



# **A Spatial Perspective on Long- Range Voice Leading and Beethoven's *Heiliger Dankgesang***

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A copy of this talk is available at  
[people.bu.edu/jyust/](http://people.bu.edu/jyust/)

# Outline

## I. Schenker, Brahms, and Keys: The Problem

1. Schenker's syllogism
2. Examples: Beethoven *Bagatelle* op. 119 no. 7  
Brahms *Cello Sonata* op. 99, subordinate theme
3. An alternative: DFT phase space and tonal regions
4. Beethoven and Brahms examples in DFT phase space

## II. DFT and Triadic Orbits

1. DFT components as sinusoidal approximations
2. Triadic orbits

## III. Beethoven's *Heiliger Dankgesang*

1. Scalar context and triadic orbits
2. The D–C motive
3. Strength and weakness

# I. Schenker, Brahms, and Keys

1. Schenker's syllogism
2. Examples: Beethoven *Bagatelle* op. 119 no. 7  
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# Schenker, Brahms, and Keys

## The problem with harmonic reduction

### The Schenkerian Syllogism

**False!**

~~*Hidden Premise: Voice leading is always a relationship between individual harmonies.*~~

*Premise:* Long-range structure is contrapuntal (based on voice-leading).

*Conclusion:* Long-range structure is *reductive*—i.e., it consists of relationships between non-adjacent harmonies.

**Problematic**

N<sup>o</sup> 7.

trill trill trill trill trill trill

*p* *p* *p* *p* *p* *p*

*schierzando*

C major ----- [ G major -----  
 I ----- V<sup>7</sup>/V ----- V Sequence

# Example:

## Beethoven *Bagatelle* op. 119 no. 7

*f* *p*

----- ] [ C major ----- ] F major -----

*p* *cresc.* *poco a*

[ D minor ----- ] F major -----  
 V<sup>7</sup> ----- V<sup>7</sup> -----



# Beethoven *Bagatelle* op. 119 no. 7

The image displays two systems of musical notation for Beethoven's Bagatelle op. 119 no. 7. The first system features a treble staff with a melodic line and a bass staff with a simple accompaniment. The tempo is marked *poco*. The second system continues the piece, with the treble staff showing more complex melodic patterns and the bass staff providing harmonic support. Dynamic markings include *al f*, *più f*, and *ff*. The piece concludes with a double bar line and a fermata over the final chord.

**C major:  
V of F major  
*not* I of C major**

# Beethoven *Bagatelle* op. 119 no. 7

Frank Samarotto (*A Theory of Temporal Placticity for Tonal Music*, PhD Diss., 1999):

The image shows a musical score for Beethoven's Bagatelle op. 119 no. 7, specifically the section labeled 'd)'. The score is in C major and 3/4 time. It features a treble and bass clef. Above the staff, measures 1, 11, 16, 20, and 26 are circled. The analysis includes several annotations: a circled '3' above measure 1, 'NT' above measure 11, a circled 'b7' above measure 16, a circled 'b7' above measure 20, and a circled '2' above measure 26. A dashed line labeled '8va' spans from measure 20 to measure 26. A dashed line labeled 'cresc.' spans from measure 16 to measure 26, ending with 'ff'. Below the staff, the chord analysis is given as: C: I IV (=5-#5-6) V b3 I b7- 6- b7 3- 4- 3 2- 1. The analysis shows a sequence of chords: I (C), IV (F), V (G), I (C), b7 (F7), 6 (E), b7 (D7), 3 (G), 4 (F), 3 (E), 2 (D), 1 (C).

Samarotto's analysis reflects only the structural **chords**, not their tonal contexts. This analysis mistakenly represents the bagatelle as **tonally closed**.



# Beethoven *Bagatelle* op. 119 no. 7

Frank Samarotto (*A Theory of Temporal Placticity for Tonal Music*, PhD Diss., 1999):

However, the higher-level progression of tonal contexts shows that the bagatelle is actually **tonally open**, in accord with its function as prelude to no. 8.

See Nicholas Marston, “Trifles or Multi-Trifle: Beethoven’s Bagatelles Op. 119, Nos 7–11.” *Music Analysis* 5/2–3 (1986)

Tonal contexts: C major - - - - - → F major

Chords: C major - - - - - → C major

Samarotto’s analysis reflects only the structural **chords**, not their tonal contexts. This analysis mistakenly represents the bagatelle as **tonally closed**.

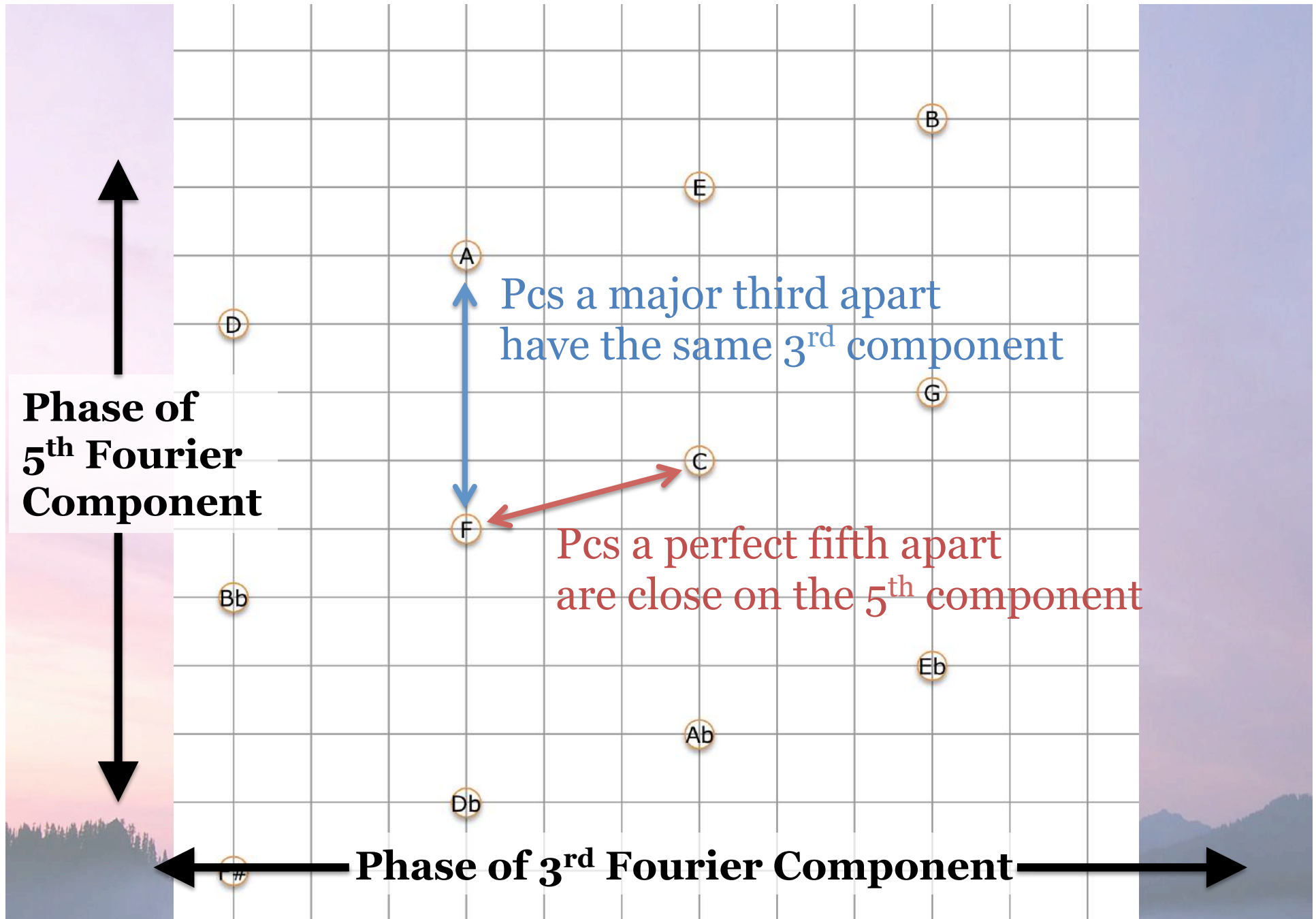
# Schenker, Brahms, and Keys

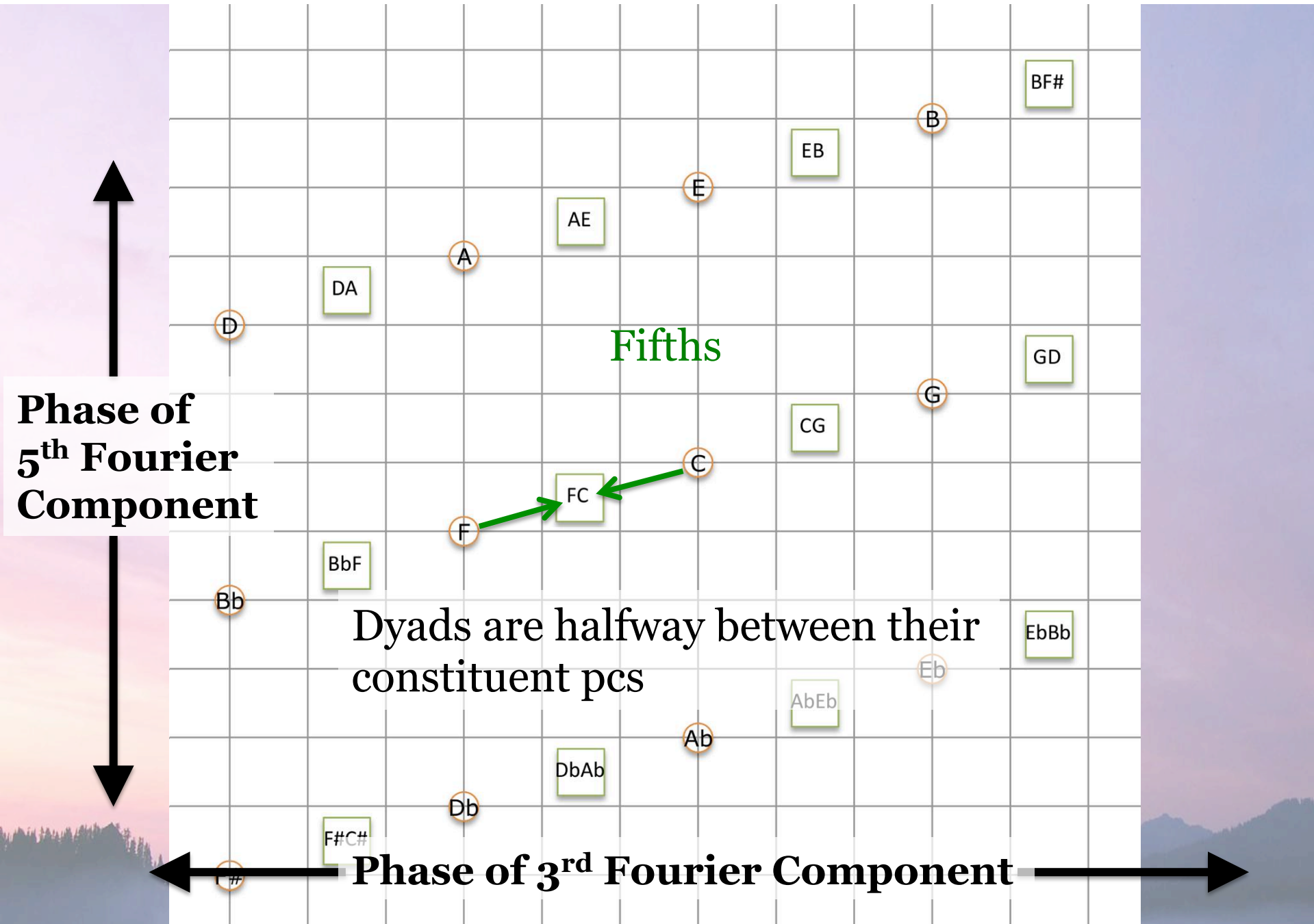
**Alternative:  
A spatial concept of tonality . . .**

