A Spatial Perspective on Long-Range Voice Leading and Beethoven's *Heiliger Dankgesang*

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Music History/Theory Colloquium
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A copy of this talk is available at
people.bu.edu/jyust/

Outline

I. Schenker, Brahms, and Keys: The Problem

1. Schenker's syllogism
2. Examples: Beethoven *Bagatelle* op. 119 no. 7
   Brahms *Cello Sonata* op. 99, subordinate theme
3. An alternative: DFT phase space and tonal regions
4. Beethoven and Brahms examples in DFT phase space

II. DFT and Triadic Orbits

1. DFT components as sinusoidal approximations
2. Triadic orbits

III. Beethoven's *Heiliger Dankgesang*

1. Scalar context and triadic orbits
2. The D–C motive
3. Strength and weakness

I. Schenker, Brahms, and Keys

1. Schenker's syllogism
2. Examples: Beethoven *Bagatelle* op. 119 no. 7
   Brahms *Cello Sonata* op. 99, subordinate theme
3. An alternative: DFT phase space and tonal regions
4. Beethoven and Brahms examples in DFT phase space

Schenker, Brahms, and Keys
The problem with harmonic reduction

The Schenkerian Syllogism

False!

~~*Hidden Premise:* Voice leading is always a relationship between individual harmonies.~~

Premise: Long-range structure is contrapuntal (based on voice-leading).

Conclusion: Long-range structure is *reductive*—i.e., it consists of relationships between non-adjacent harmonies.

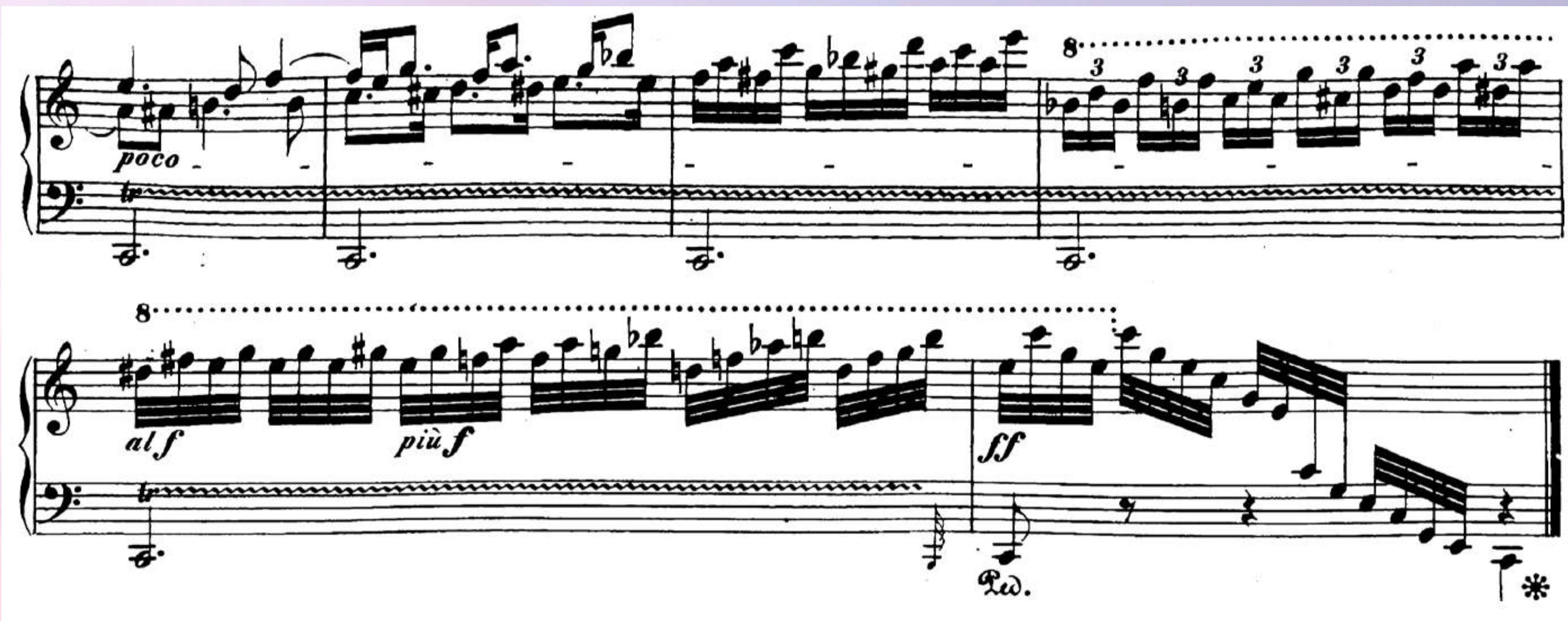
Problematic

Example:

Beethoven *Bagatelle* op. 119 no. 7

Beethoven *Bagatelle* op. 119 no. 7

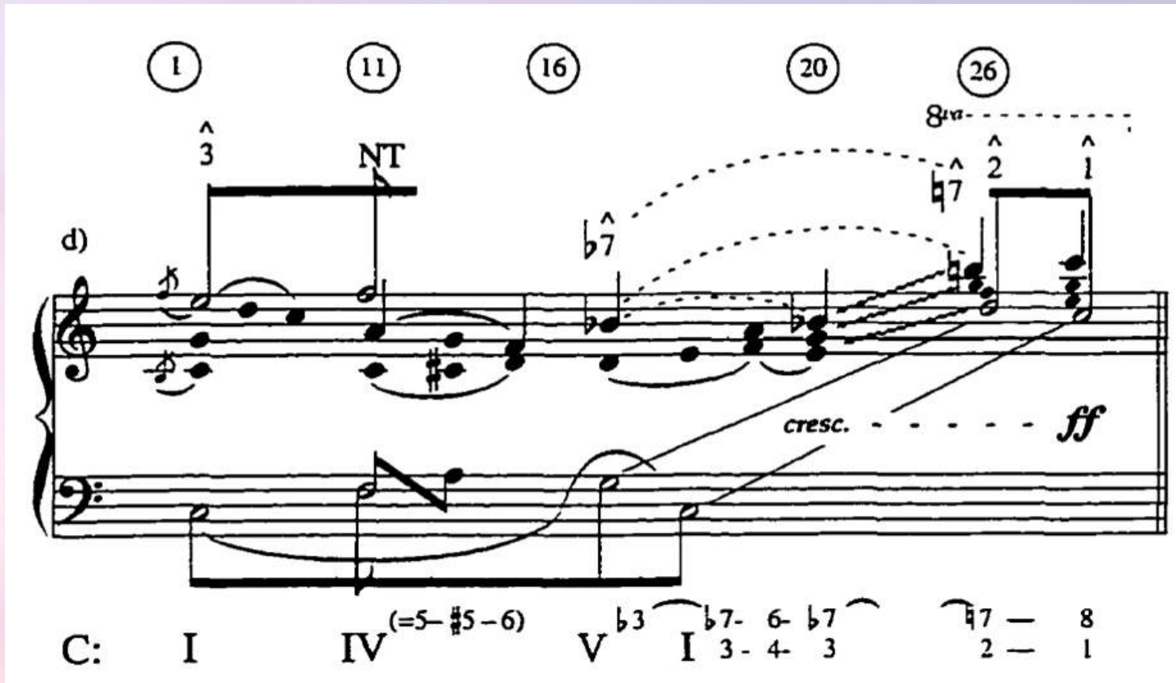
Beethoven *Bagatelle* op. 119 no. 7



C major:
V of F major
***not* I of C major**

Beethoven *Bagatelle* op. 119 no. 7

Frank Samarotto (*A Theory of Temporal Placticity for Tonal Music,* PhD Diss., 1999):



Samarotto's analysis reflects only the structural **chords**, not their tonal contexts. This analysis mistakenly represents the bagatelle as **tonally closed**.

Beethoven *Bagatelle* op. 119 no. 7

Frank Samarotto (*A Theory of Temporal Placticity for Tonal Music,* PhD Diss., 1999):

However, the higher-level progression of tonal contexts shows that the bagatelle is actually **tonally open**, in accord with its function as prelude to no. 8.

See Nicholas Marston, "Trifles or Multi-Trifle: Beethoven's Bagatelles Op. 119, Nos 7–11." *Music Analysis* 5/2–3 (1986)

Tonal contexts: C major - - - - - - - - - - - - - - - ⟶ F major

Chords: C major - - - - - - - - - - - - - - - ⟶ C major

Samarotto's analysis reflects only the structural **chords**, not their tonal contexts. This analysis mistakenly represents the bagatelle as **tonally closed**.

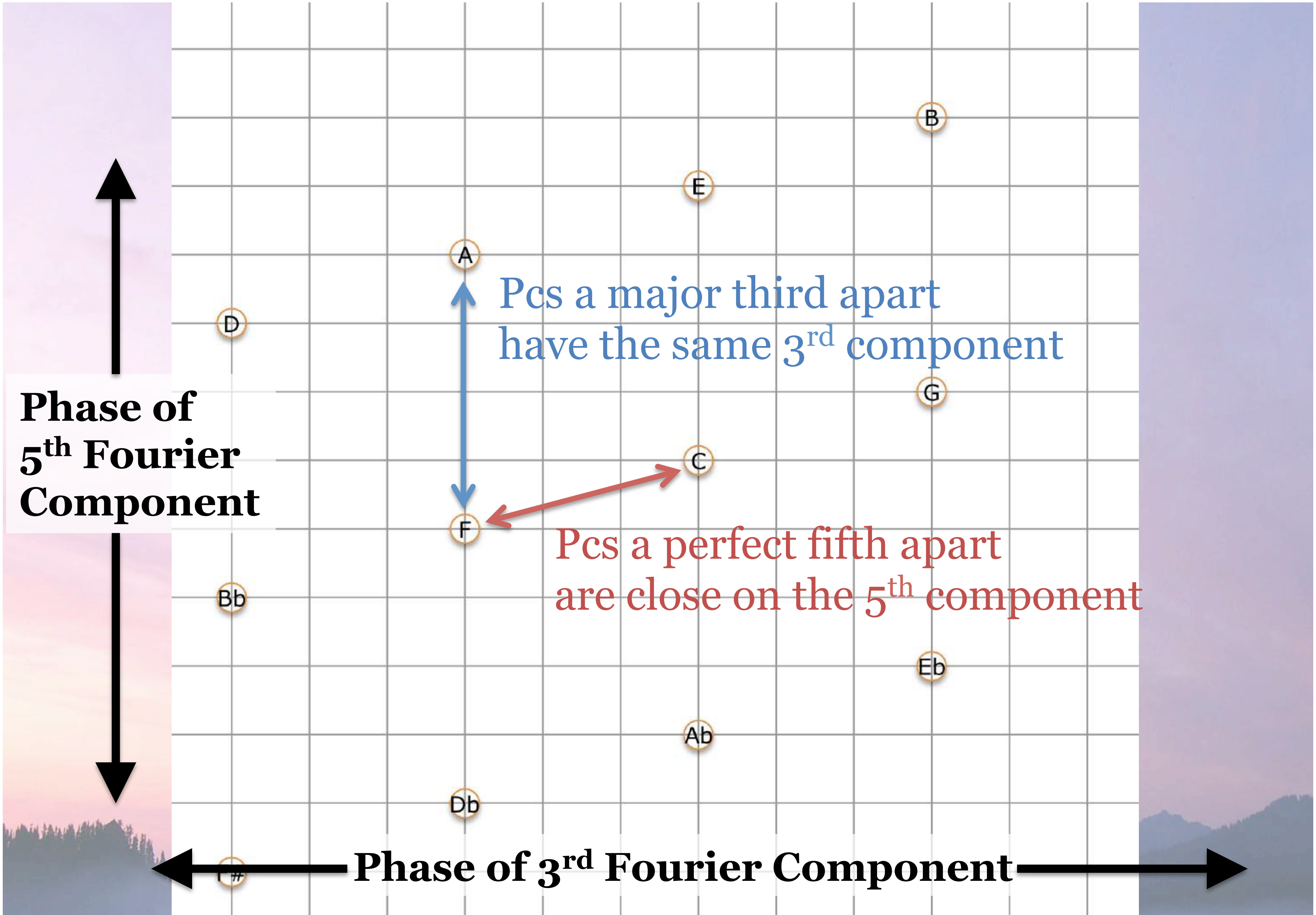
Schenker, Brahms, and Keys

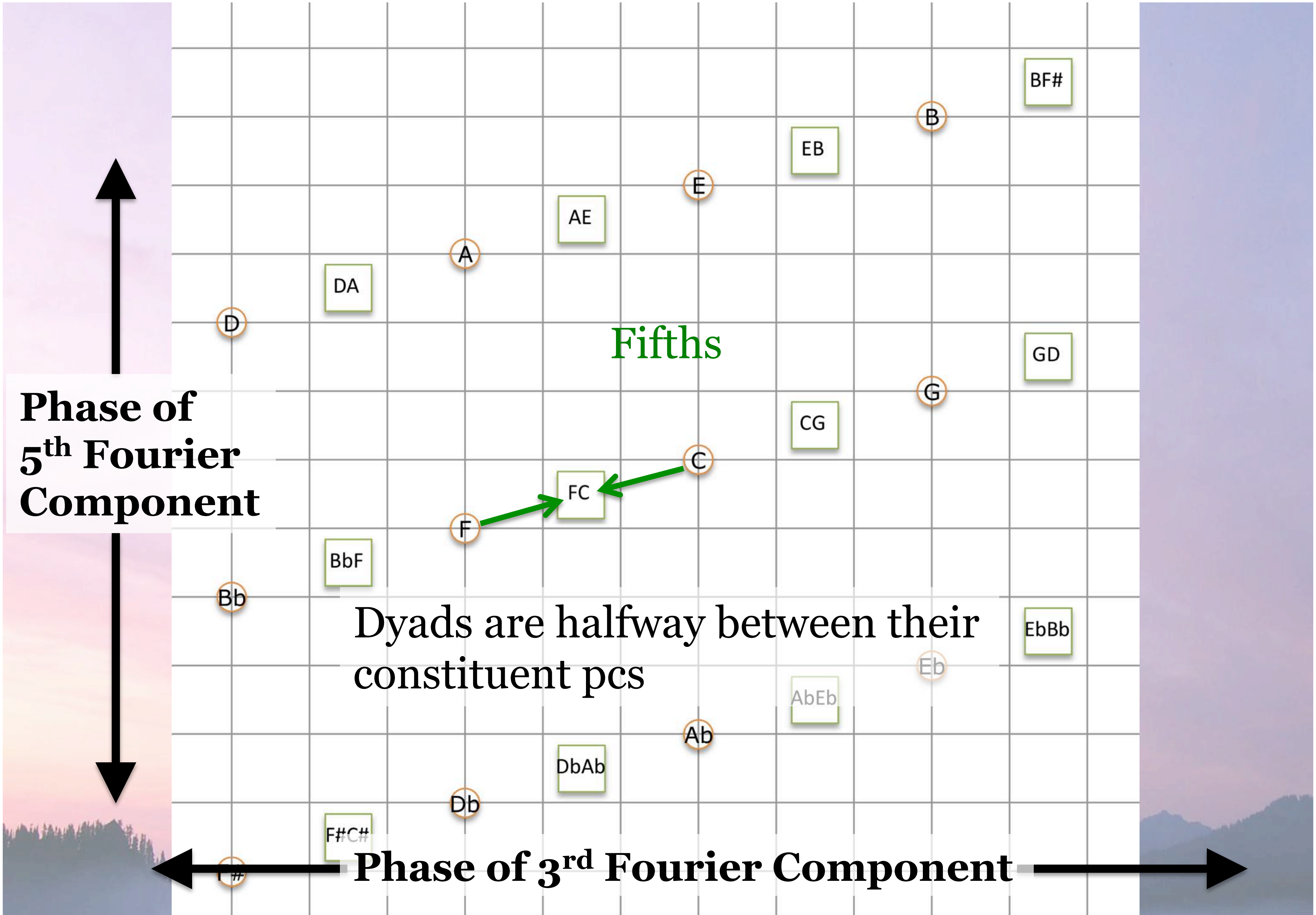
Alternative:
A spatial concept of tonality . . .

