Your Name ______________________

PY132 Physics of Motion
Practice for the Second Exam
Monday, November 11, 2006

This is a closed book exam, with no notes allowed. You may use (but not share) a calculator. Do not talk or otherwise communicate with the other students.

Show the mathematical steps that lead you to your answers. Without them, a "right" answer is unjustified. With them, a "wrong" answer may be worth partial credit.

Some equations that you might need to be reminded of:

$g = 10\,{}^{m}\!/_{s^2}$

$PE_{grav} = mgh$

$PE_{spring} = \frac{1}{2}kx^2$, $\quad F_{spring} = -kx$

$\text{impulse} = F_{avg}\Delta t = \Delta p$

$\text{Power} = W/\Delta t$

$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$\vec{F} = m\vec{a}$	$\vec{\tau} = I\vec{\alpha}$
$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
$w = Fd$	$w = \tau\theta$
	$\tau = rF\sin\theta$

$s = r\theta$, $\quad v = r\omega$, $\quad a = r\alpha$, $\quad a_c = v^2/r$

Point mass	$I = mr^2$
Ring, about axis	$I = mr^2$
Solid rod, about center	$I = \frac{1}{12}ml^2$
Solid rod, about end	$I = \frac{1}{3}ml^2$
Solid cylinder, about axis	$I = \frac{1}{2}mr^2$
Hollow cylinder, about axis	$I = mr^2$
Solid sphere	$I = \frac{2}{5}mr^2$
Hollow sphere	$I = \frac{2}{3}mr^2$

1. ______ /35 + 2. ______ /15 + 3. ______ /20 + 4. ______ /30 = Total _______ /100

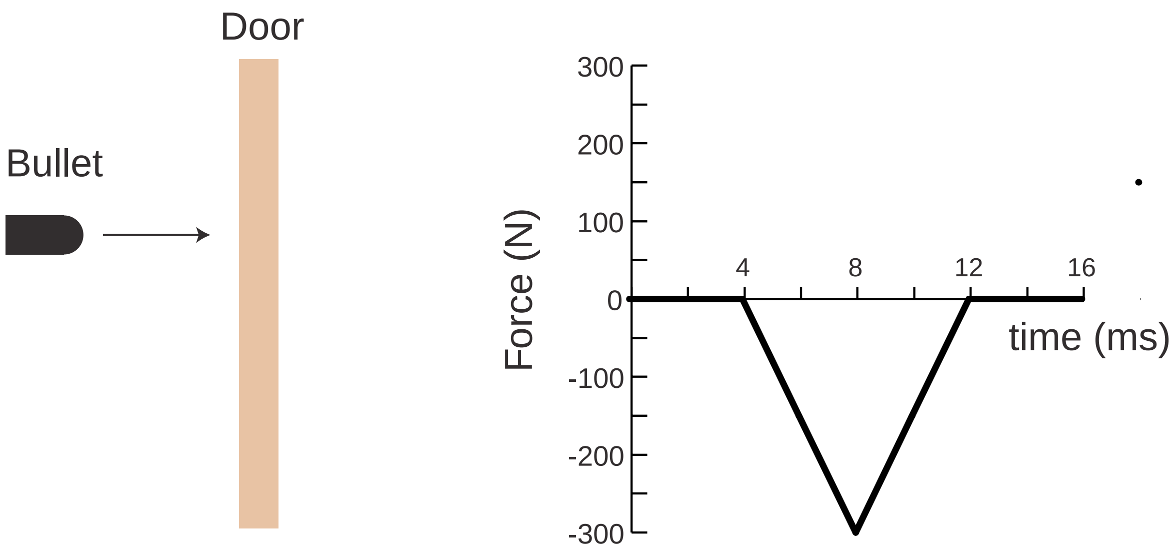
Problem 1 [25 pts]. Conceptual or short questions

1a) [8 points] Torque and work are measured in the same units of newtons times meters. Explain how torque and work are different.