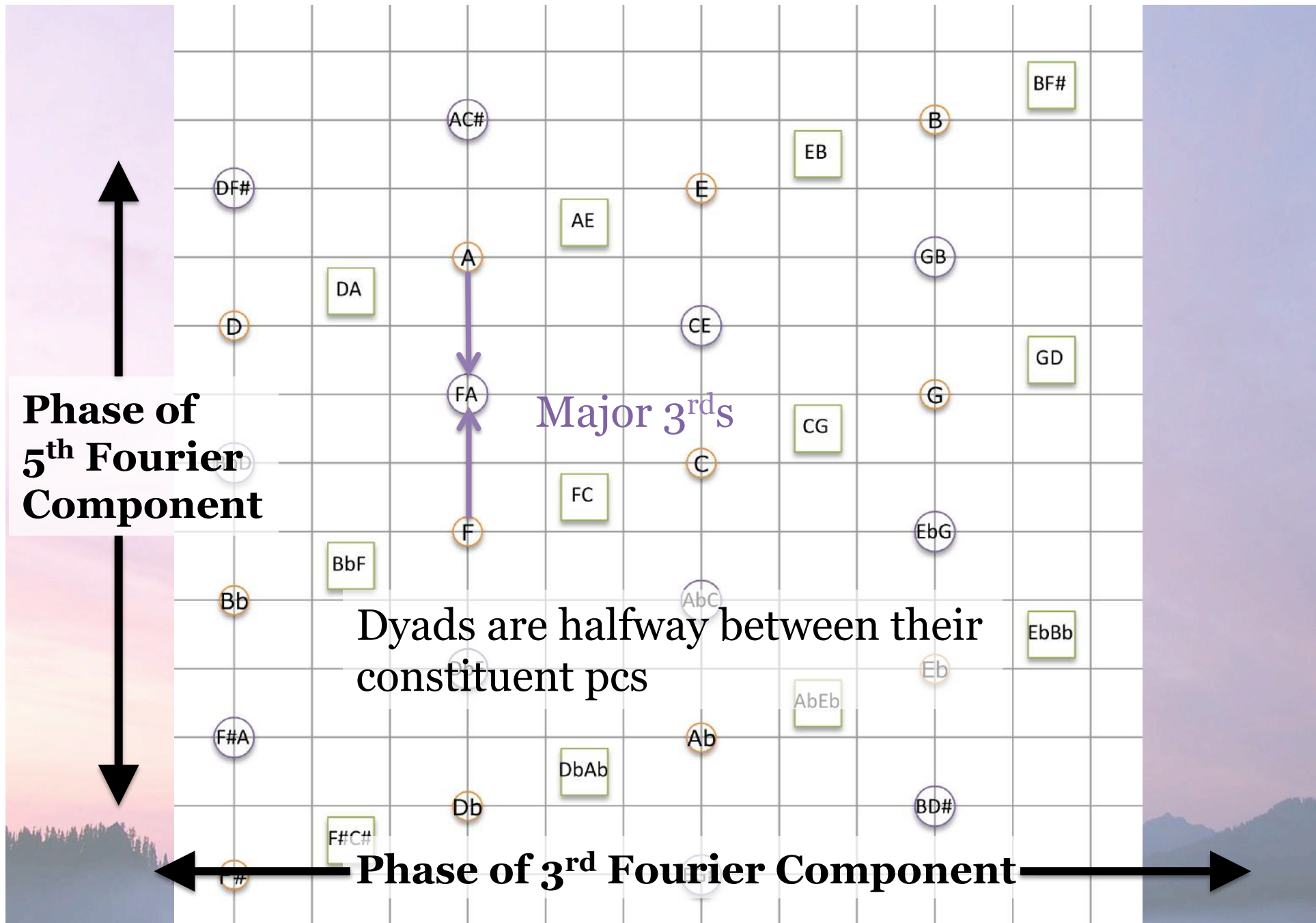
**DFT phase space**

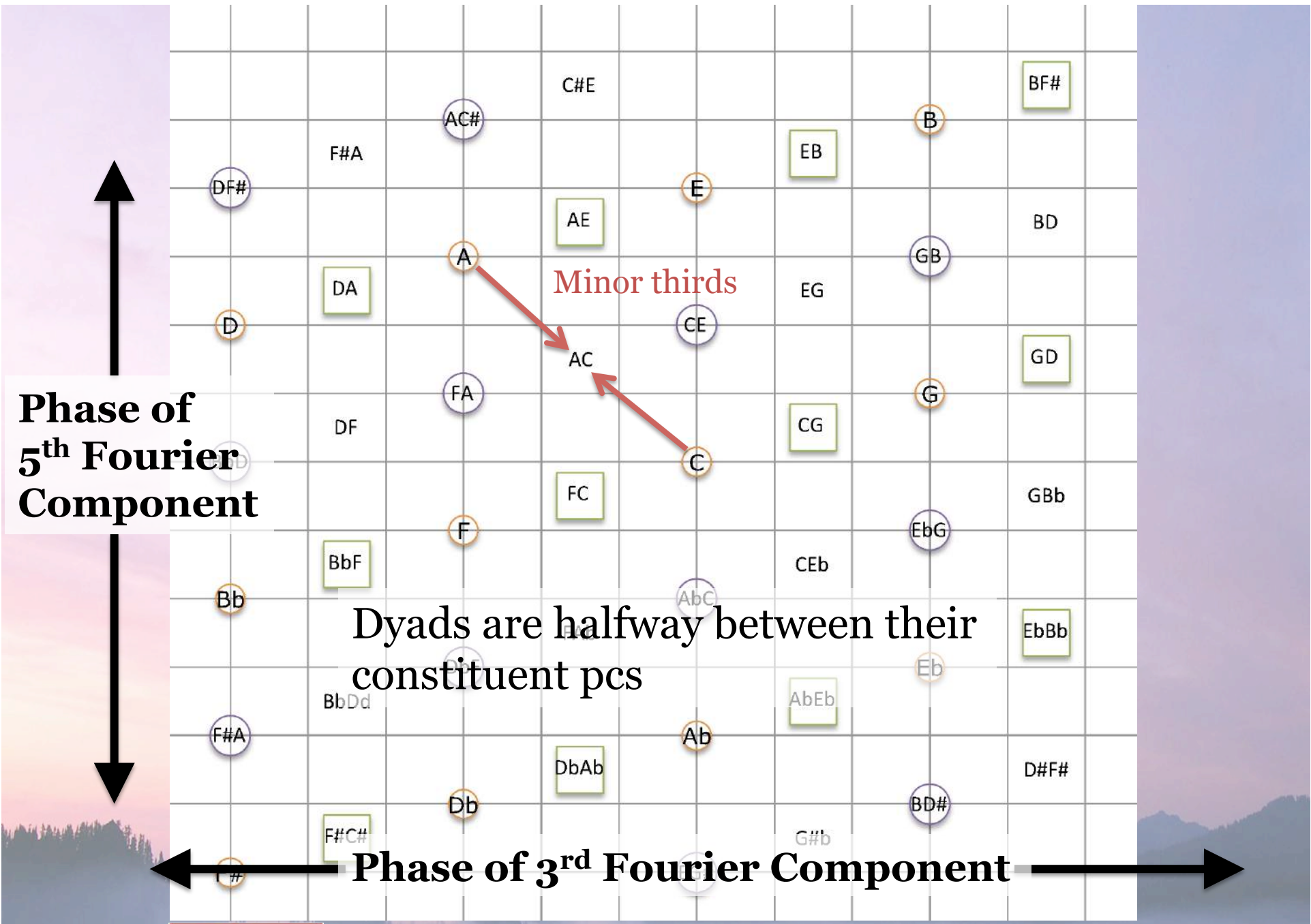
Amiot, Emmanuel. (2013). "The Torii of Phases," *Mathematics and Computation in Music, MCM 2013* (ed. Yust, Wild, & Burgoyne) 1–18.

Yust, "Schubert's Harmonic Language and Fourier Phase Space." *Journal of Music Theory* 59/1 (forthcoming). Available at <http://people.bu.edu/jyust/SchubertDFT.pdf>





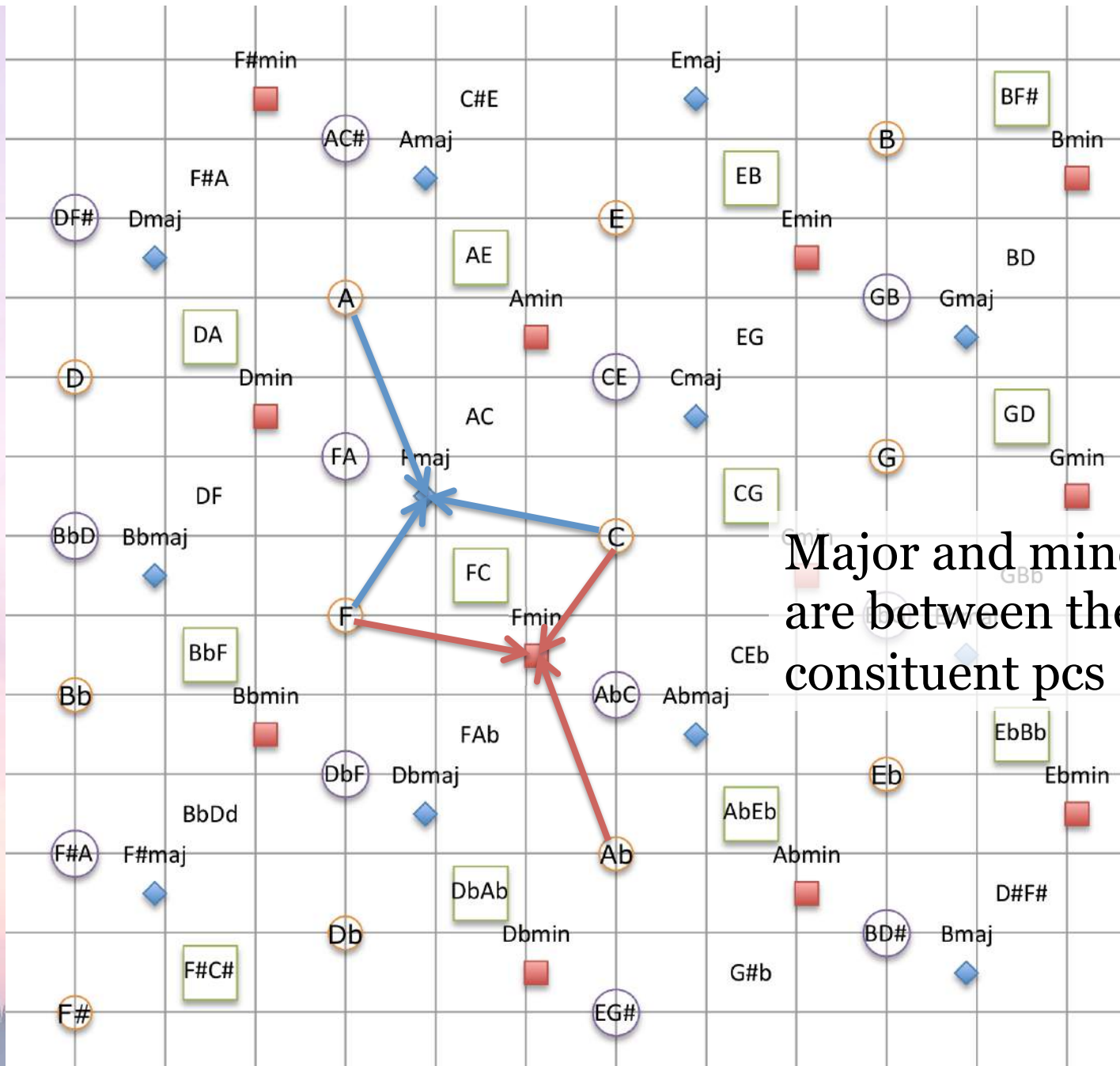




**Phase of  
5<sup>th</sup> Fourier  
Component**

Dyads are halfway between their  
constituent pcs

**Phase of 3<sup>rd</sup> Fourier Component**



Major and minor triads are between their constituent pcs

# Schenker, Brahms, and Keys

## Properties of DFT Phase Space

- Objects are pcsets, multisets, or statistical pc distributions
- Toroidal geometry
- Vertical axis indicates circle-of-fifths position
- Horizontal axis captures triadic voice-leading properties

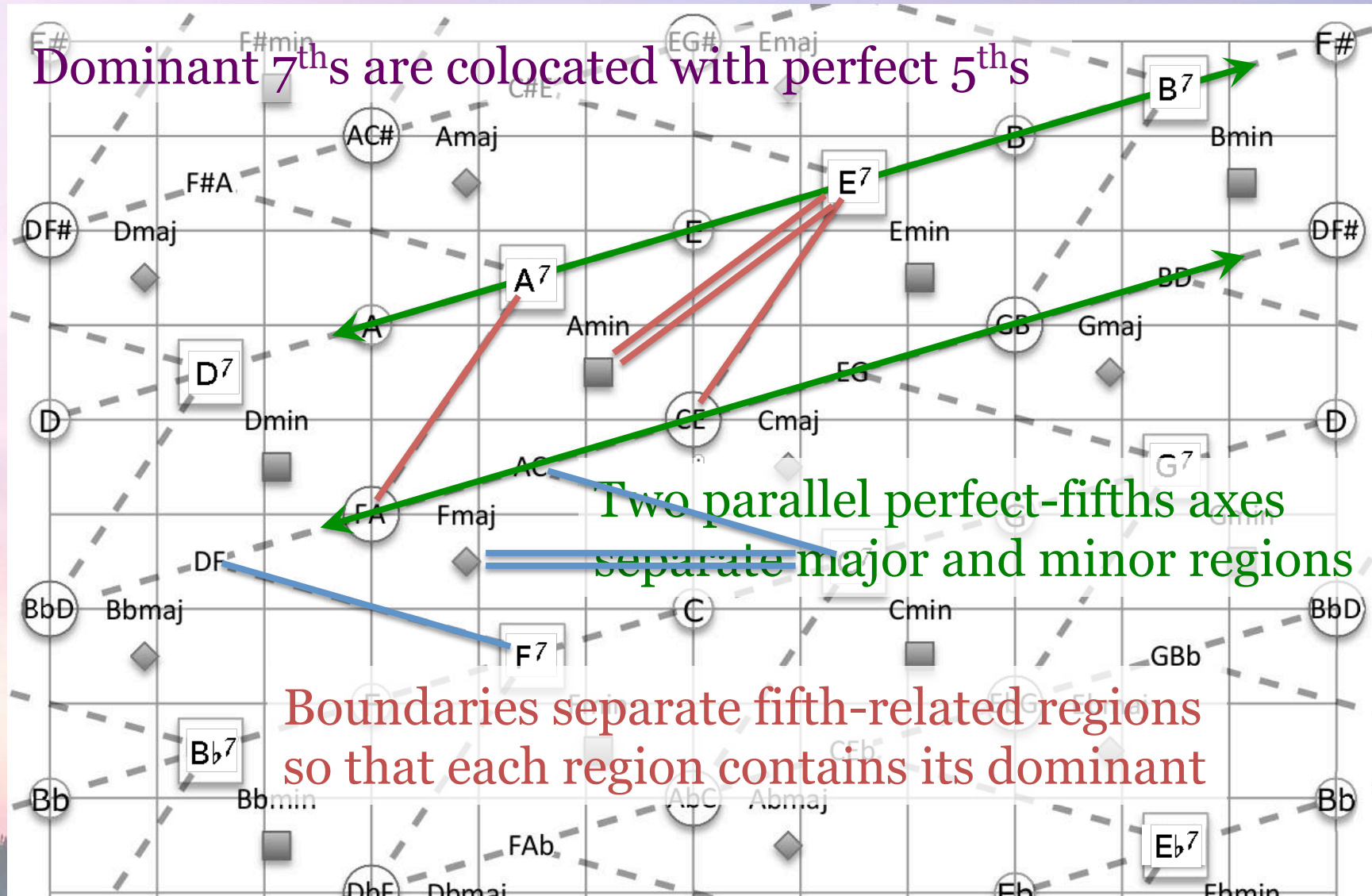


# Schenker, Brahms, and Keys

## Properties of DFT Phase Space

- Many kinds of harmonic objects exist in the space: single pcs, harmonies, scales, etc.
- Space is *continuous*—paths connect points via a potential infinite series of intermediate states (pc distributions)
- Nearness in the space (its *topology*) is based on *common pc content*.

# Tonal Regions







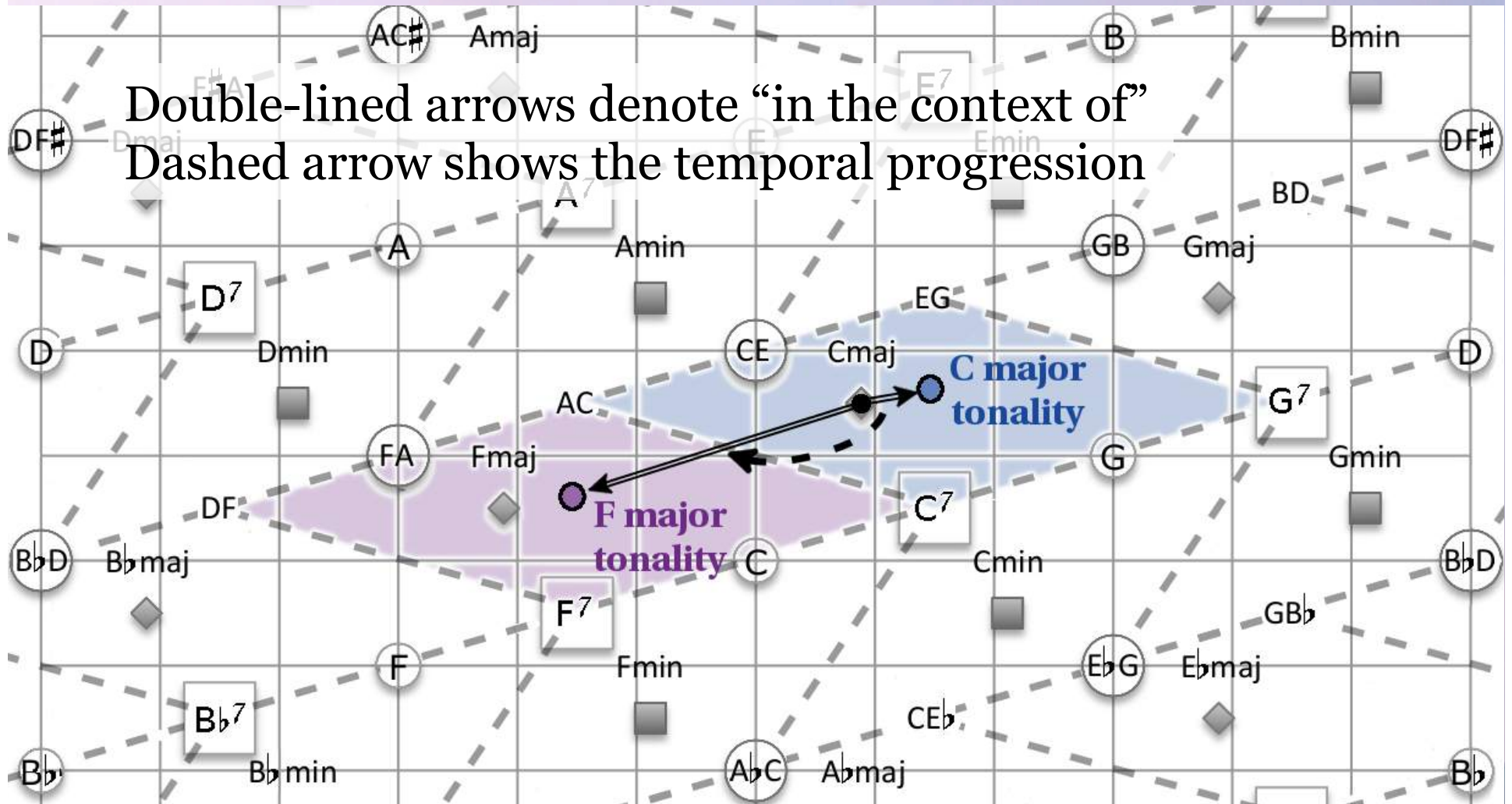
## Properties of DFT Phase Space

A **path** can represent a motion from  $A$  to  $B$ , but it can also represent “ $B$  in the context of  $A$ .”

