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LOOKING FOR THE NEWS IN THE NOISE - ADDITIONAL STOCHASTIC IMPLICATIONS OF OPTIMAL CONSUMPTION CHOICE

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ABSTRACT

In neoclassical models of consumption choice under earnings uncertainty changes in consumption programs from one period to the next are determined by new information received about future earnings over the period. This proposition suggests testing the neoclassical model by ascertaining whether new earnings information explains consumption choice through time. It also suggests that actual consumption choices imbed extractable information about the extent and time resolution of earnings uncertainty. This paper derives a fairly general theoretical relationship between properly defined innnovations in consumption (noise) and revisions in expectations of lifetime earnings (news). It also clarifies the relationship between testing for the theoretical determinants of consumption and standard Euler tests that focus on theoretical nondeterminants of consumption. The chief prediction of the paper's theoretical results, that noise exactly equals news, is tested using aggregate time series data on consumption and earnings. We find that new earnings information explains only a very small fraction of the variance of aggregate consumption innovations. On the other hand, the extent of suboptimal consumption choice appears to be of little economic significance.

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LOOKING FOR THE NEWS IN THE NOISE - ADDITIONAL STOCHASTIC IMPLICATIONS OF THE LIFECYCLE MODEL

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In neoclassical models of consumption under uncertainty optimizing agents utilize only current information about present and future prices and endowments in making current consumption decisions. This proposition has two implications. First, what was learned in the past is relevant to current behavior only in so far as past experience is incorporated in current knowledge of distributions of present and future prices and endowments. Second, given current information, new information about distributions of prices and endowments completely governs changes in the consumption program over time. The theory is thus complete, describing both what does and what does not influence consumption choices.

Recently, considerable attention has been given to testing for optimal intertemporal consumption choice in stochastic environments. In principle both implications noted above provide a basis for testing the theory. A test of the first implication is that theoretically irrelevant information is in fact irrelevant to current consumption choices. A test of the second implication is that, given current information, new information about prices and endowments fully determines the precise time path of consumption. While the two implications are closely related, one can easily construct examples of non optimizing consumption choice that satisfy tests of one implication but not of the other.

Recent empirical analysis has focused on tests of the non determinants of consumption, in particular the irrelevance of past information to current consumption choices. Hall (1978) is the first and most influential article in this literature. Hall considered consumption behavior under earnings uncertainty and demonstrated that when expected utility is maximized the marginal utility of consumption evolves as a (super or sub) martingale. Given a specification of the utility function, Hall's observation permits tests of the irrelevance of past information to contemporaneous innovations in marginal utility.[1] Contributions by Sargent (1978), Hall and Mishkin (1982), Flavin (1981), Hansen and Singleton (1983), and Mankiw, et. al. (1982) consider generalizations of Hall's tests to additional types of past information, alternative utility functions, heterogeneity in household consumption behavior, and uncertainty in interest rates as well as labor earnings.[2]

This paper departs from much of the prior literature by focusing on the second proposition of optimizing intertemporal behavior, namely that new information (news) fully explains innovations (noise) in consumption. Following Hall (1978) the paper considers the case of earnings uncertainty. Section II shows that under fairly general assumptions concerning the stochastic process governing earnings one can directly relate appropriately defined innovations in consumption to innovations (unexpected changes) in the expected discounted value of lifetime earnings. This relationship permits tests that the variance of noise equals the variance of news. Although some previous work in this area, particularly, that of Flavin (1981), imbeds, in more restrictive models, tests of this relationship in an omnibus test statistic, the test presented here is valid for arbitrary (twice continuously differentiable) concave utility functions and fairly general assumptions on the stochastic process generating earnings. The test statistic also has a transparent interpretation and can also be used to examine the fraction of the variance in consumption innovations that can be accounted for by new information on lifetime earnings. In addition, the relationship provides a measure of the uncertainty in lifetime earnings and the time resolution of that uncertainty; these measures provide a basis for assessing the welfare affects of earnings uncertainty.

Section II also clarifies: (1) The conditions under which current consumption is the only variable of value for predicting future consumption, and (2) The conditions under which the relationship between expected future consumption and current consumption is linear. Section III provides an illustration of the test that the variance of the noise in consumption is equal to the variance of the news in lifetime earnings. The example uses quarterly post war aggregate U.S. time series data on consumption and earnings, and so is comparable to most previous empirical work in this area. Our major finding is that new information about earnings explains only a small fraction of the variance of appropriately defined innovations in consumption. While the second proposition of intertemporal optimization is rejected by the data, the apparently suboptimal consumption choice appears to be of surprisingly little economic importance. One is left with the impression of very limited earnings uncertainty at the macro level, and what uncertainty exists appears of little importance to the time path of aggregate consumption. In addition, at least at the level of aggregate data, the assumptions underlying a linear relationship between expected future consumption and current consumption appear false.

II. Relating Consumption Innovations to New Information on Lifetime Earnings

In the life cycle model with earnings uncertainty considered here current consumption and plans about future consumption depend on preferences, the level of current assets, and probability distributions governing the stream of lifetime labor earnings. Revisions in consumption plans between two different periods are determined by revisions in the probability distribution of lifetime earnings associated with new information gathered between the periods. Using this fact, and assuming that distributions of revisions in the expected present value of lifetime earnings do not depend on past information, the life cycle model implies the existence of two functions, one depending on only on c_{t+1}, and the other only on c_t, where c_t denotes period t consumption. The difference between these two functions is exactly equal to the revision in the expected discounted value of lifetime earnings between periods t and t+1.