DFT phase space

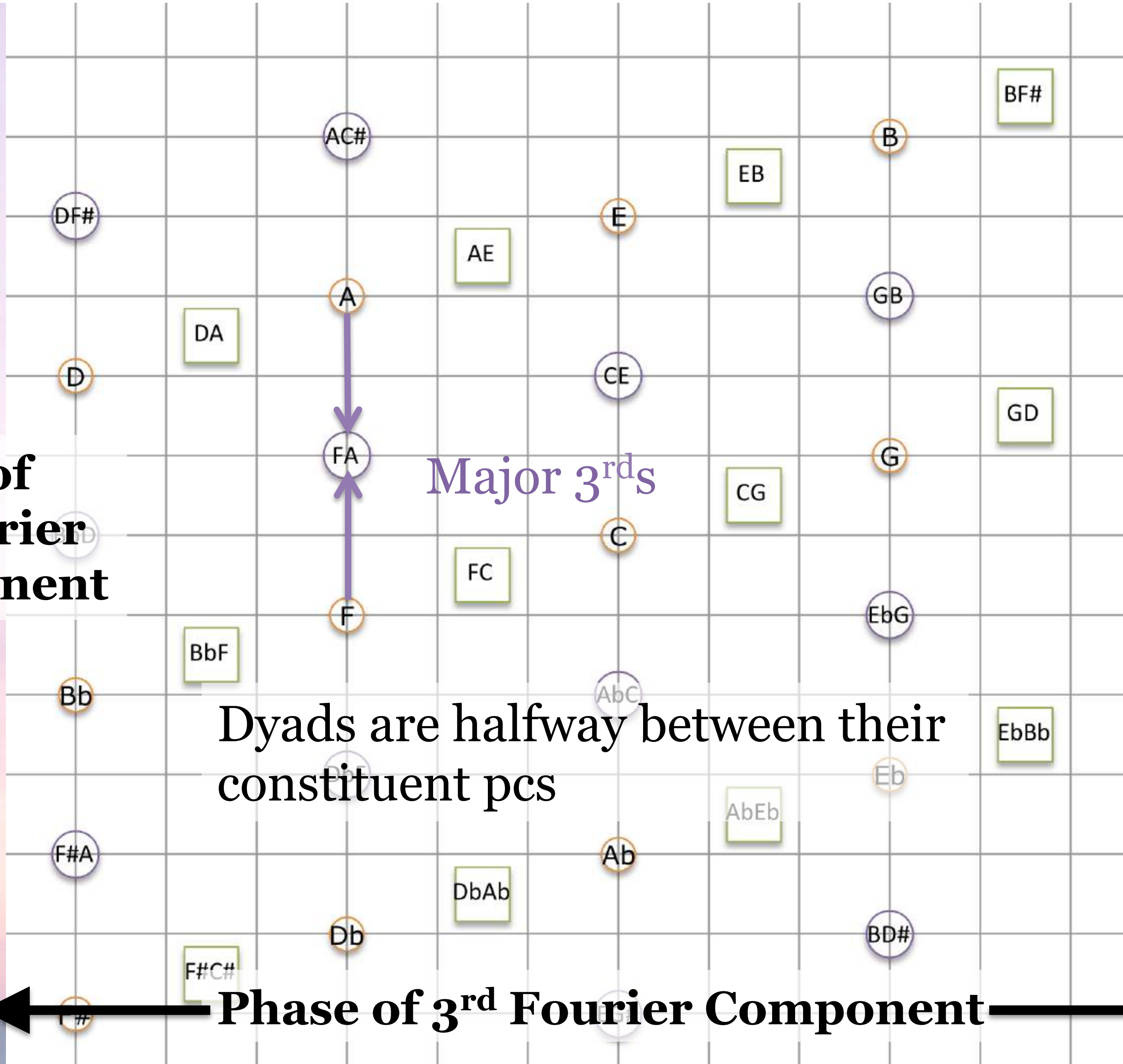
Amiot, Emmanuel. (2013). "The Torii of Phases," *Mathematics and Computation in Music, MCM 2013* (ed. Yust, Wild, & Burgoyne) 1–18.

Yust, "Schubert's Harmonic Language and Fourier Phase Space." *Journal of Music Theory* 59/1 (forthcoming). Available at http:/people.bu.edu/jyust/SchubertDFT.pdf


Phase of 5th Fourier Component
Phase of 3rd Fourier Component
Pcs a major third apart have the same 3rd component
Pcs a perfect fifth apart are close on the 5th component
B
E
A
D
G
C
F
Bb
Eb
Ab
Db



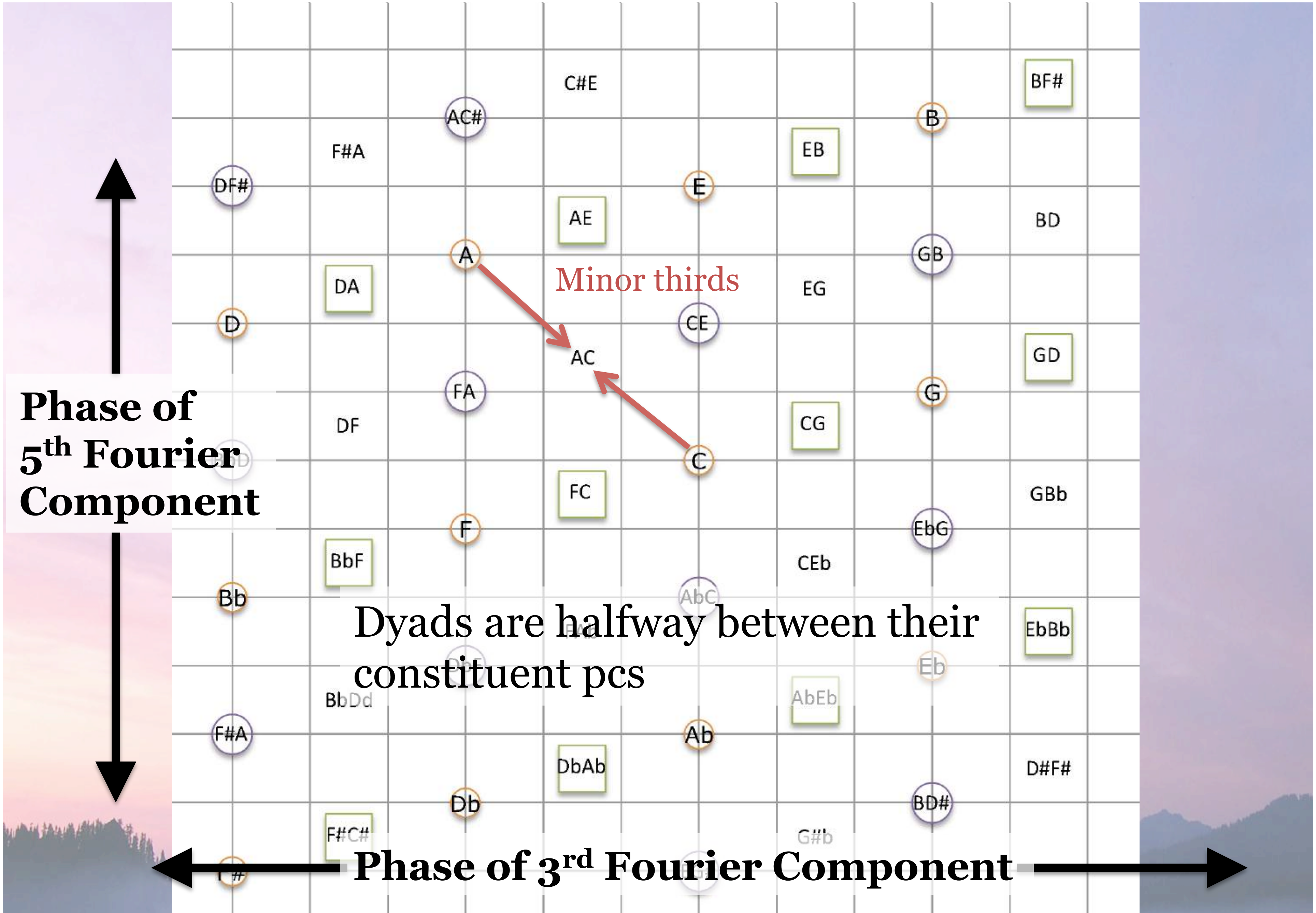
Fifths
BF#
B
EB
E
AE
A
DA
D
GD
G
CG
C
FC
F
BbF
Bb
EbBb
Eb
AbEb
Ab
DbAb
Db
F#C#
F#
Phase of 5th Fourier Component
Phase of 3rd Fourier Component
Dyads are halfway between their constituent pcs



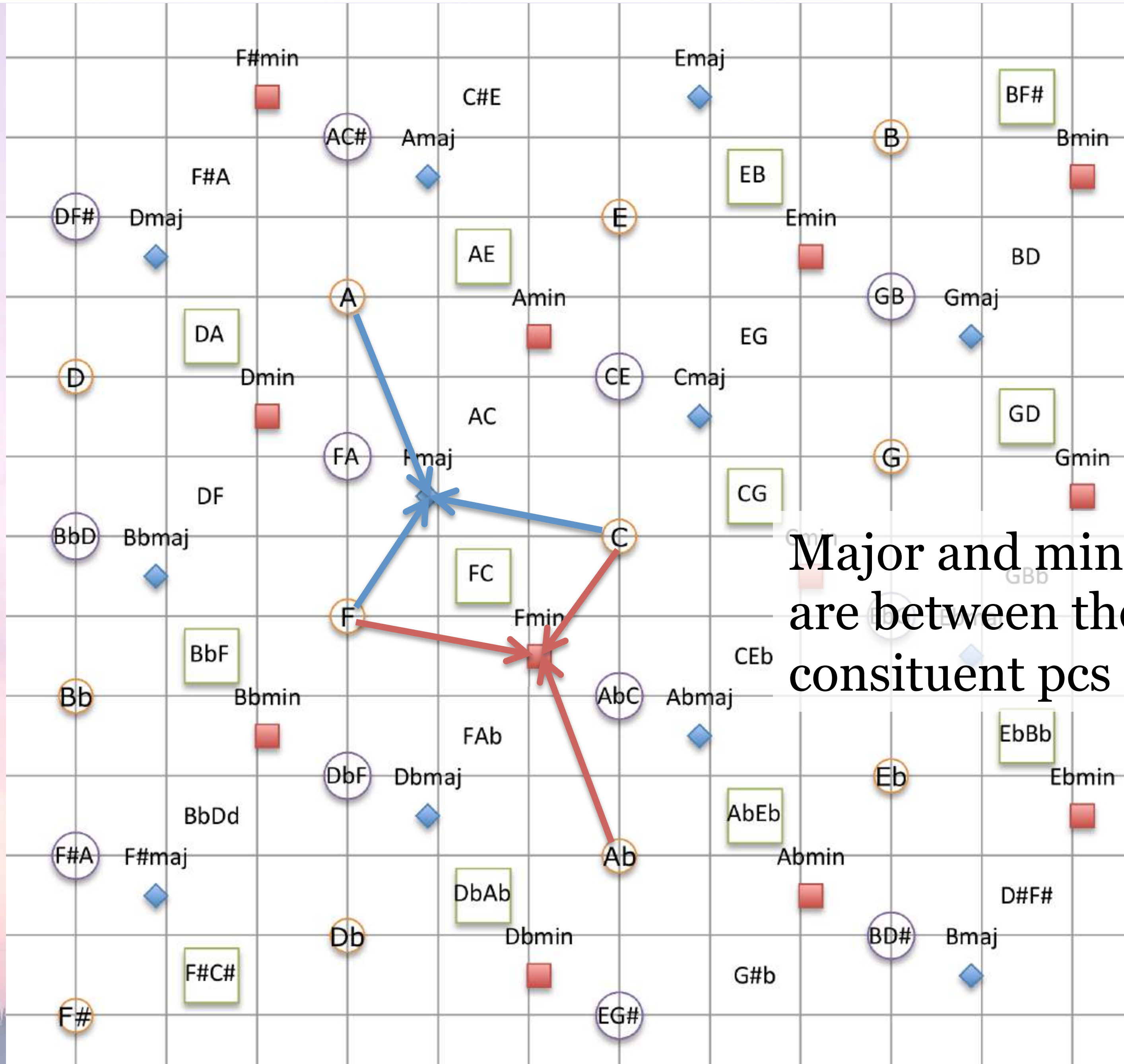
Phase of 5^{th} Fourier Component

Phase of 3^{rd} Fourier Component

Dyads are halfway between their constituent pcs



Phase of 5th Fourier Component
Phase of 3rd Fourier Component
Dyads are halfway between their constituent pcs
Minor thirds
C#E
BF#
AC#
B
F#A
EB
DF#
E
AE
BD
A
GB
DA
EG
D
CE
AC
GD
FA
G
DF
CG
C
FC
GBb
F
EbG
BbF
CEb
Bb
AbC
EbBb
Eb
AbEb
F#A
Ab
DbAb
D#F#
Db
BD#
F#C#



Major and minor triads are between their consituent pcs

Schenker, Brahms, and Keys

Properties of DFT Phase Space

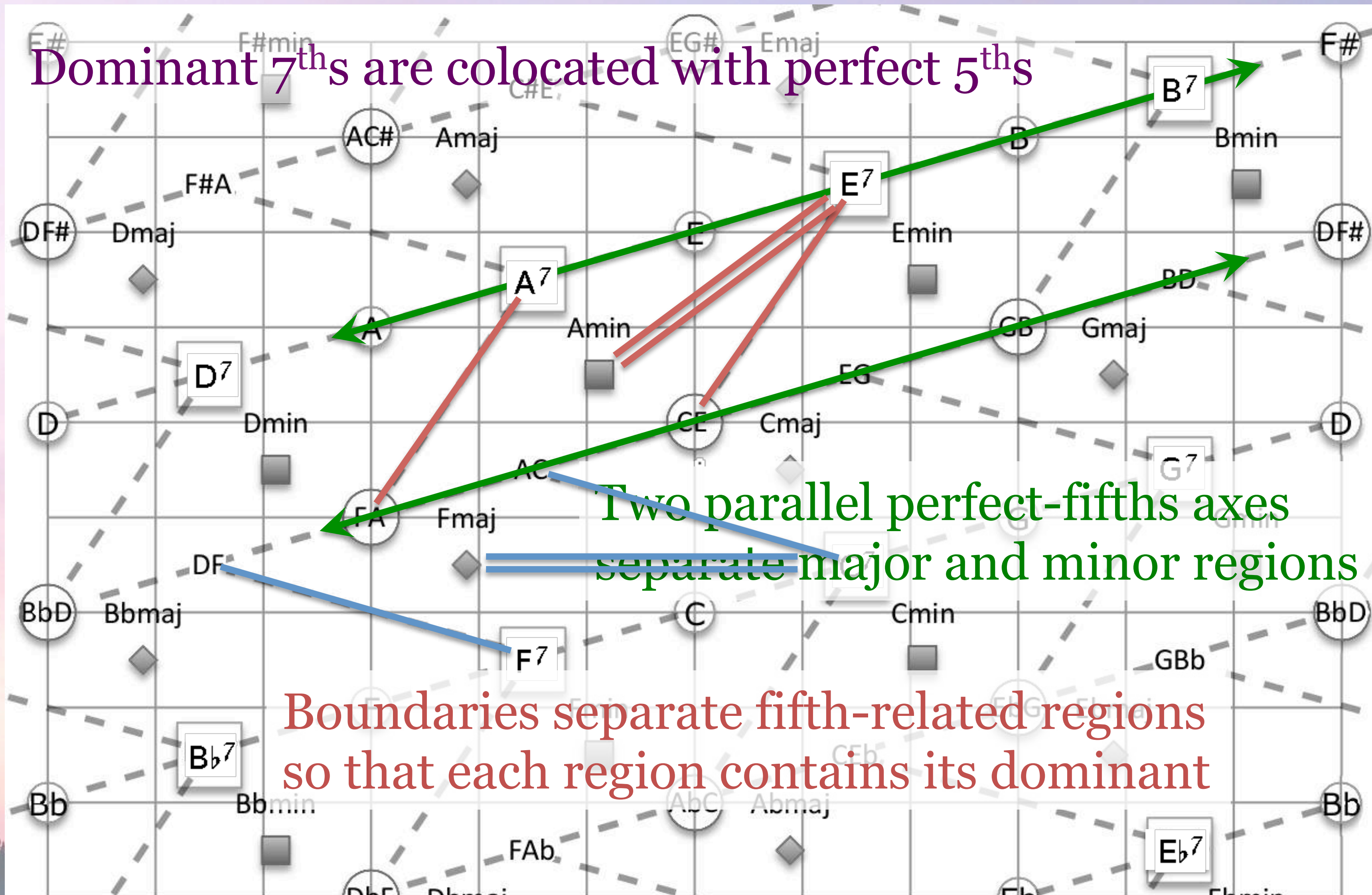
- Objects are pcsets, multisets, or statistical pc distributions
- Toroidal geometry
- Vertical axis indicates circle-of-fifths position
- Horizontal axis captures triadic voice-leading properties

Schenker, Brahms, and Keys

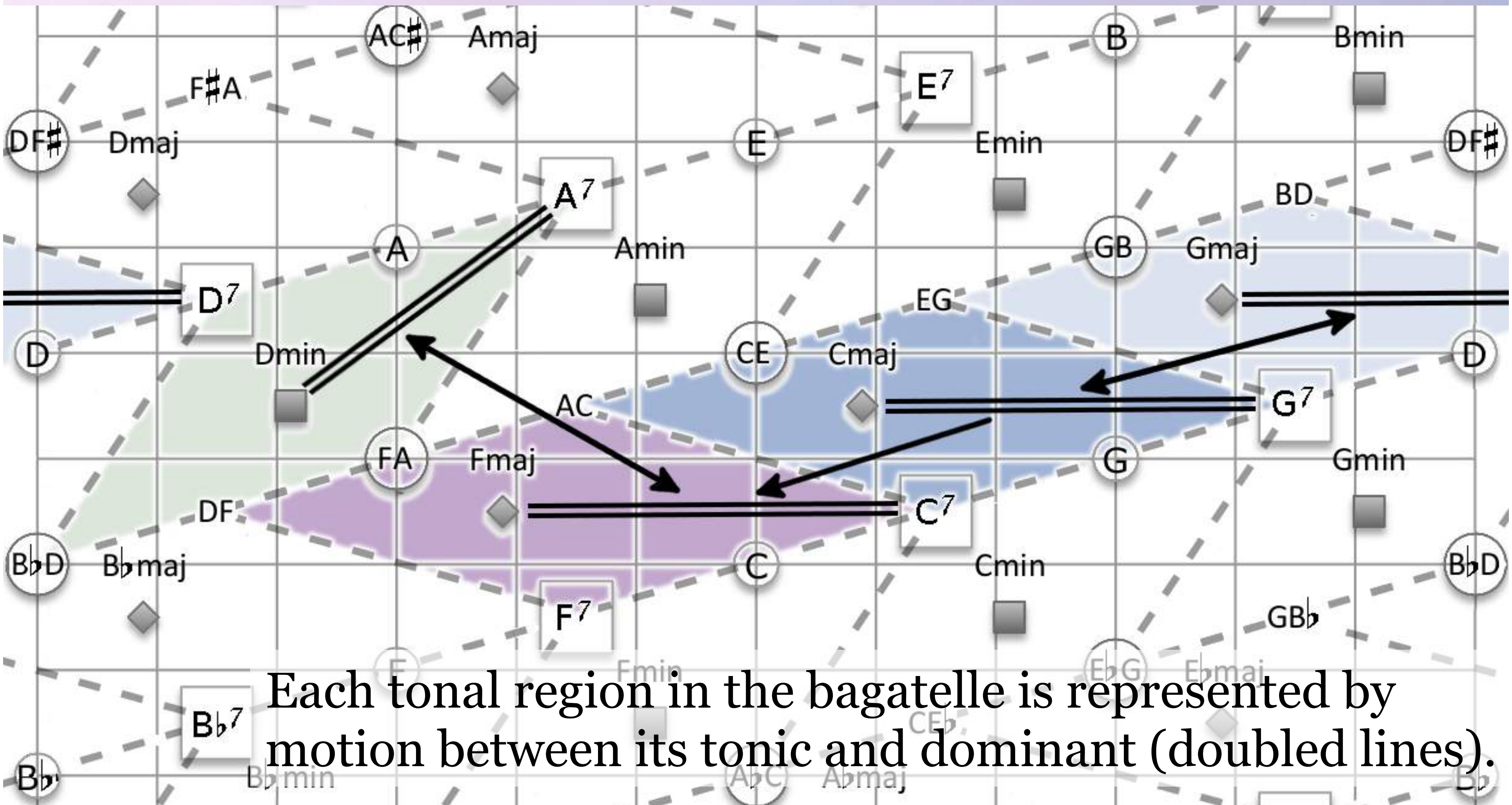
Properties of DFT Phase Space

- Many kinds of harmonic objects exist in the space: single pcs, harmonies, scales, etc.
- Space is *continuous*—paths connect points via a potential infinite series of intermediate states (pc distributions)
- Nearness in the space (its ***topology***) is based on *common pc content*.

