# Stylistic Information in Pitch-Class Distributions

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**Abstract**
This study examines pitch-class distributions in a large body of tonal music from the seventeenth, eighteenth and nineteenth centuries using the DFT on pitch-class sets. The DFT, applied over the pitch-class domain rather than a temporal domain, is able to isolate significant and salient qualities characteristic of tonal pitch-class distributions, such as *diatonicity* and *triadicity*. The data reveal distinct historical trends in tonal distributions, the most significant of these is a marked decrease in diatonicity in the eighteenth and nineteenth centuries. Comparing distributions for beginnings, endings, and whole pieces reveals a strong similarity between beginnings and whole pieces. Endings, by contrast, are more distinct in the properties of their distributions overall, and show some historical trends not shared by beginnings and whole pieces, whose differences do not appear to interact with composer date.

Keywords: Pitch-class distribution, tonality, mode, harmony, corpus analysis, discrete Fourier transform

## 1. INTRODUCTION

Pitch-class distributions are central to much research on tonal perception and expectation. According to theories advanced by Carol Krumhansl and others, listeners internalize an abstract tonal hierarchy associated with major and minor keys expressible as a distribution, a weighting of the pitch classes. These distributions, according to the theory, are observable in various kinds of listener ratings of tonal stability and expectation and in the pitch-class probability distributions that can be derived directly from symbolic corpora of tonal music. Krumhansl and Cuddy (2010) survey the existing evidence for this theory.

Early work in statistical musicology (Meyer 1957, Youngblood 1958) established the conceptual link between style and the probability of occurrence of tones and harmonies, with pitch-class distributions representing the zeroeth order. First-order transition probabilities of chords have remained a topic of active interest, though only occasionally enquiring into how these norms have changed over time. Notable here is the work of White (2013, 2014) who shows that conventional historical style-period divisions are largely inferable from chord transition probabilities, but also that aspects of harmonic function—tonic and dominant as high-frequency functional pillars—and the basic separation between major and minor mode harmonies are consistent across the 1650–1900 historical window. Quinn and Mavromatis (2011) show a strong shift towards "prograde" (authentic as opposed to plagal) harmonic motion from 17th-century chorale repertoire to Bach's.

More directly pertinent to the present study are recent analyses of historical change in pitch-class distributions. Albrecht and Huron (2014) use a data-driven method to challenge an implicit assumption of Krumhansl's theory that the tonal distributions defining major and minor keys are historically stable, and to investigate the important musicological question of how and when European music transitioned from a modal to a tonal system. Their results show a high degree of stability in the major/minor distinction and in the major and minor distributions themselves from 1600–1750, and some instability and evidence of two minor modes prior to 1600 (a period not well represented in the dataset analyzed here). Tompkins' (2017) cluster analyses of 17th and 18th century corpora are consistent with these results, but also underline the importance of genre, with secular repertoires and in particular more popular guitar-based repertoires in the 17th century showing much earlier evidence of a major/minor tonal system.

The study presented here examines the possibility that historical changes in pitch-class distributions can be better detected by using a technique, discrete Fourier transform (DFT) on pitch-class sets, developed by theorists to extract diatonicity and other tonally-relevant information from pitch-class distributions. Albrecht and Huron's pitch-class profiles, like other major/minor key profiles, are largely reflective of a basic diatonic scale and secondarily of a tonic triad. The DFT is a lossless transformation that can separate these aspects of the profile from others, and possibly reveal trends in other aspects of the profile that otherwise would be overwhelmed by trends in diatonicity.

Another question actively debated in recent literature on key-finding and pitch-class distributions is whether the key of a passage is better reflected by sampling just the beginnings and endings (as asserted by Albrecht and Shanahan [2013] and Albrecht and Huron [2014]) or by entire pieces (as assumed, e.g., by Prince and Schmuckler [2014]). The following study investigates this question by considering differences between profiles for beginnings, endings, and entire pieces.

