the 4-period framework, simplifying to 2 periods greatly reduces the computational costs of solving the problem. To facilitate comparisons of the predictions of the three models all illustrative calculations are based on a common utility function, and parameters in the 2-period model of Section III are calibrated on the basis of the 4-period models' parameterizations. In the final section, V, there is a summary of the paper's findings and suggestions for additional research on issues of precautionary savings.

II. The Unconstrained Intergenerationally Altruistic Model

In this model and in that of Section IV there are 4 periods of life. People live with certainty for the first 3 periods and survive to the fourth with probability P . Children are 1 when their parents are 3. Individuals are exogenously endowed with labor earnings, which they receive during the first 3 periods of life. Hence the present value of earning for an altruistic family is certain, and we assume in this section that families can borrow the present value of the infinite stream of earnings.

The family's utility function is taken as separable in consumption of

the parent (C_p) and the child (C_s) over time. Throughout the paper we consider the isoelastic form for utility of consumption at a point in time, $U(C)$, that is,

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad (1)$$

where γ is the coefficient of relative risk aversion. The utility of children is assumed to enter the family's expected utility function with weight θ , and future utility is assumed to be discounted by the time preference factor α .

Let W_t stand for the family's total human plus nonhuman wealth at time t . The expected utility of a family with a 1-period-old child and a 3-period-old parent at time t is a function of W_t and is denoted $V(W_t)$. Equation (2) expresses $V(W_t)$ in terms of the certain utility of the child's and parent's immediate consumption, the expected utility of their subsequent period consumption, and the expected utility of the family when the next child is born. For notational convenience the time subscript, t , is omitted wherever the time period is implicitly clear. In addition, we set $t = 1$ initially:

$$V(W_1) = \max \left\{ \frac{C_{f3}^{1-\gamma}}{1-\gamma} + \theta \frac{C_{s1}^{1-\gamma}}{1-\gamma} + \alpha P \left[\frac{C_{f4}^{1-\gamma}}{1-\gamma} + \theta \frac{C_{s2,d}^{1-\gamma}}{1-\gamma} + \alpha V(W_{3,a}) \right] \right. \\ \left. + \alpha(1-P) \left[\theta \frac{C_{s2,d}^{1-\gamma}}{1-\gamma} + \alpha V(W_{3,d}) \right] \right\}. \quad (2)$$

The first two terms on the right-hand side of (2) are, respectively, the family's utility from the parent's third- and from the child's first-period consumption. The bracketed terms multiplied by αP indicate the family's expected utility in the case the parent lives through the fourth period. The child's second-period consumption in this case is $C_{s2,a}$, and the family's full wealth at the time the initial child is a parent $W_{3,a}$. In the case the parent dies early, the child's second-period consumption and the family's resources when the child becomes a parent are, respectively, $C_{s2,d}$ and $W_{3,d}$. The bracketed terms multiplied by $\alpha(1-P)$ express the family's expected utility if the parent dies early.

The family is constrained in maximizing (2) by

$$W_1 = C_{f3} + C_{s1} + R(C_{f4} + C_{s2,a} + RW_{3,a}), \quad (3)$$

$$W_1 = C_{f3} + C_{s1} + R(C_{s2,d} + RW_{3,d}),$$

where R is one divided by one plus the interest rate. These equations simply relate the ongoing family's full future resources at the time the

child becomes a parent ($W_{3,a}$ or $W_{3,d}$) to the appropriately discounted difference between initial family resources and total family consumption during the child's first and second periods. The two constraints correspond to the two possible outcomes, namely, the parent dying early and the parent dying late.

Given the functional form for V , maximization of (2) subject to (3) is straightforward. Consider the following as a possible solution function $V(W)$: $V(W) = k[W^{1-\gamma}/(1-\gamma)]$, where k is a constant. Next replace in (2) $V(W_{3,a})$ by $k[W_{3,a}^{1-\gamma}/(1-\gamma)]$ and $V(W_{3,d})$ by $k[W_{3,d}^{1-\gamma}/(1-\gamma)]$. With this substitution $V(W_1)$ is now a homothetic function of each of the consumption demands plus $W_{3,a}$ and $W_{3,d}$. Hence the solution values for the consumption demands and for $W_{3,a}$ and $W_{3,d}$ are all proportional to W_1 . Inserting these solutions into (2) and collecting constants we have

$$V(W_1) = k \frac{W_1^{1-\gamma}}{1-\gamma} = g(P, \alpha, \theta, R, \gamma, k) \frac{W_1^{1-\gamma}}{1-\gamma}, \tag{4}$$

where $g(P, \alpha, \theta, R, \gamma, k)$ is the sum of constants, multiplying $W_1^{1-\gamma}/(1-\gamma)$ after solution values have been substituted into (2). From (4), $k = g(P, \alpha, \theta, R, \gamma, k)$, and this expression can be used to solve for k .³ Hence we have a proof by construction that $V(W) = k[W^{1-\gamma}/(1-\gamma)]$ is the form of the value function in (2).

The homotheticity of the value function (2) permits an immediate characterization of the evolution of full resources for each family through time and for the aggregate economy. The solution to (2) implies that

$$W_{3,a} = AW_1 \tag{5}$$

and

$$W_{3,d} = DW_1,$$

where A and D are constants depending on the model's parameters P, α, θ, R , and γ . For each family, expected resources when the current child becomes a parent, EW_3 , are related to initial resources by

$$EW_3 = [PA + (1 - P)D]W_1. \tag{6}$$

³ The formula for $g(\)$ is

$$g(P, \alpha, \theta, R, \gamma, k) = \{a^{1/\gamma} + \alpha^{1/\gamma}R^{(\gamma-1)/\gamma}[P(a + \alpha^{1/\gamma}R^{(\gamma-1)/\gamma}k^{1/\gamma}) + (1 - P)(\theta^{1/\gamma} + \alpha^{1/\gamma}R^{(\gamma-1)/\gamma}k^{1/\gamma})^\gamma]^{1/\gamma},$$

where $a = (1 + \theta^{1/\gamma})^\gamma$. For each set of particular values considered in the paper there is a unique value k , satisfying $k = g(\)$.