**Combination** of pcsets is highly tractable: the position of  $A + B$  is easily predictable from the path  $A \rightarrow B$ .

Averaging over *more objects* (pcs, triads) restricts the range of activity.

# Beethoven *Bagatelle* op. 119 no. 7





# Schenker, Brahms, and Keys

Brahms *Cello Sonata* Op. 99, first movement

**Brahms, Cello Sonata op. 99, 1st mvt.**

2 m. 34 45 60

F major: I  $\xrightarrow{\hspace{10em}}$  III<sup>b3</sup>  
(=a minor: III V I)

Schenker: *Der Freie Satz* Fig. 110d(2)



Schenker: C

34 cello pno HC cello

*f* *mf* *f*

C maj.: I ... IV (I) V<sup>4</sup> - 5<sub>3</sub>

40 pno Schenker: E

*mf* *f*

C maj.: I ... E minor ... i<sup>6</sup> V<sup>7</sup> i

46

*p* *f*

C maj.: ii<sup>6</sup><sub>5</sub> V<sup>6</sup><sub>5/V</sub> - iv<sup>6</sup> V<sup>4</sup><sub>3/V</sub> - IV A min.: Cad<sup>6</sup><sub>4</sub>

50 cello

A min.: vii<sup>07</sup>/V - Ger<sup>+6</sup> - vii<sup>07</sup>/V

54 pno Schenker: A

*f*

vii<sup>04</sup><sub>3</sub> - - Ger<sup>+6</sup> Cad<sup>6</sup><sub>4</sub> A min.: V<sup>7</sup> (VI ii<sup>6</sup><sub>5</sub>) V<sup>6-5</sup><sub>4-3</sub> i

C maj.: vii<sup>07</sup>/V Cad<sup>6</sup><sub>4</sub>



# Schenker, Brahms, and Keys

The image displays a musical score for Brahms' 'Lullaby' in C major, measures 34 to 58. The score is presented in two systems. The first system shows the vocal line (treble clef) and the bass line (bass clef). The vocal line features a melodic line with various intervals and rests, while the bass line provides harmonic support. The second system shows the chordal structure of the music, with chords indicated by Roman numerals and figured bass notation. The chords are: C major (I), E minor (V), C major (I), E minor (E min.), F# minor (V), C major (C maj.), D minor (IV), E minor (A min.), and C major (V). The analysis includes labels such as '(HC)', '(weak PAC)', and '(abandoned cad.)' to describe the structural status of the chords. Measure numbers 34, 40, 46, and 58 are marked above the vocal line.

C maj.: I      V (HC)      I      E min.:      V i (weak PAC)      C maj.:      IV A min.: V      i

(abandoned cad.)

**Structure is based on *large-scale voice leadings*, which must occur between **distinct musical objects (chords)**. This leads to a ***reductive*** approach.**

# Schenker, Brahms, and Keys

34 40 46 58

C maj.: I V (HC) I E min.: V i C maj.: IV A min.: V i

(weak PAC) (abandoned cad.)

**Cadential points must take precedence as structural events**

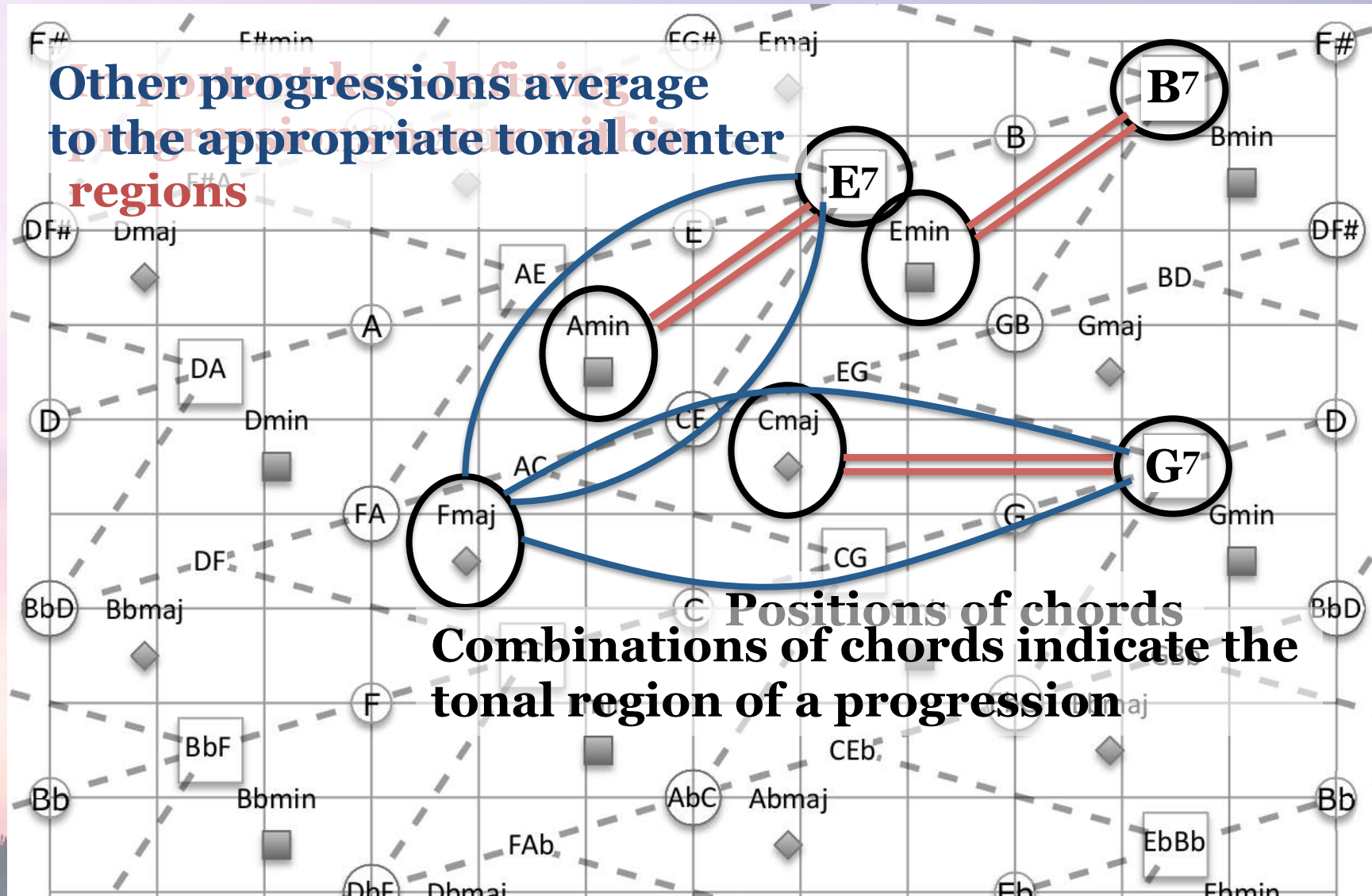
# Schenker, Brahms, and Keys

C maj.: I      V (HC)      I      E min.: V      i (weak PAC)      C maj.: IV (abandoned cad.)      A min.: V      i

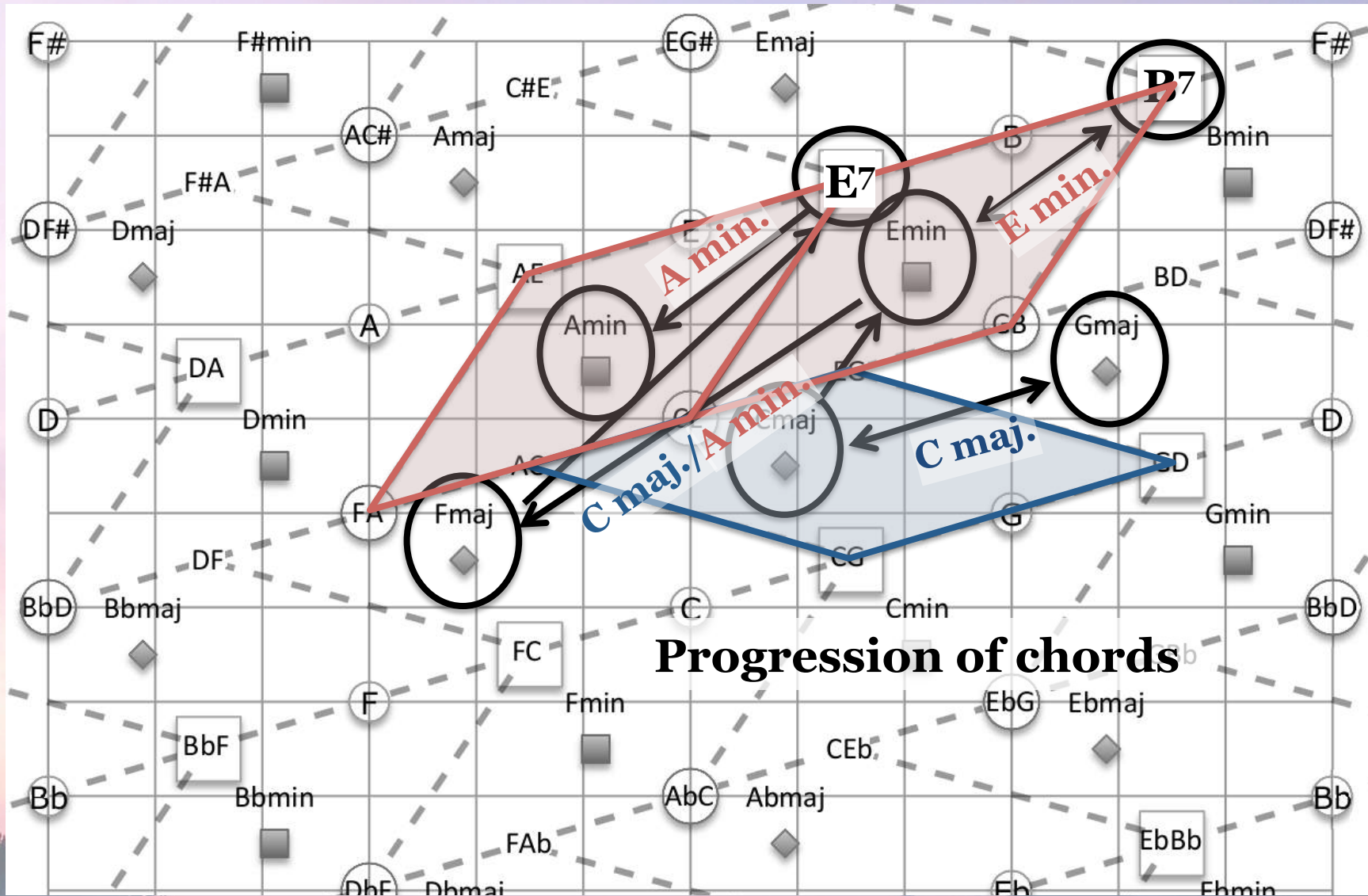
**Persistence of C major context is obscured**

**Cadential points must take precedence as structural events**

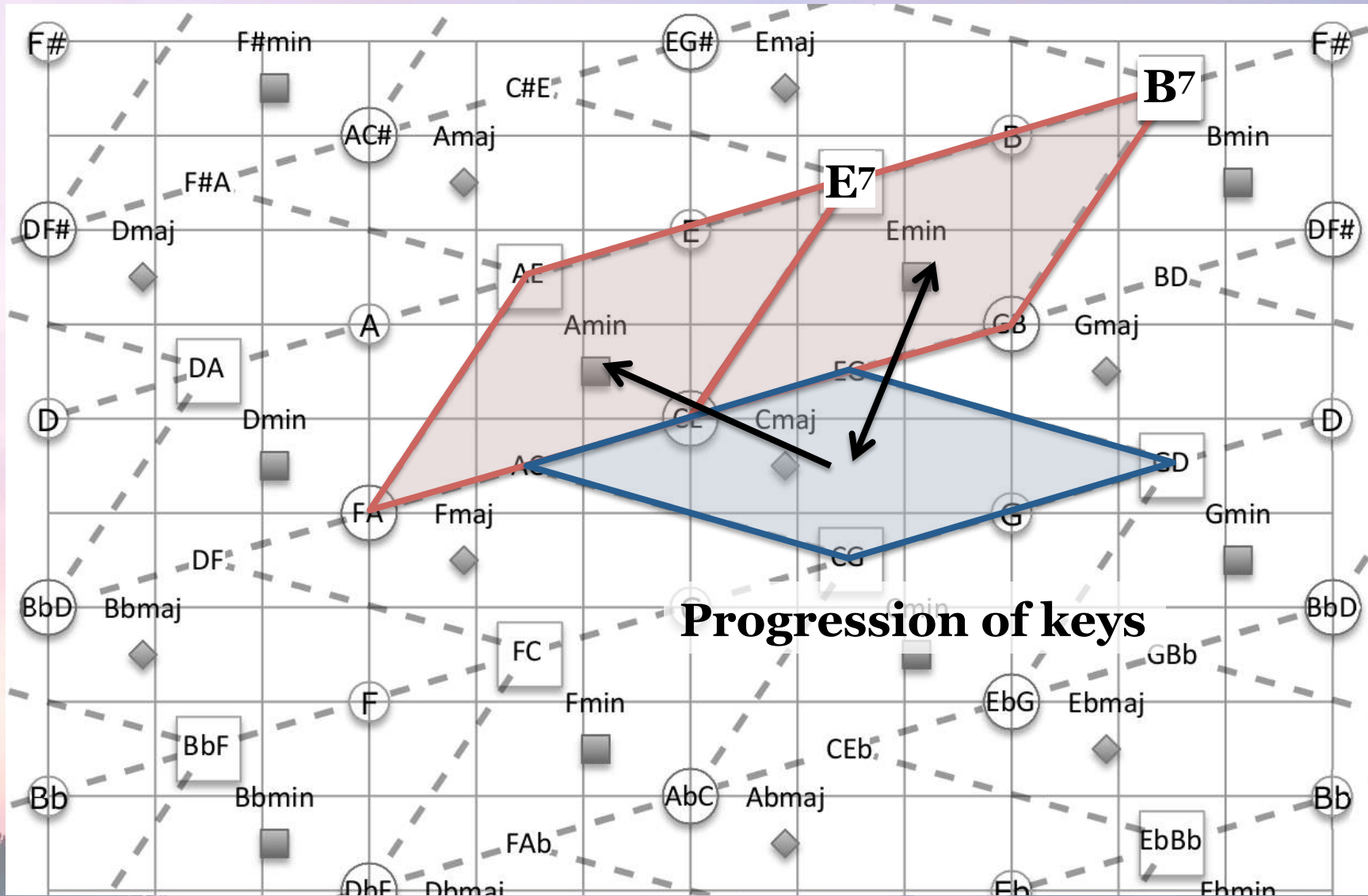
# Brahms Cello Sonata op. 99



# Brahms Cello Sonata op. 99



# Brahms Cello Sonata op. 99





## 2. DFT and Triadic Orbits

1. DFT components as sinusoidal approximations
2. Triadic orbits

# Discrete Fourier Transform on Pcsets

Lewin, David (1959). “Re: Intervallic Relations between Two Collections of Notes,” *JMT* 3/2.