The DFT method used here applies to symbolic data and is not related to the use of DFT on audio waveforms, except in the underlying mathematics. The "signal" space is the pitch-class circle, which is periodic at the octave, and the "signal" consists of the probability or weight assigned to each pitch class in the distribution. The DFT transforms the distribution into a set of twelve complex numbers that indicate the presence of periodic intervallic patterns in the distribution. This method, applied to pitch-class sets, was first proposed by Lewin (1959, 2001) as a way of understanding intervallic relationships between harmonies in non-tonal music, and subsequently promoted by Quinn (2006–7) as an index of pitch-class set similarity. The application to pitch-class distributions was independently discovered by Krumhansl (1990), who used it to plot distributions in a tonal space, and is also used by Cuddy and Badertscher (1987). Other authors have touted its efficacy for detecting harmonic change in musical audio (Harte et al. 2006), rating consonance (Bernardes et al. 2016), and as a framework for harmonic analysis of tonal and non-tonal repertoires (Amiot 2013, 2016; Yust 2015b, 2016).

Most relevant to the present work, Yust (2017) analyzes a wide range of data from previously published corpus and perceptual studies. Yust's analyses show that tonal distributions are quite consistent when viewed through the DFT, and that three DFT components (the second, third, and fifth) account for the vast share of information in distributions from tonal music. These results recommend the method for the present study, since they imply that the DFT can simplify the information in pitch-class distributions by sorting out those few parameters of apparent significance to tonality. While a data-driven method like PCA could be used to achieve a similar simplification, the DFT coefficients have established theoretical meaning, as reflected in the existing literature applying them to concepts of harmonic analysis. Furthermore, the DFT is a lossless transformation, so the observations of Yust (2017) concerning the role of the second, third, and fifth Fourier coefficients can be replicated and further validated on a different dataset by comparing the following multiple regressions on all components.

## 2. METHOD

### 2.1 Dataset

The dataset for this study is taken from the Yale Classical Archives (YCA: http://ycac.yale.edu/). The dataset and the method used to build it are described by White and Quinn (2016). Thirty-three composers were chosen spanning an approximately 350-year period. The criteria for inclusion was the presence of a minimum of five monotonic pieces in each modes in the YCA, and the coverage of the widest possible historical range. "Monotonic" means that the pieces were identified in the corpus metadata as beginning and ending in the same key, which is an essential condition for the processing of the dataset described below. A few composers whose representation was dominated by etudes were excluded, because, due to their specialized function, etudes could be expected to differ systematically in tonal features. The composers included are, in order of birthdate, Byrd (1547), Lully (1632), Pachelbel and Corelli (1653), Purcell (1659), Couperin (1668), Vivaldi (1678), Telemann (1681), Rameau (1683), J.S. Bach, Handel, and Scarlatti (1685), Zipoli (1688), Sammartini (1700), Haydn (1732), Cimarosa (1749), Mozart (1756), Beethoven (1770), Hummel (1778), Schubert (1797), Mendelssohn (1809), Chopin and Schumann (1810), Liszt (1811), Verdi and Wagner (1813), Brahms (1833), Saint-Saens (1835), Tchaikovsky (1840), Dvorák (1841), Fauré (1845), Scriabin (1872), and Rachmaninoff (1873). Non-monotonic pieces (including those that change mode) were identified using the YCA metadata and manually removed.[1] Otherwise all available pieces were used, except in the case of J.S. Bach where the chorales were removed.[2] The number of pieces for each composer was highly variable (7–544 in major, mean 94.2, median 52; 5–251 in minor, mean 43.5, median 28).

As with virtually any historical dataset of music, we must acknowledge biases and keep them in mind when interpreting results. The YCA data itself, because it is sourced from amateur enthusiasts rather than musicologists, is already heavily biased towards piano music, and instrumental music generally, and toward Austro-German composers (White and Quinn 2016). The need for monotonic pieces also acted to further exclude important opera composers such as Rossini. The possibility that some results may reflect idiosyncrasies of piano repertoire or German-speaking composers must be borne in mind, and should be investigated in future research.

The dataset includes three pitch-class distributions for each piece: one for the first 20 quarter-notes of the piece, one for the last 20 quarter-notes, and one for the entire piece. This is similar to the procedure used by Albrecht and Shanahan (2013) and Albrecht and Huron (2014), although the amount of music is usually somewhat smaller. The distribution tallies when a given pitch class is present within each quarter-note span. Since the YCA data does not include metrical information, the quarter-note spans do not necessarily correspond to beats.