Since this equation holds for all families regardless of their initial value of W_1 , per capita wealth in the economy will remain constant (assuming an arbitrary large number of families) if and only if

$$PA + (1 - P)D = 1. \quad (7)$$

Now A and D depend on the interest factor R as well as on the fixed survival probability and the fixed preference parameters. Hence, in general, equilibrium R must adjust until the values of A and D satisfy (7). Since A and D are monotonically increasing functions of R , there is a unique value of R that satisfies (7).⁴ The economy's long-run supply of savings is clearly infinitely elastic at this value of R since for larger or smaller values of R full resources (human plus nonhuman wealth) in the economy either grow indefinitely or decline to zero.

The term A is less than D , reflecting the additional demand for fourth-period parental consumption in the case the parent lives to consume C_{f4} . Since, by (7), the weighted sum of A and D equals unity, we have $A < 1$ and $D > 1$. Hence, when expected future resources equal current resources, each favorable mortality outcome reduces the family's resources, while each unfavorable mortality outcome increases its resources.

The process governing the family's accumulation and decumulation of wealth through time, with time measured in generations, can be expressed as

$$W_{t+1} = W_t e^{\varepsilon_t}, \quad (8)$$

where ε_t equals $\log A$ with probability P and $\log D$ with probability $1 - P$. Expressing the process in terms of the logarithm of family resources, we have

$$\log W_{t+1} = \log W_t + \varepsilon_t. \quad (9)$$

The expected value of ε_t is negative since

$$E\varepsilon_t = P \log A + (1 - P) \log D < \log[PA + (1 - P)D] = 0, \quad (10)$$

⁴ A and D are given by

$$\begin{aligned} A &= R^2[1 - (M/a)^{-1/\gamma}][1 - (k/a)^{-1/\gamma}], \\ D &= R^2[a - (N/\theta)^{-1/\gamma}][1 - (k/a)^{-1/\gamma}], \end{aligned}$$

where

$$a = (1 + \theta^{1/\gamma})^\gamma, \quad M = [a^{1/\gamma} + (\alpha R^{-1}v)^{1/\gamma}R]^\gamma$$

and

$$N = \{[\theta^{1/\gamma} + (\alpha R^{-1}v)^{1/\gamma}]^{1/\gamma}R\}^\gamma.$$

using the concavity of the logarithmic function and (7). Hence the logarithm of family resources is a random walk with a negative drift. Since each family's resources follow this process and, by assumption, since all families start out with the same initial resources, the distribution of each family's resources as well as the distribution of any functions of family resources, such as the logarithm, are equivalent to the economywide distributions. The fact that the logarithm of resources follows a random walk with drift indicates that there is no stationary distribution of resources in the economy. Although the mean value of family resources is constant, the distribution of family resources becomes increasingly unequal through time.

To see this, rewrite ε_t as $\varepsilon_t = -\rho + \mu_t$, where μ_t is i.i.d. with zero mean and with variance σ^2 . The expected logarithm of resources at $t + \tau$ is given by

$$E \log W_{t+\tau} = \log W_t - \tau\rho, \quad (11)$$

and the variance of the logarithm of resources at $t + \tau$ given $\log W_t$ is

$$\text{var}(\log W_{t+\tau}) = \tau\sigma^2. \quad (12)$$

Equations (11) and (12) indicate that for each successive generation the distribution of the logarithm of resources has a smaller mean and a larger variance. Not only are successive resource distributions less equal, as indicated by the increasing variance of the logarithm, but they are also increasingly skew. This is suggested by the fact that the mean of the logarithm of resources falls through time at a faster rate than its standard deviation rises. Using Chebyshev's inequality, one can show that the fraction of families whose resources in the future exceed their current values is bounded by a number that declines with each successive generation.⁵ Asymptotically, the economy's resource distribution ap-

⁵ Actually, Chebyshev's theorem provides an even stronger statement about changes in the distribution of family resources through time. Let c be an arbitrary constant. Then the probability at time t that $\log W_{t+\tau}$ exceeds c is bounded by a number that declines with τ if $\log W_t$ is less than or equal to c . If $\log W_t$ exceeds c , the bound on the probability that $\log W_{t+\tau}$ exceeds c declines with τ for $\tau \geq \log W_t - c/\gamma$. In other words, the bound of $P(\log W_{t+\tau} \geq c)$ monotonically declines with τ if the process has already reached c , and, if not, it declines monotonically once the process has had enough time to drift down to c . More formally, from Chebyshev's theorem we have

$$P(\log W_{t+\tau} \geq \log W_t - \tau\rho + \lambda\sqrt{\tau\sigma}) \leq \frac{1}{\lambda^2},$$

where λ is a positive constant. Choose τ such that $c = \log W_t - \tau\rho + \lambda\sqrt{\tau\sigma}$; then,

proaches complete inequality, with an infinitesimal fraction of families holding all the economy's resources and the rest holding no resources. This increasing inequality should be contrasted with the complete equality that arises in the case of perfect insurance under the same assumption that all families have identical initial resources. While each family consciously plans to pursue a policy that leads to immiserization with essentially 100% probability, the process of immiserization can be quite slow.

Table 1 presents calculations of the economy's reduction in savings per worker from switching to perfect insurance for a set of illustrative parameter values. Given knowledge of the equilibrium interest rate under imperfect annuity insurance determined in (7), savings per worker is calculated, assuming a Cobb-Douglas production function with a capital share equal to .3; that is, the economy's general equilibrium condition is that the marginal product of capital equals the interest rate, which provides an equation for computing the economy's stock of savings (capital) per worker. Under perfect insurance the family's budget constraint is simply that the present expected value of consumption equals the present value of resources. As is well-known, in this case R equals α in the steady state, and this equation plus the marginal productivity condition are used to solve for steady-state savings per worker under perfect insurance.⁶

In the base case, α , the time-preference factor equals .86, and P , the fourth-period survival probability, equals .6. If we think of each period as corresponding to roughly 15 years, then the .86 value corresponds to a 1% annual time-preference rate, and the .6 fourth-period survival probability is roughly equivalent to a life span of 54 years. We view

$$P(\log W_{t+\tau} \geq c) \leq \frac{1}{\lambda^2},$$

where

$$\lambda = \frac{c - \log W_t + \tau\rho}{\sqrt{\tau\sigma}}.$$

Since $\tau\rho \geq \log W_t - c$, $\partial\lambda/\partial\tau > 0$ and $\partial(1/\lambda^2)/\partial\tau < 0$.