This proposition and some of its corollaries are presented in this section. The first two corollaries consider implications of the proposition for the stochastic process generating consumption. In contrast to other results in the literature, these corollaries deal with consumption per se, which is observable, rather than the marginal utility of consumption. The third corollary concerns the relationship between the stochastic process generating consumption and that generating earnings. This corollary states that the covariance of realized lifetime earnings and the revision in consumption (defined in the proposition) equals the variance of the revision in consumption. Since the revision in consumption equals the revision in the expected discounted value of lifetime earnings, the potentially observable covariance of lifetime earnings and the revision in consumption equals the unobservable variance in revisions in expected lifetime earnings. This result is empirically useful for at least two reasons. First, one can test the life cycle model with earnings uncertainty by determining whether the variance in the revision in consumption is accounted for by the covariance of consumption revisions with realized earnings. Second, the corollary provides a method of determining the extent of household uncertainty about earnings as well as the timing of the resolution of that uncertainty.

Assumption 1 (A1) provides the model of consumption behavior that underlies our results.

A1: The consumer chooses a consumption program to max $E_t \{ \sum_{\tau=0}^{T-t} \beta^{\tau} U(c_{t+\tau}) \}$ subject to $A_t = \sum_{\tau=0}^{T-t} R^{\tau+1}(c_{t+\tau} - w_{t+\tau})$, where $U(\cdot)$ is a monotonically increasing strictly concave utility function possessing a continuous second derivative, β is a subjective discount factor, T is the known length of economic life, t is the age of the agent, c_t is consumption in period t, $w_{t+\tau}$ is labor income in period $t+\tau$, A_t is nonhuman wealth at age t, $R = \frac{1}{1+r}$, where r is the known real interest rate, and E_t denotes the expectation operator conditional on the information set at time t (where required we explicitly denote this information set by I_t).

Proposition 1 (P1) underlies the results presented in Hall (1978).3

<u>Proposition 1 (P1)</u>: (See Hall, 1978). Given A1,

$$U'(c_{t+1}) = \lambda U'(c_t) + \xi_{t+1}$$

where $E_t \xi_{t+1} = 0$ and $\lambda = \beta/R$.

Proposition 1 is the first order condition arising from expected utility maximization. It states that the expected marginal utility of consumption in period t+1 conditional on the information set in period t is a function of only consumption in period t; that is, it does not depend on any other variable in I_t. ξ_{t+1} is the discrepancy between the expected and realized marginal utility of consumption in period t+1. It is unpredictable in the sense that it does not depend on any information known at period t. Formally, given any set of variables X_t that are known at period t, and any function $f(\cdot)$, then P1 implies that $E[\xi_{t+1} \cdot f(X_t)] = 0$, where E, without a subscript, denotes the unconditional expectation operator. For a given specification of the utility function the irrelevance of past information in determining ξ_{t+1} can be tested by verifying that ξ_{t+1} and X_t are uncorrelated. This method of testing intertemporal optimization is used in most recent empirical analyses.

Note that P1 has implications only for the stochastic process generating the marginal utility of consumption. In particular, the expectation of future consumption could depend on any or all variables in the current information set without violating P1. Each of the following two assumptions is sufficient to restrict the elements of the current information set which determine the expectation of future consumption. A1 is an assumption about preferences, while A2 is an assumption about the stochastic earnings process. As indicated in corollary 2, under either assumption expected future consumption is fully determined by current consumption.

Assumption 2 (A2): The utility function is quadratic, i.e., $U(c_t) = a_0 + a_1 c_t + a_2 c_t^2$.

<u>Assumption 3 (A3)</u>: Let $\eta_{t+\tau} = (E_{t+\tau} - E_{t+\tau-1}) \sum_{j=\tau-1}^{T-t} R^{j-(\tau-1)} w_{t+j}$, and $F_{t+\tau}(\eta_{t+\tau} | I_{t+\tau-1})$ be the distribution of $\eta_{t+\tau}$ conditional on the information set in period $t+\tau-1$. Then $\{dF_{t+\tau}(\eta_{t+\tau} | I_{t+\tau-1}) = dF_{t+\tau}(\eta_{t+\tau})\}_{\tau=0}^{T-t}$.

In A3 $\eta_{t+\tau}$ is the revision in the expected discounted value of lifetime earnings arising from information that accumulates between $t+\tau-1$ and $t+\tau$. Clearly, since revisions in expectations cannot be predicted, $E[\eta_{t+\tau} | I_{t+\tau-1}] = 0$. A3 states that not only the expectation, but also the entire distribution of $\eta_{t+\tau}$ is independent of $I_{t+\tau-1}$.

Proposition 2 is central to the remainder of this paper.

<u>Proposition 2 (P2)</u>: If A1 and either A2 or A3 are satisfied, then there exist monotonically increasing continuously differentiable functions, $\delta^{(t+1)}(c_{t+1})$ and $\underset{\sim}{\delta}^{(t)}(c_t)$, such that

$$\delta^{(t+1)}(c_{t+1}) = R^{-1}\underset{\sim}{\delta}^{(t)}(c_t) + \eta_{t+1},$$

where η_{t+1} is defined in A3 and, hence, $E_t\eta_{t+1} = 0$.

