The Economic Journal, **100** (*December* 1990), 1261–1268
Printed in Great Britain

A STRATEGIC ALTRUISM MODEL IN WHICH RICARDIAN EQUIVALENCE DOES NOT HOLD*

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It is now many years since Robert Barro (1974) wrote his ingenious article showing how love of children (intergenerational altruism) can economically link current and future generations and thereby neutralise intergenerational redistribution by the government (Ricardian Equivalence). Several critics have pointed out reasons why the requirement for Ricardian Equivalence of interior transfers may not be satisfied (e.g. Barro, 1974; Drazen, 1978; Laitner, 1979 and 1988, and Feldstein, 1988). Others (Kotlikoff, 1983 and Bernheim and Bagwell, 1988) have cast doubt on the model by showing how intermarriage across Barro dynasties can lead to incredibly large groups of intragenerationally linked individuals, redistribution among whom will also be neutralised.

None of the critics has, however, questioned whether Ricardian Equivalence necessarily follows from the basic elements in Barro's study. This article does just that. It examines the strategic game between an altruistic parent and a possibly altruistic child[1] and shows, under the Extended Nash Bargaining Solution[2], that Ricardian Equivalence will almost never hold.

Barro does not make explicit the game he models between an altruistic parent and child, but in his formulation the child appears to be quite passive and simply takes whatever transfer is given. There is no scope for the child to manipulate the parent by threatening to refuse transfers that are below a specified level and/or by threatening to transfer funds to the parent if the parent is not sufficiently generous. Stated differently, there is no scope for strategies associated with statements such as 'If that's the best you can do, forget it'. The apparent restrictions on the actions of children in the Barro model become more apparent if parents not only are altruistic with respect to their children but children are also altruistic with respect to their parents. While parents and children may care for each other, they are unlikely to agree on the exact net amount to be transferred between them. For families with reciprocal altruism (presumably most families) the problem then is one of competing altruism in which parents may be trying to transfer to their children at the same time that the children are trying to transfer to their parents. In such a setting the assumption that each player simply accepts whatever is offered

* We are grateful to the National Science Foundation (grant no. SES8808362) and to the National Institute of Aging (grant no. 1P01AG05842-01) for research support. We thank Andy Weiss and a referee for helpful comments.

[1] Bernheim *et al.* (1985) also consider strategic interplay between parents and children but in a model in which altruistic parents wish to procure services from their children.

[2] Nash (1953). See also Luce and Raiffa (1957) Chapter 6 for a summary with critical comments.

seems unrealistic. Individuals seem equally empowered both to make and to refuse gifts.[3]

The next section computes both the ordinary and extended Nash Bargaining Solutions (which turn out to be the same) to a game involving a parent and child, at least one of whom is altruistic towards the other. Section II shows why the solution will almost never be neutral with respect to government redistribution between the two players. Section III discusses the differences between cooperative and noncooperative solutions to this game and suggests a way of distinguishing empirically between the two. Section IV concludes and presents ideas for future research that would expand on the framework presented here.

I. THE EXTENDED NASH BARGAINING SOLUTION IN A TWO-PERSON ALTRUISM GAME

There are two stages in this altruism game. In the second stage the players agree to maximise the product of their utility gains relative to the respective values of utility at the threat point that results from the first stage.[4] From this maximisation one can compute the indirect utility of each player as a function of the threat point. In the first stage the players choose threat strategies noncooperatively. The payoffs in the resulting two-stage game are the indirect utilities for the point resulting from any pair of threat strategies.

The Second Stage

Equation (1) expresses the Nash product N for the second stage. The terms $V_p(C_p, C_k)$ and $V_k(C_p, C_k)$ stand for the utility functions of the parent and child, respectively. Their arguments, C_p and C_k, are the respective consumptions of the parent and child. The terms $\bar{V}_p$ and $\bar{V}_k$ stand for the respective threat-point utilities of the parent and child, which are constants in this stage.

$$N(C_p, C_k) = [V_p(C_p, C_k) - \bar{V}_p]\,[V_k(C_k, C_p) - \bar{V}_k]. \tag{1}$$

To keep matters simple we assume that $V_p(\,,)$ and $V_k(\,,)$ are of the forms:

$$V_p(C_p, C_k) = u(C_p) + w(C_k) \tag{2}$$

$$V_k(C_k, C_p) = m(C_k) + n(C_p) \tag{3}$$

where the functions $u(\,)$, $w(\,)$, $m(\,)$, and $n(\,)$ are continuously differentiable, increasing, and concave.[5]

[3] Abel (1987) and Kimball (1986) rule out the refusal of gifts a *priori*. In their analyses of 'two way' altruism they develop conditions on preferences that will, in part, ensure that transfers are never refused in a noncooperative game in which each player chooses transfers taking the transfers of others as given.