⁶The intuitive explanation here is that, as the time preference factor approaches unity, families become increasingly indifferent between their own welfare and that of their far distant, indeed infinitely distant, descendants. At the limit, when α equals one, current family members are completely indifferent between consuming now and saving everything for the future consumption of their descendants. They are also indifferent with respect to allocating consumption at any point in time between children and parents, i.e., eliminating parental consumption, and, thereby, the entire concern with life-span uncertainty is a matter of indifference to families when α equals one. Since life-span uncertainty becomes an increasingly less important issue as α rises, it is not surprising that, for α equal to .99, savings with imperfect insurance is equal to savings with perfect insurance.

Table 1
The Unconstrained Altruistic Model: Percentage Savings Reduction from Switching from Family Insurance to Perfect Insurance

Percentage Savings Reduction*	Parameter Values			
	α	θ	γ	P
-6.9	.86	1	4	.6
-48.6	.10	1	4	.6
-33.4	.24	1	4	.6
-20.0	.50	1	4	.6
-3.7	.99	1	4	.6
.0	.99	1	4	.6
-6.1	.86	2	4	.6
-8.6	.86	.5	4	.6
-13.4	.86	.1	4	.6
-26.2	.86	1	10	.6
-.8	.86	1	1	.6
-.8	.86	1	.5	.6
-4.3	.86	1	4	.8
-7.8	.86	1	4	.4
-6.1	.86	1	4	.2
-75.1	.24	.5	10	.5
-62.8	.50	.5	10	.5

* Calculations assume a Cobb-Douglas production function with capital's output share equal to 30%.

“children” in the model as very young adults who are initially roughly age 20 and who are parents of adult children at roughly age 50. Hence the 54-year life span corresponds to a real-world adult life span starting at 20 and ending, on average, at 74. Base-case values of γ and θ are four and one, respectively.

Adopting the base-case parameters, the altruistic economy's total savings is 7% larger with imperfect family annuity insurance than with perfect annuity insurance. This figure is quite sensitive to the time-preference rate. *Ceteris paribus*, lowering α to .5, which corresponds to a 4.1% annual time-preference rate, implies a much smaller stock of savings but a 20% difference between savings under family and perfect annuity insurance. For α equals .24, equivalent to a 10% annual time-preference rate, there is a 33% savings reduction. The reduction is 49% for α equals .10. Alternatively, assuming α equals .99, there is essentially no difference between savings under family and perfect insurance arrangements.⁷

The sensitivity of savings changes to the weight on the child's utility, θ , is surprisingly small. As θ rises, more weight is placed on the

⁷ By “rough convergence” we mean that economic choice variables were identical to at least the second digit between iterations. For a range of intermediate values of W_{f3} the calculated consumption terms are identical to five digits between iterations. While we believe that more accurate values of the V_s and V_n functions could be obtained, the computation costs of achieving the additional accuracy is considerable; solving (18) for any one of the 80 values of W_{f3} in any one of the 12 iterations requires rather extensive computation.

consumption of the child, and, in the limit, as θ approaches infinity, life-span uncertainty does not affect utility since families only care about their childrens' consumption, which is not subject to life-span risk. Hence as θ rises, R approaches α , and the savings reduction from perfect insurance goes to zero. While table 1 indicates larger precautionary savings the smaller the value of θ , lowering the value of θ from 1 to .5 raises precautionary savings by less than 2%. The precautionary savings response to larger values of γ , the relative risk-aversion coefficient, also has the expected positive sign. The magnitude of the savings difference is quite sensitive to γ . For values of γ below one, precautionary savings is less than .5% of the total; on the other hand, it is 26% for γ equals 10, assuming base-case values of other parameters. Finally, precautionary savings appears to be maximized for values of P between .4 and .6. This is also intuitive since uncertainty disappears from the model when P equals zero or unity. The bottom two rows of table 1 consider combinations of parameters chosen to raise the level of precautionary savings. For α equals .24, γ equals 10, and θ and P both equal to .5, there is a 75% reduction in the economy's wealth stock associated with switching from family to perfect annuity insurance. This figure declines only slightly, to 63%, if α equals .5 and the other parameters take these specified values.

Table 2 illustrates the transmission of inequality through time. The first generation of families is assumed to have identical resources. The table reports the fraction of resources held by the poorest 10%, 30%, 50%, and 90% of families. Since the model's 2 periods between generations correspond to roughly 30 years, the resource distribution after 10 generations corresponds in real time to roughly 300 years. In the base case the increase through time in inequality is extremely slow, with the richest 10% holding only 20% of total resources after 200 generations. The time preference factor, α , appears to be the most critical parameter for affecting the rate at which inequality increases through time. The transition to complete inequality is much faster if α equals .5 and the other parameters are held at their base-case values. Lowering α to .5 produces roughly the same degree of inequality within 10 generations that arises after 200 generations when α equals .86. After 200 generations (6,000 years in real time) of the α -equals-.5 process, the 10% richest families own almost three-quarters of all resources. If α equals .24 (roughly a 10% annual time-preference rate), the 10% richest families own close to 30% of all resources after 10 generations. This compares with 19% for α equals .5 and 12% for α equals .86. A time-preference rate of 10% per annum is quite high but is within the range estimated empirically (Hausman 1979). In addition the associated equilibrium interest rate of 8.5% computed on a yearly basis, while high, is not implausible.