The proof of P2 presented in the appendix shows that $\delta^{(t+1)}(c_{t+1})$ is equal to the expected discounted sum of consumption expenditures between t+1 and the end of the planning horizon conditional on the information set in period t+1; $\underset{\sim}{\delta}^{(t)}(c_t)$ is the expectation of this same variable conditional on the information set in period t.[4] The proposition states that these expectations can be written as functions of only c_{t+1} and c_t, respectively. It follows from the budget constraint that the revision in the expectation of the discounted value of consumption expenditures must be equal to the revision in the expectation of the discounted value of lifetime earnings (i.e., η_{t+1}).

It is worth comparing P1 and P2. Both propositions establish the existence of two functions, one dependent only on c_{t+1} and one only on c_t, such that the difference between them is "unexpected"; that is, both differences have an expectation, conditional on the information set in period t, of zero. In P2, however, this difference is precisely the revision in the expected discounted value of lifetime earnings. P1, in itself, does not provide information on the source of $\xi_{t+1} = U'(c_{t+1}) - \lambda U'(c_t)$, nor does it indicate anything about the properties of ξ_{t+1} except that $E_t\xi_{t+1} = 0$. It should be clear, however, that ξ_{t+1} is determined by η_{t+1}. In fact, given A3, there is a one to one relationship between the realizations of the two random variables.[5]

We first use proposition 2 to clarify two properties of the stochastic process generating consumption, and then discuss how it can be used to investigate the stochastic relationship between consumption and earnings. Corollary 1 is an immediate consequence of P2 and provides sufficient conditions for Hall's (1978) statement that "no variable apart from current consumption should be of any value in predicting future consumption."

<u>Corollary 1</u>: If A1, and either A2 or A3 are satisfied, then there exists a (monotonically increasing and continuously differentiable) function $g_t(c_t)$, such that[6]

$$c_{t+1} = g_t(c_t) + u_{t+1}$$

where

$$E_t u_{t+1} = 0.$$

The assumptions underlying P2 and Corollary 1 are quite general, requiring no explicit specification of the utility function or stochastic process generating earnings. As a consequence, the function $g_t(\cdot)$ could be quite complicated. Corollary 2 notes, however, that if the utility function either displays constant absolute risk aversion, as specified in A4, or is quadratic, then $g_t(\cdot)$ is linear,

A4: The utility function exhibits constant absolute risk aversion, i.e, $U'(c_t) = Be^{-\gamma c_t}$ $(B, \gamma > 0)$.

Corollary 2 (proved in the Appendix): Provided A1 and either A2, or A3 and A4, are satisfied, then $c_{t+1} = \alpha_{0t} + \alpha_1 c_t + \alpha_{2t}\eta_{t+1}$.

Many of the tests of proposition 1 presented in the literature assume $g_t(c_t)$ is linear in c_t (e.g., Hall (1978), Hall and Mishkin (1982), and Flavin (1981)). Corollary 2 indicates that those results are somewhat more general than noted by Hall (1978), who justified linearity by quadratic utility, since $g_t(c_t)$ will also be linear if A3 and A4 are satisfied. Note also that if the assumptions underlying this corollary are valid, the revision in expected discounted value of lifetime earnings is simply proportional to the difference between c_{t+1} and a linear function of c_t. Hence, under the assumptions of the corollary, the η_{t+1} revisions defined in P2, can be identified in a straightforward manner, and this identification does not require any additional information on the sequence of distribution functions $\{F_{t+1}(\eta_{t+1})\}$. Our final corollary concerns the relationship between η_{t+1} and the revisions in the expected discounted value of the consumption program. This corollary requires only the more general assumptions underlying proposition 2.

Corollary 3: Let $L_t = \sum_{\tau=0}^{T-t} R^{\tau} w_{t+\tau}$ and $r_{t+1} = \delta^{(t+1)}(c_{t+1}) - R^{-1}\underset{\sim}{\delta}^{(t)}(c_t)$; and assume A1 and either A2 or A3. Then

$$E_{(j)}[r_{t+1}L_t] = E_{(j)}[r_{t+1}^2] = E_{(j)}[\eta_{t+1}^2]$$

for $j < t$.

L_t is the realized discounted value of labor earnings between t and the end of the planning horizon. It can be partitioned into the revision between t+1 and the end of the planning horizon in the expected discounted value of lifetime earnings, $(L_t - E_{t+1}L_t)$, the period t+1 revision in that expectation that occurred because of information accumulated between t and t+1, η_{t+1} (recall that $E_{t+1}L_t - E_tL_t = \eta_{t+1}$), and the period t expected discounted value of lifetime earnings (E_tL_t); that is

$$L_t = L_t - E_{t+1}L + \eta_{t+1} + E_tL_t.$$

Provided the assumptions underlying proposition 2 are correct, the revision in the expected discounted value of the consumption program, i.e., $r_{t+1} = \delta^{t+1}(c_{t+1}) - R^{-1}\underset{\sim}{\delta}^{t}(c_t)$, just equals η_{t+1}. Corollary 3 follows from noting that $L_t - E_{t+1}L_t$ cannot be correlated with any variable in I_{t+1} including η_{t+1} , while η_{t+1} cannot be correlated with any variable in I_t

including $E_t L_t$.