[4] There is an extensive literature beginning with Nash (1950) justifying the product-of-the-utility gains solution. See Roth (1979) for a recent survey.

[5] These forms for the utility functions $V_p(\,,)$ and $V_k(\,,)$ are consistent with the parent (the child) caring about his own consumption and the utility of the child (the parent). For example, use (3) to write the following expression: $C_k = m^{-1}[V_k - n(C_p)]$. The insertion of this expression into (2) yields $V_p(C_p, C_k) = H_p(C_p, V_k) = u(C_p) + w(m^{-1}[V_k - n(C_p)])$.

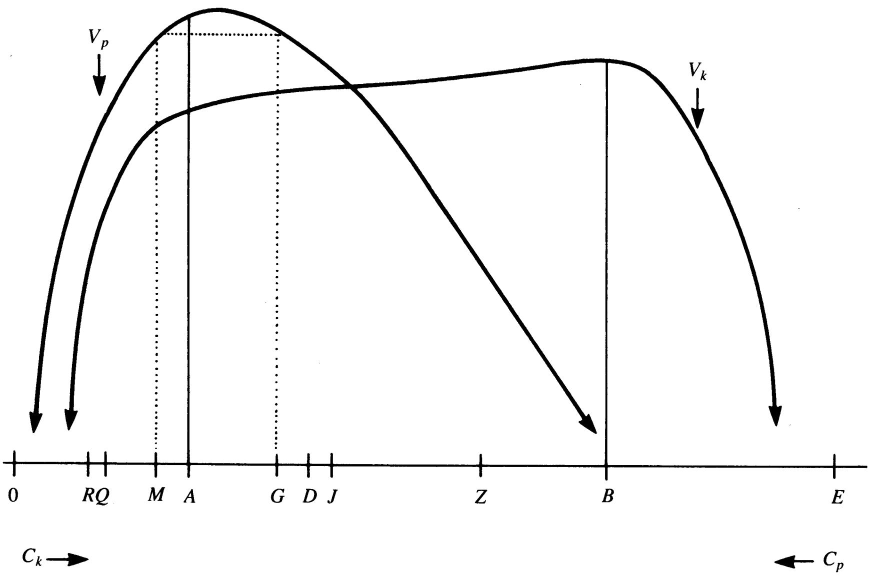

Fig. 1.

The expression for N is maximised subject to the collective parent-and-child budget constraint:

$$C_p + C_k = E_p + E_k = E, \tag{4}$$

where E_p and E_k are the endowments of the parent and child, respectively, and subject to the constraint that both factors in brackets on the right-hand side of (1) be non-negative. Any solution to this maximisation problem satisfies:

$$[u'(C_p) - w'(C_k)]\,[m(C_k) + n(C_p) - \bar{V}_k] + [m'(C_k) - n'(C_p)]\,[u(C_p) + w(C_k) - \bar{V}_p] = 0. \tag{5}$$

There are two different ways equation (5) could be satisfied. One way is for both terms in (5) to be zero (i.e. at least one of the factors of each term to be zero). This can occur, for instance, if both parent and child remain at the threat point or if the factors involving derivatives are both zero. The second way is for the ratio of $\partial V_p(C_p, E - C_p)/\partial C_p$ (the first factor in square brackets in (5)) to $\partial V_k(E - C_p, C_p)/\partial C_p$ to equal minus the ratio of the parent's utility gain to the child's utility gain.

Fig. 1 depicts $V_p(C_p, E - C_p)$ and $V_k(C_k, E - C_k)$ under the assumptions that: (i) $u'(0) = w'(0) = m'(0) = n'(0) = \infty$ and (ii) $x > y$ where x and y are defined by $u'(x) = w'(E - x)$ and $n'(y) = m'(E - y)$. In the Fig., C_k is measured from left to right on the horizontal axis and C_p from right to left, their sum being fixed at E. The first assumption ensures that the parent's and child's most preferred allocations (their respective bliss points) lie between 0 and E on the horizontal axis. The second assumption ensures that the parent's (child's) bliss point involves more consumption by the parent (child) than does the child's (parent's) bliss point. Points A and B indicate allocations corresponding to the bliss points of the parent and child, respectively. Any allocation lying between points A and B, such as Z, is a Pareto-optimal allocation.