1b) [9 pts] When you take the battery out of a clock, it takes 30 seconds for the second hand to completely come to a stop (uniformly decelerating). The second hand is 0.10 m long.

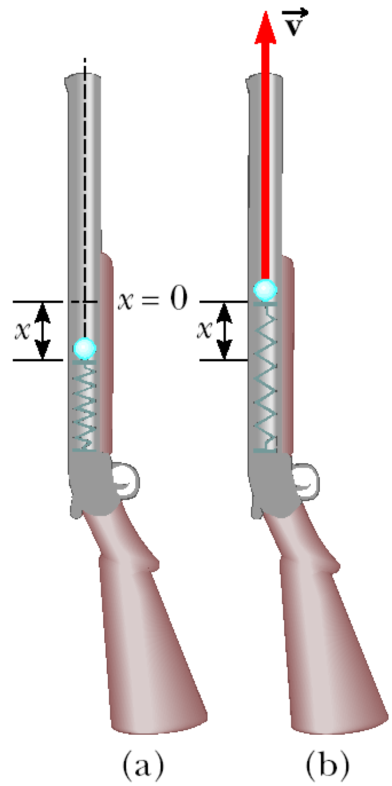
How fast (linear speed) is the tip of the second hand going 10 seconds after you take out the battery? **Show your work.**

1c) [8 pts] A 10 gram bullet is fired at a door with a speed of 400 m/s. The force on the bullet as it goes through the door is shown on the graph. How fast is the bullet going when it exits the door? **Show your work!** 10 gram = 0.010 kg.



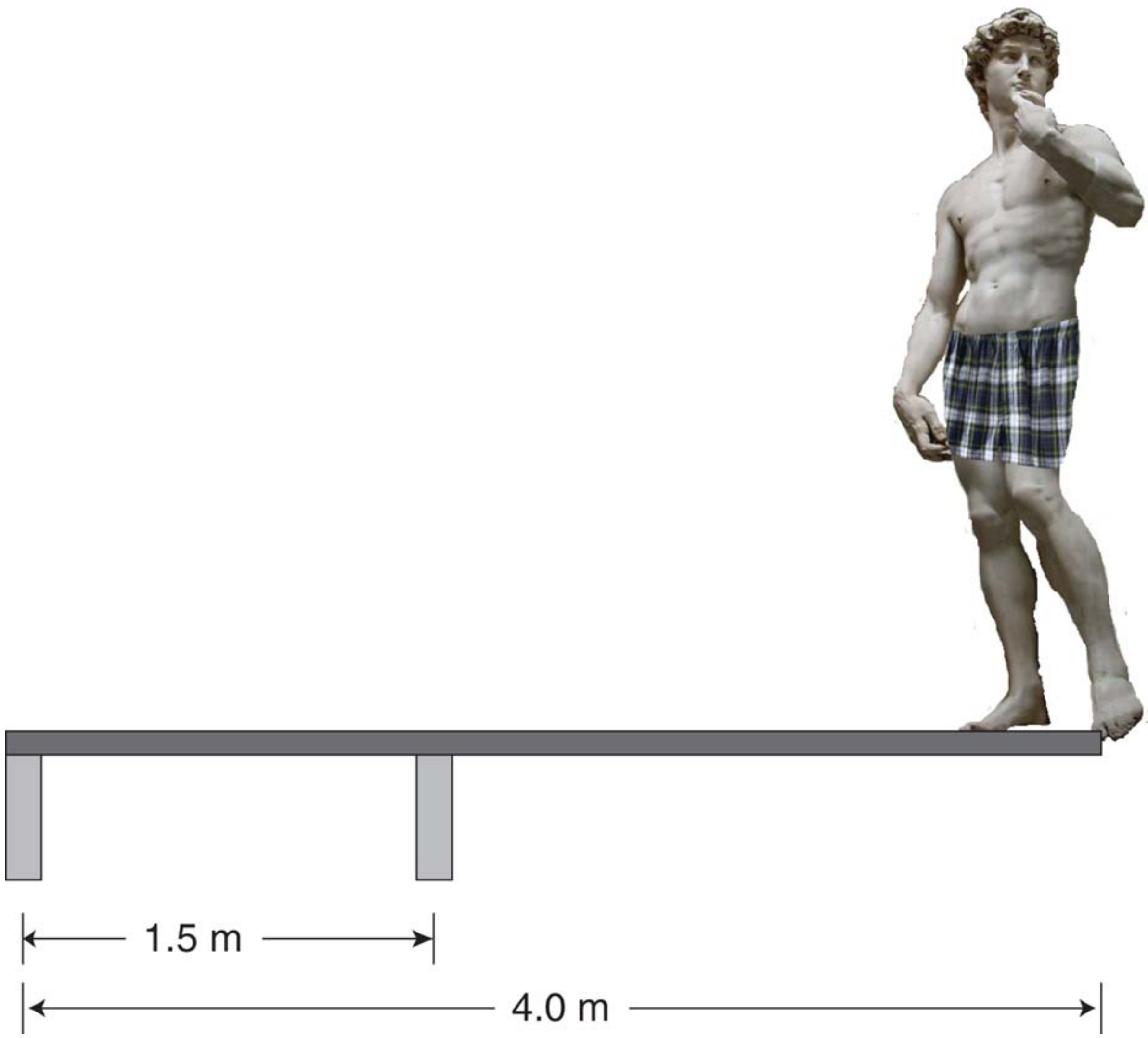
Problem 2. **[25 points] The spring in a toy gun.**