——— (2001). “Special Cases of the Interval Function between Pitch Class Sets X and Y.” *JMT* 45/1.

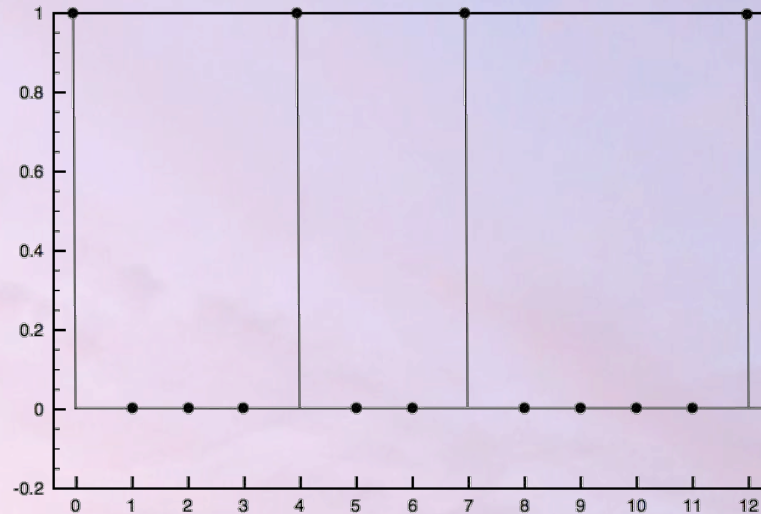
Quinn, Ian (2006–2007). “General Equal-Tempered Harmony,” *Perspectives of New Music* 44/2–45/1.

Amiot, Emmanuel (2013). “The Torii of Phases.” *Proceedings of the International Conference for Mathematics and Computation in Music, Montreal, 2013* (Springer).

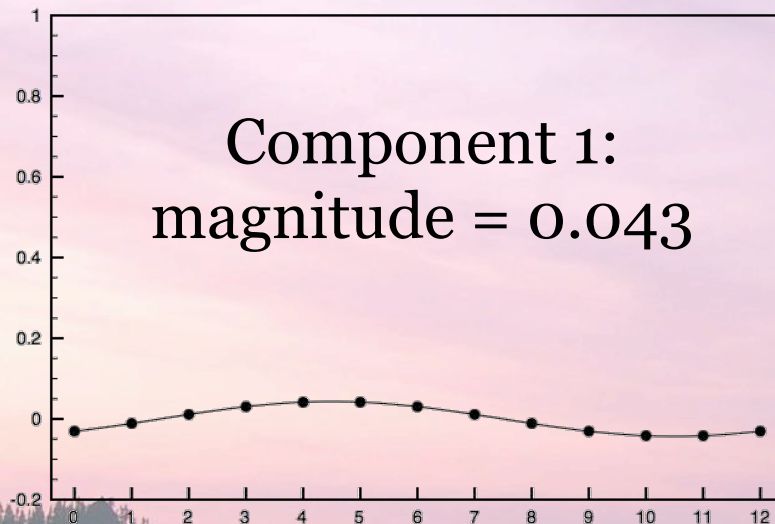


# DFT components

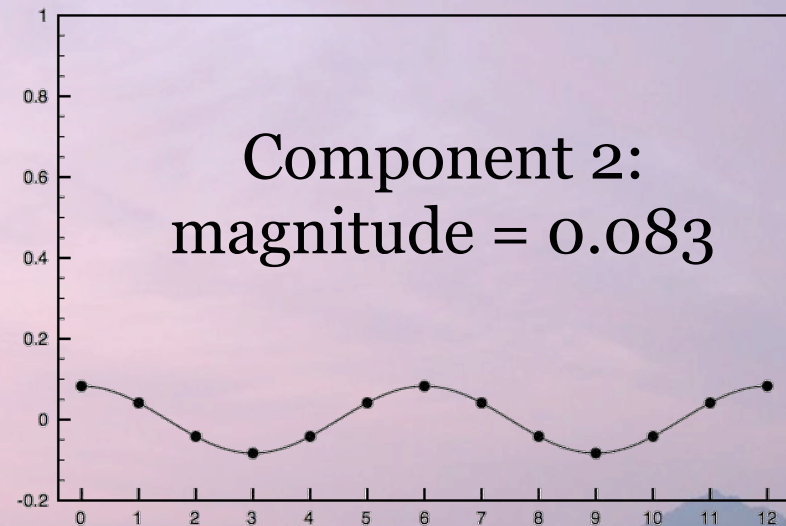
C major triad



=



+

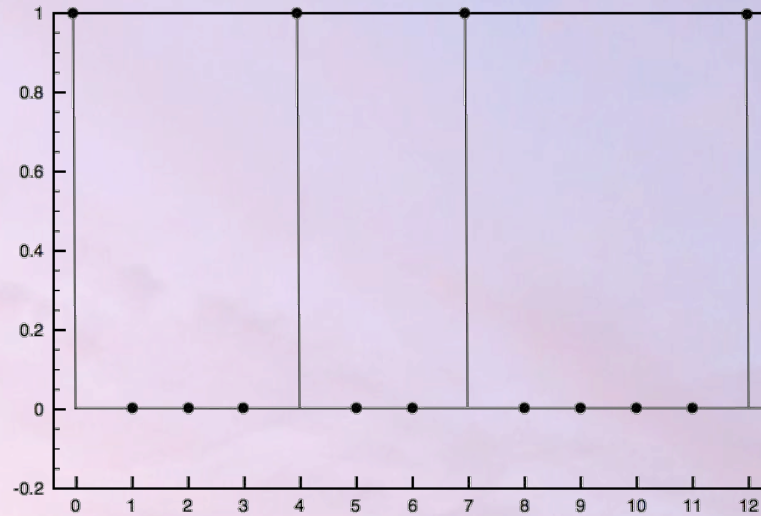


+

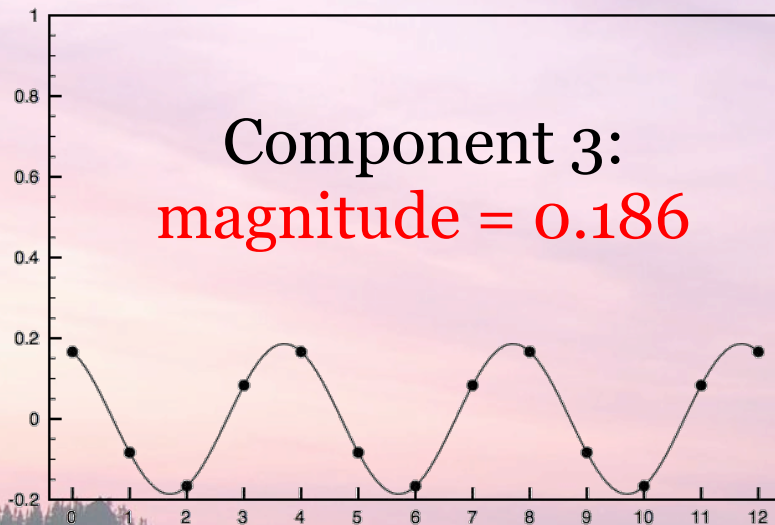
...

# DFT components

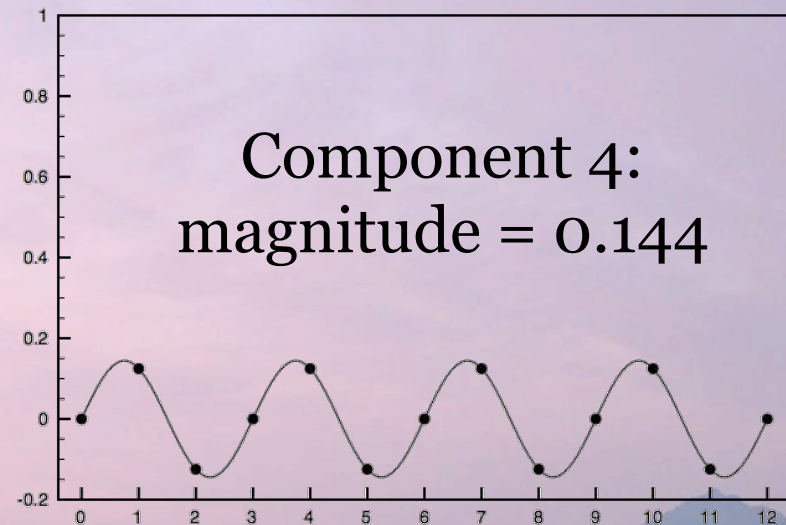
C major triad



=



+

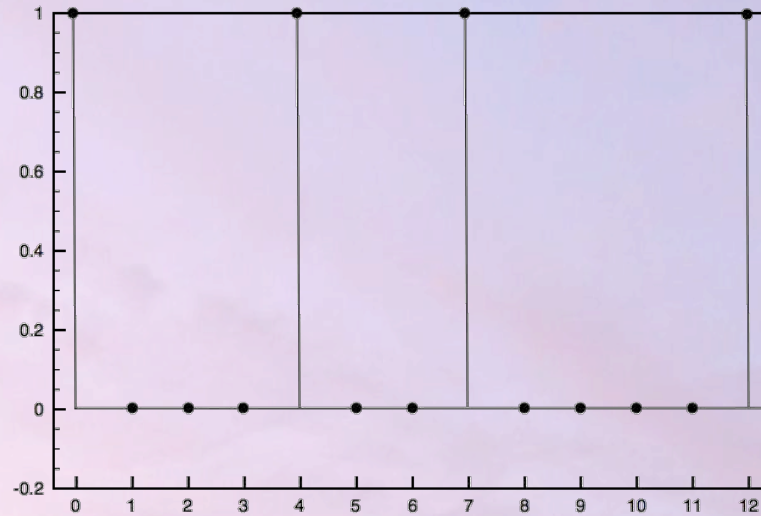


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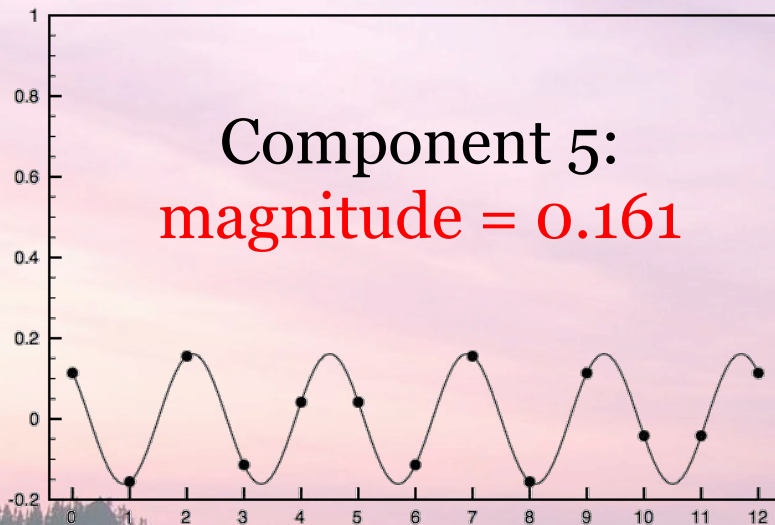
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# DFT components