As described below, the first-stage game leads to the determination of a threat-point allocation on the horizontal axis. The threat values of the parent's and child's utility, $\bar{V}_p$ and $\bar{V}_k$, correspond to the values of $V_p(\,,\,)$ and $V_k(\,,\,)$ evaluated at this threat-point allocation. If the result of the first-stage game is a threat allocation such as Z that lies between A and B, the solution to the second stage is for the players to consume the allocation Z. This is a simple consequence of Pareto optimality and corresponds to the first instance of the first type of solution to equation (5). In contrast, if the first-stage game leads to a threat allocation to the right of B or to the left of A, such as R, N can be increased beyond the value obtained by consuming the allocation R. The solution in this case occurs at a point like D, where the ratio of the slope of V_p to that of V_k equals minus the ratio of the parent's utility gain to the child's utility gain. (The point D necessarily lies between points A and B because to the left of A and to the right of B the slopes of V_p and V_k have the same sign, making their ratio nonnegative.) Note that the point D resulting from R is uniquely determined: between A and B the absolute value of the ratio of the slopes increases in C_k, while the ratio of the utility gains falls.

In the case that the parent's bliss point lies to the right of the child's bliss point, threat-point allocations lying between the two bliss points will again be decisive in that each player will consume at the threat-point allocation. For points to the left of the child's bliss point and to the right of the parent's bliss point the bargaining solution will again map to a point between the two bliss points. In the case that the child does not care about the parent ($n(C_p) = 0$, violating assumption (i)), the graph corresponding to Fig. 1 is similar except that V_k rises monotonically between $C_k = 0$ and $C_k = E$.

In the case that the two bliss points coincide there is only one Pareto optimum (at the coincident bliss points), and the solution to (5) involves both derivative factors being zero.

The First Stage

We model the first-stage game strategies for the parent and child as choices of how much to offer each other and how much to accept from each other. These should be thought of as maxima in all cases; for instance, if the parent offers α and accepts β, while the child offers γ and accepts δ, then $\min(\alpha, \delta) - \min(\beta, \gamma)$ is the net transfer from parent to child. Since the two players start out with the total endowment and nothing is wasted, the result of the first-stage game is simply a reallocation (here representable as a point on the horizontal axis in Fig. 1).

The solution to the first stage game is quite simple. The equilibrium threat-point allocation always turns out to be just the point of initial endowments of the players. This is an immediate consequence of the fact that each player can enforce the initial endowment point as the outcome of the first stage. Since for each solution to the first stage game, the second stage results in a Pareto optimum, any move from the endowment point in the first stage must be worse overall for one of the players who will, therefore, veto it.

II. THE FAILURE OF RICARDIAN EQUIVALENCE

Ricardian Equivalence means that any government transfer is subsequently undone by the private actions of parent and child, so that the government's redistribution has no real consequences. This property almost never holds for the model considered above. To see this examine again Fig. 1. Any government redistribution that results in an endowment between A and B is not altered. A government redistribution that leaves the endowment to the left of point A or leaves the endowment to the right of point B, while it leaves unchanged the direction of the bargained net transfer, will, nonetheless, change the threat point in stage 1 and, therefore, the solution.

In the Fig., government redistribution from the parent to the child that moves the endowment point, and therefore the threat point, from point R to point Q moves the solution from point D to point J. In the Fig., point J lies to the right of D (though it need not in general). Hence, the government policy is successful in increasing both the consumption and welfare of the child; i.e. in this example, private transfers do not fully offset the government's transfers.

Private transfers may, however, more than fully offset government redistribution. As an example, if the government redistributes from the parent to the child by moving the endowment from point R to point M, the solution will move from point D to a point between A and G. The solution must lie to the left of G because to the right of G the parent is made worse off than remaining at the threat point M. Compared with point D, a solution to the left of point G involves smaller net transfers to the child: i.e. private transfers more than offset the government's transfers. From the Fig. it is clear that, in general, if the government redistributes enough to the net recipient of private transfers, but not so much as to move the endowment point into the region between A and B, the private response will more than offset the government policy. Thus, if the government takes away too much of the net transferee's bargaining leverage, the net transferee will end up worse off.