One significant shortcoming of the YCA data is that, due to many examples being entered at the keyboard, note offsets often occur earlier than indicated in the score (deClerq 2016). Since this typically happens around the sixteenth-note level, the data collection procedure of summing over quarter-note spans rather than exact durations should largely eliminate this flaw.

Distributions are transposed to C, using the key identifications from the YCA, the DFT applied the DFT and normalized, dividing by $f_0$ (which reflects cardinality) and multiplying by 100 for convenience. The normalization ensures that all pieces are weighted equally, rather than being weighted by length. The DFT yields twelve complex numbers, $f_0$–$f_{11}$, but five of these ($f_7$–$f_{11}$) are redundant, and the imaginary part of $f_6$ is always zero. The resulting useful information therefore consists of five complex numbers, $f_1$–

[1] Pieces by Byrd listed as modulating were retained because they appear to be Picardy-third endings rather than genuine changes of mode

[2] Previous studies, especially Albrecht & Shanahan (2013), show that the chorales are idiosyncratic due especially to their often short length and rapid harmonic rhythm, which is particularly important when isolating beginnings and endings (see below). There are also a large number of them in the corpus, so they would disproportionately affect the results for Bach if included.

$f_5$, and one real number, $f_6$. These are averaged for each composer.

### 2.2 Interpreting the DFT

Previous research (e.g., Quinn 2006–7, Amiot 2016, Yust 2016) has recommended interpreting the DFT by first assessing the "spectrum" of a pitch-class set or distribution—which DFT components are the largest, i.e. the farthest from zero in the complex plane. This indicates whether certain kinds of periodicity characterize the set or distribution. Then one may consider the phases, or direction from the origin, of the most important components, which give transposition-dependent information. For instance, music based on diatonic materials will have a large $f_5$, and the phase of $f_5$ will then correspond to the average key signature. Yust (2017) shows that distributions from tonal music appear to be quite consistent in concentrating the energy of their spectra primarily in $f_5$ and secondarily in $f_2$ and $f_3$, while Cuddy and Badertscher (1987) show that the same is typically true for probe-tone profiles obtained from listener responses to simple tonal stimuli.

We will consider the possibility of locating stylistic information in all of the DFT components in this study. Let us consider the interpretation of each of these in turn as they have been explained by theorists such as Quinn (2006) and Amiot (2016, Ch. 4):

($f_1$) Sometimes referred to as "chromaticity," this component indicates the concentration of the distribution in a specific location on the pitch-class circle. It is low for pitch-class sets of relatively even spacing, such as typical tonal chords and scales. The term "chromaticity" can be misleading in a sense: $f_1$ does *not* indicate a tendency to use all twelve pitch-classes, which results in a relatively *flat* distribution. In flat distributions, all qualities are depressed relative to the cardinality, $f_0$. Since $f_0$ is used to normalize, flatter distributions will be observable as decreases in all components.

($f_2$) This component balances the distribution on a half-octave periodicity, and therefore measures the weight of a prominent tritone or (in a more diatonic context) fifth. It has therefore been dubbed *dyadicity* (Yust 2017). Yust (2015a, 2016) also refers to it as "quartal quality" in non-tonal contexts, because it is largest for chords made out of stacked perfect and augmented fourths.

($f_3$) This is the most important component for major and minor triads and therefore has been used along with $f_5$ to construct tonal spaces (Krumhansl 1990, Amiot 2013, Yust 2015b) and may be referred to as *triadicity*. In tonal contexts, it weights a distribution's position in a stack of major and minor thirds, and hence can indicate a weighting towards the subdominant or dominant side of a particular key. In non-tonal contexts, it indicates similarity to an augmented triad or hexatonic scale (Amiot 2016, 2017).

($f_4$) This component weighs the three diminished seventh chords. In twentieth-century music it indicates octatonicity (Yust 2016). In tonal contexts, it is strongest for seventh chords and may also differentiate distributions by which of three harmonic functions is most prominent, along the lines of a Riemannian or "axis" system (Lendvai 1979).

($f_5$) This gives the *diatonicity* of a distribution, its weight and position on the circle of fifths, and is of principal significance in tonal distributions.

($f_6$) This is one-dimensional, and weighs one whole-tone collection against the other.