While risk premiums in uncertainty models are not invariant to the specified number of periods, it may be of some interest to know the

Table 2
Resource Inequality in the Unconstrained Altruistic Model
after Successive Generations

	Generation				
	1	10	50	200	∞
Base case:					
Resource share of poorest 90%	.90	.88	.83	.80	.00
Resource share of poorest 50%	.50	.45	.34	.30	.00
Resource share of poorest 30%	.30	.26	.16	.15	.00
Resource share of poorest 10%	.10	.08	.06	.04	.00
Coefficient of variation of resource distribution	.00	.12	.27	.57	∞
Base case except $\alpha = .5$:					
Poorest share of poorest 90%	.90	.81	.62	.27	.00
Poorest share of poorest 50%	.50	.33	.16	.03	.00
Poorest share of poorest 30%	.30	.17	.07	.01	.00
Poorest share of poorest 10%	.10	.04	.01	.00	.00
Coefficient of variation of resource distribution	.00	.45	1.23	5.13	∞

welfare gain from perfect insurance in this model. Assuming baseline parameter values, the lack of perfect insurance has a small—less than 1%—welfare cost as measured by the percentage increase in resources with family insurance required to obtain the expected utility level under perfect insurance. As expected, this figure is highly sensitive to the degree of risk aversion. *Ceteris paribus*, raising γ to 10 raises the equivalent variation to almost 16%.

To summarize these findings, in the unconstrained intergenerationally altruistic model, switching from family to perfect insurance reduces aggregate savings by either a modest or a significant amount, depending, primarily, on the time-preference rate and the degree of risk aversion. The time-preference rate is critical for determining the rate at which the resource distribution becomes more unequal. If one accepts a relatively high time-preference rate, the model suggests an important potential contribution of imperfect annuity arrangements in increasing inequality.

III. The Constrained Altruistic Model

The solution to the unconstrained altruistic model involves, in the long run, essentially all families holding close to zero future resources, meaning they are in debt by an amount equal to the present value of their certain future labor earnings. In contrast, this section examines the

behavior of the altruistic economy under the assumption that families cannot borrow against their future earnings. For this problem the state variable corresponding to equation (2) is no longer full future resources; rather, it is current nonhuman wealth, A_t . To simplify the numerical calculations required to solve this problem we considered a 2-period version of the Section II model. Each child lives for 1 period with certainty and survives with probability P for 2 periods. Hence when children are born their parents may be dead. The family's expected utility in the case the parent is alive, EU_a , is given by

$$EU_a(A_1) = \frac{C_{f2}^{1-\gamma}}{1-\gamma} + \theta \frac{C_{s1,a}^{1-\gamma}}{1-\gamma} + \alpha V(A_{2,a}), \quad (13)$$

where A_1 is the family's initial nonhuman wealth. The term $A_{2,a}$ is the value of such assets at the time the grandchild is born given that the grandfather lives for 2 periods. If the parent is dead when the child is born, the family's expected utility, EU_d , is

$$EU_d(A_1) = \theta \frac{C_{s1,d}^{1-\gamma}}{1-\gamma} + \alpha V(A_{2,d}), \quad (14)$$

where $A_{2,d}$ is the family's assets when the grandchild is born given that the grandfather dies at the end of period 1.

The expected utility function, $V(A)$, is defined by

$$V(A) = PEU_a(W) + (1-P)EU_d(A). \quad (15)$$

If the initial father (the second child's grandfather) is alive, the family maximizes (13) subject to

$$A_{2,a} = \frac{(A_1 - C_{f2} - C_{s1,a})}{R} + e \quad (16)$$

and

$$C_{f2} + C_{s1,a} \leq A_1.$$

In (16), e is the child's first-period earnings. The constraint that $C_{f2} + C_{s1,a}$ not exceed A_1 implies that family assets are never smaller than e . The corresponding constraint in maximizing (14) is

$$A_{2,d} = \frac{(A_1 - C_{s1,d})}{R} + e \quad (17)$$

and

$$C_{s1,d} \leq A_1.$$

The value function, $V(A)$, as well as the consumption demands, C_{f2} , $C_{s1,a}$, and $C_{s1,d}$, were computed numerically using the contraction properties of the maximization problem (Denardo 1967). Specifically, we started with a guess of the function $V(A)$ and used this function to solve for $EU_a(A_1)$ and $EU_d(A_1)$. From (15) these $EU_a(A_1)$ and $EU_d(A_1)$ functions provide a new guess of the $V(A)$ function. This iteration was repeated until the $V(A)$ function converged to its fixed-point solution.

While there is a critical interest rate above which the economy's wealth increases without bound, there is a unique distribution of family assets for each interest rate below this critical value. Corresponding to each of these stochastic steady-state asset distributions is an aggregate stock of assets. Hence the introduction of liquidity constraints in the altruistic model produces (1) a supply schedule of aggregate savings that increases with the interest rate and (2) a nondegenerate wealth distribution. The equilibrium interest rate in this model is no longer determined solely by household preferences; rather, it is determined by equating the household supply of savings to firm's demands for savings (capital).

Assuming a Cobb-Douglas production function with a capital share of .3, a time preference factor α of .742 (corresponding to a 1% annual time-preference rate), and values of θ and γ equal to one and four, respectively, the constrained economy's equilibrium steady-state capital-output ratio is roughly 30% larger than it is in the corresponding unconstrained model. For these parameter values the reduction in aggregate savings from switching to perfect insurance is 50%. This number should be contrasted with the 7% figure in the first row of table 1. The percentage reduction in wealth in this model is highly sensitive to the value of α . At α equals .25 (a 4.7% annual time-preference rate) the percentage reduction is 95%. At α equals .057 (a 10% assumed time-preference rate) the percentage reduction is 99%. The per capita wealth stock is much more sensitive to α in the unconstrained perfect insurance version of this model than it is in the constrained family insurance version. This is expected since, for values of α above a critical level, the family is always constrained, and assets in each family equal e , their lower bound (see [16] and [17]).

The lower bound on a family's nonhuman wealth also means a lower equilibrium interest rate than with no borrowing constraints. Since families always save at least as much with liquidity constraints as they do without these constraints, an interest rate equal to or greater than the unconstrained equilibrium value would imply explosive wealth accumulation in the constrained case. The lower equilibrium value of the interest rate in the constrained model reduces the savings incentives of all agents in the economy, particularly those with the greatest wealth, and, therefore, reduces the dispersion in the distribution of wealth. For the baseline parameter values the richest 10% of families hold less than

15% of economywide resources. Inequality in wealth, while limited in all our constrained economy calculations, declines rapidly with increases in α .