Eden and Pakes (1981) appear to be the first to utilize the fact that the revision in consumption expenditures should contain information on changes through time in the expected discounted value of lifetime earnings. They note that the total variance in the individual's expected discounted value of lifetime earnings at time t is just $\sum_{j=1}^{T-t} R^{2(j-1)} E\eta_{t+j}^2$, and that the sequence $\{E\eta_{t+j}^2\}$ provides a measure of the age profile of the realizations of the variance in lifetime earnings. The article by Eden and Pakes (1981) assumes a quadratic utility function, and uses only information on consumption expenditures to estimate $E\eta_t^2$. Corollary 3 provides the analogue of the Eden and Pakes result for an arbitrary concave utility function and indicates that there are, in principal, two unbiased estimates of $E\eta_{t+1}^2$. The latter fact can be quite useful in estimating these variances from flawed data.[7]

The fact that there are two unbiased estimators of $E\eta_{t+1}^2$ provides a way of testing whether the lifecycle model with earnings uncertainty does indeed account for the variance in the revision in consumption expenditures; for if it does, the value of $r_{t+1}L_t$ should be very close to that of r_{t+1}^2. To be more precise we shall allow for an additional disturbance process to affect r_{t+1}. Specifically, we introduce a sequence of independent random variables $\{v_t\}$, whose joint distribution is assumed to be independent of the joint distribution of earnings (and whose realizations cannot, therefore, be accounted for by the lifecycle model with earnings uncertainty) and write,

$$r_{t+1} = \eta_{t+1} + v_{t+1} \tag{1}$$

Then the ratio

$$\rho^2 = \frac{Er_{t+1}L_t}{Er_{t+1}^2} = \frac{E\eta_{t+1}^2}{E\eta_{t+1}^2 + Ev_{t+1}^2}\,, \tag{2}$$

provides the fraction of the variance in r_{t+1} explained by the model. If ρ^2 is large, then the revision in the expected discounted value of lifetime earnings accounts for a large fraction of the variance in r_{t+1}, and if ρ^2 is close to one then one cannot reject the hypothesis that the lifecyle model with earnings uncertainty accounts for all the observed variance in r_{t+1} (note that the realization of ρ^2 could lie outside the unit interval). The next section uses aggregate U.S. time series data to illustrate this test of the determinants of consumption innovations.

III. Testing the Relationship Between Earnings News and Consumption Noise

The method of implementing the test of whether the revision in consumption expenditures can be accounted for by the lifecycle model with earnings uncertainty depends on the type of data available, particularly whether the data is micro panel or aggregate time series. Most previous tests that the revision in marginal utility is unpredictable (Propostion 1) have used aggregate time series data. For comparability we also use aggregate time series data. The use of aggregate time series data does, however, require us to make some additional simplifying assumptions. In particular, we ignore issues of aggregation over individuals, assume T (the planning horizon) approaches infinity, and assume the stochastic process generating earnings is (strictly) stationary and normal.[8] These assumptions simplify the testing procedure considerably.

The assumption of stationarity allows us to write the earnings process as an infinite autoregression with an independent and identically distributed disturbance. This disturbance is proportional to η_{t+1}, the revision in expected lifetime earnings between t and t+1 (see Anderson (1971)) and the definition of η_{t+1} in A3); that is,

$$w_{t+1} = \sum_{\tau=0}^{\infty} w_{t-\tau}\gamma_{\tau} + \varepsilon_{t+1} ,$$

and

$$\eta_{t+1} = \Theta\varepsilon_{t+1} , \tag{3}$$

where $\{\varepsilon_{\tau+1}\}$ is a sequence of independently and identically distributed random variables.9

Given stationarity of the earnings process, it is assumed that as $T \to \infty$ the function $\delta^{t+1}(\cdot)$ converges (pointwise) to the function $\delta^*(\cdot)$, a function that can be expressed as the n^{th} order polynomial

$$\delta^*(c_t) = \sum_{c=0}^{n} m_i c_t^i . \tag{4}$$

Note that if $m_i = 0$, for $i > 2$, the assumption of a quadratic or constant absolute risk aversion utility function (Corollary 2), and the corresponding linear predictor function for c_{t+1} used in previous analyses (e.g., Flavin (1981), Hall and Mishkin (1982)), is valid.

To obtain the system of equations to be estimated, we use the fact that $\underset{\sim}{\delta}^t(c_t) = R^{-1}[\delta^t(c_t) - c_t]$ (see the appendix). Equation (5a) is derived from this fact and (17), (3), (4) and the definition of r_{t+1} presented in Corollary 3, while equation (5b) comes directly from (3). This produces the system

$$c_{t+1} = k_0 + \sum_{i=1}^{n} k_i c_t^i - R^{-1} \sum_{i=2}^{n} k_i c_{t+1}^i + \theta m_1^{-1} \varepsilon_{t+1} + m_1^{-1} v_{t+1} \qquad (5a)$$

$$(5)$$

$$w_{t+1} = \sum_{\tau=0}^{\infty} \gamma_\tau w_{t-\tau} + \varepsilon_{t+1} \qquad (5b)$$

where $k_0 = m_0 (R^{-1}-1)/m_1$, $k_1 = R^{-1}(m_1 - 1)/m_1$, and $k_i = m_i/m_1 (i=2,\ldots,n)$.