The launching mechanism of a toy gun consists of a spring of unknown spring constant, as shown in the adjacent figure. If the spring is compressed a distance of 0.120 m and the gun fired vertically as shown, the gun can launch a 20.0-g projectile from rest to a maximum height of 20.0 m above the starting point of the projectile. Neglecting all resistive forces, determine (a) the spring constant and (b) the speed of the projectile as it moves through the equilibrium position of the spring (where x = 0), as shown in part (b) of the figure.



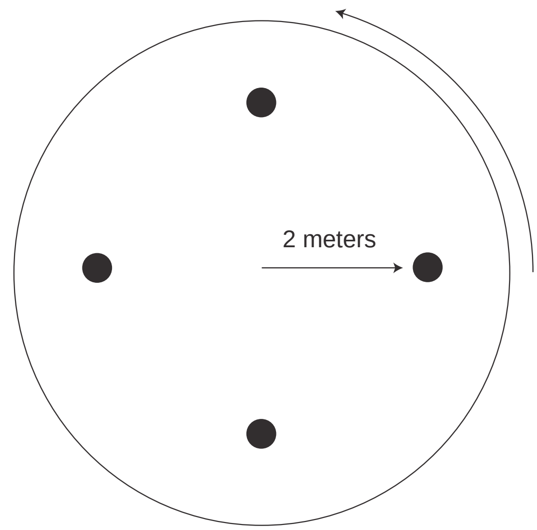
Problem 3 [25 pts]. David is standing on the end of a heavy diving board. What are the magnitude and direction of the forces exerted on the board by the two posts?

The board is a uniform plank weighing 100 N, and it is attached to the two posts at the far end and 1.5 meters from the far end, as shown. David weighs 400 N.



Problem 4 [25 pts].

In the park, four children each with a mass of 40 kg, are riding on a merry-go-round: a flat freely rotating platform 3 meters in radius. Initially, it is rotating with an angular velocity of 0.1 revolutions per second. The children are all standing 2 meters from the center.



(a) What is the centripetal acceleration of one of the children?

(b) What coefficient of friction is needed to keep the children from flying off?

(c) Two children step in to 1 meter from the center and two children step out to 3 meters from the center. Does the merry-go-round speed up or slow down? You must show your reasoning, mathematically, to get credit!

Problem 1 [25 pts]. Conceptual or short questions

1a) [8 points] Torque and work are measured in the same units of newtons times meters. Explain how torque and work are different.