IV. The Selfish Life-Cycle Model

The family insurance model, in which each member acts solely out of self-interest, is more complicated. In this case all transfers that occur are the result of ex ante selfish bargains. A 4-period framework is required here to permit these intergenerational risk-sharing arrangements. When the bargaining takes place, the child is age 1 and the parent is age 3. The parent has 1 period more of certain life followed by 1 period of uncertain life. The agreement reached by parent and child can be thought of as the parent buying an annuity from the child. In return for some money in period 3 the child promises to offer a specified level of support for the parent in period 4 in the event that the parent lives that long. Equivalently, the deal can be arranged such that the child gives the parent some money before period 3 in return for being made beneficiary of the will of the parent. The timing and labeling of payments in these arrangements is not critical; what is critical is that the child share the risk of the parent's life span. Both these arrangements involve such risk sharing. If the parent dies prior to period 4, the child ends up receiving in present value more money than he (she) pays. Alternatively, if the parent dies late, the child pays more money in present value than he (she) receives. There is also no requirement that those risk-pooling arrangements be explicitly stated or written down. Kotlikoff and Spivak (1981) describe mechanisms for enforcing implicit family annuity contracts.

Both the parent and the child can be made better off by striking a bargain. However, there is an indeterminacy as to how the gains from trade should be divided. One can imagine an arrangement under which the parent's utility in the bargain is just the same as if no deal had been struck and, therefore, all the gains from trade go to the child. Alternatively, one can arrange the risk sharing such that all the gains from trade go to the parent. An additional complication is that the child, in striking an arrangement with the parent, considers the third-period bargain he (she) will make with his (her) own child. The expected utility from the future bargain is denoted \hat{V} and depends on the child's level of third-period wealth (human plus nonhuman) $W_{3,3}$, that is, $\hat{V} = \hat{V}(W_{3,3})$. Since we assume that successive children all earn identical amounts with certainty in the first 3 periods of their lives, the resources of the grandchild, with whom the child will bargain, is suppressed as an argument of \hat{V} .

The frontier of the utility possibilities space with intergenerational bargaining is located by solving the following. Maximize

$$\frac{C_{f3}^{1-\gamma}}{1-\gamma} + \frac{\alpha PC_{f4}^{1-\gamma}}{1-\gamma} + \frac{\theta C_{s1}^{1-\gamma}}{1-\gamma} + \theta P \left[\frac{\alpha C_{s2,a}^{1-\gamma}}{1-\gamma} + \alpha^2 \hat{V}(W_{s3,a}) \right] + \theta(1-P) \left[\frac{\alpha C_{s2,d}^{1-\gamma}}{1-\gamma} + \alpha^2 \hat{V}(W_{s3,d}) \right] \quad (18)$$

subject to

$$C_{f3} + C_{s1} + R(C_{f4} + C_{s2,a}) + R^2 W_{s3,a} = W_{s1} + W_{f3} \quad (19)$$

and

$$C_{f3} + C_{s1} + RC_{s2,d} + R^2 W_{s3,d} = W_{s1} + W_{f3}. \quad (20)$$

As in (2), C_{f3} and C_{f4} are the parent's certain and contingent consumption levels in periods 3 and 4, respectively; C_{s1} is the child's first-period consumption, and $C_{s2,a}$ and $C_{s2,d}$ are the child's respective second-period consumption levels contingent on the parent being alive or dead in period 4. Children initially have no nonhuman assets. Each child's certain present value of labor earnings is W_{s1} , and his (her) parent's third-period human plus nonhuman wealth is W_{f3} . Finally, $W_{s3,a}$ and $W_{s3,d}$ are the respective third-period wealth levels of the child, which he or she uses in bargaining with the grandchild, contingent on the parent being alive or dead in period 4.

Problem (18) involves maximizing a weighted sum of the two participants' expected utility where the weight θ , applied to the child's utility, potentially ranges from zero to infinity. The child considers his or her consumption in periods 3 and 4 under two eventualities: either the parent dies early and the child does not have to pay off on the annuity insurance agreement (this is reflected in the final term of [18], which is weighted by the $[1 - P]$ possibility of its occurrence), or the parent dies late and, hence, the child does have to pay off on the annuity insurance (the fourth term in [18]). As stated, the $\hat{V}(W)$ function gives the expected utility the child experiences from his third- and fourth-period consumption discounted to period 3 of his life as a function of his wealth in period 3.

Problem (18) has two budget constraints; total consumption plus savings for the child's third period equals total initial wealth of the parent and child under both lifetime possibilities for the parent. The weight θ reflects the terms of trade in this bargaining problem. In general one would expect θ to be a function of the resources of both the parent and the child, W_{f3} and W_{s1} , respectively. However, since W_{s1} is constant in our analysis, we express $\theta = \theta(W_{f3})$.

Solving problem (18) for different values of θ traces out the utility possibility frontier for family deals shown in figure 1. Obviously, not all values of θ will generate outcomes that are in the core. We have

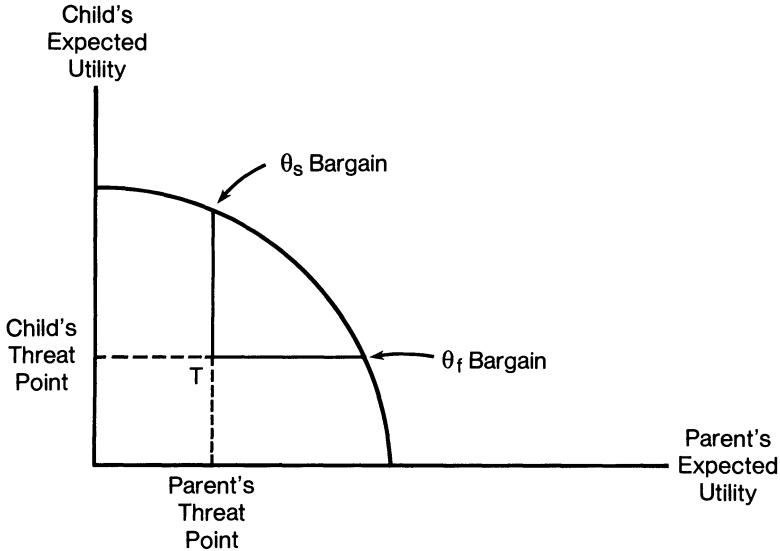


FIG. 1.—Utility possibilities frontier

labeled as θ , the critical value for θ for which the parent receives none of the gains from trade (i.e., the child receives all gains from trade). The term θ_f is defined symmetrically with the parent getting all the surplus. The point T is the threat point, indicating the parent's and child's expected utility levels if they fail to bargain with each other. As is clear from problem (18), figure 1 depends on the respective resources of the father and the son, W_{s1} and W_{f3} , and on the function $\hat{V}_s(\cdot)$.