Tonal Regions



Beethoven *Bagatelle* op. 119 no. 7



Each tonal region in the bagatelle is represented by motion between its tonic and dominant (doubled lines).

Properties of DFT Phase Space

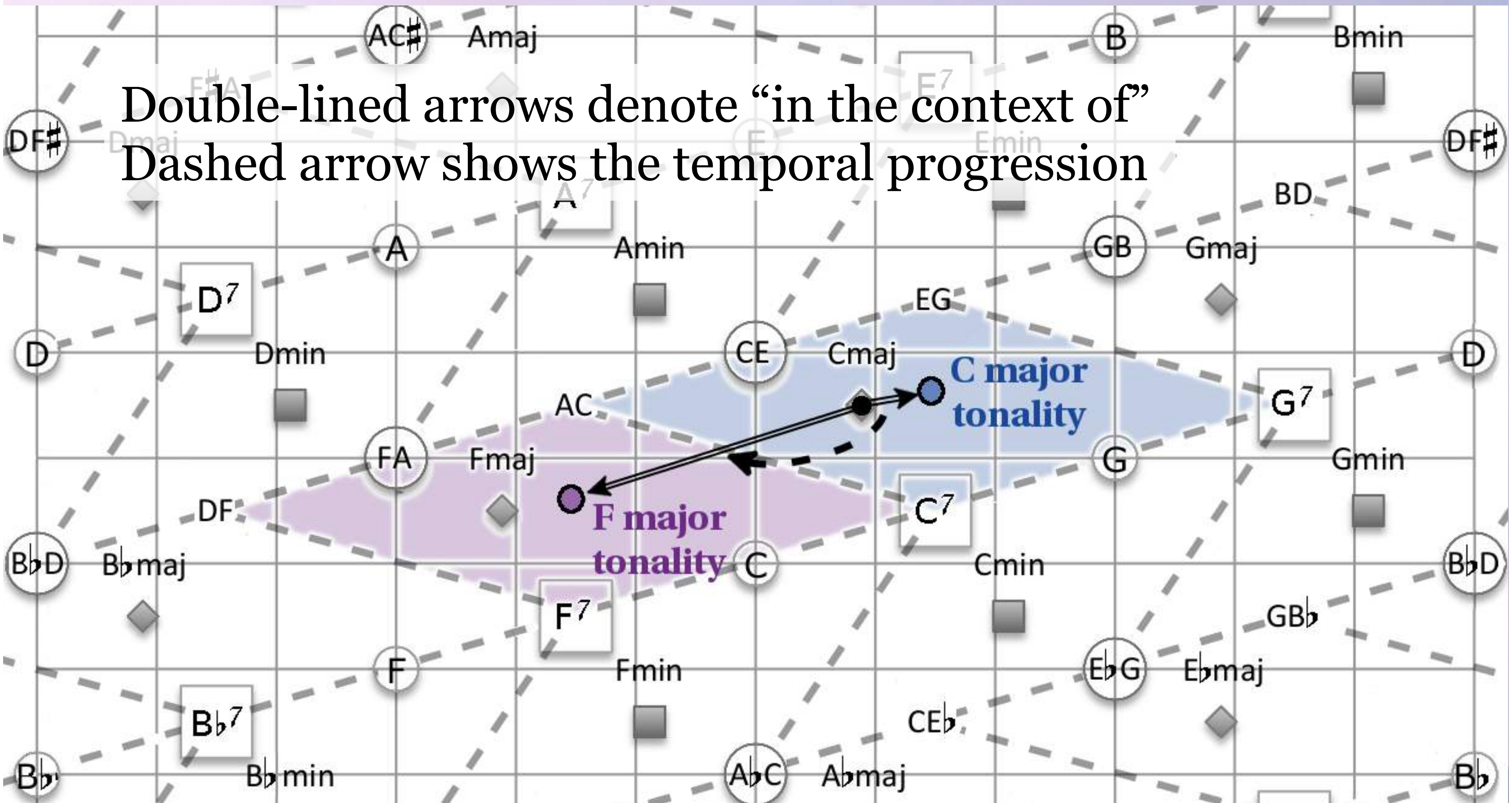
A ***path*** can represent a motion from A to B, but it can also represent "B in the context of A."

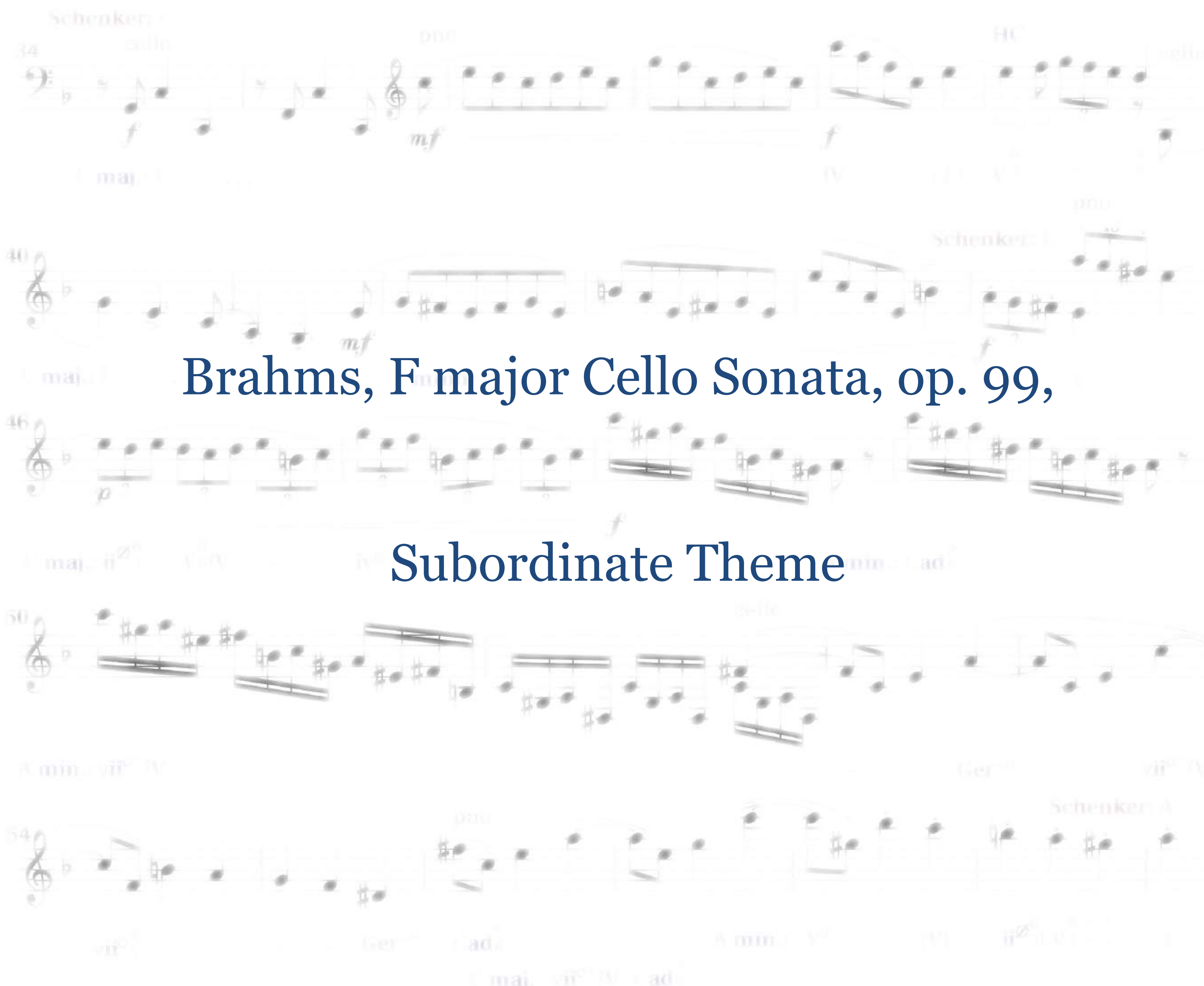
Combination of pcsets is highly tractable: the position of $A + B$ is easily predictable from the path $A \rightarrow B$.

Averaging over *more objects* (pcs, triads) restricts the range of activity.

Beethoven *Bagatelle* op. 119 no. 7

Double-lined arrows denote "in the context of"
Dashed arrow shows the temporal progression



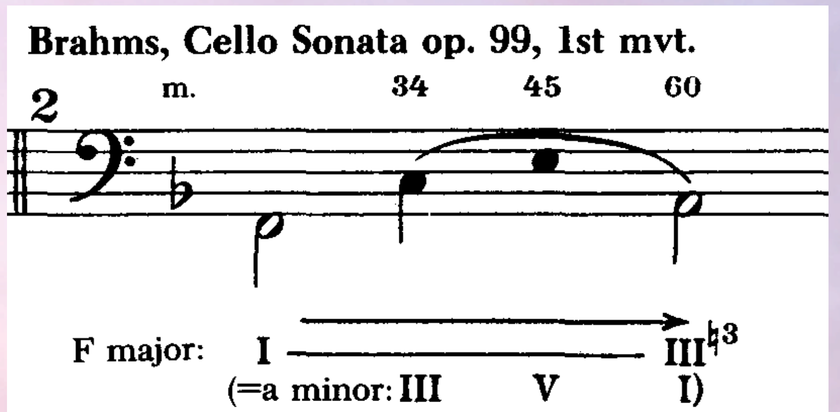


Brahms, F major Cello Sonata, op. 99,

Subordinate Theme

Schenker, Brahms, and Keys

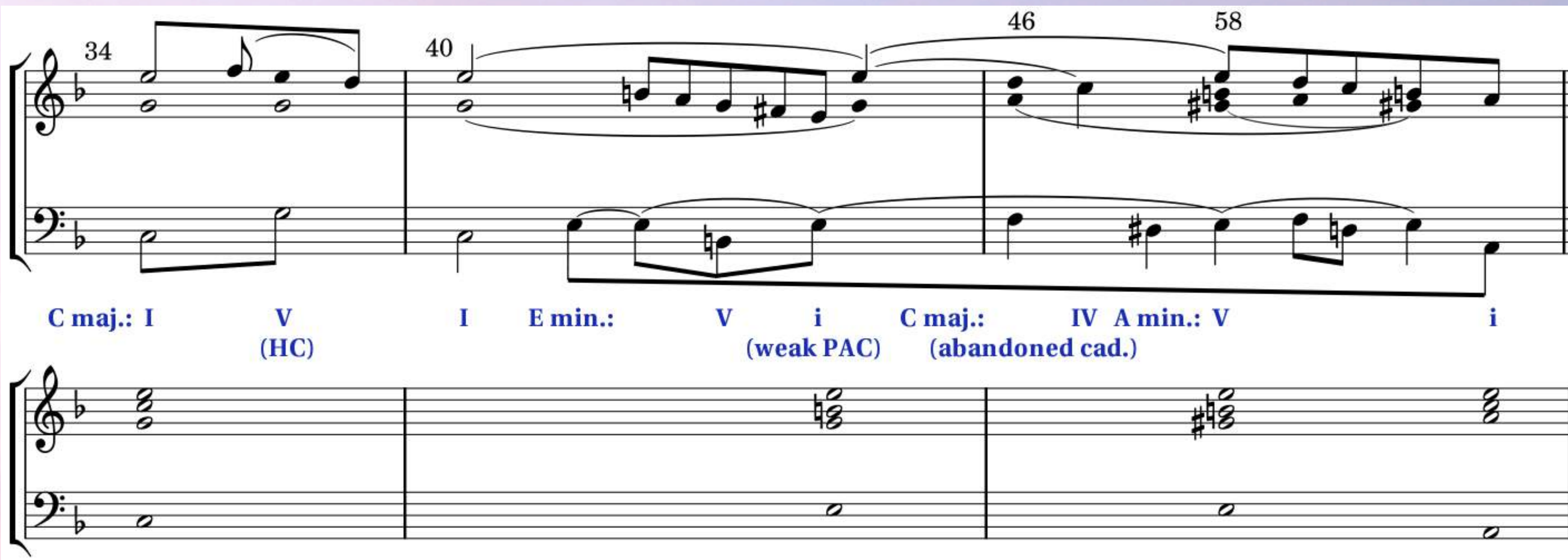
Brahms *Cello Sonata* Op. 99, first movement



Schenker: *Der Freie Satz* Fig. 110d(2)

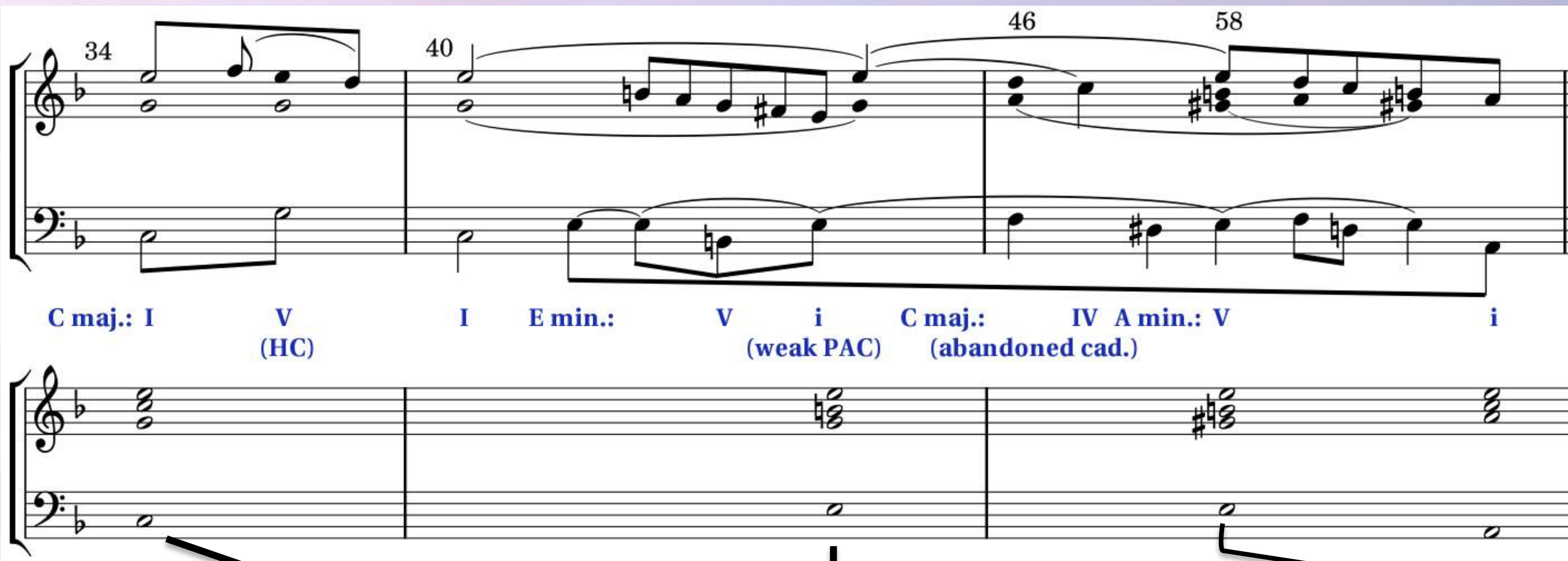

Schenker: C
34
cello
pno
HC
cello
C maj.: I . . .
IV
(I)
V 6 4 – 5 3
40
pno
Schenker: E
C maj.: I . . .
E minor . . .
i6
V7
i
46
C maj.: ii∅6 5
V6 5/V
–
iv6
V4 3/V
–
IV
A min.: Cad6 4
50
cello
A min.: vii°7/V
–
Ger+6
–
vii°7/V
54
pno
Schenker: A
vii°4 3
–
–
Ger+6
Cad6 4
A min.: V7
(VI
ii∅6 5) V6 4 – 5 3
i
C maj.: vii°7/V Cad6 4

Schenker, Brahms, and Keys



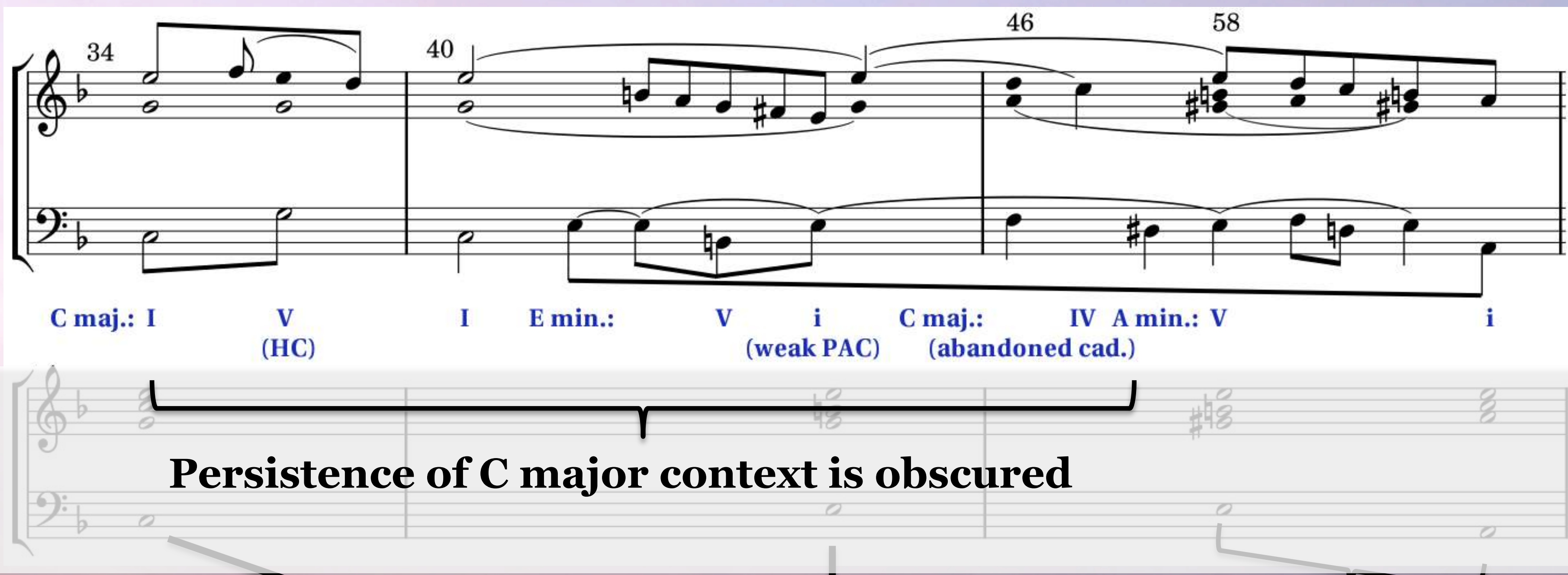
Structure is based on *large-scale voice leadings*,
which must occur between **distinct musical objects (*chords*).**
This leads to a ***reductive*** approach.