Torque is rotational
(but work can be rotational or linear)

Work is a <u>scalar</u>
Uses the parallel Force component
× distance "travelled"

Torque is a vector
Uses the perpendicular force component
× radius arm of force from rotational center

1b) [9 pts] When you take the battery out of a clock, it takes 30 seconds for the second hand to completely come to a stop (uniformly decelerating). The second hand is 0.10 m long.

How fast (linear speed) is the tip of the second hand going 10 seconds after you take out the battery? **Show your work.**

I need $V = \omega r$
⇒ I need $\omega(t)$
$\omega(t) = \omega_0 + \alpha t$
⇒ I need α
$\alpha = \frac{\Delta\omega}{\Delta t}$
⇒ I need $\Delta\omega$
$\Delta\omega = \omega_i - 0$
⇒ I need ω_i

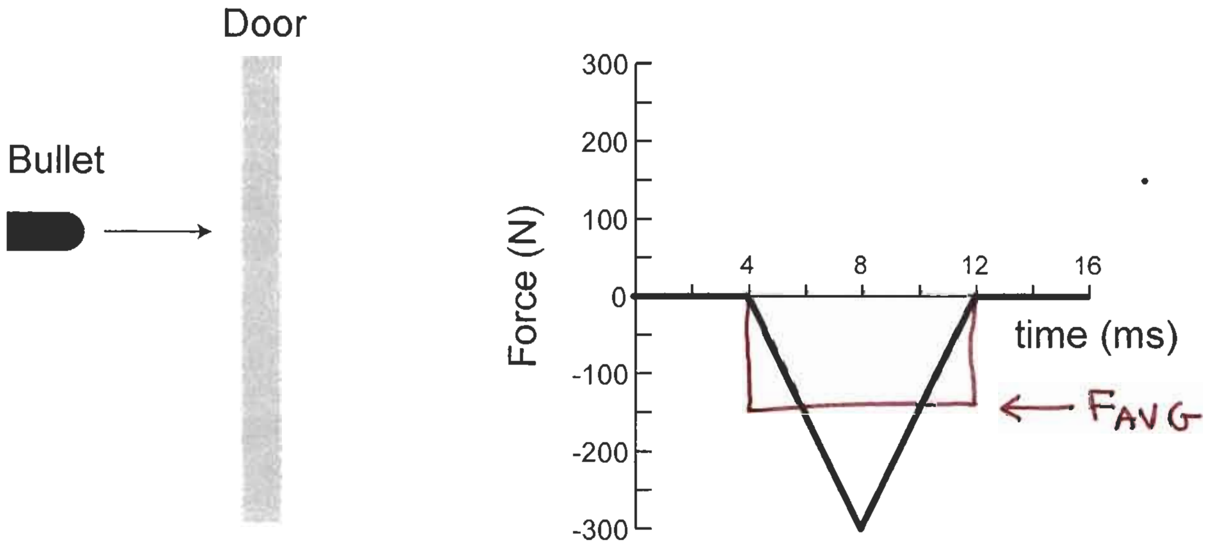
$\omega_i = \frac{2\pi \text{ rad}}{60s} = 0.105 \frac{\text{rad}}{s}$

$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0.105 \frac{\text{rad}}{s}}{30s} = 0.0035 \text{ rad}/s^2$

$\omega = 0.105 \frac{\text{rad}}{s} - (0.0035 \frac{\text{rad}}{s^2})(10s) = 0.07 \text{ rad}/s$

$V = \omega r = 0.07 \frac{\text{rad}}{s} \times 0.1m = \boxed{0.007 \text{ m}/s}$

1c) [8 pts] A 10 gram bullet is fired at a door with a speed of 400 m/s. The force on the bullet as it goes through the door is shown on the graph. How fast is the bullet going when it exits the door? **Show your work!** 10 gram = 0.010 kg.