Each of these components (except $f_6$) is a complex number, with real and imaginary parts, which is best interpreted by converting to polar coordinates, magnitude (distance from the origin) and phase (angle). However, statistical analysis can only be done in the standard complex coordinates, since phase values are cyclic. Therefore, the conversion to polar coordinates is done only after the statistical analysis for the purpose of interpreting the results. Following Yust (2015a), phases are converted from the standard 0–2π scale to a 0–12 scale, denoted "Ph$_n$."

### 2.3 Examples

Figures 1 gives a few examples of beginnings of pieces in the corpus, with distributions transposed to C in Fig. 2, to illustrate aspects of the DFT. The first (Haydn Symphony no. 95) is an example of a typical minor-key distribution. Only the notes of the harmonic minor scale are represented, and the notes of the tonic triad are more heavily weighted, with $\hat{6}$ and $\hat{7}$ the weakest. The other two distributions are unusual in one way or another. Purcell's Jig uses a lot of minor dominant chords, so that $\hat{2}$ and $\hat{5}$ (and also $\hat{6}$) are weighted higher than the tonic, and the natural form of $\hat{7}$ is somewhat more prominent than the leading tone. Liszt's opening emphasizes the tonic triad, but otherwise is fairly chromatic, tending to not favour a particular scale, except around $\hat{2}$–$\hat{3}$.

Figure 3 shows the sizes of each of the six Fourier components for these distributions, normalized. Haydn's distribution, because it is more concentrated in a smaller number of pitch classes, is higher overall. Liszt's flatter distribution is lower overall. All distributions have a large $f_5$, reflecting concentration on the circle of fifths. Liszt's, however, has almost as much $f_3$, due to the emphasis on triad over scale. Purcell's is unusual in that $f_2$ and $f_4$ are stronger than $f_3$. This comes from the prevalence of non-tonic functions, which reinforces $f_4$ by putting greater emphasis on $\hat{2}$, $\hat{4}$, and $\hat{6}$ (the central diminished triad of the scale) and the weighting of structural fifths $\hat{1}$–$\hat{5}$–$\hat{2}$–$\hat{6}$ over thirds (reinforcing $f_2$).

The transposition-dependent information of the DFT is in the phases, and these are most significant when components are large. Therefore Figure 4 shows just phases of components 2–5. These are shown in 2-d toroidal plots, starting with the strongest components, $Ph_3$ and $Ph_5$. The $Ph_{3/5}$-space is the same as Krumhansl's (1990) key-finding space, and typical major/minor key profiles occur close to their respective major/minor triads, which are shown in the figure. A correlational key-finding algorithm (like those of Aarden [2003], Krumhansl [1990], Temperley [2007], or Sapp [2011]) would