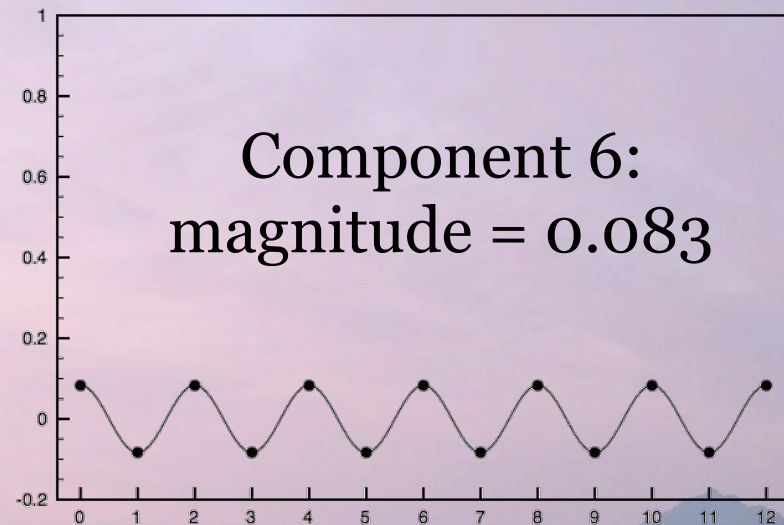
C major triad



=



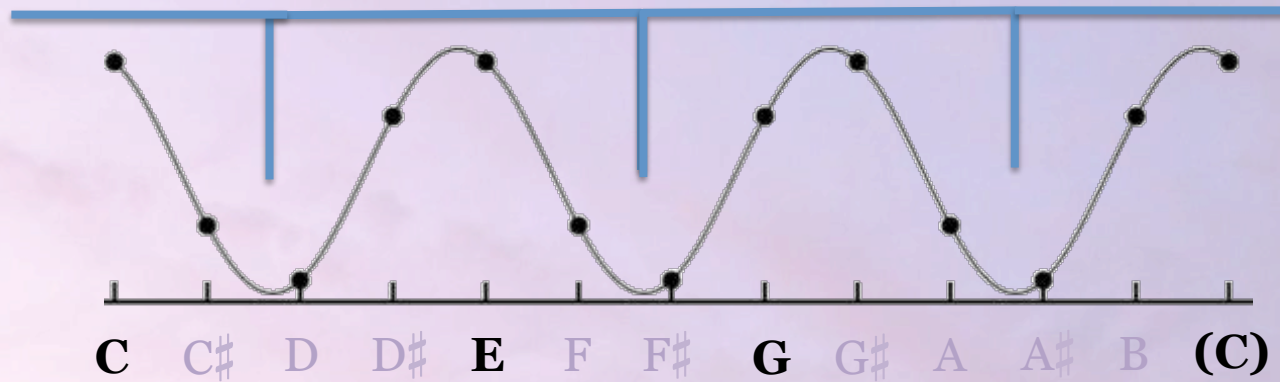
+



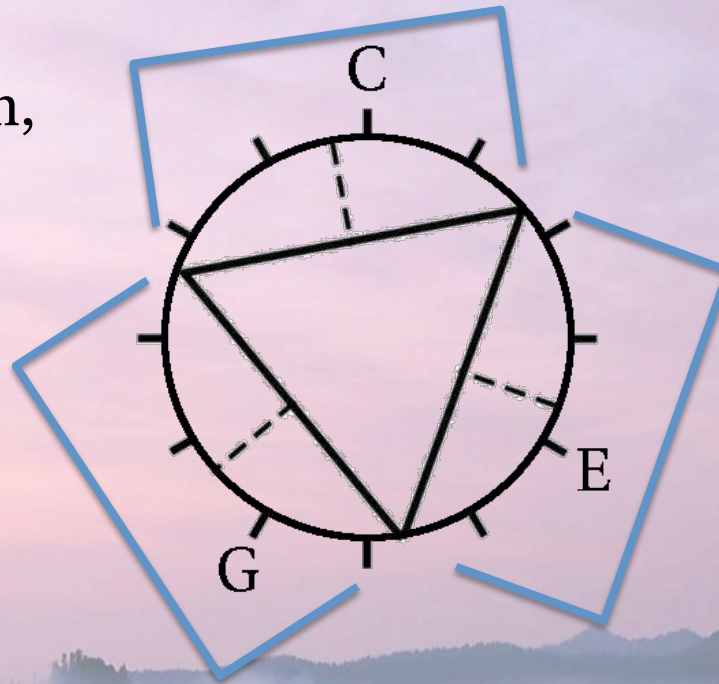
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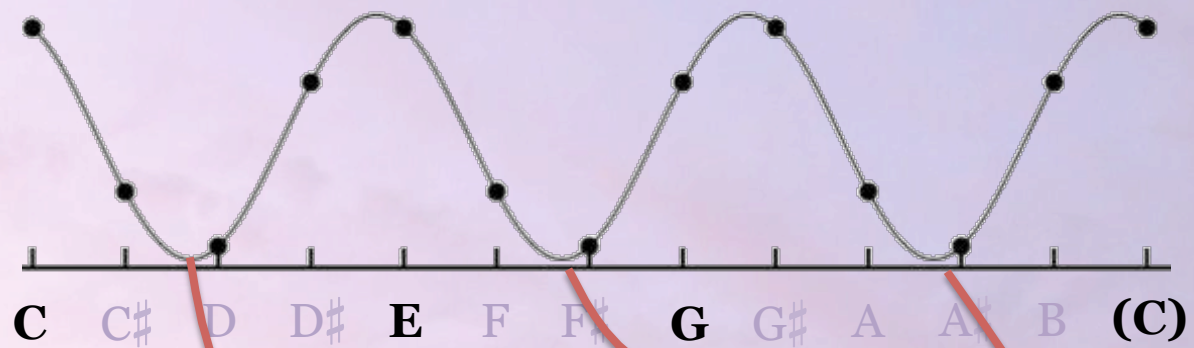
# Triadic Orbits



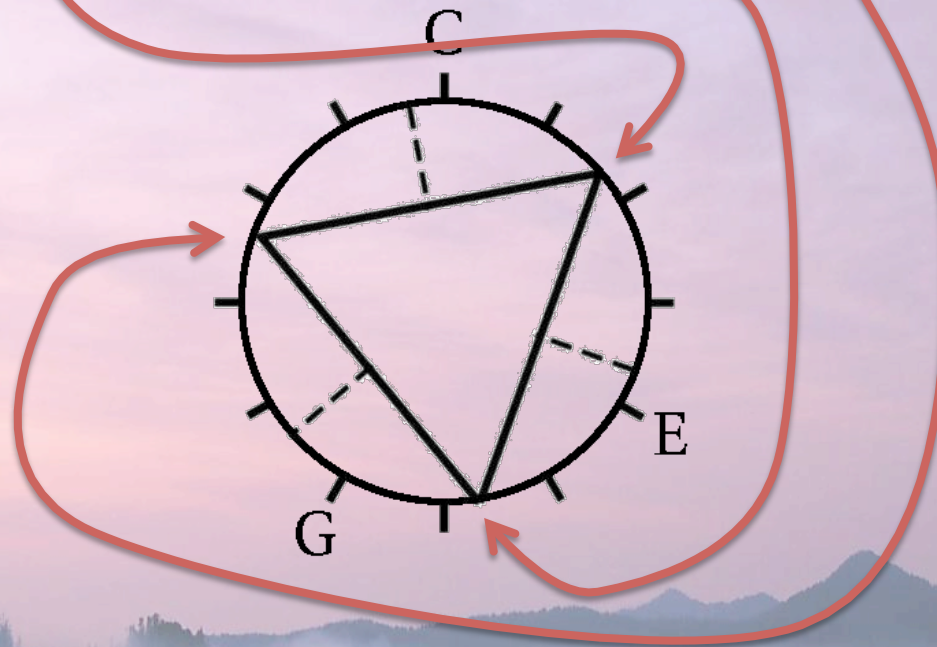
The triadic orbits go from trough to trough, and group pcs that may be considered displacements of those in the triad



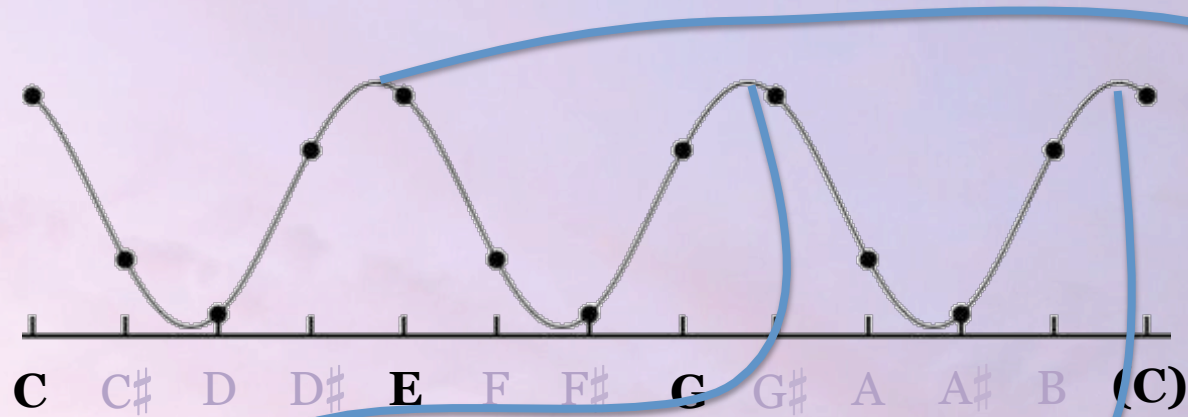
# Triadic Orbits



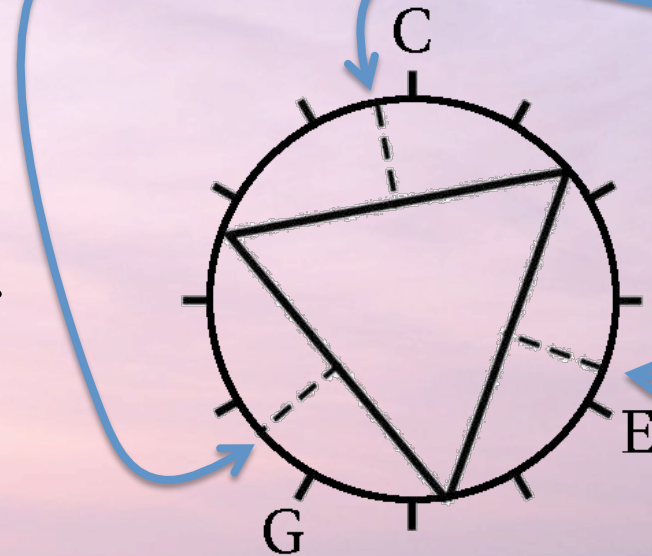
The troughs of the sinusoid are the boundary points



# Triadic Orbits

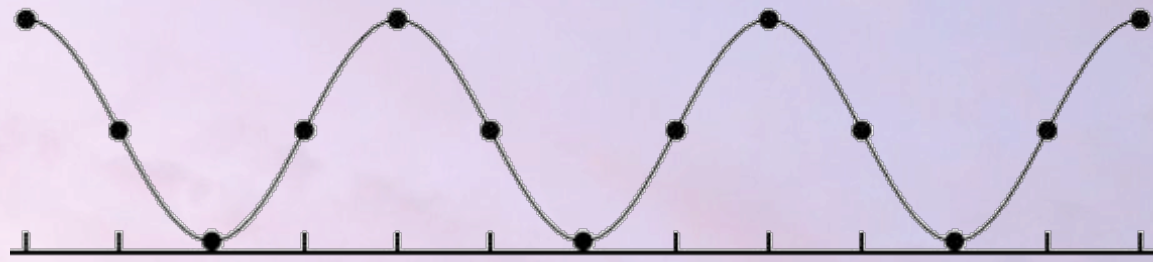


The peaks of the sinusoid are shown by the dashed lines.



# Triadic Orbits

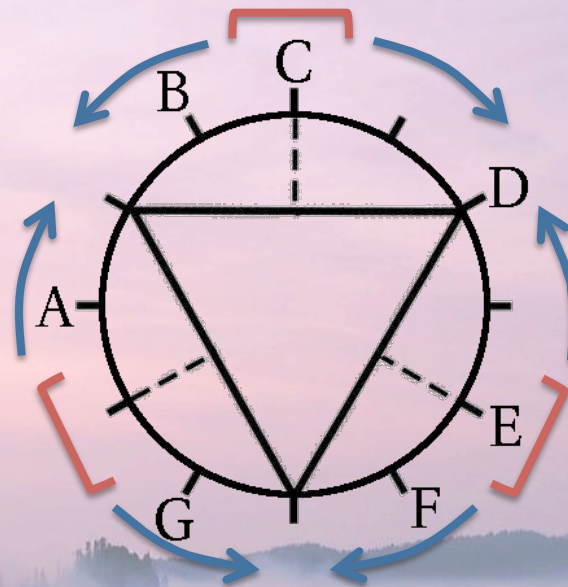
Any pc-distribution can define a set of triadic orbits, including **scales**.



**C diatonic:** C C# D D# E F F# G G# A A# B (C)



**Periphery: Unstable upper and lower neighbors**  
**Regions of stability**



Pitch classes at the center of a region are more stable  
 Pitch classes at the periphery of regions are unstable  
 displacements

# Triadic Orbits

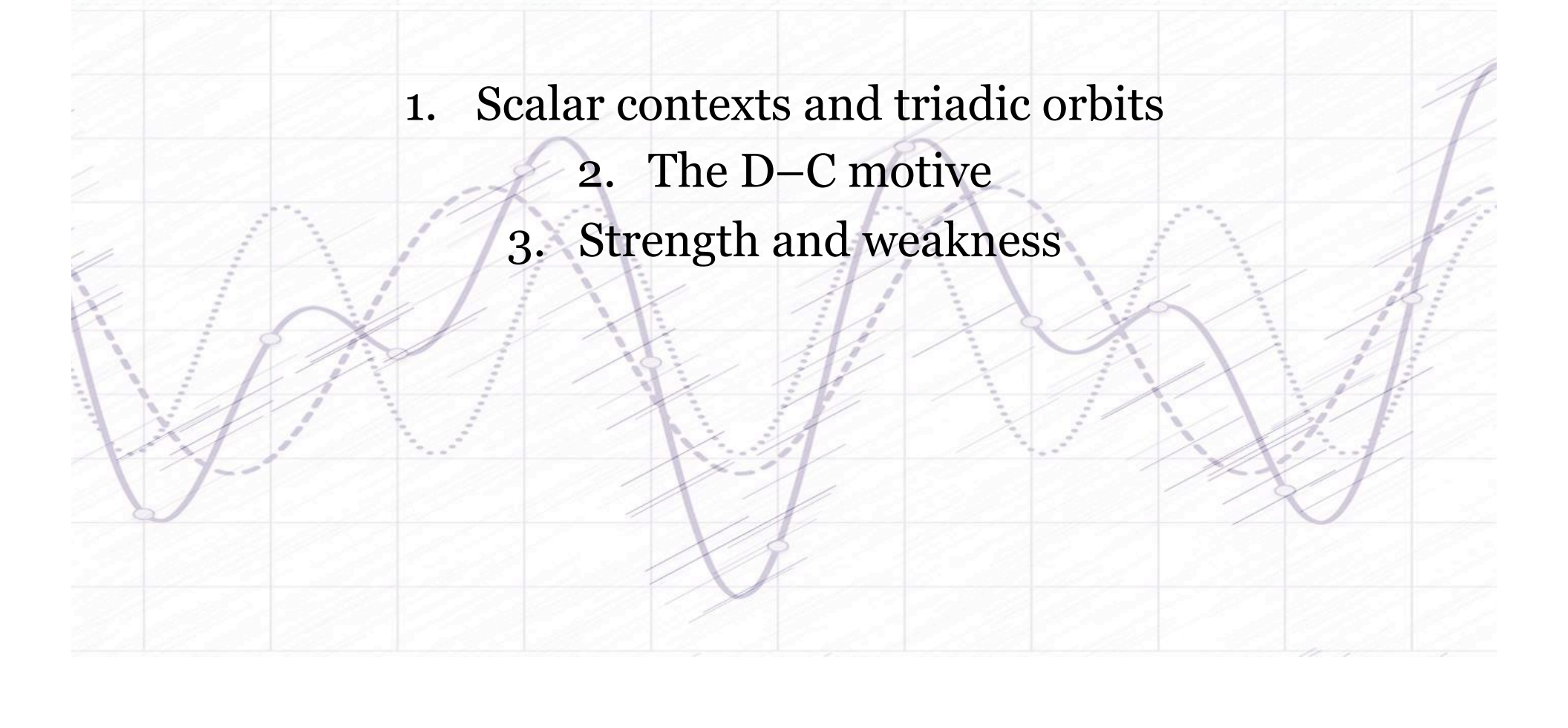
Triadic orbits as a **voice leading** property:

- Clockwise and counter-clockwise movement of triadic orbits corresponds to ascending or descending efficient voice leading between triads.
- The spatial relationship of one harmonic object to its context indicates its voice-leading stability and whether it resolves with upward or downward voice leading in the context.
- The triadic voice-leading properties of the 3<sup>rd</sup> DFT component apply to sets of *any cardinality*, not just triads, on the basis of common pc content with consonant triads.