Schenker, Brahms, and Keys



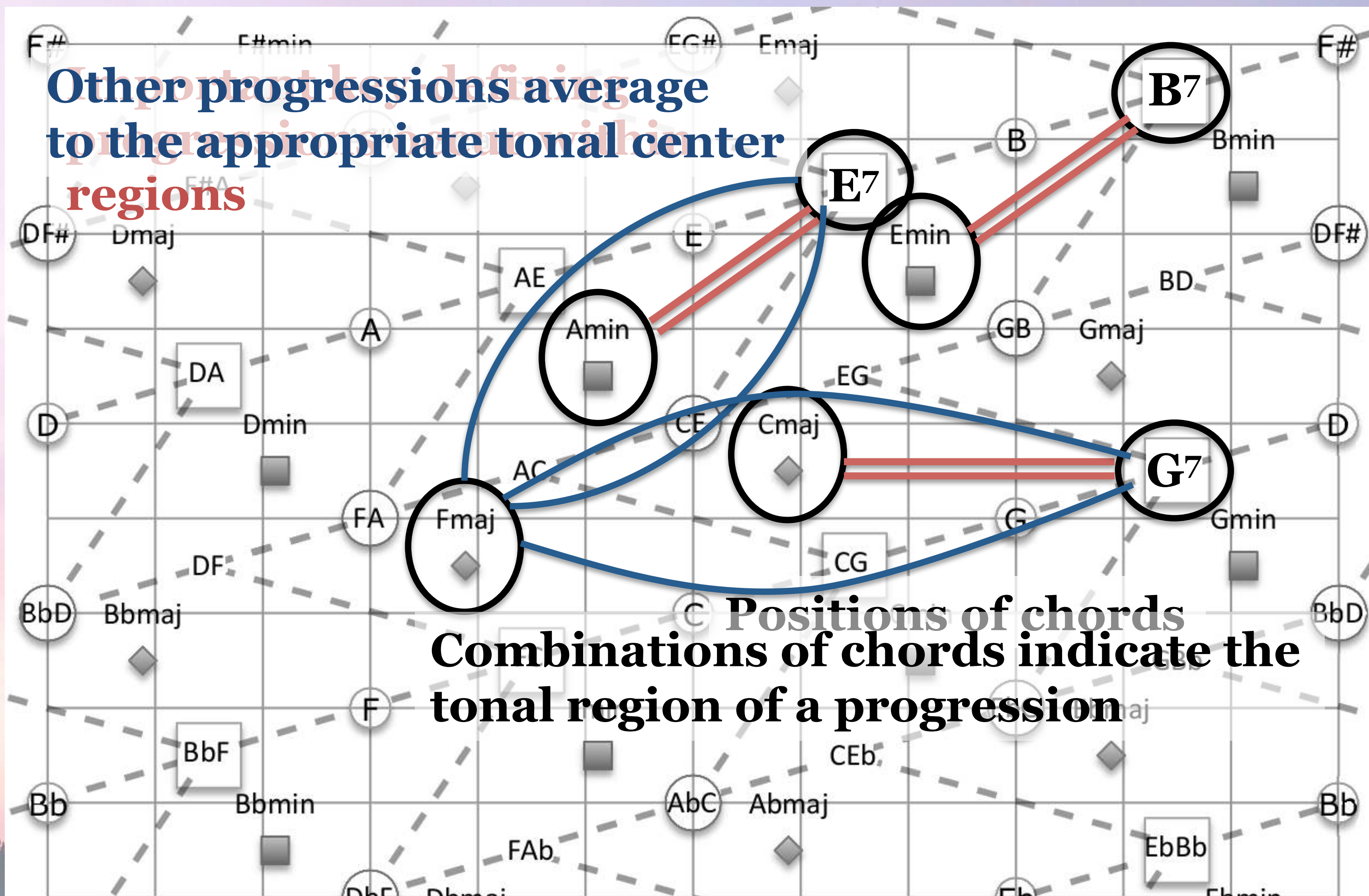
Cadential points must take precedence as structural events

Schenker, Brahms, and Keys

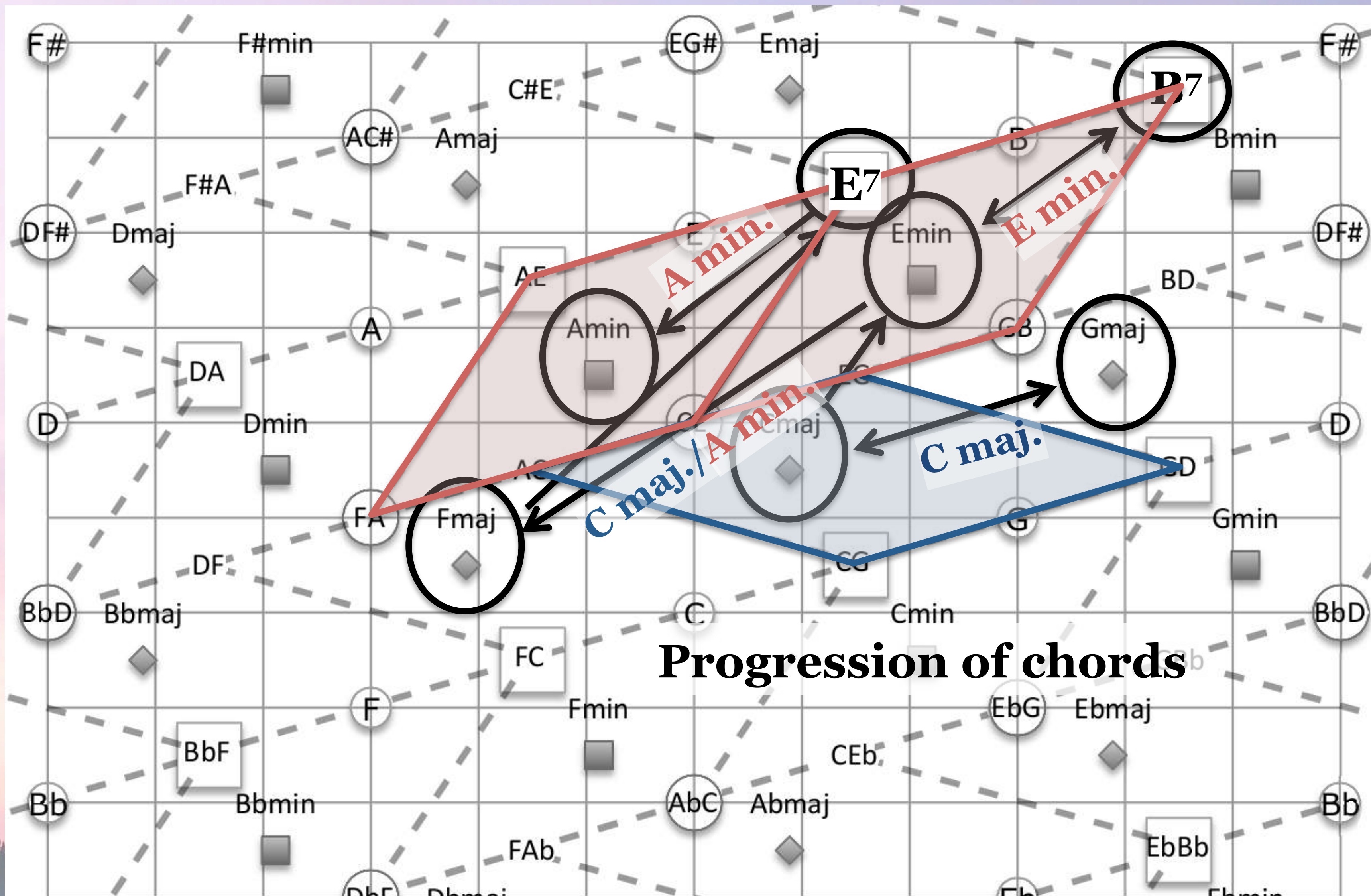


Cadential points must take precedence as structural events

Brahms *Cello Sonata* op. 99

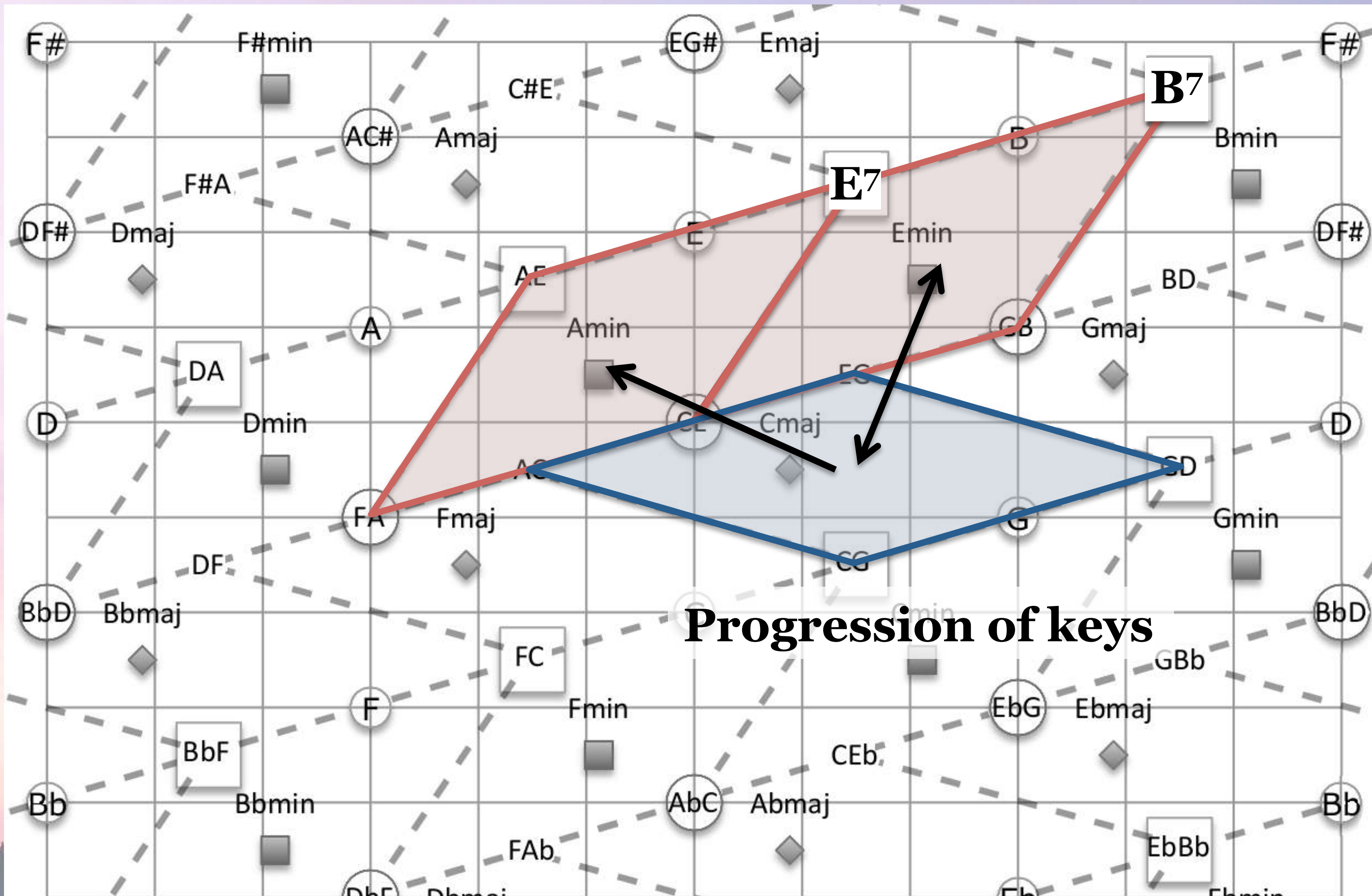


Brahms *Cello Sonata* op. 99



Progression of chords

Brahms *Cello Sonata* op. 99



2. DFT and Triadic Orbits

1. DFT components as sinusoidal approximations
2. Triadic orbits

Discrete Fourier Transform on Pcsets

Lewin, David (1959). "Re: Intervallic Relations between Two Collections of Notes," *JMT* 3/2.

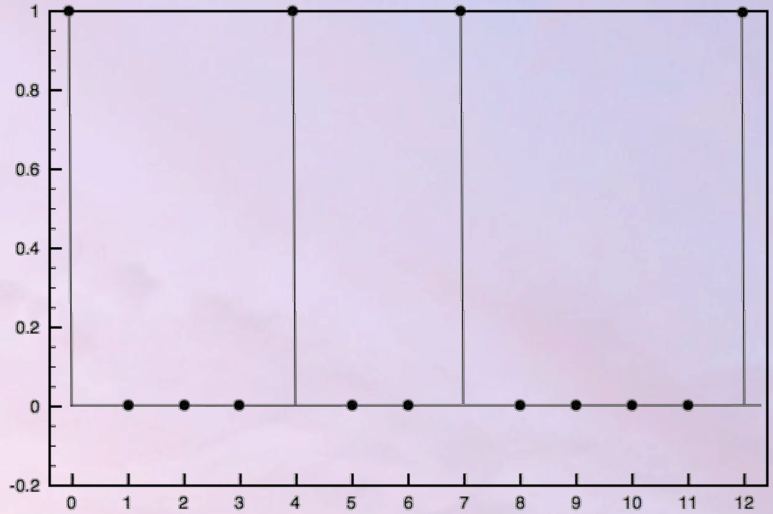
——— (2001). "Special Cases of the Interval Function between Pitch Class Sets X and Y." *JMT* 45/1.

Quinn, Ian (2006–2007). "General Equal-Tempered Harmony," *Perspectives of New Music* 44/2–45/1.

Amiot, Emmanuel (2013). "The Torii of Phases." *Proceedings of the International Conference for Mathematics and Computation in Music, Montreal, 2013* (Springer).