Note that, if the model is correct, the coefficient of c_t^i in equation (5a) should be opposite in sign, and a bit smaller (in absolute value) than the coefficient of $c_{t+1}^i (i=2,\ldots,n)$ with the difference determined by R. Thus, for $i \geq 2$ we can obtain an estimate of R, and for $i > 2$ we can test the model's implications by testing if the coefficient of c_t^i equals R^{-1} times the coefficient of c_{t+1}^i.

Since both ε_{t+1} and v_{t+1} are determinants of c_{t+1} they will, in general, be correlated with powers of that variable. Therefore, consistent estimates of the coefficients in equation (5a) require the use of instruments for $c_{t+1}^i (i=2,\ldots,n)$. Clearly, the assumptions of the model imply that $E(c_t^i v_{t+1}) = E(c_t^i \varepsilon_{t+1}) = E(w_{t-\tau} v_{t+1}) = E(w_{t-\tau} \varepsilon_{t+1}) = 0$ for $i, \tau \geq 0$. Equation (5a) is therefore estimated by two-stage least squares using current and lagged earnings and powers of current consumption as instruments. Equation (5b) is estimated by ordinary least squares. Let e_{t+1}^c and e_{t+1}^w be the estimated residuals from the consumption and earnings equations, respectively, that is

$$e^c_{t+1} = c_{t+1} - \hat{k}_0 - \sum_{i=1}^{n} \hat{k}_i c^i_t - \sum_{i=2}^{n} R^{-1}\hat{k}_i c^i_{t+1}, \text{ and}$$
$$e^w_{t+1} = w_{t+1} - \sum_{\tau=0}^{\infty} w_{t-\tau}\hat{\gamma}_\tau, \tag{6}$$

where a circumflex over a variable indicates its estimated value. Then, letting $S(x,y)$ represent the sample covariance of x and y,

$$r^2_{e^c e^w} = \frac{S(e^w,e^c)^2}{S(e^w,e^w)^2 S(e^c,e^c)^2} \xrightarrow{p} \frac{\theta^2 E\varepsilon^2}{\theta^2 E\varepsilon^2 + Ev^2} = \rho^2 \tag{7}$$

where $\xrightarrow{p}$ reads converges in probability, and the last equality follows from the fact that $\eta = \theta\varepsilon$ (equation 3) and the definition of ρ^2 (equation 2). That is, the r^2 from the residuals of the two equation system in (5) provides us with a consistent estimate of ρ^2, the fraction of the variance in the revision in the expected discounted value of consumption expenditures that is accounted for by the lifecyle model with earnings uncertainty.

<u>The Data</u>

The data used in this study are National Income Accounts (NIA) quarterly observations of consumption of nondurables and services and quarterly NIA observations of wages and salaries.[10] There are 147 observations corresponding to the first quarter of 1947 through the third quarter of 1983. All observations were expressed in percapita terms and converted to 1972 dollars using a weighted average of the NIA nondurables deflator and the NIA services deflator, with the fixed weight determined by the average share of nondurables consumption in total consumption of nondurables plus services. Since our empirical approach assumes stationarity in earnings, we detrended wages and salaries with the trend path estimated by regressing the logarithm of wages and salaries against a constant and time.

Empirical Results

Table 1 presents the coefficients from estimating equation (5a) assuming first through fourth order polynomial functions for $\delta^*(\cdot)$ (equation (4)). Estimation of the linear model is by OLS, while the second, third, and fourth order models are estimated by two stage least squares.

The higher order terms in each of the regressions are highly significant suggesting that the linear model posited by Flavin (1981) and Hall and Mishkin (1982) is inappropriate. The appropriateness of a higher order model is also suggested by a test of the linearity of the function $g_t(\cdot)$ of Corollary 1. Specifically, we regressed c_{t+1} on successive higher order polynomials of c_t. In the regression of c_{t+1} on c_t and c_t^2 the coefficient of c_t^2 has a t ratio of -2.30 which is significant at the 5 percent level. The F statistics for the inclusion of two higher order terms is 2.65; it is 1.72 for the inclusion of three higher order terms. These values are marginally below their respective 5 percent critical F values of 3.00 and 2.60.

Note that acceptance of the higher order model has implications for one's views about the extent of earnings uncertainty at the macro level. As indicated in Table 1 the ratio of the standard error of the regression to the mean value of real percapita consumption declines rapidly as the number of higher order terms is increased. For the fourth order model the ratio is .00003, suggesting a triviably small degree of uncertainty influencing aggregate per capita consumption.

Recall that if the model is appropriate the coefficient of c_t^i equals minus R^{-1} times the coefficient of C_{t+1}^i for $i \geq 2$ (see equation (5a)). Looking at the unconstrained parameters estimates in Table 1 it is clear that they are close to satisfying these constraints. However, a formal test of these constraints clearly rejects them; the observed value of the $F(2, 131)$ test statistic is 21.36. This occurs because the fourth order model has a near perfect fit, making even those alternatives that are close to the null hypothesis very powerful. The estimate of R^{-1}, that is of one plus the annual real interest rate, obtained from the constrained 4th order model has the reasonable value of 1.032 with a standard error of .018.