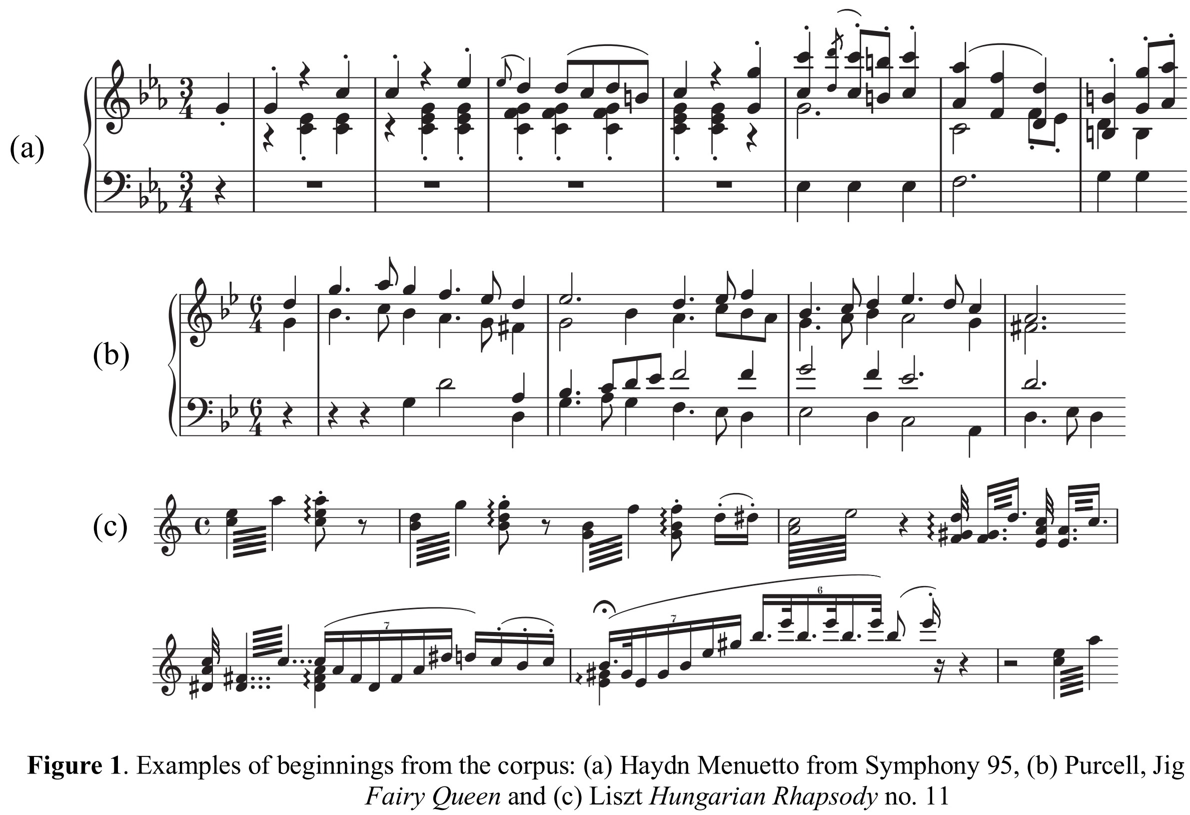



**Figure 1**. Examples of beginnings from the corpus: (a) Haydn Menuetto from Symphony 95, (b) Purcell, Jig from *The Fairy Queen* and (c) Liszt *Hungarian Rhapsody* no. 11

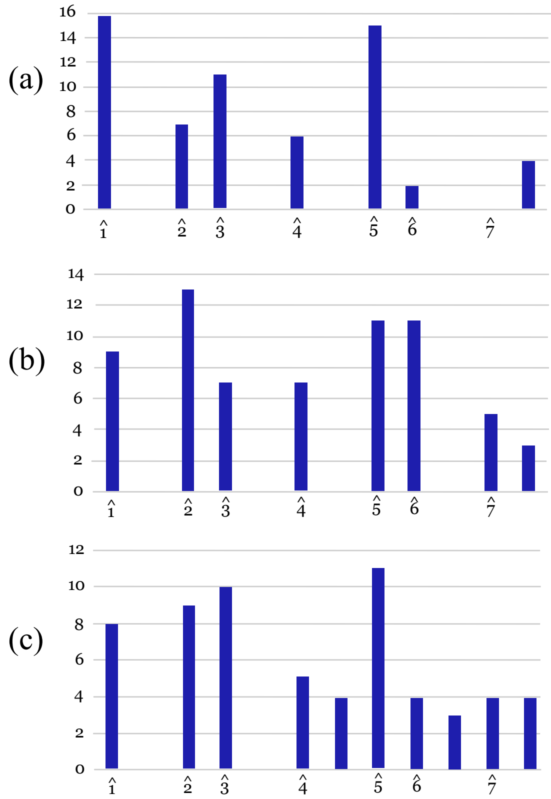



**Figure 2**. Distributions from the beginnings of (a) Haydn Symphony 95 Menuetto, (b) Purcell Jig, and (c) Liszt Hungarian Rhapsody no. 11