DFT components

C major triad



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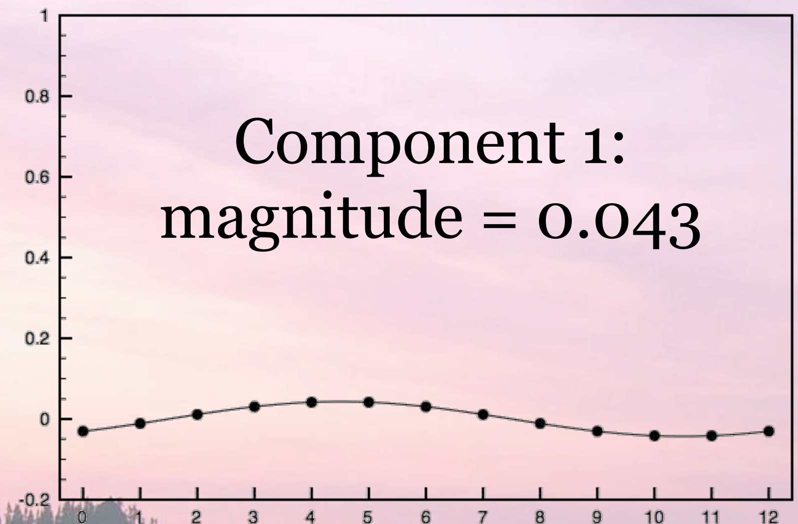

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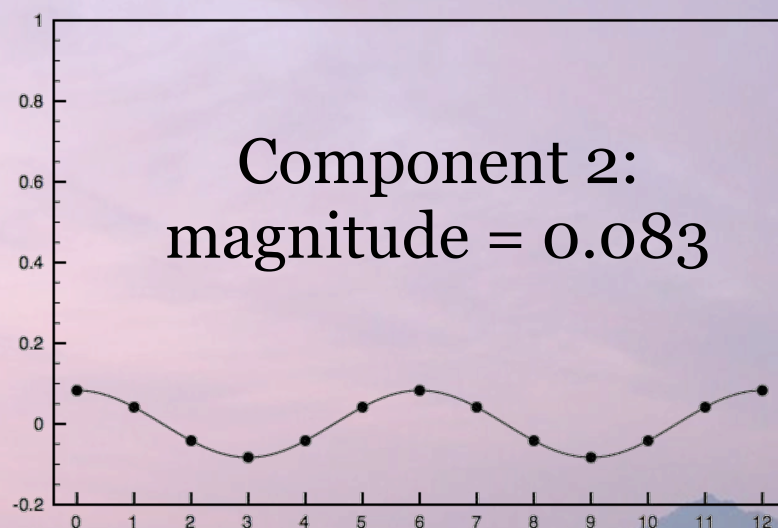

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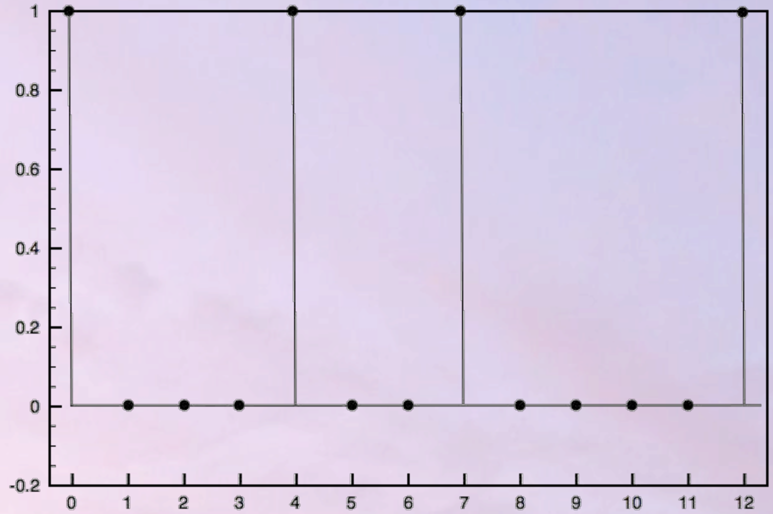
DFT components

C major triad



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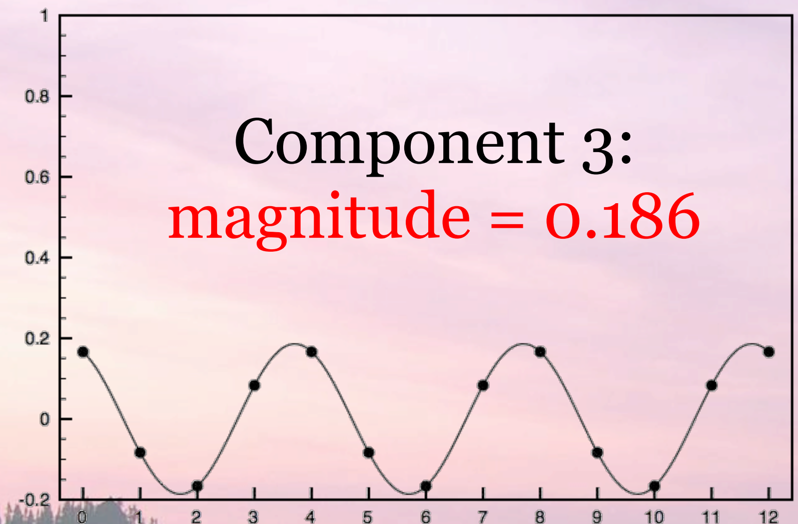

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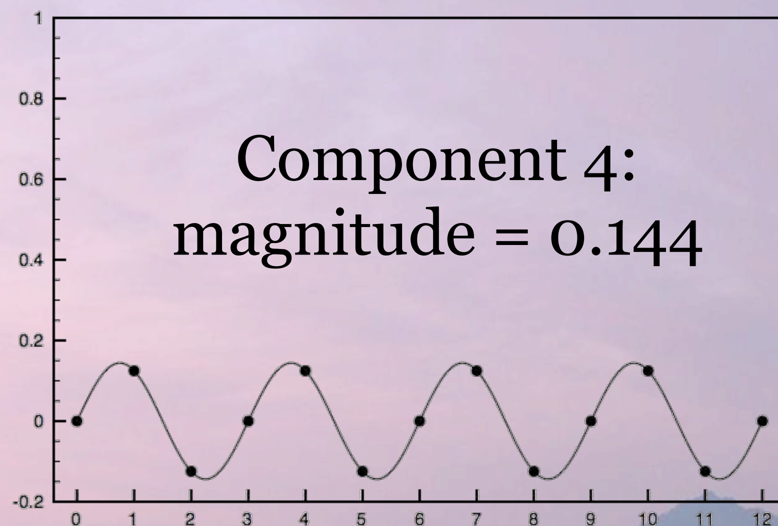

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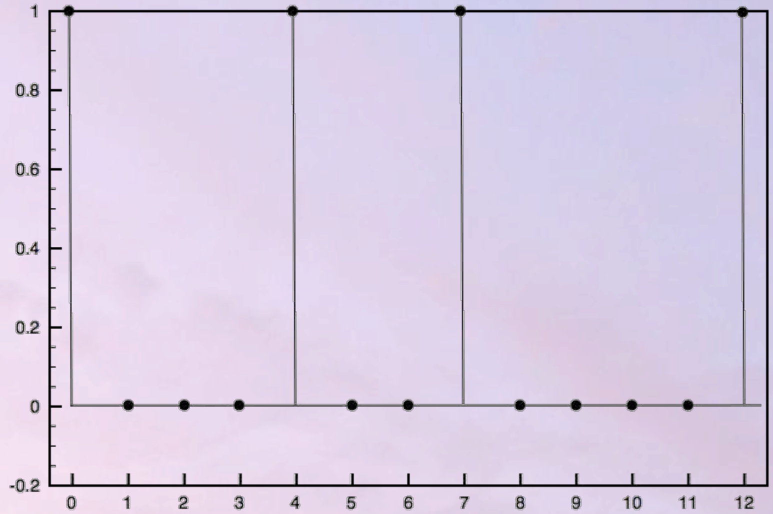
DFT components

C major triad



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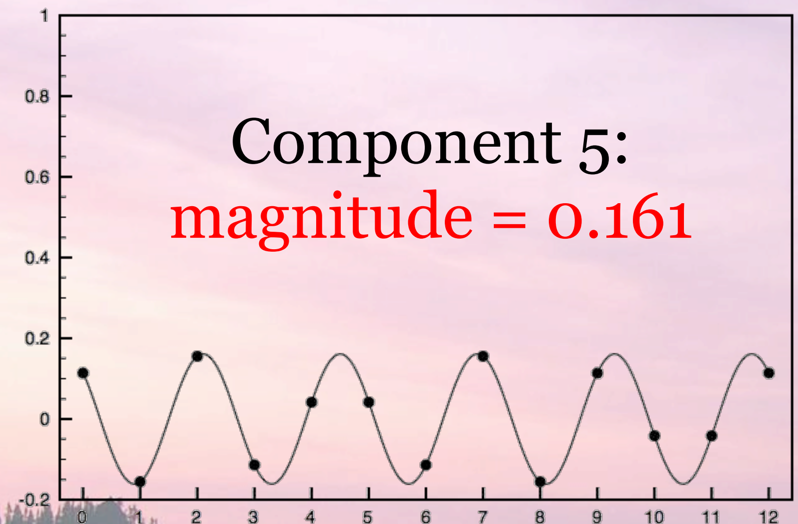

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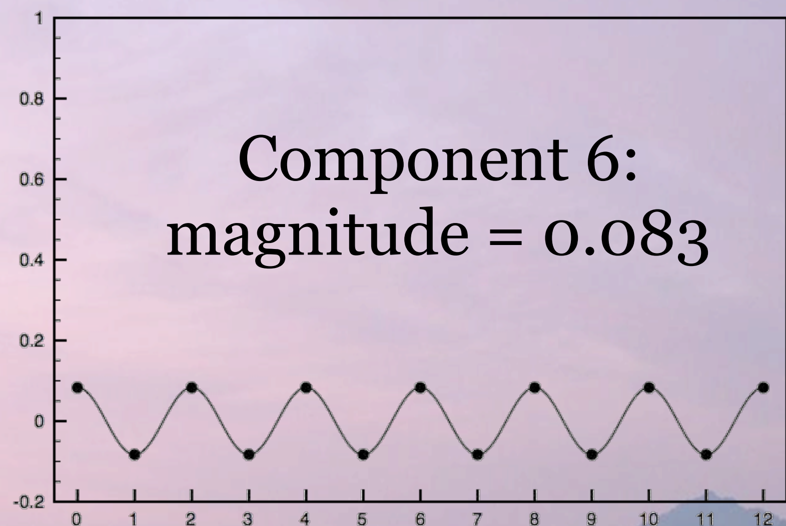

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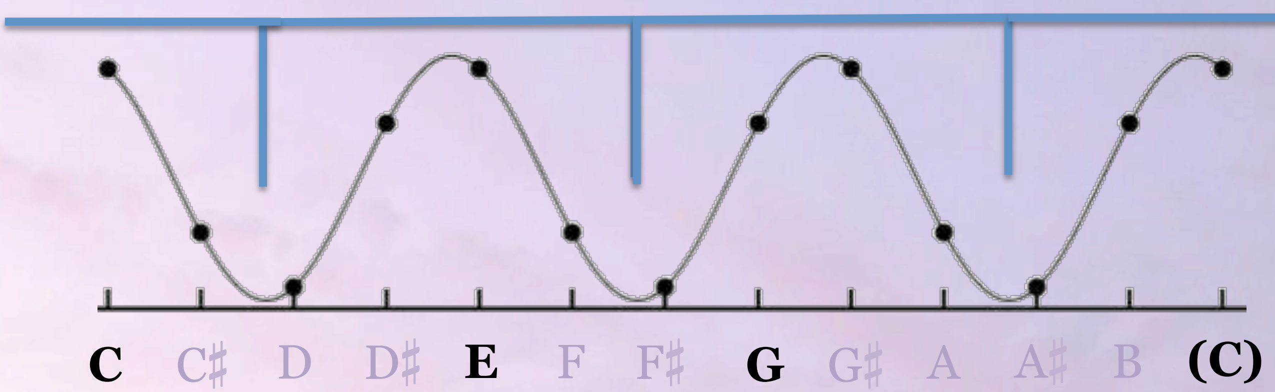
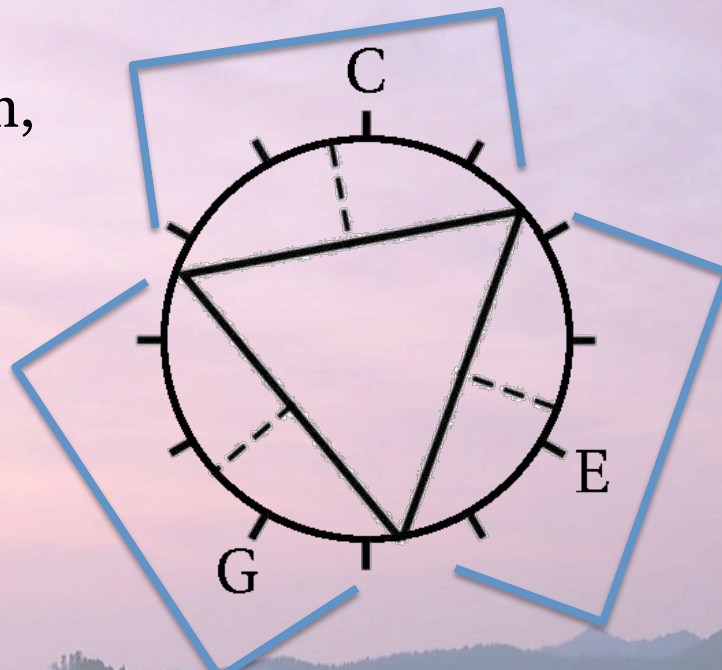
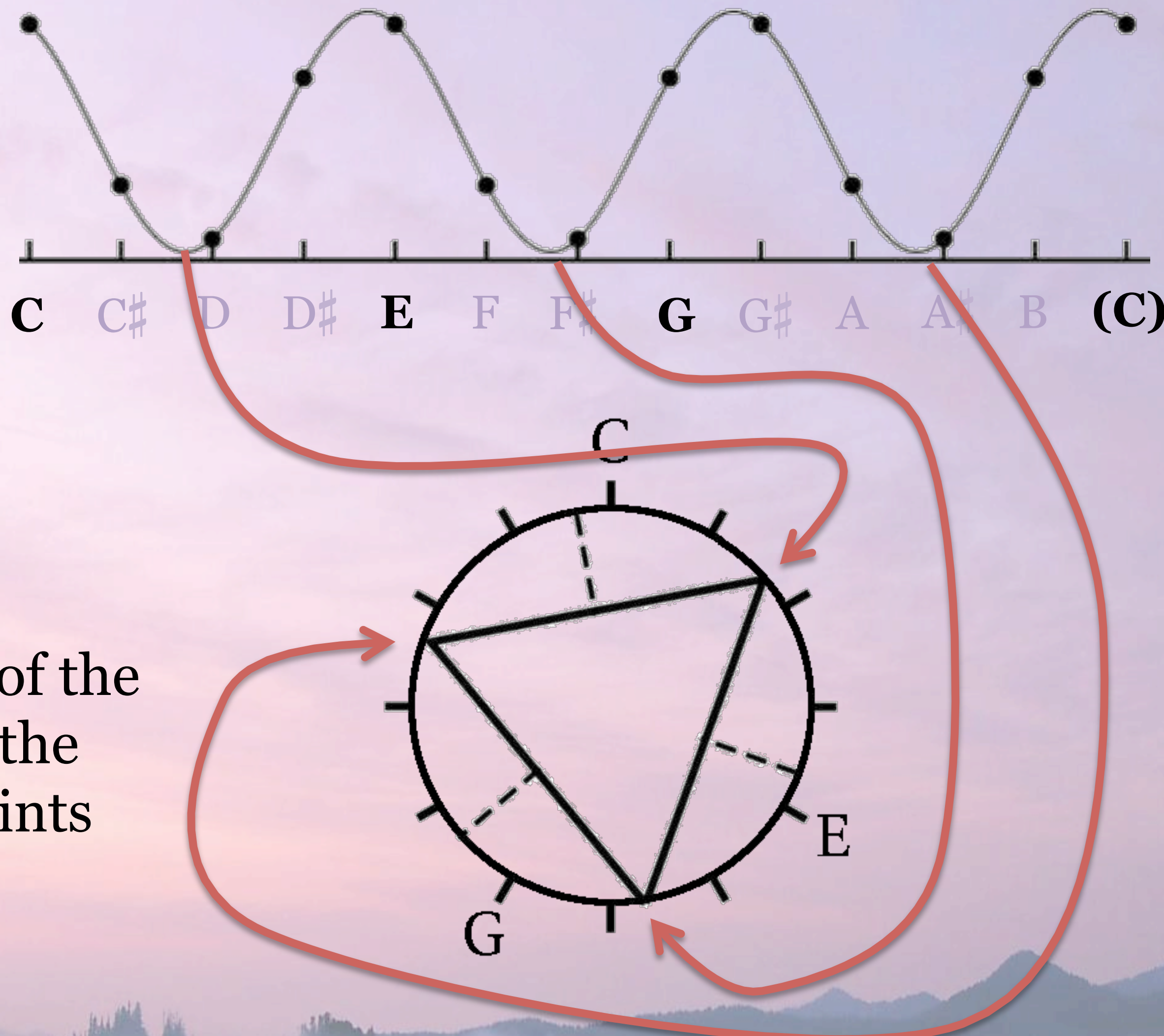
Triadic Orbits



The triadic orbits go from trough to trough, and group pcs that may be considered displacements of those in the triad