While the innovation to consumption may be economically insignificant, at least at the macro level of aggregation, a separate issue examined in Table 2 is the extent to which consumption innovations are explained by new information about earnings. Table 2 provides the estimated fractions of the variance in consumption innovations (noise) explained by earnings information (news) (See equation (2)). As indicated by equation (7) this ratio is equal to the squared correlation coefficient between the residual in the consumption regression (equation (5a)) and the residual in the earnings autoregression (equation (5b)). Equation (5b) was estimated using eight lagged values of quarterly earnings. We also conducted the analysis using four rather than eight lags of earnings in the autoregression and obtained results essentially identical to those reported in Table 2.

All of the ratios reported in Table 2 are quite small. In the first order, linear model the innovation in earnings explains less than a fifth of the innovation in consumption. For the higher order models "news" is two percent or less of "noise". Only in the first order model is the estimated ratio of news to noise statistically significantly different from zero.

Summary and Conclusion

This paper's chief contribution is to suggest a method of examining the determinants of consumption when consumption decisions are based on expected utility maximization and lifetime earnings is uncertain. Our analysis also clarifies the relationships between innovations in marginal utility (the error in the standard Euler equation approach) and fundamental unexpected changes in perceptions of future lifetime earnings. We believe our results can be used not only to test the optimality of consumption choice under earnings uncertainty, but also to identify the extent and structure of this uncertainty. Applying these results to micro panel data should be particularly fruitful since they permit comparison across demographic and occupational groups in the magnitude and time resolution of earnings uncertainty. Much of the uncertainty in earnings in the cross section is, of course, averaged out in macro data. Indeed the application, presented here, of our theoretical results to time series data suggests strikingly little earnings uncertainty at the macro level. In addition, new information about earnings has little or no bearing on consumption innovations in contradiction to the standard neoclassical model of consumption choice under earnings uncertainty.

Footnotes

1. King (1982) stresses the explicit specification of a utility function required for this test, and points out that statistical analysis of the martingale properties of marginal utility involves joint tests of particular preference structures as well as intertemporal optimization given those preferences.

2. One difficulty in evaluating these studies as a group is that they involve repeated use of much the same data for consumption and leisure choices while using a variety of different time series that incorporate past information; in a finite sample innovations in marginal utility will be significantly correlated with a multitude of time series that represent past information, and one will surely find many such series in repeated searches.

3. This and the following proposition assume that all consumption paths having positive probability are feasible, i.e., they satisfy the budget constraint; and that consumption is strictly positive. These assumptions permit borrowing over the lifecyle, but require repayment of all debts by the end of period T.

4. The proof of the proposition is constructive in that it provides a method of calculating these functions from the utility function and probability distributions of revisions in the expected discounted values of lifetime earnings.

5. Inverting P1 to solve for c_{t+1} and substituting the resulting expression into P2 yields: $\delta^{(t+1)}[U'^{-1}(\lambda U'(c_t) + \xi_{t+1})] - R^{-1}\underset{\sim}{\delta}^{(t)}(c_t) = \eta_{t+1}$, and

$$\frac{\partial \xi_{t+1}}{\partial \eta_{t+1}} = 1/(\delta^{(t+1)'}[U'^{-1'}(\cdot)]) < 0, \text{ since } \delta^{(t+1)'} > 0, \text{ and } U'^{-1'} < 0.$$

6. P2 and the implicit function theorem imply the existence of a monotonically increasing continuously differentiable function $Q^{(t+1)}(\cdot)$ such that $c_{t+1} = Q^{(t+1)}\{R^{-1}\underset{\sim}{\delta}^{(t)}(c_t) + \eta_{t+1}\}$. The function $g_t(c_t)$ is constructed by integrating $Q^{t+1}(\cdot)$ over the probability measure, $dF_{t+1}(\eta_{t+1})$.

7. In particular we could add quite general error processes to both consumption, and to earnings, and still derive consistent estimates of the average of $E\eta^2_{t+1}$ among individuals in different groups. We are currently pursuing this line of research in related work on micro data.

8. Strictly speaking the assumption of stationarity is not necessary since, under mild regularity conditions on the boundedness of the variance of the earnings process, the fact that $R < 1$, implies that, as T grows, the difference, $\sum_{\tau=0}^{T} R^{\tau}w_{j+\tau} - \sum_{\tau=0}^{\infty} R^{\tau}w_{j+\tau}$, converges, in mean square, to zero. That is, if we formed $L_j^{(T)} = \sum_{\tau=0}^{T} R^{\tau}w_{j+\tau}$, then, by choosing T large enough, we can insure that the difference $r_jL_j^{(T)} - r_jL_j^{(\infty)}$ is smaller than any positive ε with probability one. On the other hand the larger is T, the less data is available from the means $(J^{-1}\sum_{j=0}^{J} r_jL_j^{(T)}$ and $J^{-1}\sum_{j=0}^{J} r_j^{L})$ and, the larger will be the standard error of our estimate of ρ^2 [equation 2]. We actually tried to form these means empirically for the special case of quadratic or constant absolute risk aversion utility functions (see Corollary 2), but it became clear that sufficiently large T results in the loss of too many degrees of freedom.