IMPULSE - MOMENTUM

$F_{AVG}\Delta t = \Delta p = m\Delta V$

$(150N)(8\times10^{-3}s) = (0.010\,kg)\,\Delta V$

$\Delta V = 120\ m/s$

so $V_{final} = V_{initial} - \Delta V = 400\ m/s - 120\ m/s$

$= 280\ m/s$

or I could use area under curve

$\frac{1}{2}\,b\cdot h$

$\frac{1}{2}(8\times10^{-3}s)(300N)$

same result!!

Greg's Question/Comment

What about $F = ma$?

$-150\ N = 0.010\ kg \times a$

$a = 15000\ m/s^2$

$v = v_0 - at$

$= 400\ m/s - (15000\ m/s^2)(8\times10^{-3}s)$

$= 280\ m/s$ ✓✓

Problem 2. [25 points] The spring in a toy gun.

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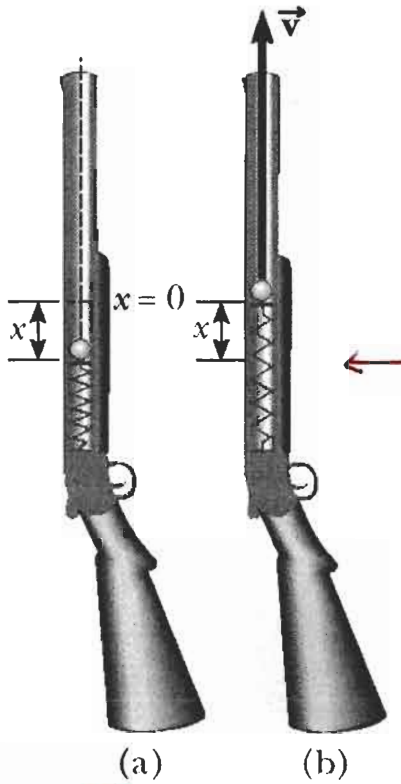

see underline

$\leftarrow y = 0$

See also Student Solution Manual

$$(KE + PE_g + PE_s)_{initial} = (KE + PE_g + PE_s)_{final}$$

$$0 + mgh + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$k = \frac{2mgh}{x^2} = \frac{2\,(0.020\,\text{kg})(10\,\text{m/s}^2)(20\,\text{m})}{(0.12\,\text{m})^2} = \boxed{556\ \text{N/m}}$$

= N

(a)

(b) $(KE + PE_g + PE_s) = mgh$

$$\frac{1}{2}mv^2 + 0 + 0 = mgh$$

$$v = \sqrt{2gh} = \sqrt{2 \cdot 10\,\text{m/s}^2 \cdot 20\,\text{m}}$$

$$= \boxed{20\,\text{m/s}}$$

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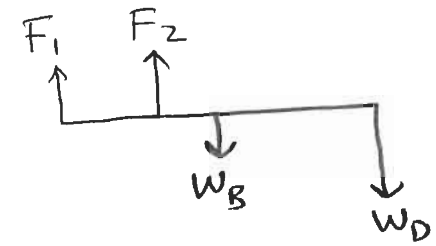
EQUILIBRIUM $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$

Free body diagram



we will find size and direction of F_1 and F_2

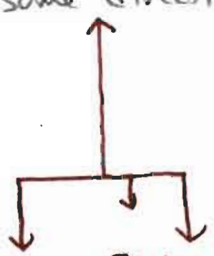
$\sum F$: $F_1 + F_2 - W_B - W_D = 0$

$F_1 + F_2 - 500\ N = 0$

$\sum \tau$: choose rotation around F_2 position so torque due to F_2 is 0

$- F_1 \cdot (1.5 m) - 100 N \cdot (0.5 m) - 400 N \cdot (2.5 m) = 0$

NOTE: ALL TORQUES are in the same direction!