There can be isolated instances where the government's transfer is exactly offset; here Ricardian Equivalence holds. Also in the case $A = B$ the unique Pareto-optimal allocation is the solution in the second stage no matter what transfers occur; hence, for this case Ricardian Equivalence holds.

III. COMPARING THE EXTENDED NASH BARGAINING SOLUTION TO THE NON-COOPERATIVE EQUILIBRIUM

The noncooperative equilibrium in this same static framework exhibits Ricardian Equivalence in more cases than the isolated ones above.[6] In the one-

[6] Non-cooperative models are examined by Carmichael (1982), Burbridge (1983), Weil (1987), Abel (1987), and Kimball (1986).

stage noncooperative equilibrium each player takes the other player's maximum offer and acceptance as given and chooses his best response. In terms of Fig. 1 if the endowment point lies between points A and B, the only noncooperative equilibrium is the same as the cooperative solution, resulting in each player consuming his endowment. If the endowment lies to the left of point A the following describes all the equilibria: The parent offers to transfer just enough funds to the child to move the allocation to the parent's bliss point A and the parent accepts nothing. The child offers nothing and accepts the amount of the parent's offer (or more). The obvious analogy holds to the right of B. Starting from endowments to the left of A (or to the right of B), government redistributions that keep the endowment to the left of A (or to the right of B) leave unchanged the equilibrium outcome. The players still move to the same bliss point with the same utility payoffs. For these cases Ricardian Equivalence holds.

One might argue that the noncooperative solution is more plausible than the cooperative solution. Rather than agree to play the cooperative game, why does the potential net transferor not simply call the other player's bluff. For example, if the endowment lies to the left of point A, why does the parent not simply tell the child 'take it or leave it,' and why does the child not simply take it. One answer is that since the child knows the parent's altruistic utility, the child calls the parent's bluff. This assumes the parent has no last-mover advantage.

Another answer may be that the child cares about the bargaining process as well as the outcome. If the child feels he is being told 'take it or leave it,' he may leave it because he resents being treated in that manner. The child may also feel a loss of pride in accepting a transfer, so that transfer may need to be conveyed to the child in a manner that preserves the child's pride.

In the model presented here the noncooperative equilibria happen to be Pareto optimal, so Pareto improvements do not justify the cooperative solution. However, extensions of the model lead to cases in which the noncooperative equilibrium is not Pareto optimal. As an example, take the case in which the parent has two children each of whom cares about the parent, but who are not altruistic toward each other. Also suppose that the parent is not altruistic toward the children. In this case the noncooperative equilibrium, if it involves both children transferring to the parent, will not be Pareto optimal (see Nerlove *et al.*, 1984). In making their transfers to the parent, each child ignores the external benefit to the other. As a consequence, the noncooperative equilibrium involves too little being transferred to the parent.

One way to test empirically the cooperative model against the non-cooperative model is to determine whether the distribution of consumption among family members who are parties to net transfers depends on the distribution of initial resources among these members. For example, suppose one had a sample of parents each transferring to his child. According to the cooperative model presented here, the distribution of endowments between the parent and child will affect the distribution of consumption between the parent and child. Such is not the case in the noncooperative model.

IV. IDEAS FOR FUTURE RESEARCH AND CONCLUSION

This article demonstrates that Ricardian Equivalence does not necessarily hold in models with altruistic transfers once one takes into account the strategic behaviour of recipients as well as donors. The model we have used to make this point is, however, static and highly stylised. It does not take into account that parents and children can bargain over many periods and that their bargaining positions may depend on their life expectancies. It also takes a particular view of both the bargaining process and the strategies available to the players (although more realistic alternatives are not obvious). Finally, it does not consider how the bargaining outcome is affected by the presence of more than one child and/or more than one parent.

Specifying alternative noncooperative and cooperative strategic-altruism models in finer detail may represent a fruitful line of research. We suspect that for the most part such models will not, however, satisfy Ricardian Equivalence because, as in the cooperative model of this paper, interior transfers can result from non-interior strategic postures and because non-interior strategies (e.g., accepting nothing) are likely to be aspects of equilibria.

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Date of receipt of final typescript: June 1990

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