Since we consider a stationary environment in which tastes and endowments of children remain unchanged, we will limit ourselves to stationary bargaining solutions. That is, we assume that the \hat{V} function will be the same for the bargaining of each successive pair of generations. An implication of stationarity is that the parent's expected utility in (18), expressed as a function of his wealth, W_{f3} , equals the child's expected utility function, \hat{V} , when the child becomes a father. An immediate property of stationarity is that the child reaches the same deal with his child as his parent did with him if respective resources are the same. More formally, a stationary solution is defined as a bargaining function $\theta(W_{f3})$ and an expected utility function $V(W)$ such that if C_{f3}^* , C_{f4}^* are optimal values of consumption derived from solving problem (18), where $V(W_{s3})$ is substituted for $\hat{V}(W_{s3})$, then $V(W_{f3}) = [1/(1 - \gamma)]C_{f3}^{*1-\gamma} + \alpha P[1/(1 - \gamma)]C_{f4}^{*1-\gamma}$.

Solving problem (18) involves searching for a fixed-point function V and an associated $\theta(W_{f3})$ function that produces outcomes that are in the core. We consider and compute three solutions to problem (18). In

the first solution, denoted θ_s , the child receives all the gains from trade; furthermore, all successive bargains involve children receiving all gains from trade. In the second solution, denoted θ_f , the initial and all the successive fathers receive all gains from trade. In the third solution the gains from trade are always divided between child and son according to Nash's (1983) two-person bargaining solution.

In the θ_s solution parents receive their threat-point level of expected utility. This is the expected utility received by the parent if he acts on his own and is given by the solution to (21). Maximize

$$\frac{C_{f3}^{1-\gamma}}{1-\gamma} + \frac{\alpha PC_{f4}^{1-\gamma}}{1-\gamma} \quad (21)$$

subject to

$$C_{f3} + RC_{f4} = W_{f3}.$$

Denote $V_s(W_{f3})$ as the maximum utility that the parent with wealth W_{f3} can achieve on his own by solving (21). The term $V_s(W_{f3})$ is, thus, the indirect utility function when no deal is struck and is given by

$$V_s(W_{f3}) = k \left(\frac{W_{f3}^{1-\gamma}}{1-\gamma} \right),$$

where

$$k = R^{\gamma-1} \{ [1 + (\alpha P)^{1/\gamma}] R^{(\gamma-1)/\gamma} \}^{\gamma}.$$

Naturally, $V_s(W_{f3})$ is the minimum the parent is willing to accept in an annuity bargain with his child. The term V_s is also the expected utility function of the child in the θ_s bargain with his own child. Replacing \hat{V} with V_s in (18) and choosing θ_s for each value of W_{f3} such that $V_s(W_{f3}) = C_{f3}^{*1-\gamma}/(1-\gamma) + \alpha[PC_{f4}^{*1-\gamma}/(1-\gamma)]$ provides a proof by construction that V_s is a fixed-point function for the θ_s problem. In addition the computed values of θ_s for different values of W_{f3} determine the function $\theta_s(W_{f3})$. While parents, in this θ_s bargain, receive their threat-point levels of expected utility, their actual pattern of consumption differs from what they would choose on their own; C_{f3}^* is smaller and C_{f4}^* greater than the respective solution values to problem (21).

Although the V_s function was obtained analytically, finding an analytic expression for \hat{V} for other bargaining solutions is generally not possible. For the θ_f and Nash (denoted θ_n) solutions an iterative technique described below is used to find fixed-point functions and their associated θ functions. Both the θ_f and the θ_n solutions require specifying the child's threat point. Given our assumption of a cooperative, efficient

solution to parent/child bargaining, the child, if he (she) fails to bargain with the parent, can credibly assert to the parent that he (she) will be able to reach a deal with his child. The child's threat point, EU_s^T , is the solution to problem (22); it involves the child's consuming C_{s1} and C_{s2} in his first 2 periods, respectively, and bargaining with his child in period 3 based on third-period wealth, W_{s3} .

Maximize

$$EU_s^T = \frac{C_{s1}^{1-\gamma}}{1-\gamma} + \frac{\alpha C_{s2}^{1-\gamma}}{1-\gamma} + \alpha^2 \hat{V}(W_{s3}) \tag{22}$$

subject to

$$C_{s1} + RC_{s2} + RW_{s3} = W_{s1}. \tag{23}$$

In the case of $\hat{\theta}_f$ bargaining, \hat{V} is replaced by V_f in (22) as well as in (18). The θ_f solution proceeds by first guessing a function, V_f . Next we solve (22) to determine the son's threat-point utility, EU_s^T . Given the guess of V_f and the derived value of EU_s^T , θ is chosen in (18) such that the son's expected utility in the solution to (18) equals EU_s^T . This last calculation is repeated for different values of W_{f3} , thereby generating a function, $\theta_f(W_{f3})$. In addition to computing a θ_f function on the basis of the initial guess of V_f , the solution to (18) based on $\theta_f(W_{f3})$ determines the father's expected utility in the bargain. The maximizing values of $C_{f3}^{*1-\gamma}/(1-\gamma) + P\alpha[C_{f4}^{*1-\gamma}/(1-\gamma)]$ for different values of W_{f3} provide an expected utility function for the parent in the θ_f bargain with the child. This function is used as the next guess of the V_f function, and the calculations are repeated. The iteration proceeds until the guess of the V_f function equals the parent's expected utility as a function of W_{f3} , that is, until we have found a function, V_f , which is a fixed point of the mapping described.