9. If the process generating earnings has a convergent autoregressive representation, then θ can be expressed as a function of the autoregressive

coefficients (the γ_τ), and one could impose, or test, this constraint. In the empirical work the value of θ varied with the order of the autoregression we assumed, though the estimated variance of the disturbance from the wage equation, and its covariance with the residual in the consumption equation (see below) did not vary significantly. This is another example of the familiar result that the residuals formed after estimating a stationary process do not vary much with the precise form of the process estimated, though other properties of the estimated process may vary substantially. Since our theoretical results are independent of the precise form of the earnings process, we thought it best to leave θ unconstrained.

10. We also used NIA observations on compensation of employees as the earnings variable. None of the empirical results were materially affected by using employee compensation rather than wages and salaries.

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Appendix

Proof of P2: Let $\{c_{j+\tau}^{(j)}\}_{\tau=0}^{T-j}$ be the optimal consumption program for period j. Since this program must satisfy the budget constraint in year j,

$$E_j \sum_{\tau=0}^{T-j} R^{\tau} c_{j+\tau}^{(j)} = A_j R^{-1} + E_j \sum_{\tau=0}^{T-j} R^{\tau} w_{j+\tau}.$$

Using this condition for period t and t+1, and the fact that $A_{t+1} = A_t R^{-1} + w_t - c_t$, one can show that

$$\delta_{t+1} = R^{-1} \underset{\sim}{\delta}_t + \eta_{t+1}, \tag{A1}$$

where $\delta_{t+1} = E_{t+1} \sum_{\tau=0}^{T-(t+1)} R^{\tau} c_{t+1+\tau}^{(t+1)}$, and $\underset{\sim}{\delta}_t = \delta_t - c_t$. The term δ_{t+1} equals the expected discounted value of current and future consumption conditional on the information set in period t+1 and is, therefore, a function of I_{t+1}; i.e., $\delta_{t+1} = \delta_*^{(t+1)}(I_{t+1})$; and $\underset{\sim}{\delta}_t = \underset{\sim}{\delta}_*^{(t)}(I_t)$. To prove the proposition, it suffices to prove the following lemma.

Lemma: If A1 either A2 or A3 is satisfied then for t=1,...,T-1, $\underset{\sim}{\delta}_*^{(t)}(I_t) = \underset{\sim}{\delta}^{(t)}(c_t)$; with $\underset{\sim}{\delta}^{(t)}(c_t)$ monotonically increasing and continuously differentiable in c_t.

Proof: If the utility function is quadratic (A2), then the lemma follows directly from Proposition 1 and the definition of $\underset{\sim}{\delta}_t$, for quadratic utility implies that $E_t[c_{t+\tau}^{(t)}] = (\lambda^{\tau} - 1)\alpha_o + \lambda^{\tau} c_t$ for $\tau \geq 0$; where $\lambda = \beta/R$, and α_0 is determined by the parameters of the utility function. If A2 is not satisfied but A3 is, the lemma is proved by induction. Thus assume $\delta_*^{(j+1)}(I_{j+1}) = \delta^{(j+1)}(c_{j+1})$, the latter function being monotonically increasing continuously differentiable in c_{j+1}. Then equation (A1) and the implicit function theorem imply the existence of a continuously differentiable monotonically increasing function $Q^{j+1}(\cdot)$ such that

$$c_{j+1} = Q^{(j+1)}(\underset{\sim}{\delta}_j R^{-1} + \eta_{j+1}). \tag{A2}$$

Also from Proposition 1,

$$U'(c_{j+1}) = \lambda U'(c_j) + \xi_{j+1} \text{ with } E_j \xi_{j+1} = 0. \tag{A3}$$

Substituting (2) into (3) and taking expectations we have

$$H^{(j+1)}(\underset{\sim}{\delta}_j, c_j) = \int [U'\{Q^{(j+1)}(\underset{\sim}{\delta}_j R^{-1} + \eta_{j+1})\}] dF_{j+1}(\eta_{j+1}) - \lambda U'(c_j) = 0, \tag{A4}$$

with $H^{(j+1)}_{\underset{\sim}{\delta}_j} = R^{-1} \int U''\{Q^{(j+1)'}(\underset{\sim}{\delta}_j + \eta_{j+1})\} dF_{j+1}(\eta_{j+1})$, which is negative and continuous in $\underset{\sim}{\delta}_j$ by virtue of the continuity of $U''(\cdot)$ and $Q^{j+1'}(\cdot)$; and $H'_{c_j} = -\lambda U''(c_j)$, which is positive and continuous in c_j. The implicit function theorem therefore implies the existence of a monotonically increasing continuously differentiable function $\underset{\sim}{\delta}^{(j)}(c_j)$ such that $\underset{\sim}{\delta}_j = \underset{\sim}{\delta}^{(j)}(c_j)$. Since $\delta_j = \underset{\sim}{\delta}_j + c_j$, it follows that $\delta_*^{(j)}(I_j) = R[\underset{\sim}{\delta}^j(c_j) + c_j] = \delta^{(j)}(c_j)$, is also monotonically increasing and continuously differentiable in c_j. To complete the inductive argument one need only observe that $\delta_*^{(T)}(I_T) = c_T$, and construct $\underset{\sim}{\delta}^{T-1}(c_{T-1})$ from equations (A2), (A3), and (A4) substituting T-1 for j.