### ***3. Heiliger Dankgesang***

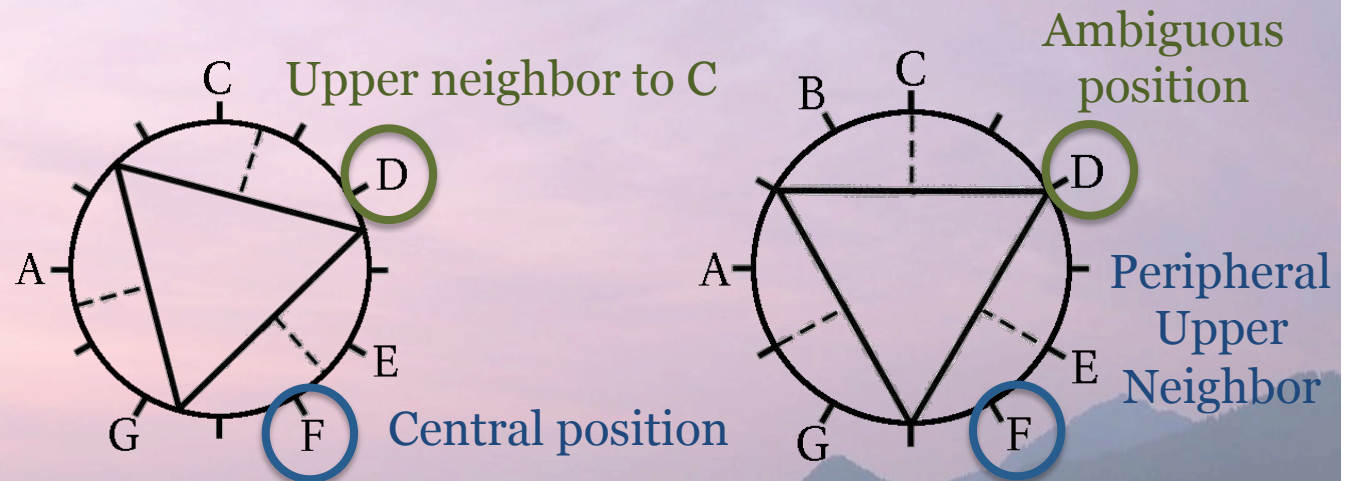
1. Scalar contexts and triadic orbits
  2. The D–C motive
  3. Strength and weakness
- 

# Heiliger Dankgesang: Scalar Contexts

**First intonation and chorale**

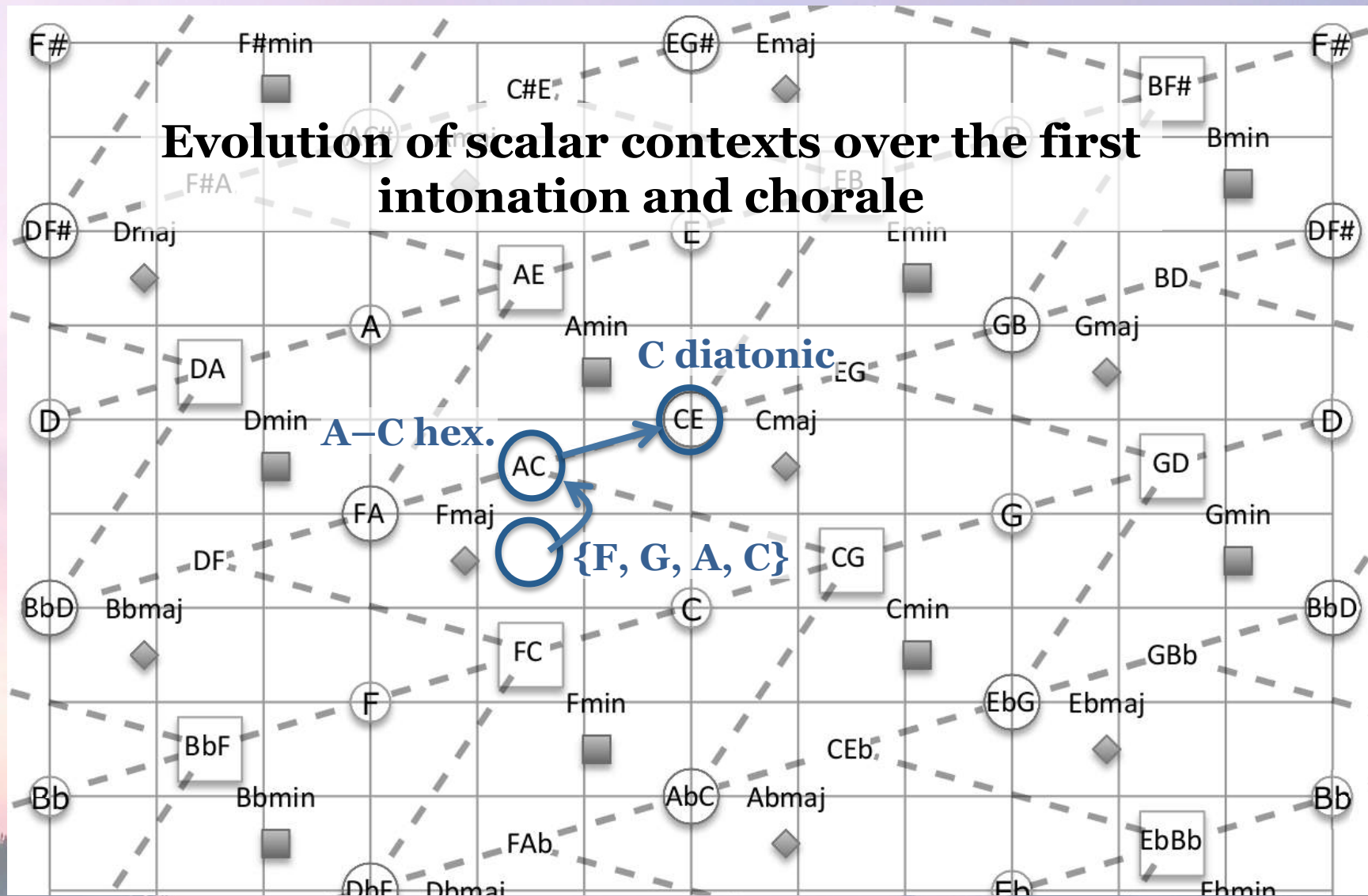
**C-A hexachord**

**C diatonic**



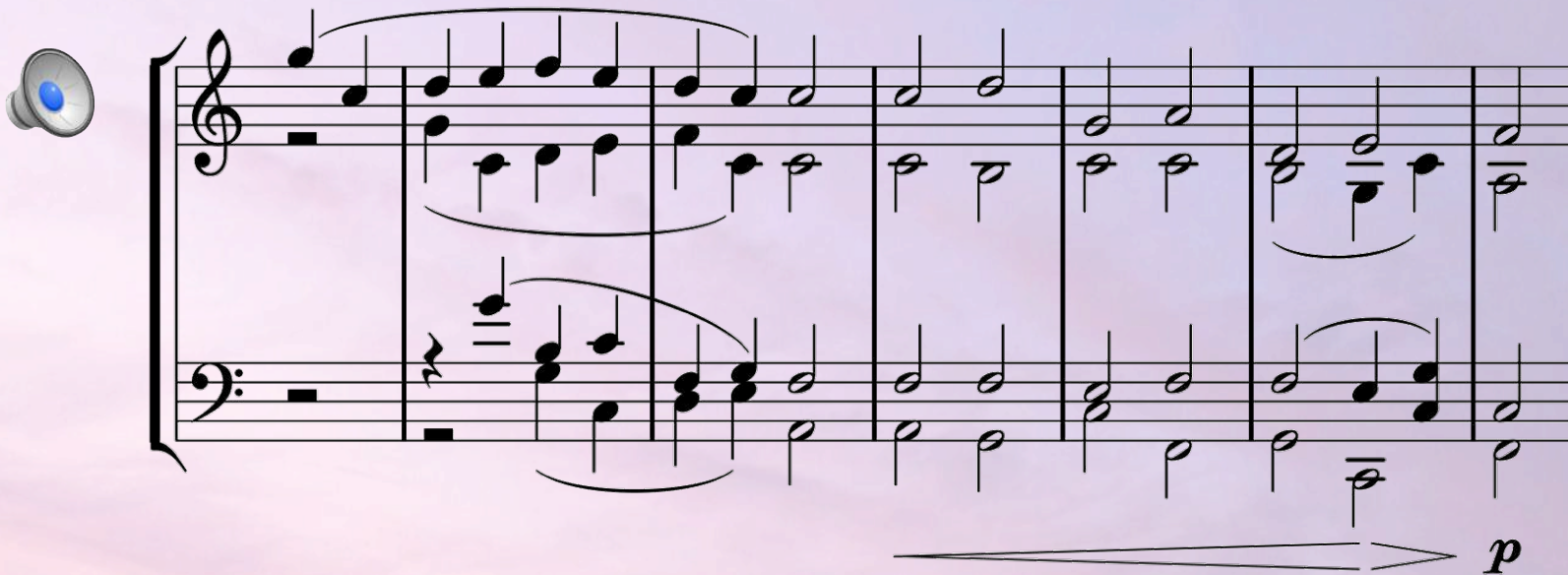
Recording: Muir Quartet, *Ecoclassics*

# Heiliger Dankgesang: Scalar Contexts

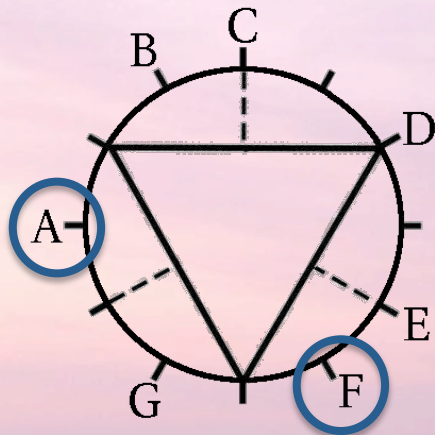


# Heiliger Dankgesang: Scalar Contexts

## Chorale phrase 4:



A musical score for a chorale phrase in G major, consisting of 8 measures. The score is written for a four-part setting (Soprano, Alto, Tenor, Bass) on a grand staff. The melody is primarily scalar, moving stepwise. A speaker icon is located to the left of the first measure. A dynamic marking of *p* (piano) is placed at the end of the phrase, with a hairpin indicating a crescendo leading to it.



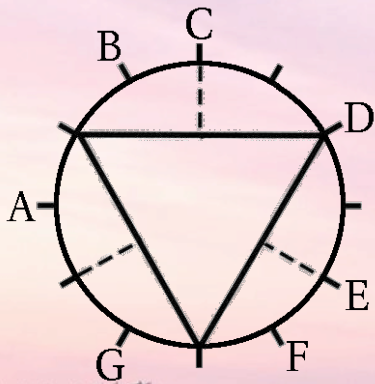
Ending high in the orbit gives the effect of suspension



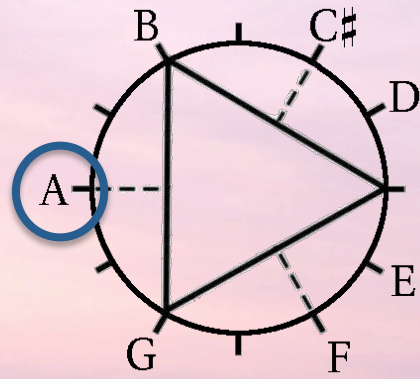
# Heiliger Dankgesang: Scalar Contexts

Neue Kraft fühlend  
Andante

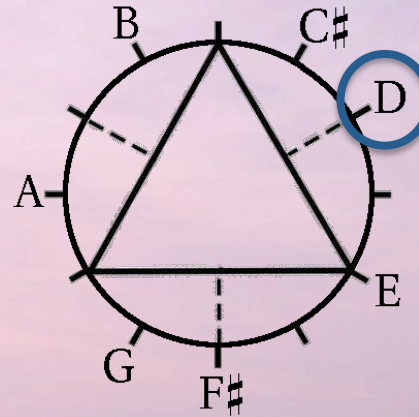
The musical score consists of two staves: a piano part on the left and a violin part on the right. The piano part begins with a piano (*p*) dynamic and features a melodic line with a trill (*tr*) in the final measure. The violin part starts with a forte (*f*) dynamic and includes a trill (*tr*) in the first measure. The score is in 3/8 time and includes various dynamics such as *p*, *f*, and *tr*.