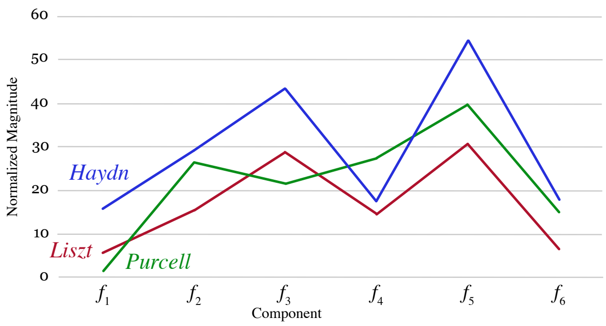



**Figure 3**. DFT spectra for the distributions in Fig. 2

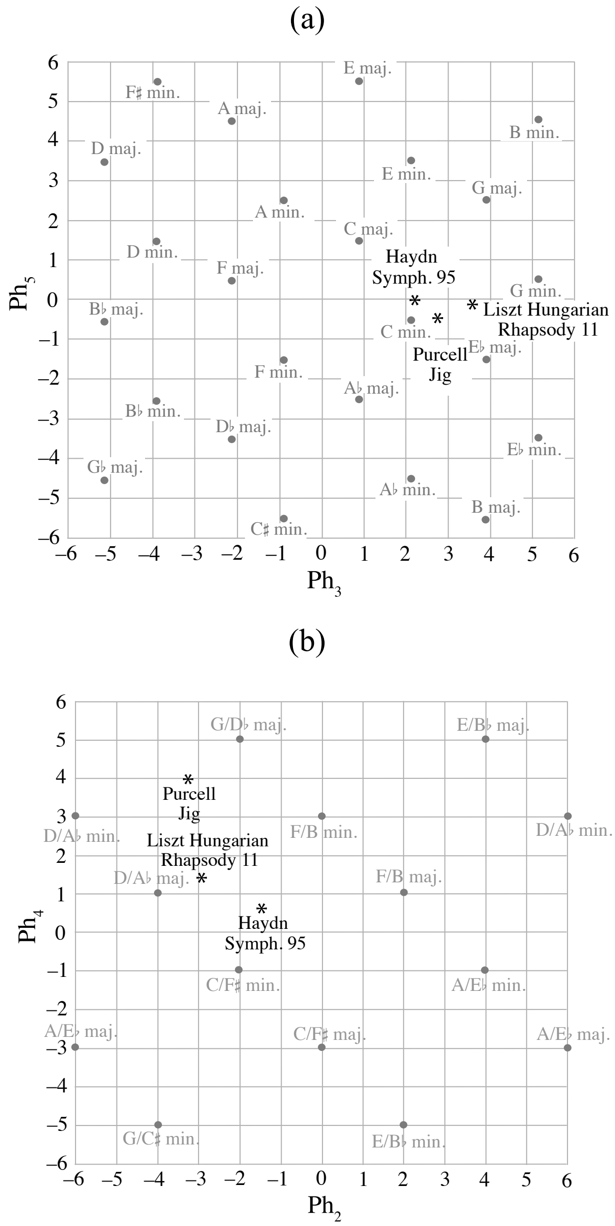



**Figure 4**. Phase space plots for the distributions

essentially locate the nearest key to the distribution in this space. Despite the unusual features of Purcell's distribution, which place it somewhat closer to the dominant and relative major, a correlational algorithm would have no trouble identifying its key (which, n.b., is transposed to C). The Liszt distribution is more ambiguous. Still, despite being quite unusual, it is still very close to the typical $Ph_5$ value for minor-key distributions, and not too far off the typical $Ph_3$ value.

The $Ph_{2/4}$ plot illustrates the greater volatility of $Ph_4$ in particular: the values for the three distributions are widely dispersed and not especially close to the tonic triad. The $Ph_2$ values are more stable, though, comparable to $Ph_3$. Note that the space is invariant under tritone transposition, so it would not be suitable for key-finding. However, key finding with just $Ph_2$ and $Ph_5$ would not fare much worse than with $Ph_3$ and $Ph_5$. Considering the distributions at hand, it would do somewhat better on the Liszt, somewhat worse on the Purcell.

### 2.4 Statistical Procedure

For each composer, an average distribution was calculated for the given mode (major or minor) and position (beginning, ending, or all), normalized, and converted via the DFT. A series of 11 multiple regressions was then run on these 198 data points, one for each possible dependent variable (the real and imaginary parts of $f_1$–$f_5$ and the real part of $f_6$) that included the factors of composer's birthdate, mode (2 levels), and position (3 levels), and all possible interactions between these. After a first battery of regressions revealed that the influence of date was likely to be non-linear, a quadratic factor on date and its interactions to the initial model were added.