Two points are worthy of note here. First the proof clarifies the roles of assumptions 2 and 3 in the text in deriving proposition 2. If the utility function is quadratic (assumption 2) the both $Q^{j+1}(\cdot)$ and $U'(\cdot)$ are linear. In that case equation (A4) involves integrating over a linear function of η_{j+1}, so that $H^{j+1}(\cdot)$ depends on the distribution of η_{j+1}, only through $E_j\eta_{j+1}$, which is zero by construction. For quadratic utility then the proposition is true regardless of whether the distribution of η_{j+1} depends on any variables in I_j. If the utility function is not quadratic, then equation (A4) involves integrating over a convex function of η_{j+1}. The integral will then depend on higher order moments of η_{j+1}, and though $E_j\eta_{j+1}=0$, the conditional variance, say, of η_{j+1} may depend on variables in I_j. Thus, without either quadratic utility or assumption 3, $H^{j+1}(\cdot)$ will be a function of more variables in I_j then c_j, and neither proposition 2 nor the statement that E_jc_{j+1} is only a function of c_j alone are true. The second point is that the proof is constructive in the sense that given any $U(\cdot)$, and any sequence $\{dF_{j+1}(\eta_{j+1})\}$, the proof explains exactly how to construct $\{\delta^{j+1}(c_{j+1})\}$ and $\{\underset{\sim}{\delta}^j(c_j)\}$.

Proof of Corollary 2:

If A2 is satisfied then Corollary 2 follows directly from the proof of Proposition 2. To prove the corollary when A3 and A4 are satisfied we first use an inductive argument to show that $\{\delta^{(t+1)}(c_{t+1}) = \psi_{0,t+1} + \psi_{1,t+1}c_{t+1}\}_{t=1}^{T-t}$ and then derive the implied relationship between c_{t+1} amd c_t. Assuming $\delta^{(t+1)}(c_{t+1}) = \psi_{0,t+1} + \psi_{1,t+1}c_{t+1}$, equation (2) in the proof of Proposition 2 becomes,

$$c_{t+1} = \frac{1}{\psi_{1,t+1}}(\underset{\sim}{\delta}_t R^{-1} + \eta_{t+1} - \psi_{0,t+1}).$$

Substituting this equation into (3) and solving (4) for $\underset{\sim}{\delta}_t$ yields:

$$\underset{\sim}{\delta}_t = R\psi_{1,t+1}c_t + \frac{R\psi_{1,t+1}}{\gamma}\log\left(\frac{k_{t+1}}{\lambda}\right) + \psi_{0,t+1}R,$$

where $k_{t+1} = \int e^{-\frac{\gamma}{\psi_{1,t+1}R}\eta_{t+1}} dF_{t+1}(\eta_{t+1})$. Noting that $\delta_t = (\underset{\sim}{\delta}_t + c_t) = \delta^{(t)}(c_t)$, and that $\delta^T(c_t) = c_t$, completes the inductive argument. Clearly this argument implies that the sequences $\{\psi_{0,t}\}$ and $\{\psi_{1,t}\}$ are determined by the recursions $\psi_{1,t} = \psi_{1,t+1}R + 1, \psi_{0,t} = \frac{R\psi_{1,t+1}}{\gamma}\log\frac{k_{t+1}}{\lambda} + \psi_{0,t+1}R$, with initial conditions $\psi_{1,T} =$; and $\psi_{0,T} = 0$. This solution and equation (1) in the proof of Proposition 2, imply the corollary.

Table 1

Regression Results: First Order Through Fourth Order Consumption Models*

Variable	First Order Model	Second Order Model	Third Order Model	Fourth Order Model
Constant	-3.417 (5.973)	-2.914 (8.818)	14.986 (12.825)	-22.224 (10.845)
C_t	1.006 (.002)	1.006 (.007)	.988 (.014)	1.040 (.017)
C^2_t		-.184E-3 (.805E-5)	-.405E-3 (-.158E-4)	-.583E-3 (.115E-4)
C^2_{t+1}		.183E-3 (.843E-5)	.407E-3 (.180E-4)	.557E-3 (.908E-5)
C^3_t			-.521E-9 (.380E-10)	.140E-8 (.459E-10)
C^3_{t+1}			.521E-9 (.406E-10)	-.133E-12 (.475E-10)
C^4_t				-.123E-12 (.631E-14)
C^4_{t+1}				.116E-12 (.677E-14)
Ratio of Standard Error of Regression to Mean Value of Consumption	.00542	.00116	.00024	.00003

*Two stage least squares estimates of equation (5a). $W_{t-\tau}$ and c_{it}(τ=0, 1, ..., 7 and i = 1, ..., N) are used as instruments. There are 129 observations. Numbers in parentheses below coefficient estimates are estimated assymptotic standard errors. E1 is 10.

Table 2

Estimated Ratios of News to Noise and Estimated Assymptotic Standard Errors

Model	Ratio of News to Noise	Standard Error of Ratio
First Order	.181	.077
Second Order	.511E-3	.084
Third Order	.743E-2	.084
Fourth Order	.020	.050