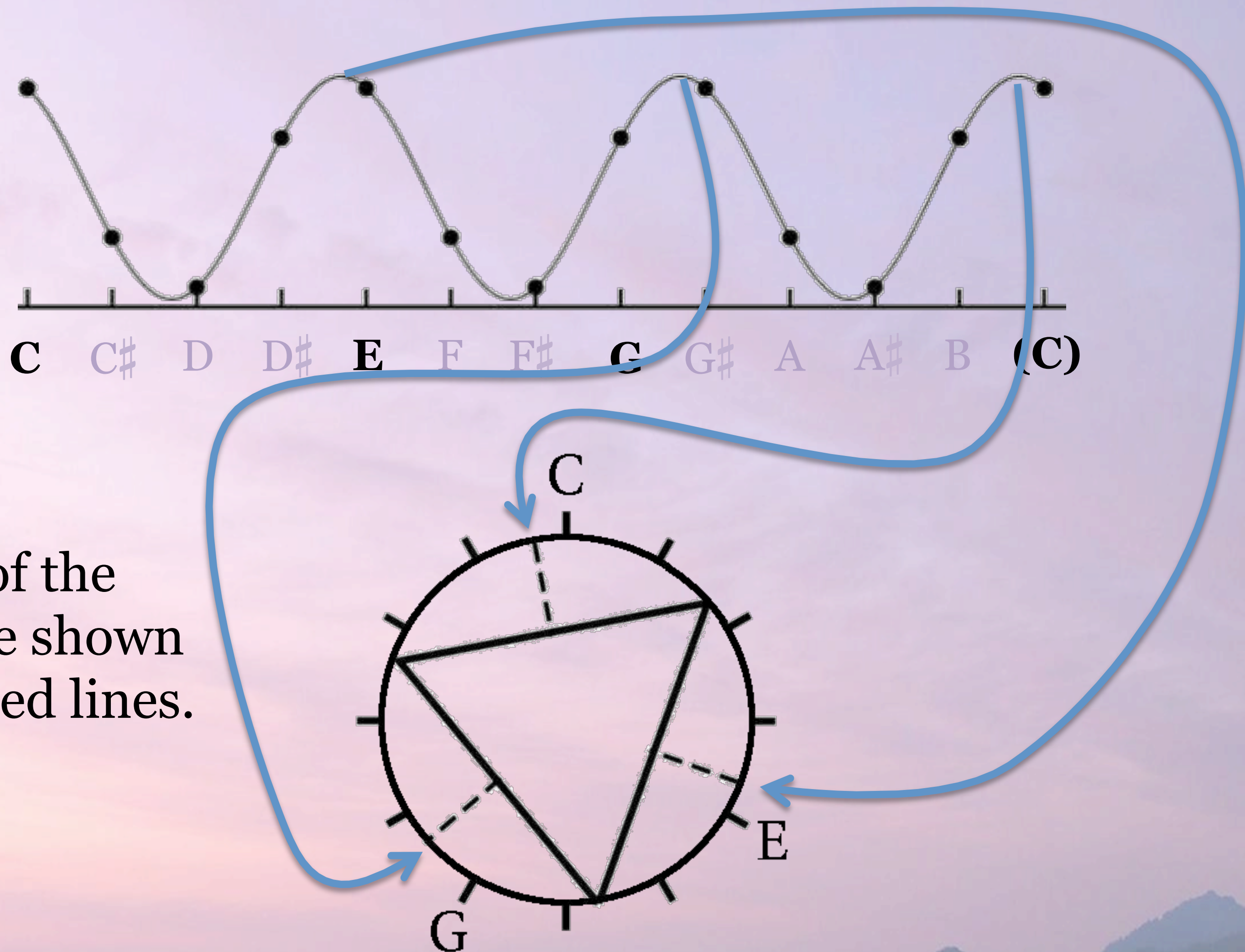


Triadic Orbits



The troughs of the sinusoid are the boundary points

Triadic Orbits

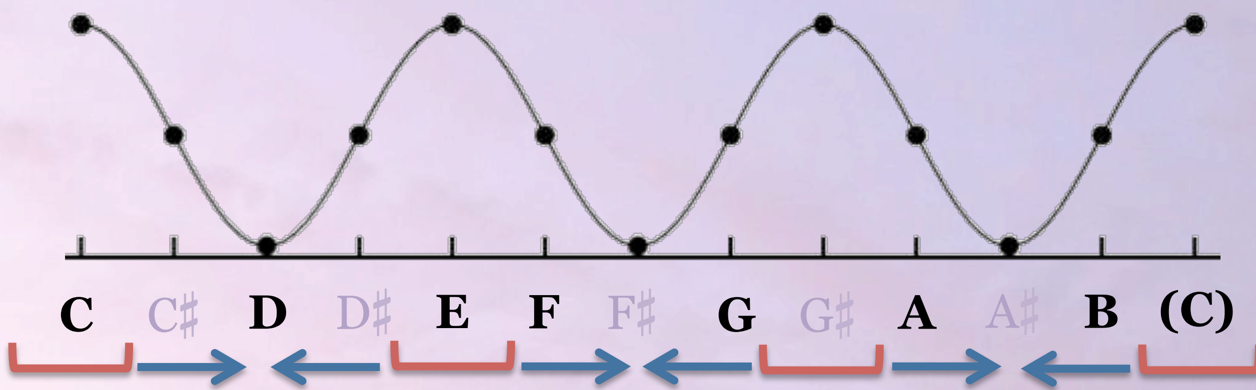


The peaks of the sinusoid are shown by the dashed lines.

Triadic Orbits

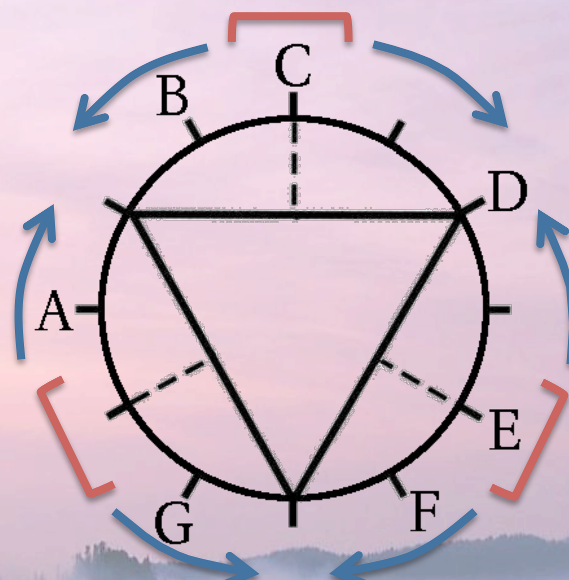
Any pc-distribution can define a set of triadic orbits, including **scales.**

C diatonic:



Regions of stability

Periphery: Unstable upper and lower neighbors



Pitch classes at the center of a region are more stable

Pitch classes at the periphery of regions are unstable displacements

Triadic Orbits

Triadic orbits as a **voice leading** property:

- Clockwise and counter-clockwise movement of triadic orbits corresponds to ascending or descending efficient voice leading between triads.
- The spatial relationship of one harmonic object to its context indicates its voice-leading stability and whether it resolves with upward or downward voice leading in the context.
- The triadic voice-leading properties of the 3rd DFT component apply to sets of *any cardinality*, not just trichords, on the basis of common pc content with consonant triads.

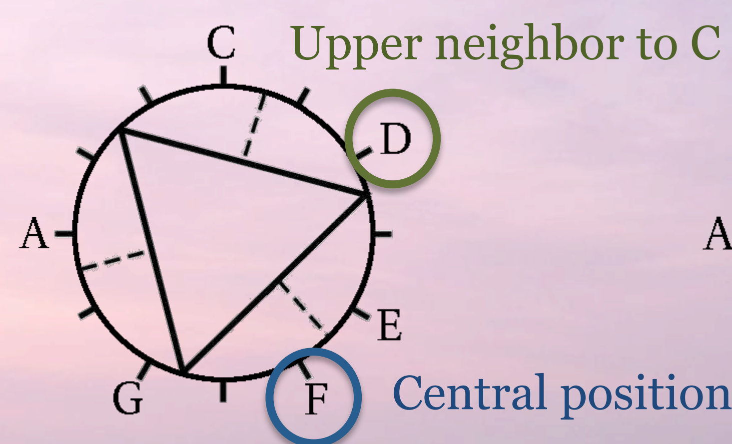
3. *Heiliger Dankgesang*

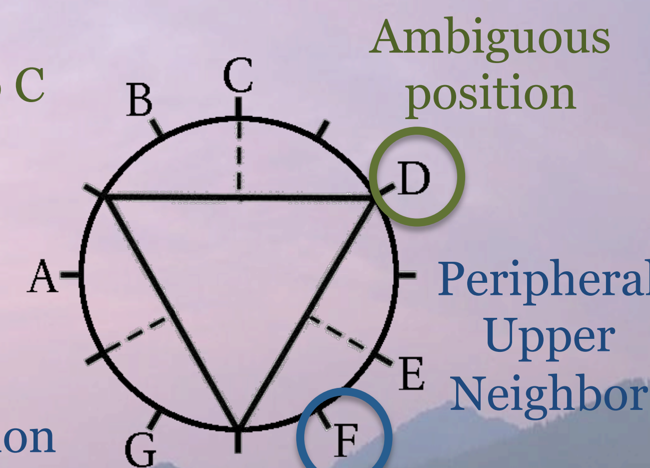
1. Scalar contexts and triadic orbits
2. The D–C motive
3. Strength and weakness

Heiliger Dankgesang: Scalar Contexts

C–A hexachord | **C diatonic**

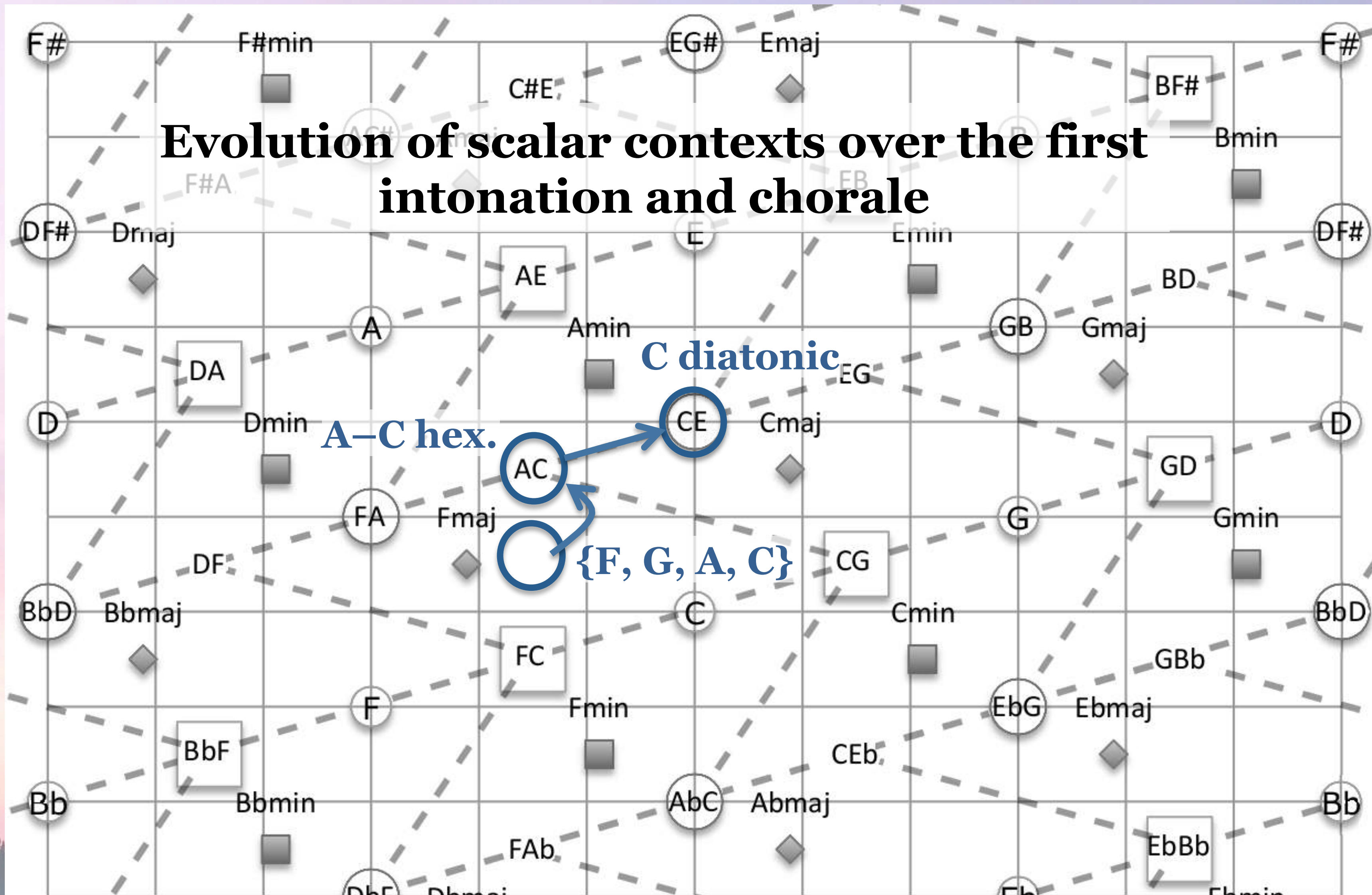
First intonation and chorale





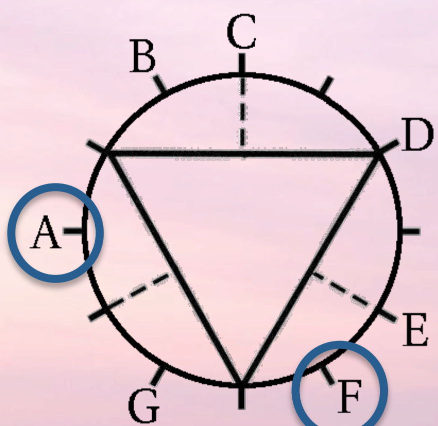
Recording: Muir Quartet, *Ecoclassics*

Heiliger Dankgesang: Scalar Contexts



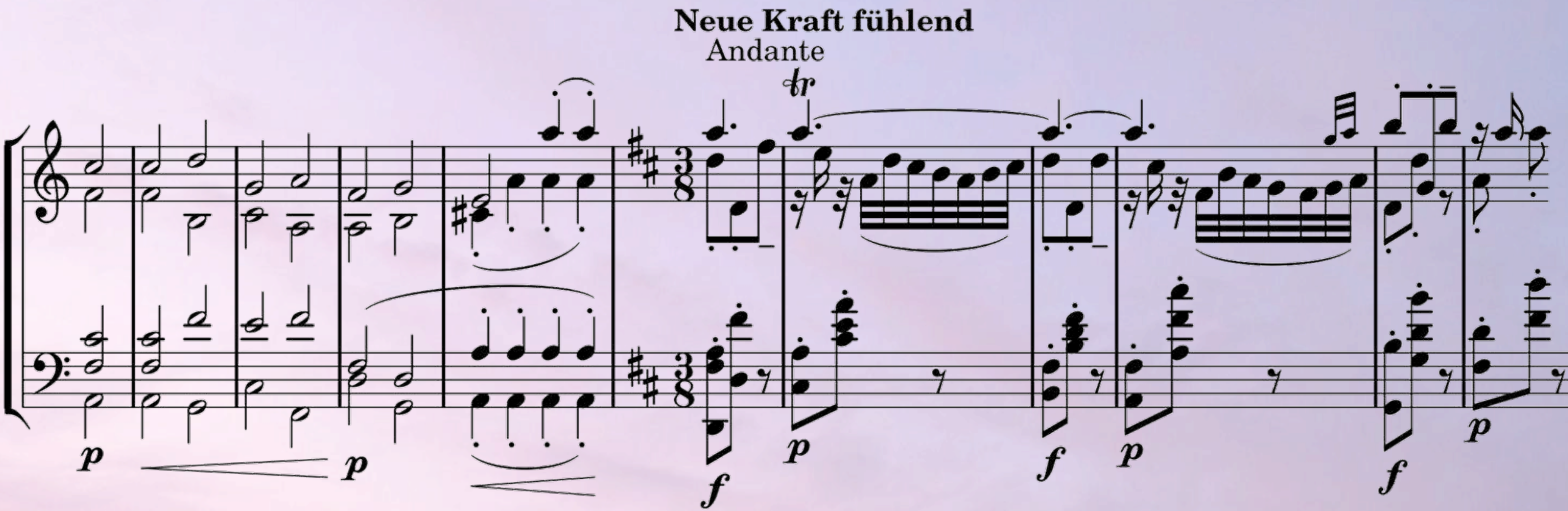
Heiliger Dankgesang: Scalar Contexts

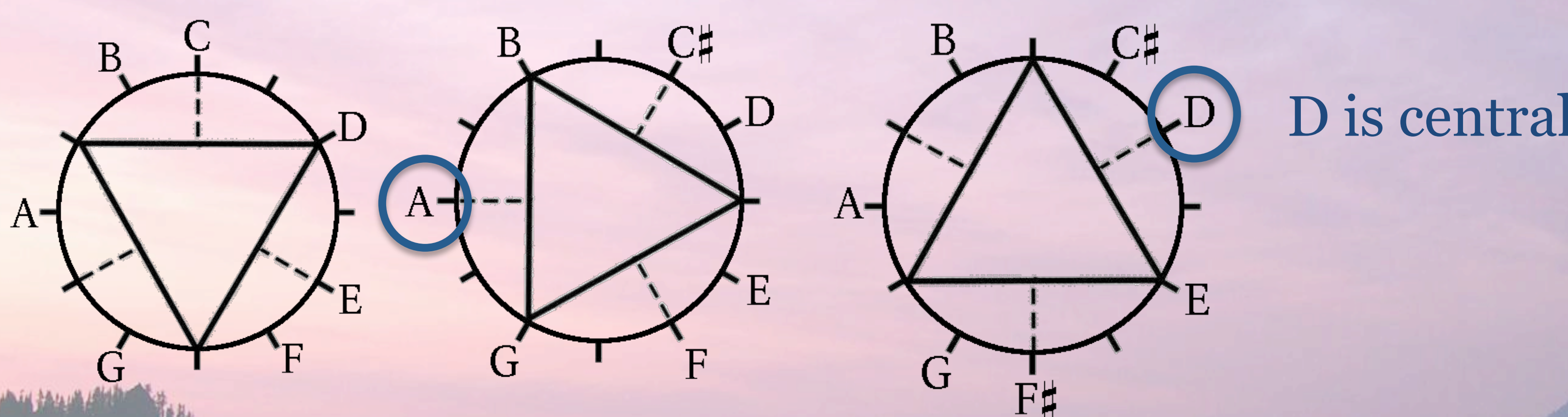
Chorale phrase 4:



Ending high in the orbit gives the effect of suspension

Heiliger Dankgesang: Scalar Contexts



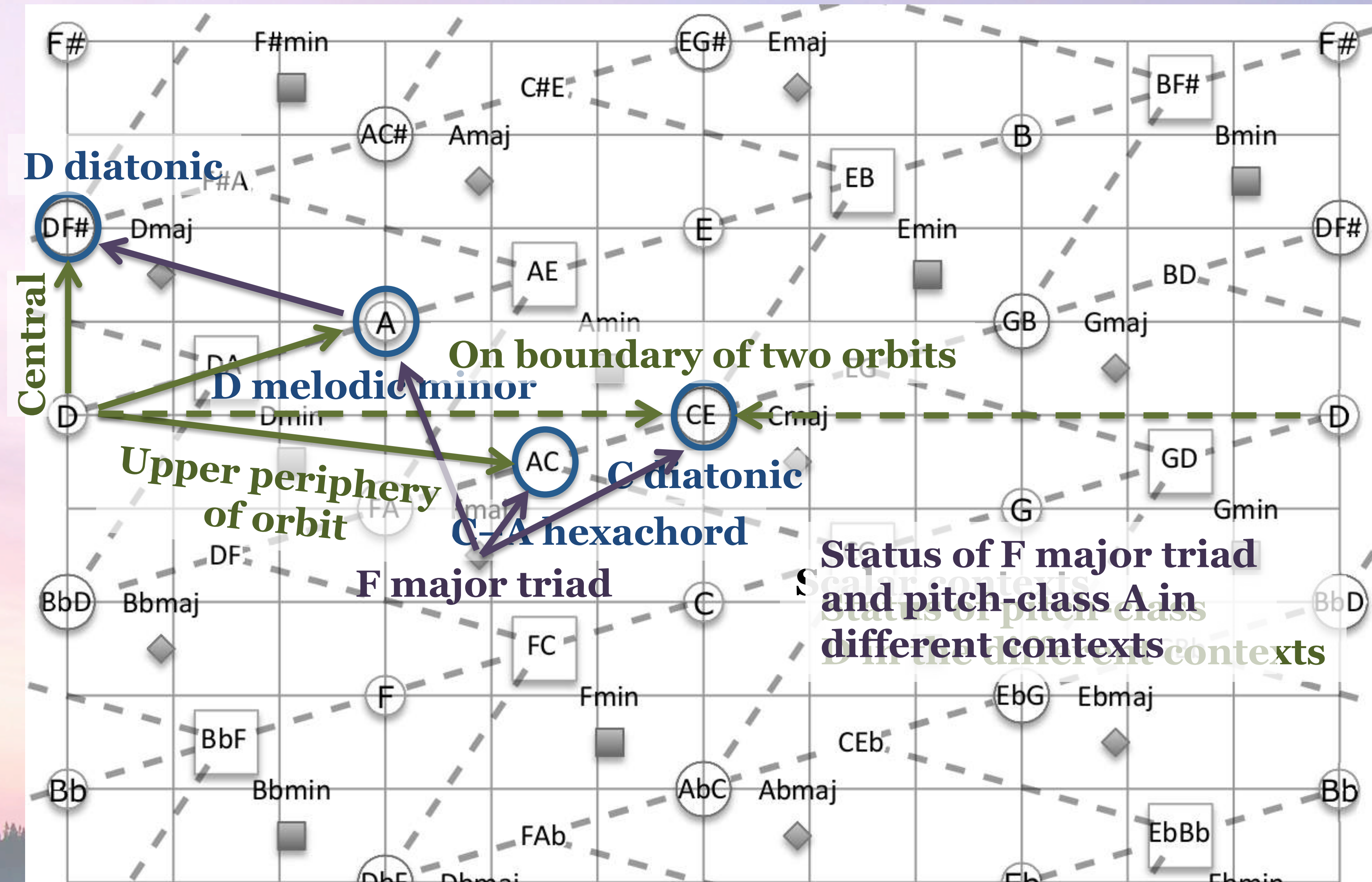


C diatonic

A is central

D diatonic

Heiliger Dankgesang: Scalar Contexts

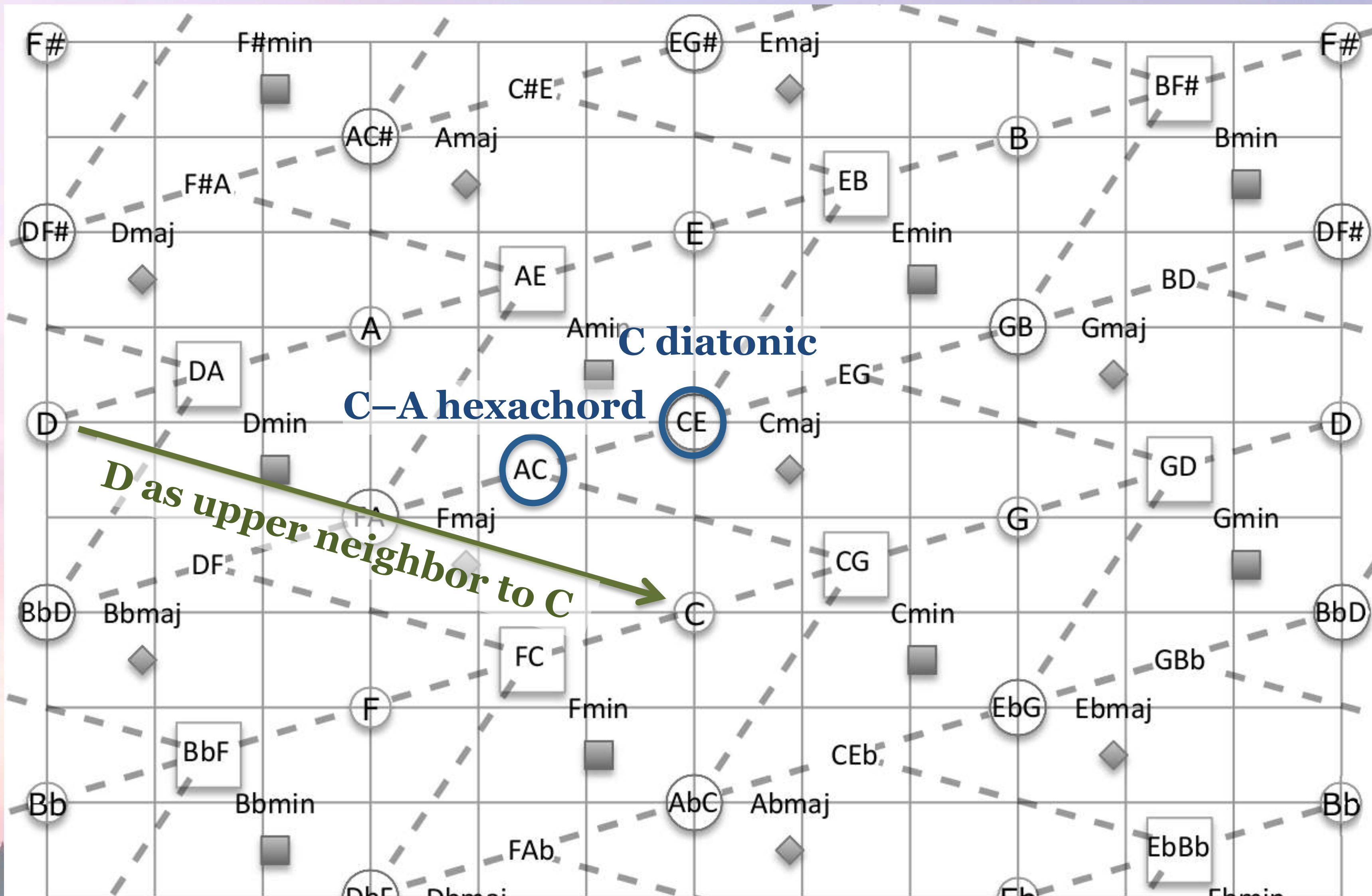


Heiliger Dankgesang: Motivic D–C

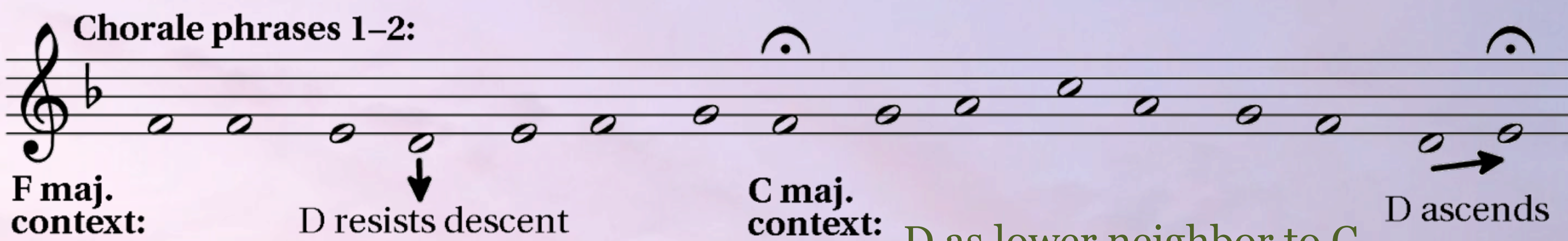
Final form of the intonation:

D–C

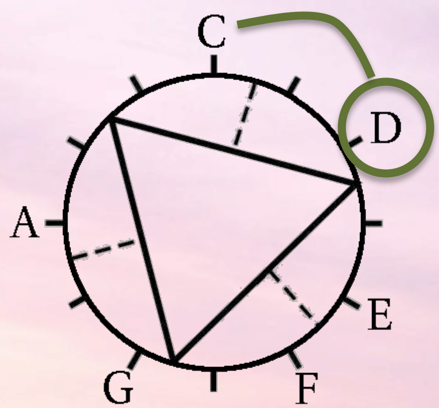
Heiliger Dankgesang: Motivic D–C



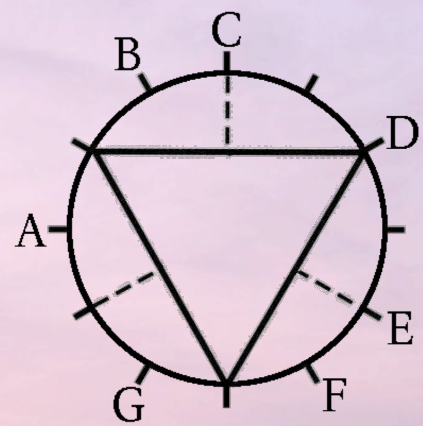
Heiliger Dankgesang: Motivic D–C



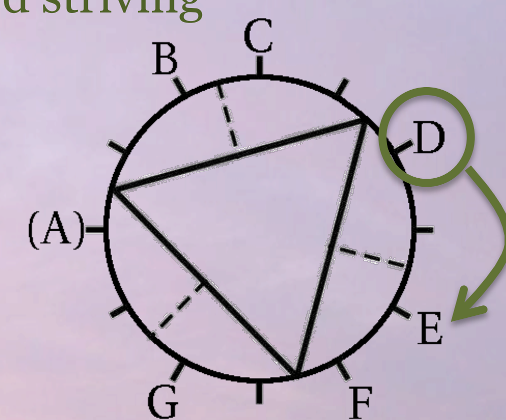
D as lower neighbor to C, upward striving



C–A hexachord



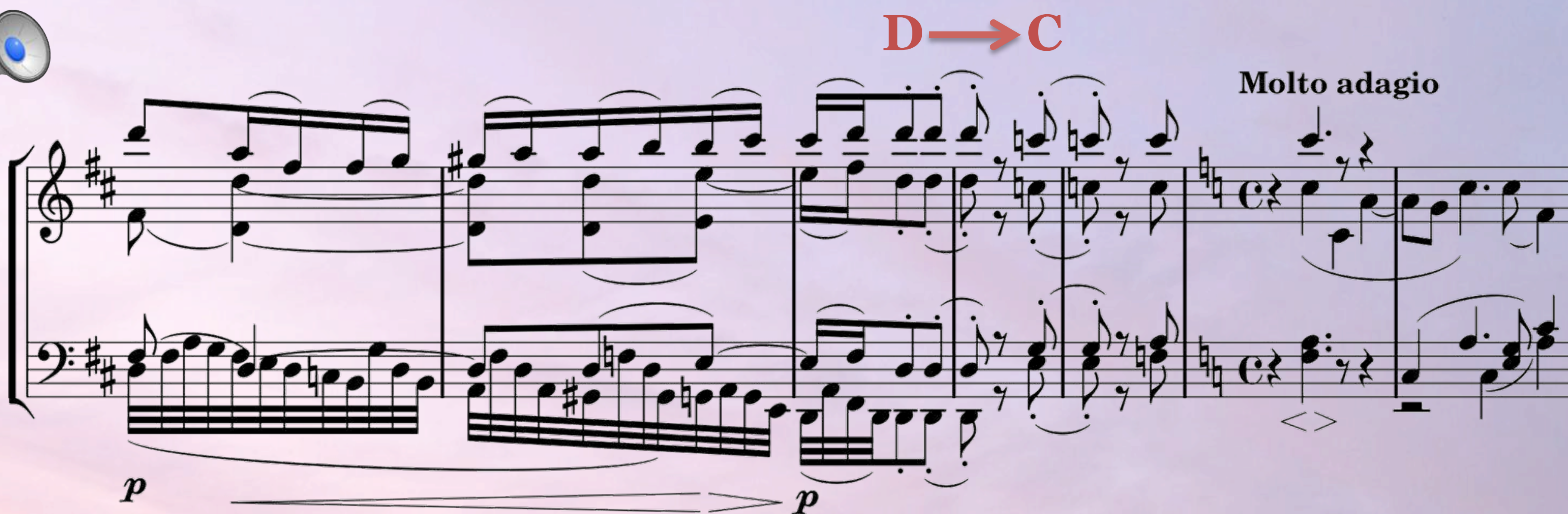
C diatonic

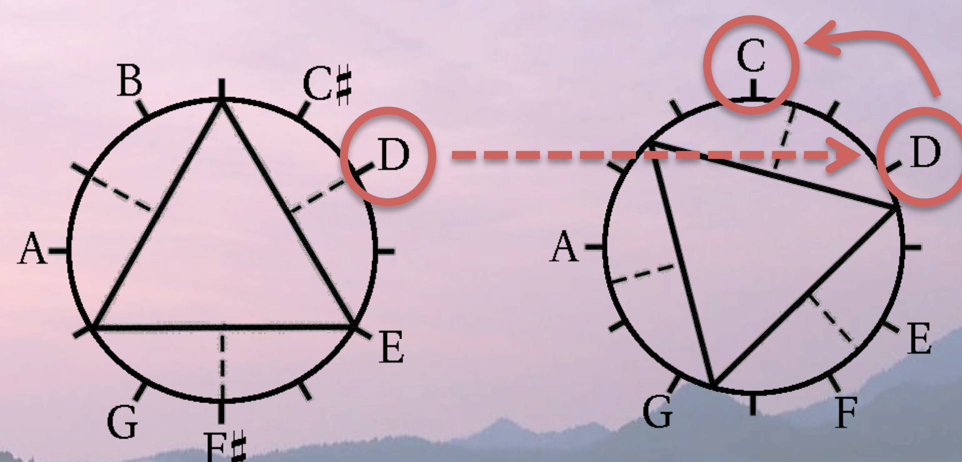


C maj. tonality

Heiliger Dankgesang: Motivic D–C

End of *Neue Kraft* section



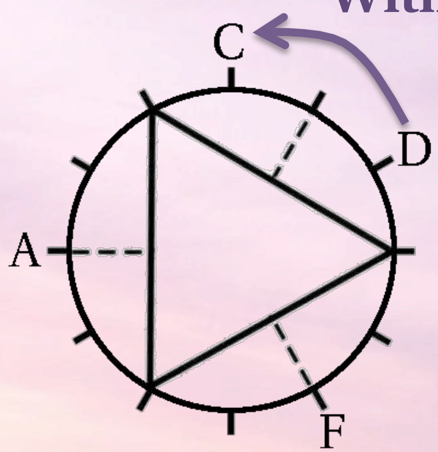


Weakness and Strength

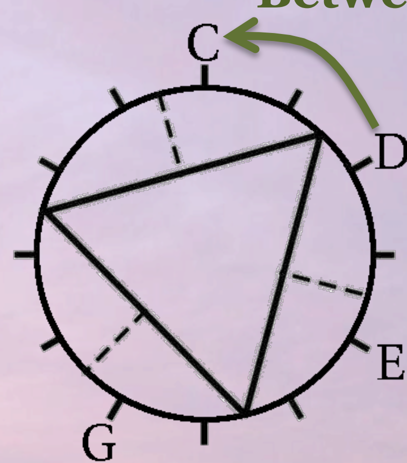
In an F major context, the step C–D is a weak neighbor motion