In the Nash bargaining case a very similar solution technique is applied. The Nash solution involves choosing θ in (18) to maximize the quantity $(EU_f - EU_f^T)(EU_s - EU_s^T)$, where EU_f and EU_s are the expected utilities obtained by the parent and child, respectively, and EU_f^T equals V_s , the parent's threat point. To find V_n , the Nash fixed-point function, we again choose an initial guess of V_n and solve (22) to find EU_s^T . We also solve (21) to find EU_f^T . Next the guessed value of V_n is substituted for \hat{V} in (18), and θ_n is chosen to maximize $(EU_f - EU_f^T)(EU_s - EU_s^T)$. Repeating this last step for alternative values of W_{f3} generates a function, $\theta_n(W_{f3})$, as well as an expected utility function of the father arising from Nash bargaining. This latter function is used as the second guess of the V_n function. The iteration continues until we find a fixed-point function, V_n . In this bargaining solution, as in the previous θ_s solution, the $\theta_f(W_{f3})$ and $\theta_n(W_{f3})$ functions calculated in the last round of the iteration

correspond to the correct bargaining functions for the functions V_f and V_n , respectively.

The V_s function is used as the initial guess of the V function for the θ_f and Nash bargaining solutions. In each iteration we computed the solution to (18) for 80 different values of W_{f3} . We then fit a fifth-order polynomial in W_{f3} to these points and used the resulting regression as the guess of V in the next iteration. The iterative procedure for determining V converged roughly by the eighth iteration;⁸ 12 iterations each were used for both the θ_f and the Nash cases.

Savings in the Life-Cycle Model under Imperfect Family and Perfect Insurance Arrangements

As described more fully in Kotlikoff, Shoven, and Spivak (1983), the stochastic steady state of the life-cycle economy has a stationary and bounded savings distribution. The distributions and stocks of wealth for each of the bargaining cases were computed numerically as follows. First we calculated the functions $W_{s3,a}^a(W_{f3})$ and $W_{s3,d}^a(W_{f3})$ for each of the three solutions to problem (18). These two functions indicate the third-period wealth of a child (when the child becomes a parent) whose parent brings W_{f3} to the bargain and who dies, respectively, either early or late. Starting with 100,000 families with the same initial value of W_{f3} , we randomly assigned these two functions to each family in each generation according to the probability P of survival to period 4. The distribution of W_{f3} stabilized after roughly eight generations. Unlike the case of unconstrained altruistic economies, there is a nonzero positive lower bound as well as an upper bound on family resources.⁹ The degree of inequality in parental asset holdings (W_{f3}) generated by this model is rather moderate. The upper bound for W_{f3} is only 1.7 times the lower

⁸ The explanation for the upper bound is the following. As successive parents continue to die early, their children bring more (but not proportionately more) resources into their own third period. When the parent dies early, his or her child enjoys an increase in nonhuman wealth but no change in human wealth. Hence, when the child's nonhuman wealth rises by a given percentage, his or her total wealth rises by a smaller percentage. Hence, even if a 2-period-old child increased his (her) W_{f3} proportionately in response to an increase in total resources available at the beginning of period 2, the increased nonhuman wealth received in period 2 would lead to a less than proportional increase in W_{f3} .

⁹ The absolute size of these economies' wealth stocks may appear small in comparison to the level of earnings or income in a particular period. However, such stock-flow ratios must be adjusted for the fact that flows in this model are received over a period that corresponds roughly to 15 years. In the case of the third and probably the most realistic earnings profile in table 4, the ratio of wealth to one-fifteenth of a period's labor earnings is 6.9 in the case of the θ_f bargain. A wealth-to-earnings ratio of 6.9 is somewhat greater than that observed in the United States.

bound for our baseline parameters under θ , bargaining. The poorest 50% of parents have close to 45% of all parental assets, and the richest 10% have 18% of all assets. These figures proved highly robust to a wide range of parameter values.

Table 3 compares steady-state per capita wealth stocks in the different insurance regimes under alternative assumptions about age-earnings profiles. Each of the age-earnings profiles has a present value of 10, which is received with certainty over the course of the first 3 periods. Since the child's resources are identical in each of these cases, the consumption decisions of the child and parent are the same for each of these earnings paths. Hence the difference in stocks of wealth by row in table 3 are simply a function of the timing of the receipt of labor income. The earnings profile in the third row appears closest to the shape of real-world profiles.

The percentage reductions in wealth from moving to perfect insurance reported in table 3 are very large. For the earnings profile in the third row, the long-run wealth reduction is 59%, starting from the θ , (children-take-all) stochastic steady state. It is 41% in the case of an initial Nash bargaining equilibrium and 35% when the initial equilibrium involves θ_r (parents-take-all) bargain.

In contrast to the altruistic models of Sections II and III, a major factor explaining the savings reduction in this model is the implicit intergenerational transfer associated with providing perfect annuity insurance. In the family insurance equilibrium the expected inheritance of children is positive since children would not otherwise be willing to accept the additional consumption risk arising from the parent-child annuity arrangement. Once perfect insurance is available, parents as a collective group will consume what they would otherwise have be-

Table 3
Wealth per Capita and Percentage Long-Run Decline
in Wealth from Switch to Perfect Insurance

Age-Earnings Profile	Perfect Insurance: Wealth	θ_r Bargain (Parents Receive All Gains from Trade)		Nash Bargaining Solution		θ , Bargain (Children Receive All Gains from Trade)	
		Wealth	Percentage Wealth Decline	Wealth	Percentage Wealth Decline	Wealth	Percentage Wealth Decline
(10, 0, 0, 0)	12.7	13.9	8.6	14.2	10.6	15.9	20.1
(5.0, 5.8, 0, 0)	7.7	8.9	13.5	9.2	16.3	10.9	29.4
(3.3, 3.9, 4.5, 0)	2.2	3.4	35.3	3.7	40.5	5.4	59.3
(3.0, 5.8, 2.7, 0)	3.4	4.6	26.1	4.9	30.6	6.6	48.5
(2.0, 5.8, 3.5, 0)	.7	2.9	75.9	3.2	78.1	3.9	82.0

NOTE.—Table assumes $\gamma = 4$, $P = .6$, and $\alpha = R = .86$.

queathed, on average, to their children. This negative resource transfer from younger children to older parents reduces the economy's savings because the parents, with shorter life spans, have higher marginal consumption propensities than have their children. This intergenerational transfer explains why savings falls significantly in the life-cycle model from introducing perfect insurance regardless of the value of γ . Recall that, in the altruistic model of Section II, introducing perfect insurance lowers savings only very slightly for $\gamma < 1$.