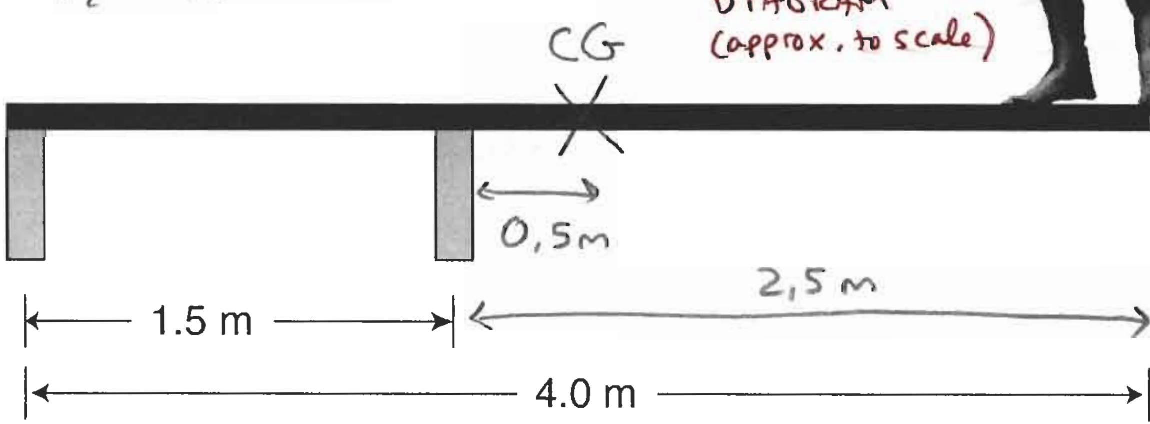


$F_1 = -700\ N$

plug into $\sum F$ equations

$F_2 = +1200\ N$

FINAL FREE BODY DIAGRAM (approx. to scale)



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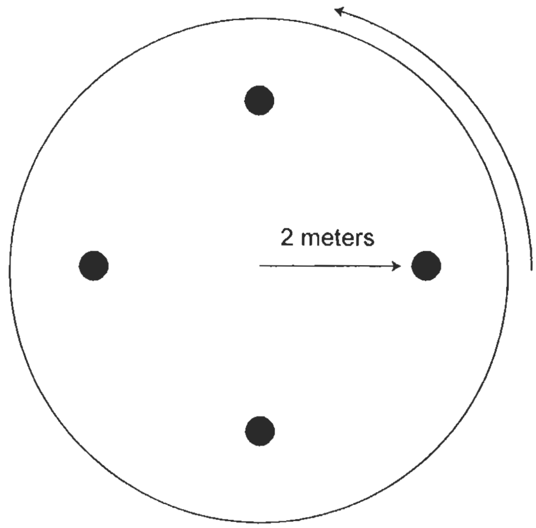

(a) What is the centripetal acceleration of one of the children?

(b) What coefficient of friction is needed to keep the children from flying off?

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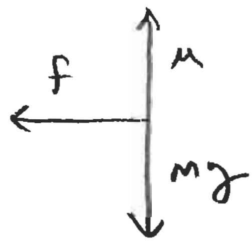
(a) $a_c = v^2/r$ Need v!

$v = \omega r$

$= \left(0.1 \frac{rev}{s}\right)\left(\frac{2\pi\ rad}{rev}\right)(2m) = 1.26\ m/s$

$\boxed{a_c = 0.79\ m/s^2}$

(b)



$f = \mu N = m a_c$

$\mu m g = m a_c$

$\mu = \frac{a_c}{g} = \frac{0.79\ m/s^2}{10\ m/s^2} = \boxed{0.079}$

very small, so merry-go-round is safe!

(c) CONSERVATION of ANGULAR MOMENTUM

$L = I\omega$

INITIAL: $I = 4MR^2$

FINAL: $I = 2M\left(\frac{R}{2}\right)^2 + 2M\left(\frac{3R}{2}\right)^2$

$= 5MR^2$

$I_i \omega_i = I_f \omega_f$

$\omega_f = \left(\frac{4MR^2}{5MR^2}\right)\omega_i = \frac{4}{5}\omega_i \Rightarrow$ **slows down**

NOTE: can't say how much unless we know I of merry-go-round