In a C major context, it is a strong completion of passing motion

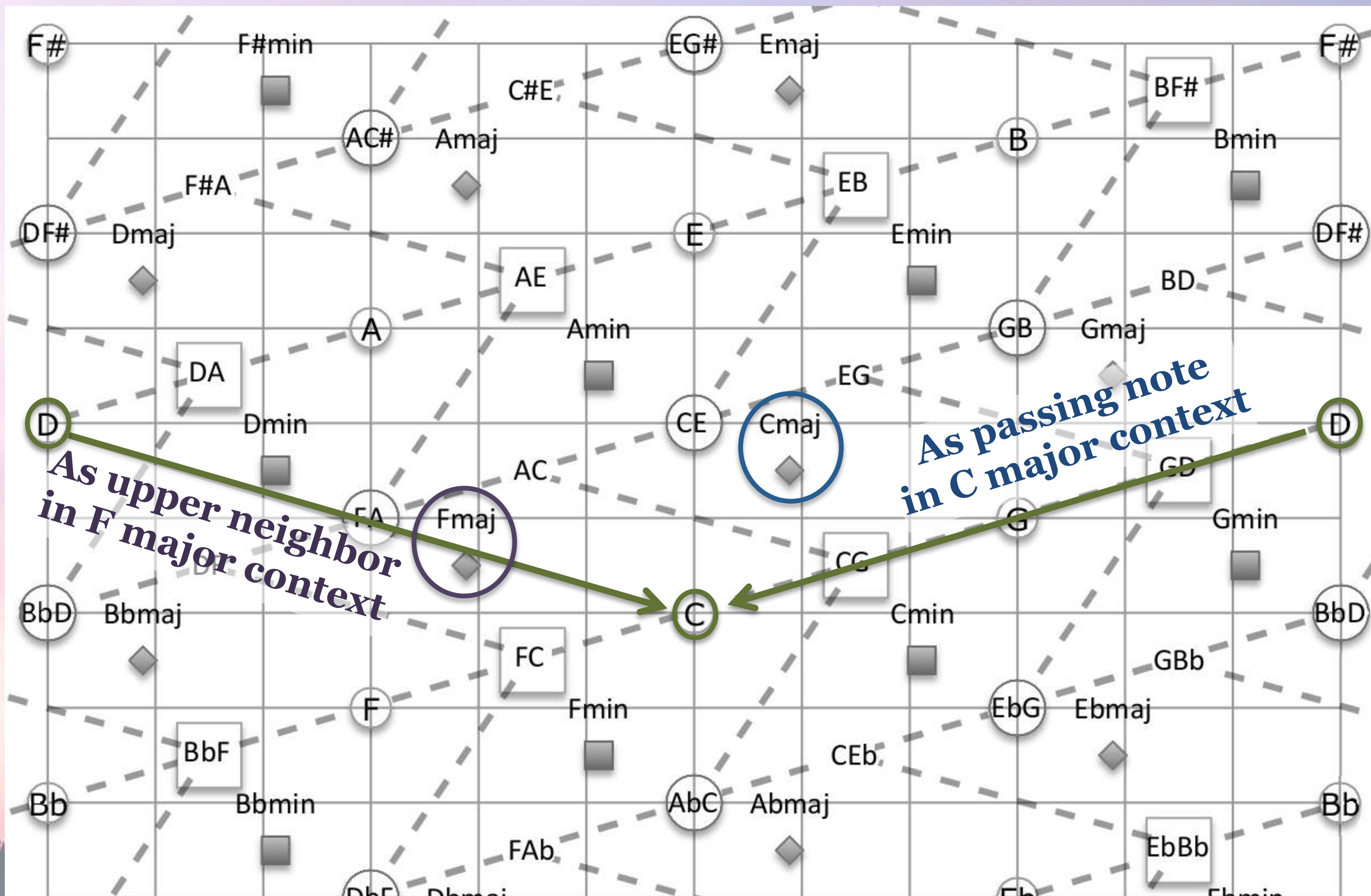
Within orbit



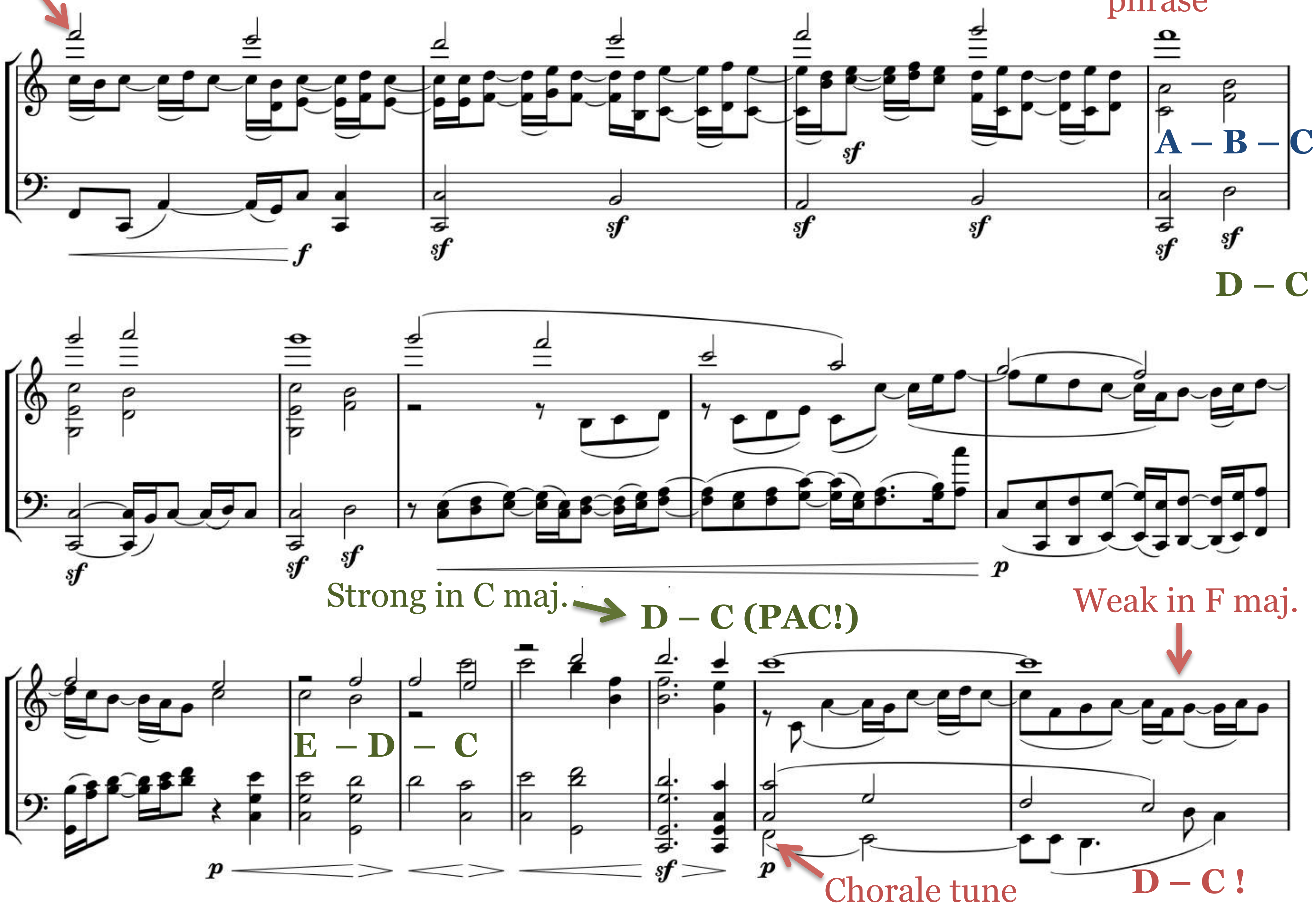
Between orbits



Weakness and Strength



Heiliger Dankgesang: Final Chorale



Conclusions

- DFT phase space effectively reflects tonal process at multiple **levels of structure.**
- It does so through processes of **combination** rather than **reduction**.
- Relating levels through combination better reflects the **traditional notion of keys**.
- Motions in DFT phase space can be construed as a kind of **voice leading** through the idea of **triadic orbits**.
- Triadic orbits also have hermeneutic value in showing the **gravitational forces** that color tones and distinguishing **strong** and **weak** melodic motions.

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A Spatial Perspective on Long-Range Voice Leading and Beethoven's *Heiliger Dankgesang*

Jason Yust, Boston University

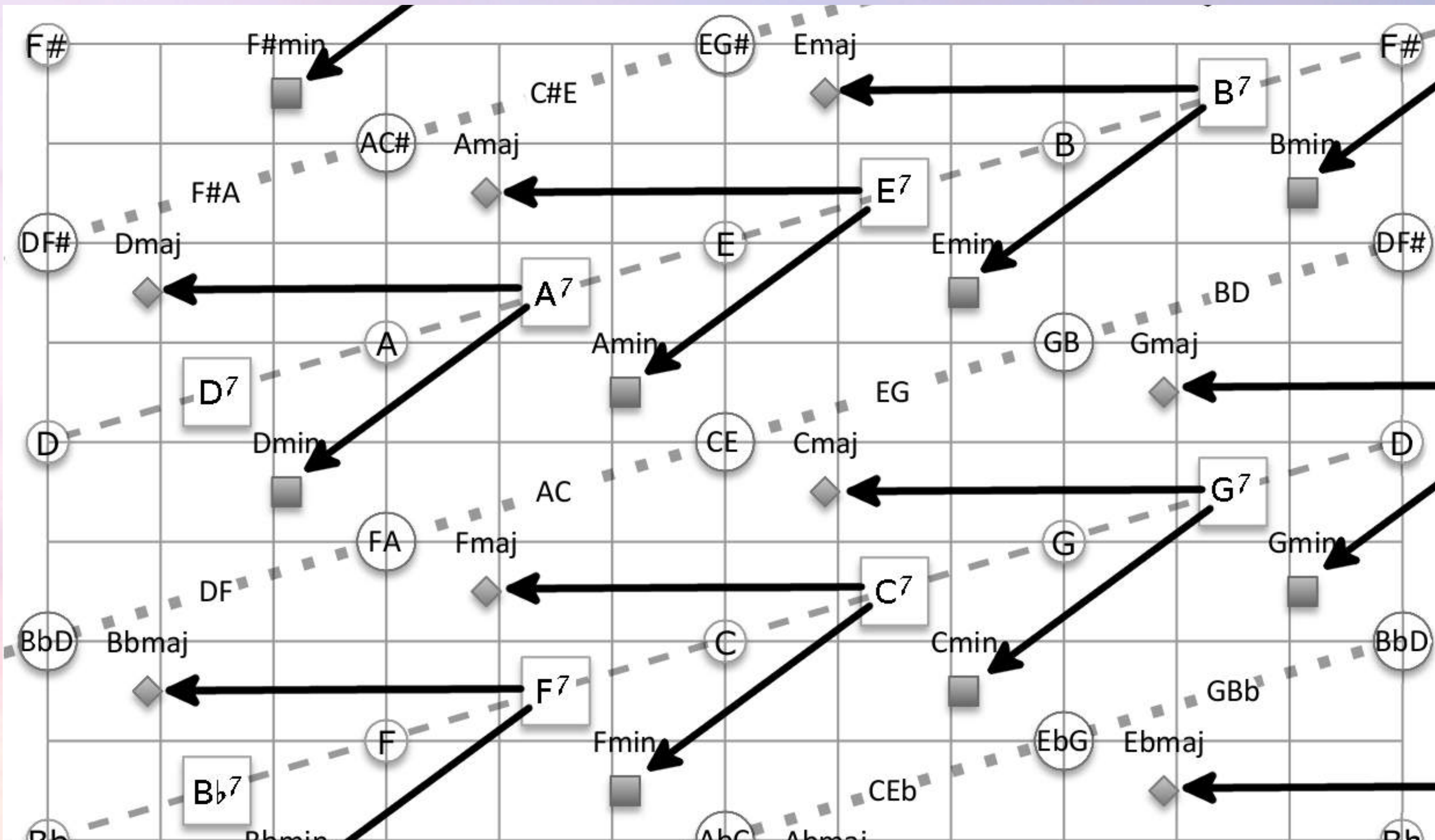
Presentation to the University of Connecticut
Music History/Theory Colloquium
March 6, 2015

A copy of this talk is available at
people.bu.edu/jyust/

Appendices:

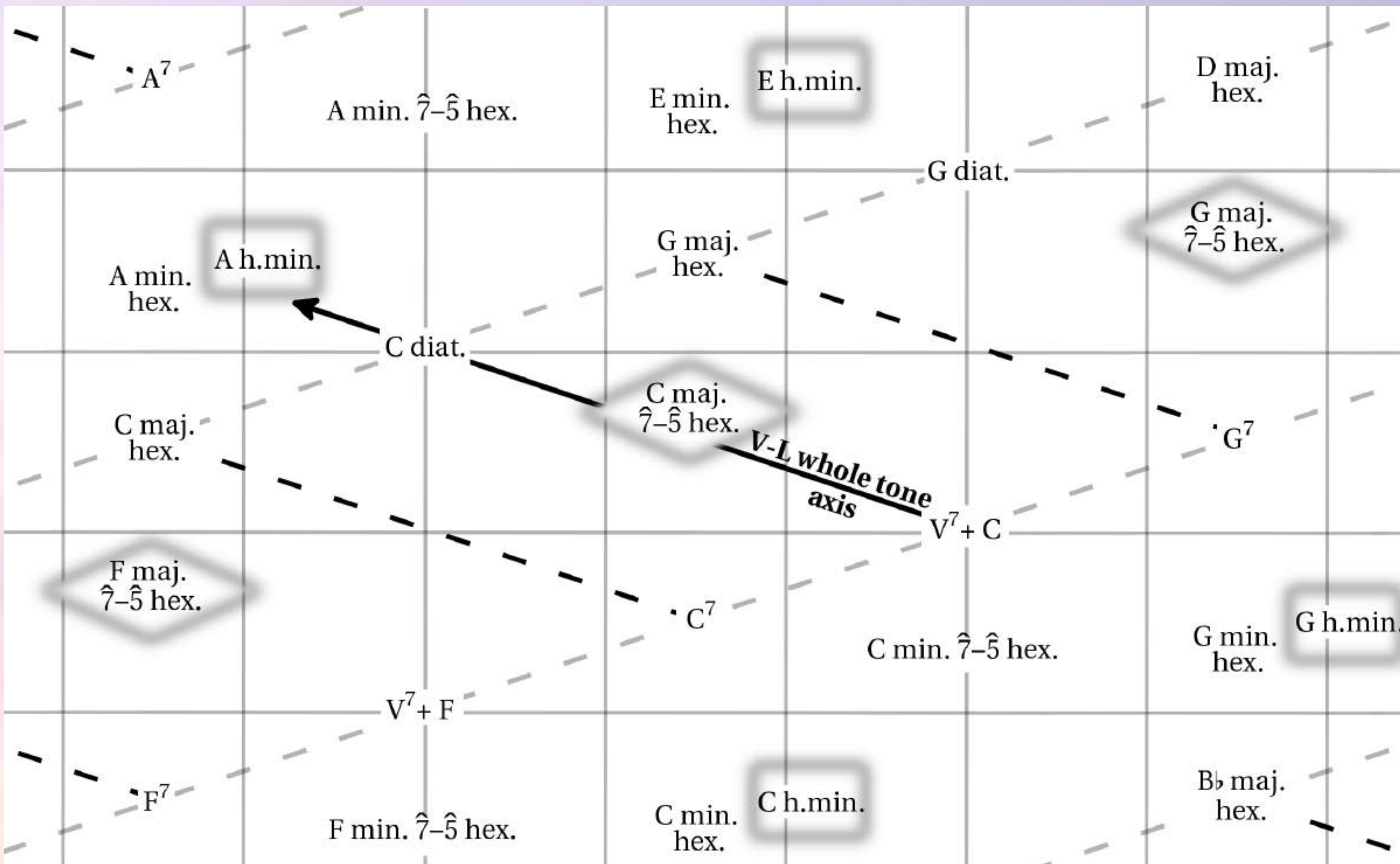
A1: Derivation of tonal regions

Derivation of Tonal Regions



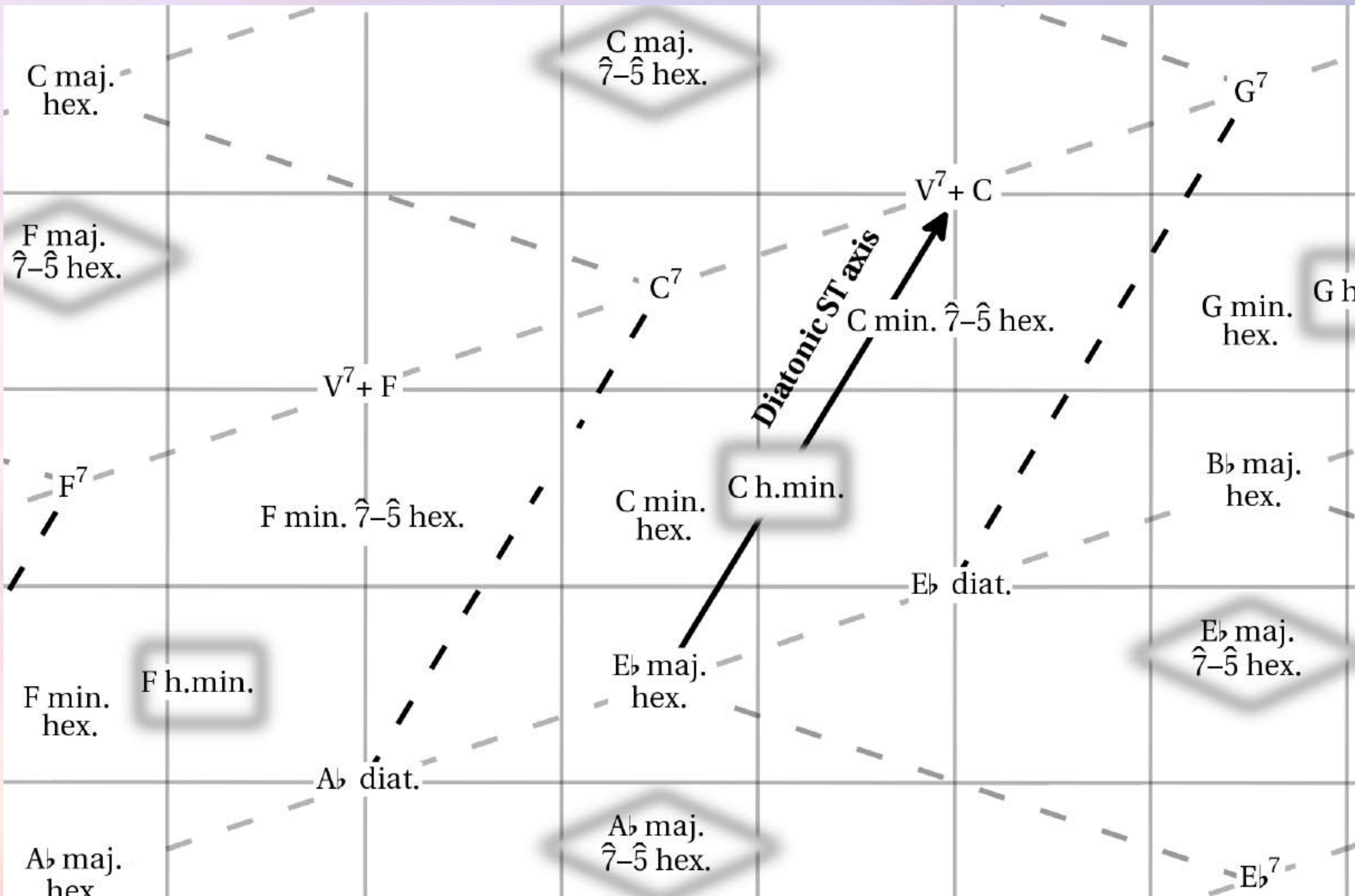
Boundaries between major and minor follow the circle of fifths through diatonic scales and dominant sevenths / individual pcs.

Derivation of Tonal Regions



A characteristic hexachord is at the center of the major regions. Boundaries between fifth-related major regions are parallel to an axis that approximately passes through this hexachord.

Derivation of Tonal Regions



Minor region boundaries are parallel to an axis that approximately passes through the harmonic minor scale