The values in table 3 are highly sensitive to the shape of the age-earnings profile. The smallest percentage wealth reduction arises when all earnings are received in the first period; in this case wealth falls by 20.1%, starting from the θ_s bargain, and by 13.9%, starting from the θ_f bargain.

The percentage change in wealth appears relatively insensitive to variations in the degree of relative risk aversion, γ . For example, reducing γ from 4 to 1.5 lowers the percentage decline in wealth under row 3's earnings profile and initial θ_s bargaining from 59.3% to 50.7%. Raising γ to 8 increases the value to 63.2%. Under table 3's first age-earnings profile the percentage wealth reductions starting from θ_s economies are 15.1, 20.1, and 22.9 for values of γ equal to 1.5, 4, and 8, respectively.

There is considerably more sensitivity to changes in the fourth-period survival probability P ; however, the sensitivity depends on the choice of earnings profile. For example, lowering P from .6 to .3, which reduces the expected age of death from roughly 74 to roughly 69, converts the 59.3% θ_s reduction (row 3, table 3) to 83.6%. The same reduction in P raises table 3's row-1 θ_s value from 20.1% to only 23.4%.

Unlike the previous two sections, the analysis here is partial equilibrium, that is, the wage and interest rate are assumed fixed. Since the wealth reduction from introducing fair annuity insurance would increase rates of return and depress wages, which stimulates savings, the general equilibrium differences between family and perfect insurance are likely to be considerably smaller than these partial equilibrium differences.

The large differences in wealth stocks in partial equilibrium between the perfect insurance and family insurance regimes suggest that steady-state welfare could actually be lower in the case of perfect insurance. This is indeed possible. Under θ_f (children-take-all) bargaining, and assuming that γ equals 1.5, the expected utility of even the child of the poorest parent exceeds the uniform, steady-state expected utility under perfect insurance. Starting from a situation of zero insurance, achieving the perfect insurance expected utility level requires a 7% increase in resources; achieving the expected utility of the child with the poorest parent in the θ_s stochastic steady state requires an 8% increase in lifetime resources, starting from this benchmark regime. Attaining the level of welfare of the child whose parent in the θ_f steady state has the maximum potential wealth requires a corresponding 12% increase in resources.

Another question raised by table 3 is the extent to which imperfections in annuity markets can fully explain observed intergenerational transfers. Kotlikoff and Summers (1981) invoked the assumption of perfect insurance arrangements in estimating that roughly 80% of private U.S. wealth corresponds to accumulated inheritances of those currently alive. This assumption that annuity insurance is fairly well developed in the United States can be defended by pointing to social security and other government annuities, private pensions, old age labor earnings that are partly contingent on survival, and the potential for family risk sharing involving multiple members. Still it is interesting to ask how their calculation turns out when it is applied to the two-member family insurance economy described above. Their technique involves subtracting accumulated consumption from accumulated earnings for each cohort and then summing across cohorts to get a total wealth stock. This "life cycle" wealth is then compared with actual wealth holdings. If agents in the economy are selfish and annuity arrangements are perfect or very close to perfect, computed and actual aggregate wealth will be identical or extremely close to one another.

The two-person family regime is, however, quite far from that of perfect insurance. As described here, this imperfection produces a stochastic steady state in which observed consumption profiles often exceed what could be financed from one's own labor earnings even under perfect insurance. Hence in this economy, subtracting for all cohorts accumulated consumption, part of which is financed by past intergenerational transfers, from accumulated earnings produces an underestimate of the economy's actual wealth. For the θ , bargain, with γ equals 4 and with table 3's row-3 earnings profile, the underestimate is close to 90% of actual wealth. Since Kotlikoff and Summers's calculation understates U.S. wealth by 80%, imperfections in annuity markets appear potentially capable of fully explaining actual U.S. intergenerational transfers.

V. Summary and Conclusions

This paper has explored the effect of perfecting annuity insurance on savings and inequality using two standard neoclassical models—the intergenerationally altruistic model and the nonaltruistic life-cycle model. While the modeling is intentionally structured to produce the maximum precautionary savings, the results are, nonetheless, quite surprising. Partial equilibrium calculations based on the life-cycle model suggest very substantial reductions in national wealth associated with improving annuity insurance. Indeed, in the life-cycle model the reduction in savings from the annuity insurance associated with social security could be larger than the savings reduction arising from social security's "pay as you go" method of finance. In addition, imperfections in annuity

insurance appear potentially capable of explaining a sizable fraction of observed intergenerational transfers in the United States.

In the intergenerationally altruistic model, perfecting annuity insurance can also significantly lower national savings, but the results here are much more sensitive to precise parameter values as well as to assumptions about liquidity constraints.

In the presence of family annuity insurance each of the models provides a theory of the distribution of resources. While the life-cycle model suggests a rather limited dispersion in family resources in the stochastic steady state, the altruistic model, absent liquidity constraints, generates increasing inequality through time. Asymptotically, the altruistic model's resource distribution is completely skew. In the long run, virtually all families hold close to zero resources, and an infinitesimal fraction of families hold all the economy's wealth.

The results of this analysis suggest the importance of studying other motives for precautionary savings, such as uncertainty with respect to future earnings and future health status. A variety of government programs, including disability and unemployment insurance, welfare, and progressive taxation, may be greatly affecting the extent of precautionary savings in the economy. To the extent that such savings is highly responsive to government insurance programs, the government may wish to offset their adverse savings effects not by reducing its provision of insurance but by using alternative policy instruments that raise savings without increasing economic uncertainty.

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