C diatonic



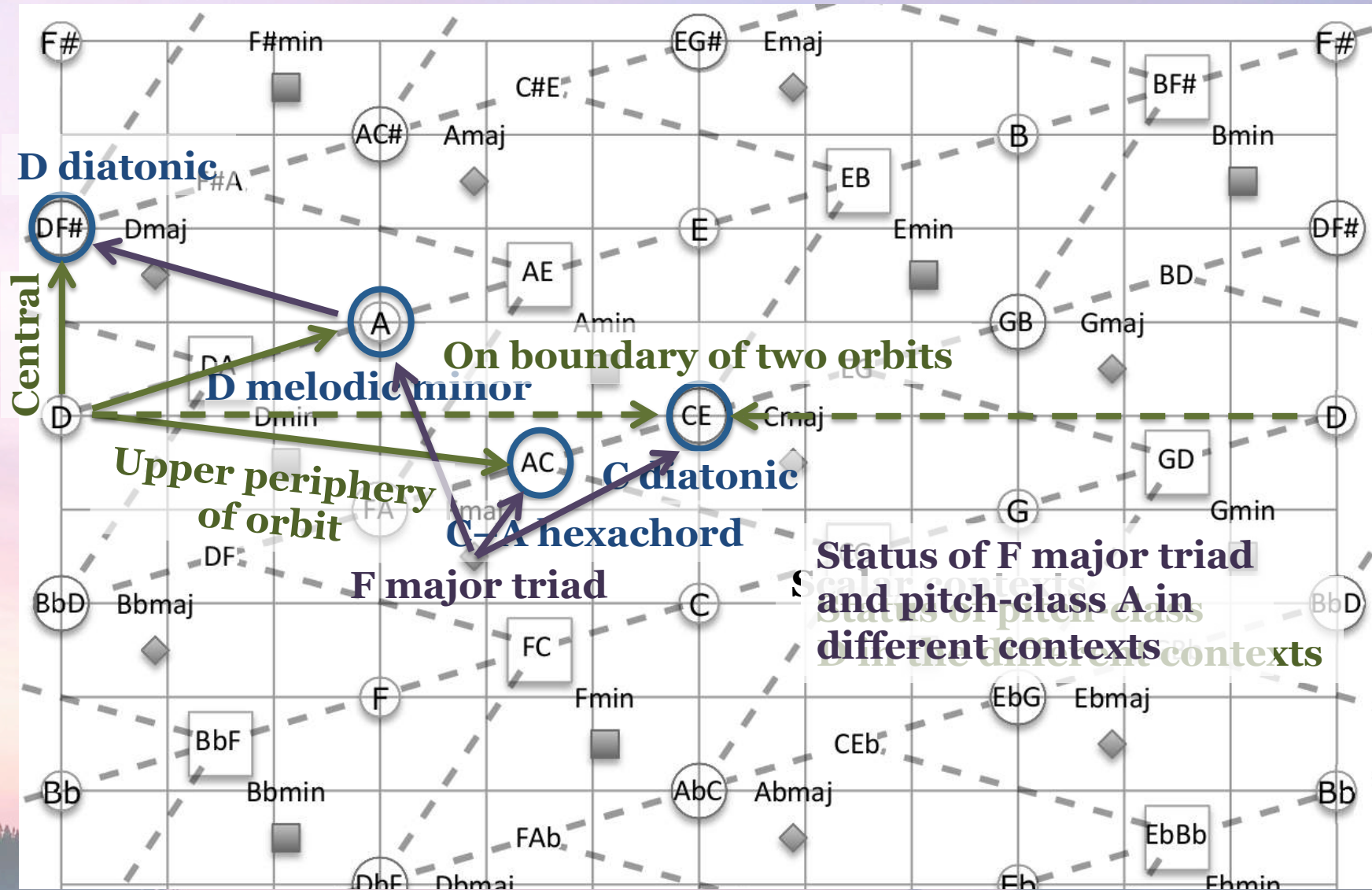
A is central



D diatonic

D is central

# Heiliger Dankgesang: Scalar Contexts



# *Heiliger Dankgesang*: Motivic D–C

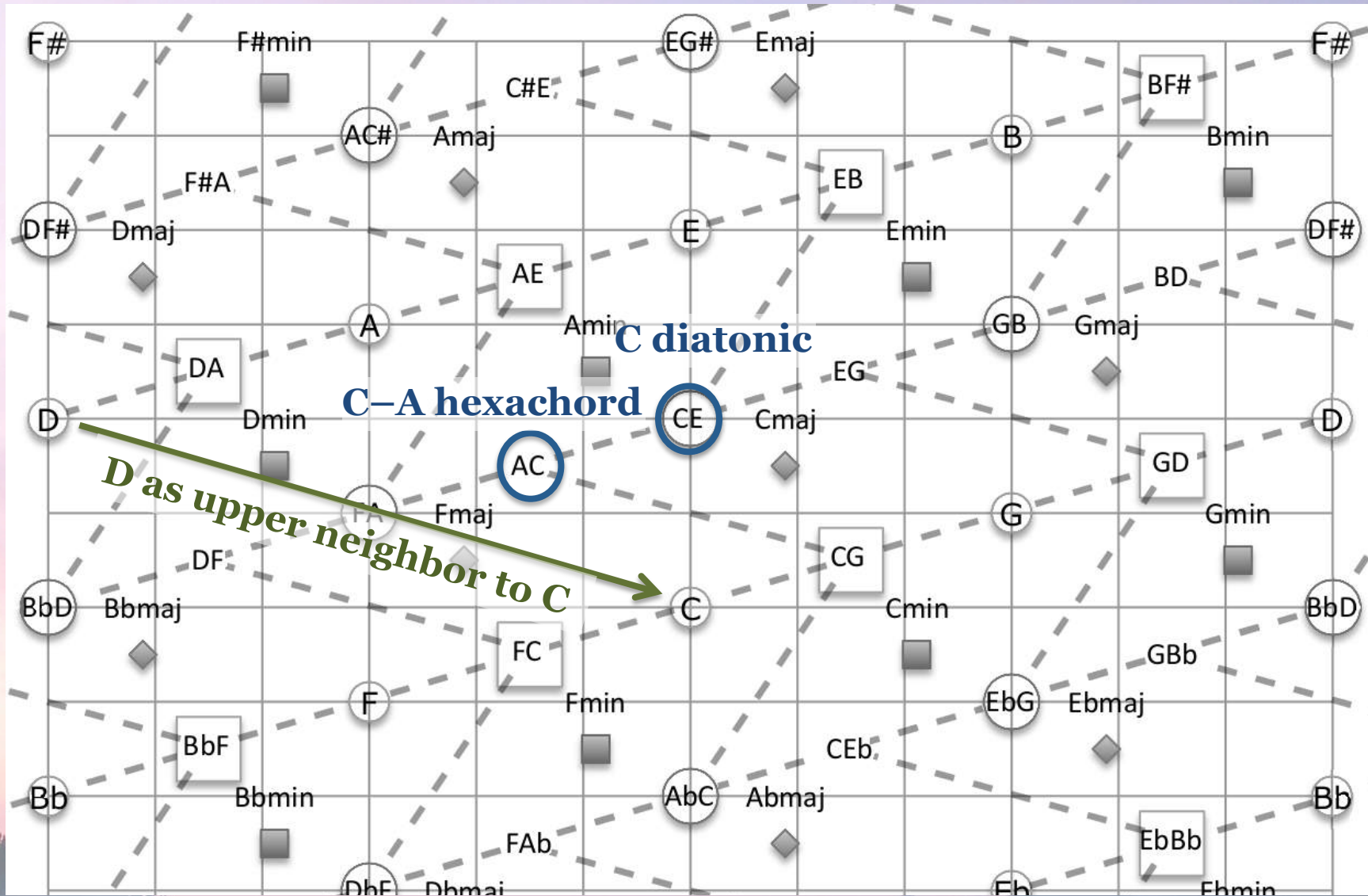


**Final form of the intonation:**



**D–C**

# Heiliger Dankgesang: Motivic D-C





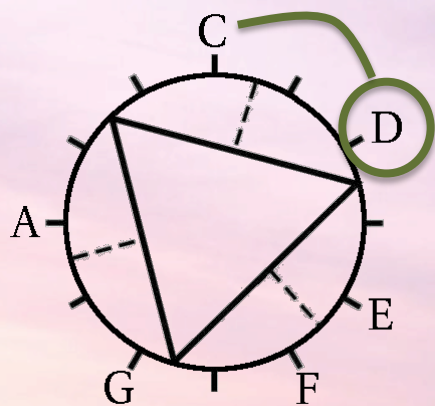
# Heiliger Dankgesang: Motivic D–C

Chorale phrases 1–2:

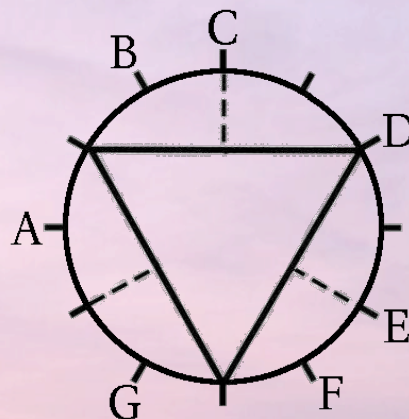
**F maj. context:** D resists descent

**C maj. context:** D as lower neighbor to C, upward striving

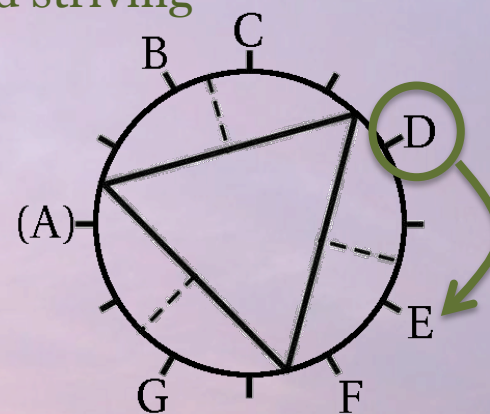
D ascends



C–A hexachord



C diatonic



C maj. tonality

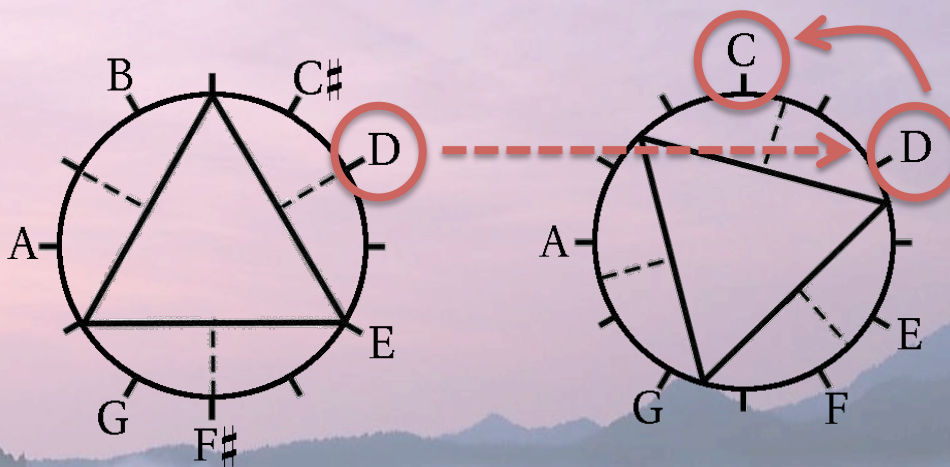
# Heiliger Dankgesang: Motivic D-C

End of *Neue Kraft* section



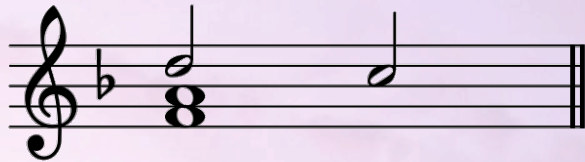
D → C

Molto adagio

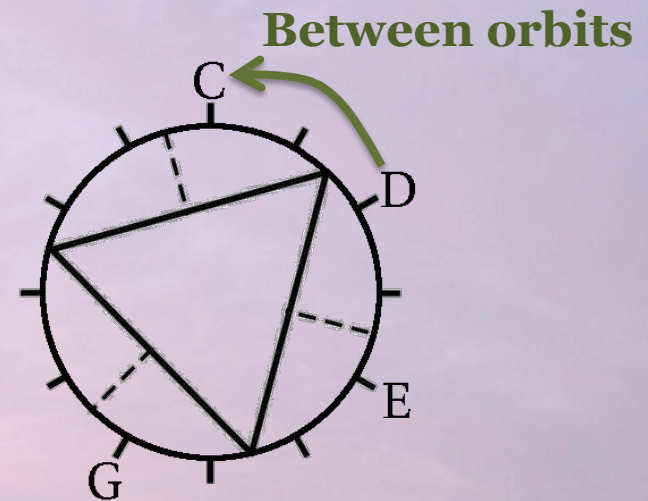
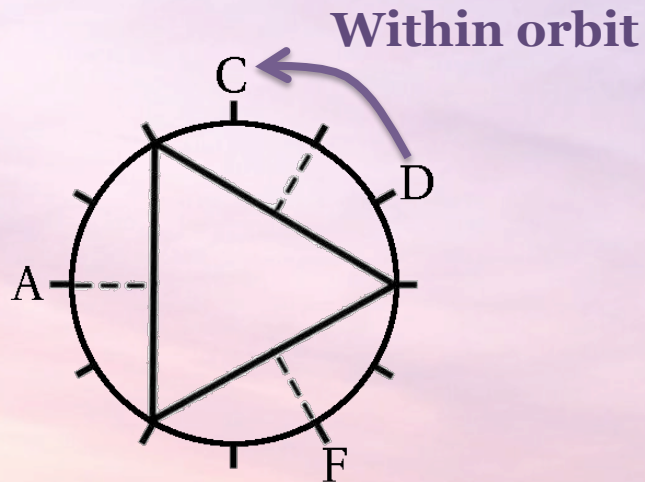


# Weakness and Strength

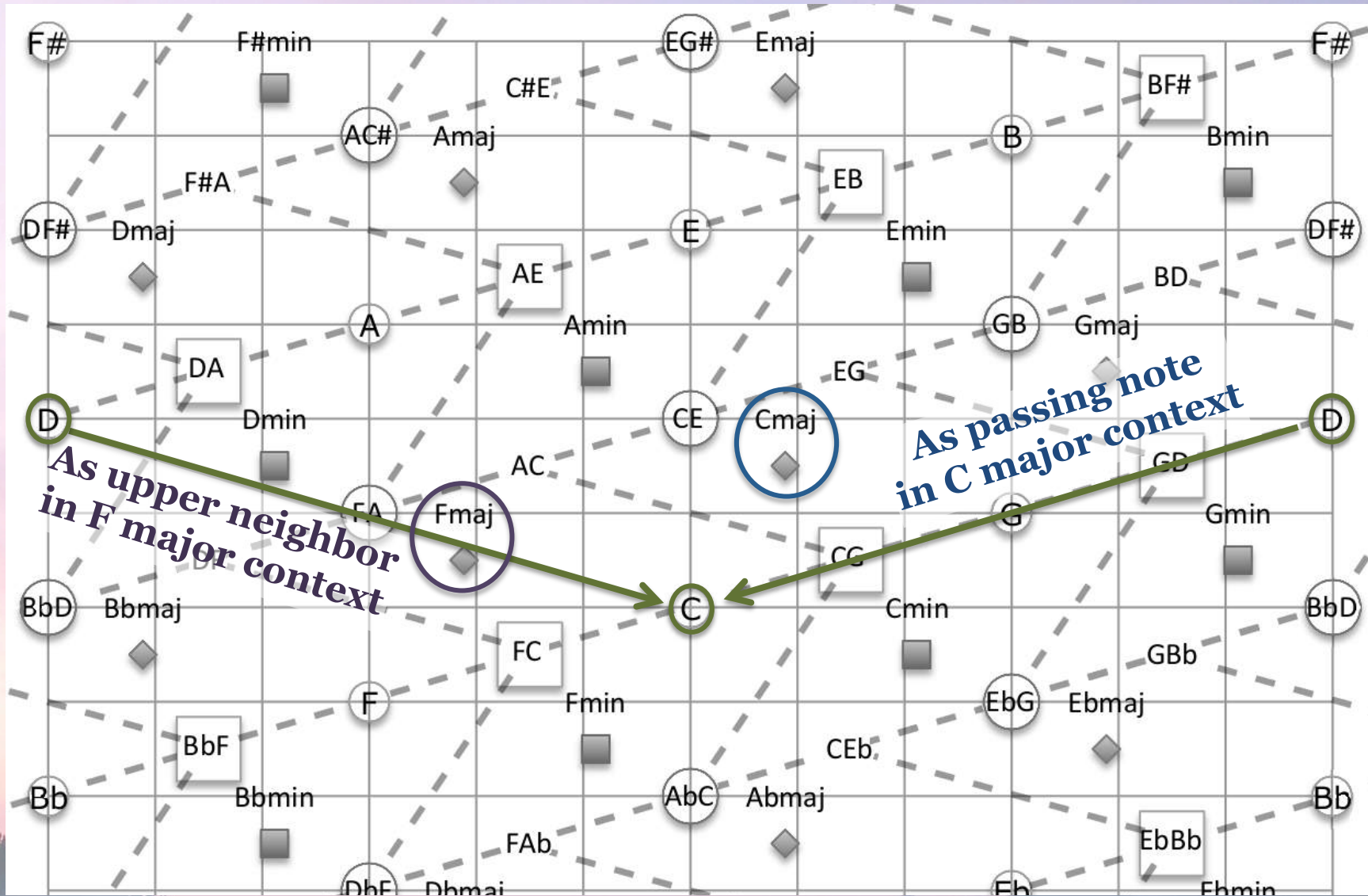
In an F major context, the step C–D is a weak neighbor motion



In a C major context, it is a strong completion of passing motion



# Weakness and Strength



# Heiliger Dankgesang: Final Chorale

Chorale tune



End of chorale phrase

*f* *sf* *sf* *sf* *sf* *sf* *sf* *sf*

A - B - C  
D - C

*sf* *sf* *sf* *p*

Strong in C maj. → D - C (PAC!)

Weak in F maj.

E - D - C

*p* *sf* *p*

Chorale tune D - C!

# Conclusions

- DFT phase space effectively reflects tonal process at multiple **levels of structure**.
- It does so through processes of **combination** rather than **reduction**.
- Relating levels through combination better reflects the **traditional notion of keys**.
- Motions in DFT phase space can be construed as a kind of **voice leading** through the idea of **triadic orbits**.
- Triadic orbits also have hermeneutic value in showing the **gravitational forces** that color tones and distinguishing **strong** and **weak** melodic motions.

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# **A Spatial Perspective on Long-Range Voice Leading and Beethoven's *Heiliger Dankgesang***

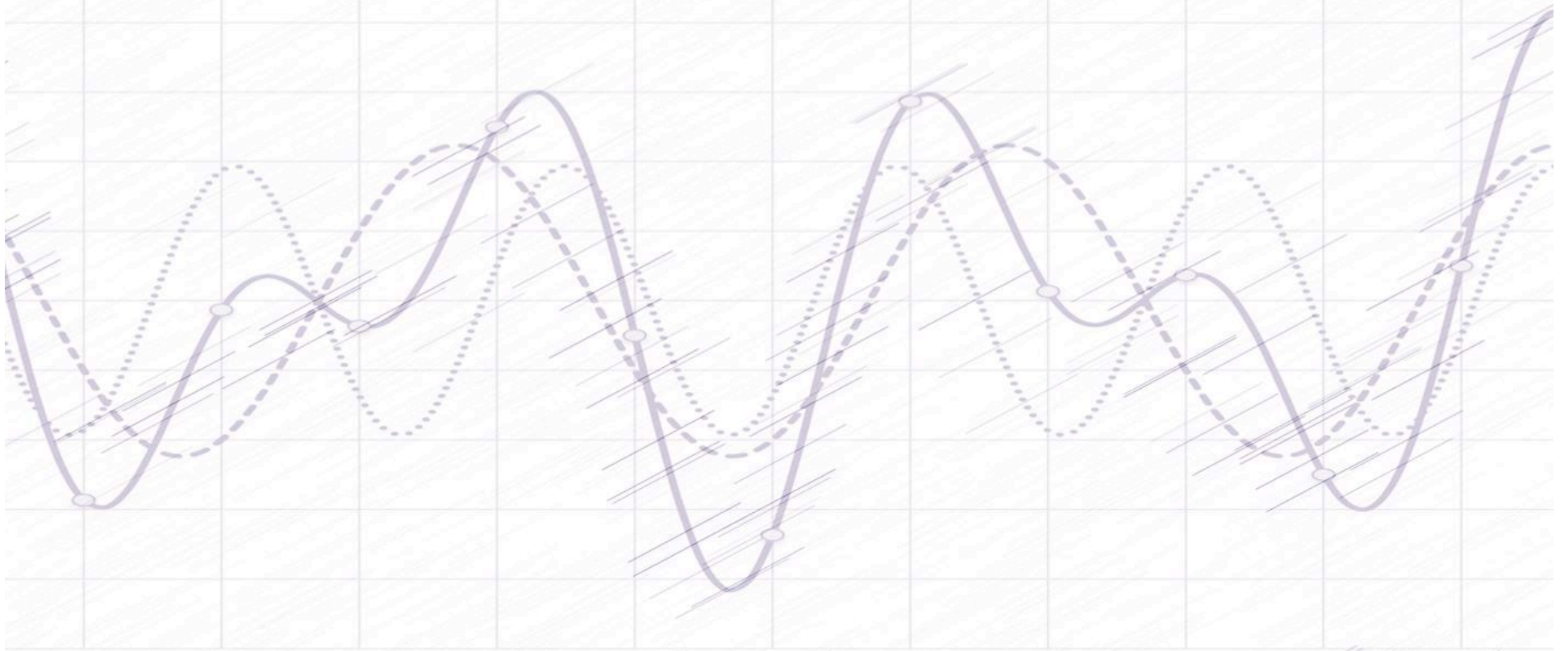
Jason Yust, Boston University

Presentation to the University of Connecticut  
Music History/Theory Colloquium  
March 6, 2015

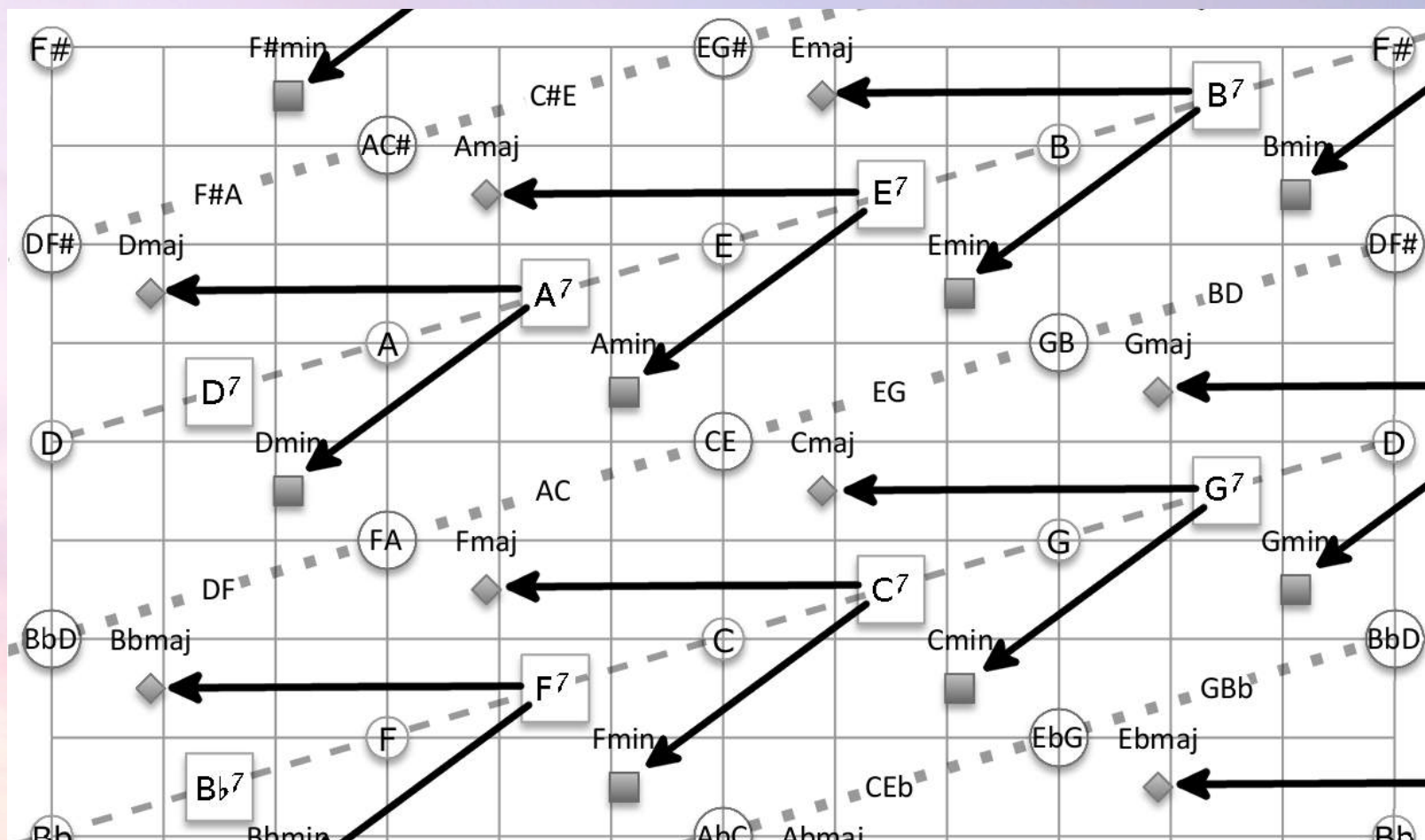
A copy of this talk is available at  
[people.bu.edu/jyust/](http://people.bu.edu/jyust/)

# Appendices:

## A1: Derivation of tonal regions

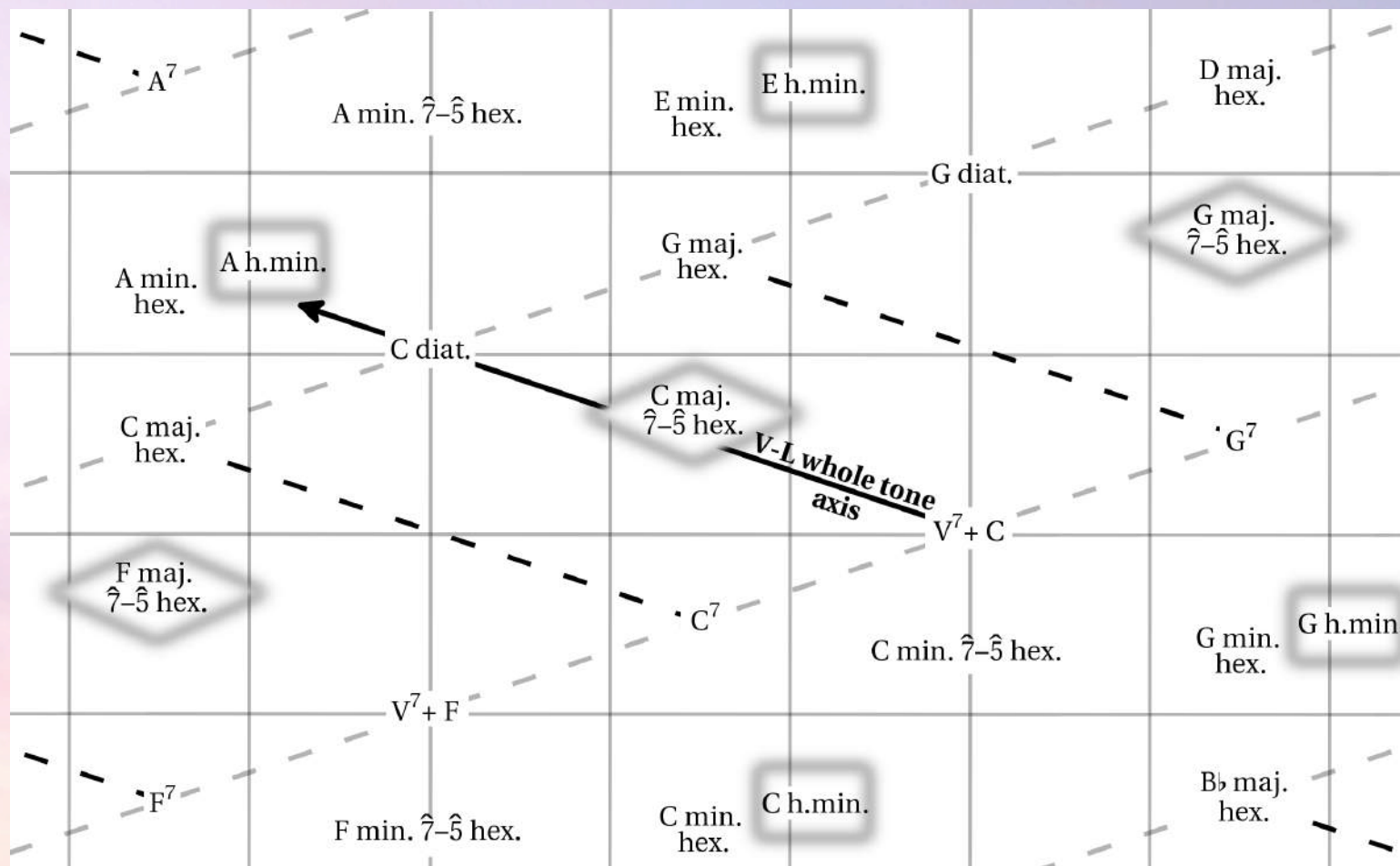


# Derivation of Tonal Regions



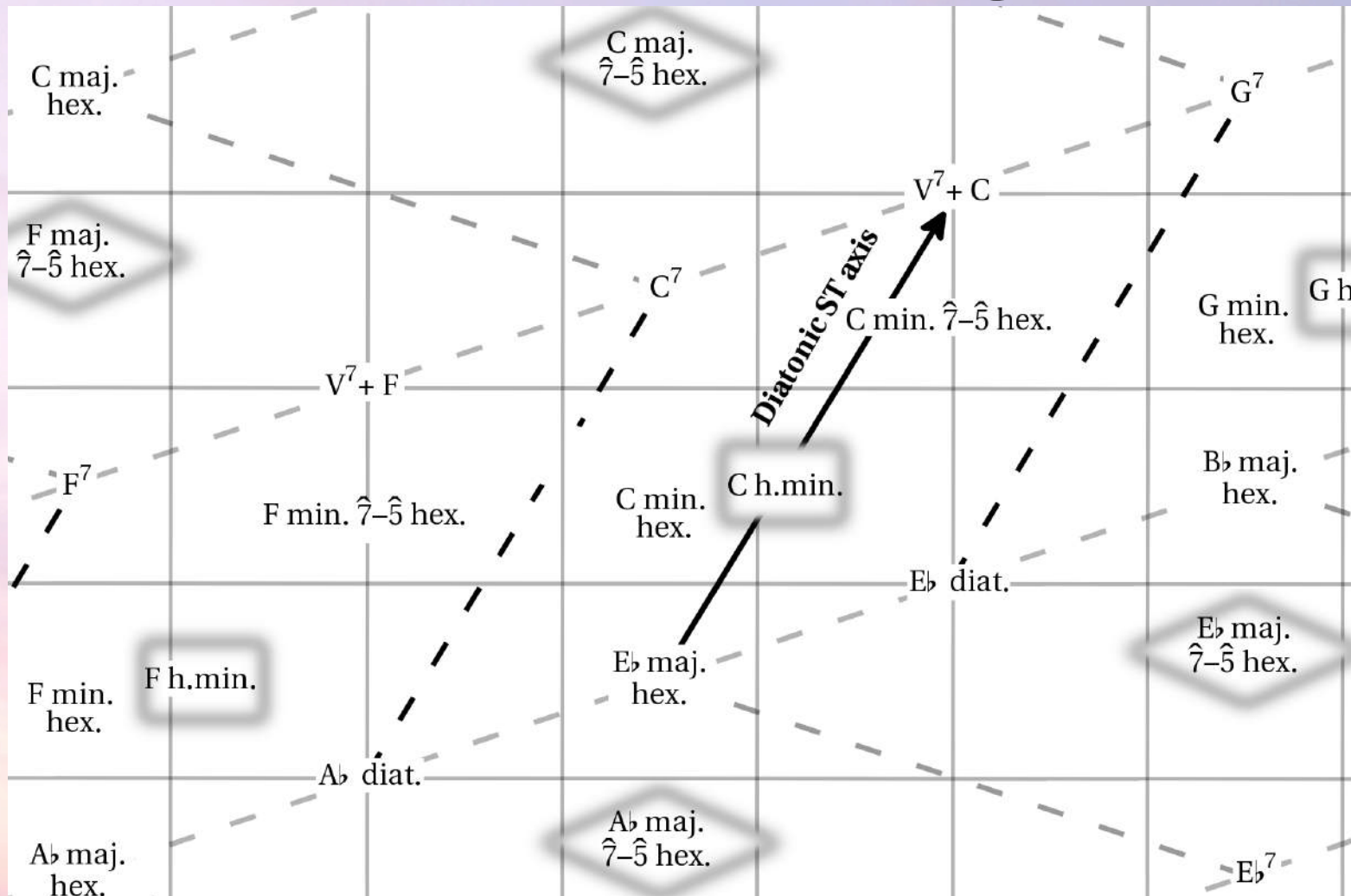
Boundaries between major and minor follow the circle of fifths through diatonic scales and dominant sevenths / individual pcs.

# Derivation of Tonal Regions



A characteristic hexachord is at the center of the major regions. Boundaries between fifth-related major regions are parallel to an axis that approximately passes through this hexachord.

# Derivation of Tonal Regions



Minor region boundaries are parallel to an axis that approximately passes through the harmonic minor scale