Starting from a full model on each parameter, factors were removed one at a time in a stepwise procedure from each of these regression equations based on a $p < .01$ criterion. A factor was not removed if it was involved in a significant interaction, and date was not removed if date$^2$ remained in the model, and similarly for interactions involving date and date$^2$. Position was divided into two Boolean factors, beginnings and endings, and these could be removed separately. Factors were removed for the real and imaginary parts of a given component simultaneously, and the criterion for the removal of a factor was the smaller of the two $p$ values (for the real and imaginary parts). Thus, a significant result in *either* the real or imaginary coordinate was sufficient to retain a factor, and the significance value for a factor is always considered to be the lower of the two $p$ values for that factor.

After determining a regression equation, the observed and predicted data points where converted into polar coordinates (magnitude and phase), which are of more theoretical interest than the real and imaginary parts. This step was done last because phase is cyclic, so one cannot run an ordinary regression on it.

## 3. RESULTS

### 3.1 Regression

Table 1 shows the $R^2$ values of the resulting regression equations averaged between the real and imaginary parts. The regressions were highly predictive of $f_5$, $f_2$, $f_3$, and $f_4$, less so of $f_1$ and $f_6$. Figure 5 shows the coefficients of the regression equations and effect sizes in the form of non-normalized weights, with date converted to number of centuries with the average (1748) set to zero, so that its standard deviation was close to 1 and balanced. This gives a rough idea of which effects were the most substantial. All of the simple factors were significant on $f_2$–$f_5$, except for endings on $f_2$. Furthermore, all interactions with mode were significant except date × mode and date$^2$ × mode on $f_1$. Interactions between date and position were limited to endings; date × beginnings interactions were eliminated from all models.

| $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ |
|---|---|---|---|---|---|
| 0.399 | 0.817 | 0.785 | 0.767 | 0.821 | 0.629 |

**Table 1.** $R^2$ values for the resulting regressions, averaged between real and imaginary models.

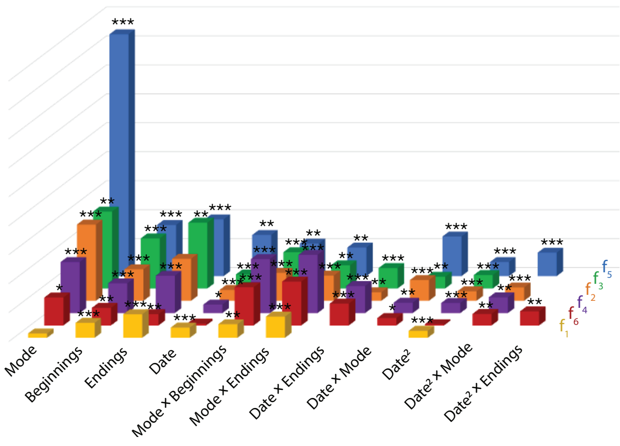



**Figure 5**. Effect sizes from the regressions, with Date scaled to number of centuries before or after 1748. * $p < .05$, ** $p < .01$, *** $p < .001$

When considering the effect sizes involving mode in Fig. 1, it must be borne in mind that they are dependent upon the choice of transpositions, because modes are largely distinguished by phase. C major and C minor have especially large phase differences in $f_5$, which leads to a very large simple effect of mode. Were the transpositions C major and A minor chosen, on the other hand, the difference in $f_2$ would be very large and the $f_5$ difference would be much smaller.

### 3.2 Magnitudes for whole-piece data

Figures 6a–b show the observed and predicted magnitudes for each component divided by composer, for just the whole-piece data. These results agree with the observation of Yust (2017) that tonal distributions are dominated by $f_5$, followed by $f_3$ and then $f_2$, with little representation of the remaining components ($f_1$, $f_4$, and $f_6$). The predictability of a component appears to relate to its size, with higher $R^2$s in Table 1 for components more prominent in Fig. 1, but with qualifications: $f_2$ gets higher $R^2$s than $f_3$, although $f_3$ has a consistently larger presence in the distributions. Also, regressions for $f_4$ and $f_6$ get higher $R^2$s than would be expected from the magnitudes of these components.

The concentration of energy in $f_5$ testifies to the efficacy of using the DFT to discover stylistic trends observable in distributional data.

(a)

(b)

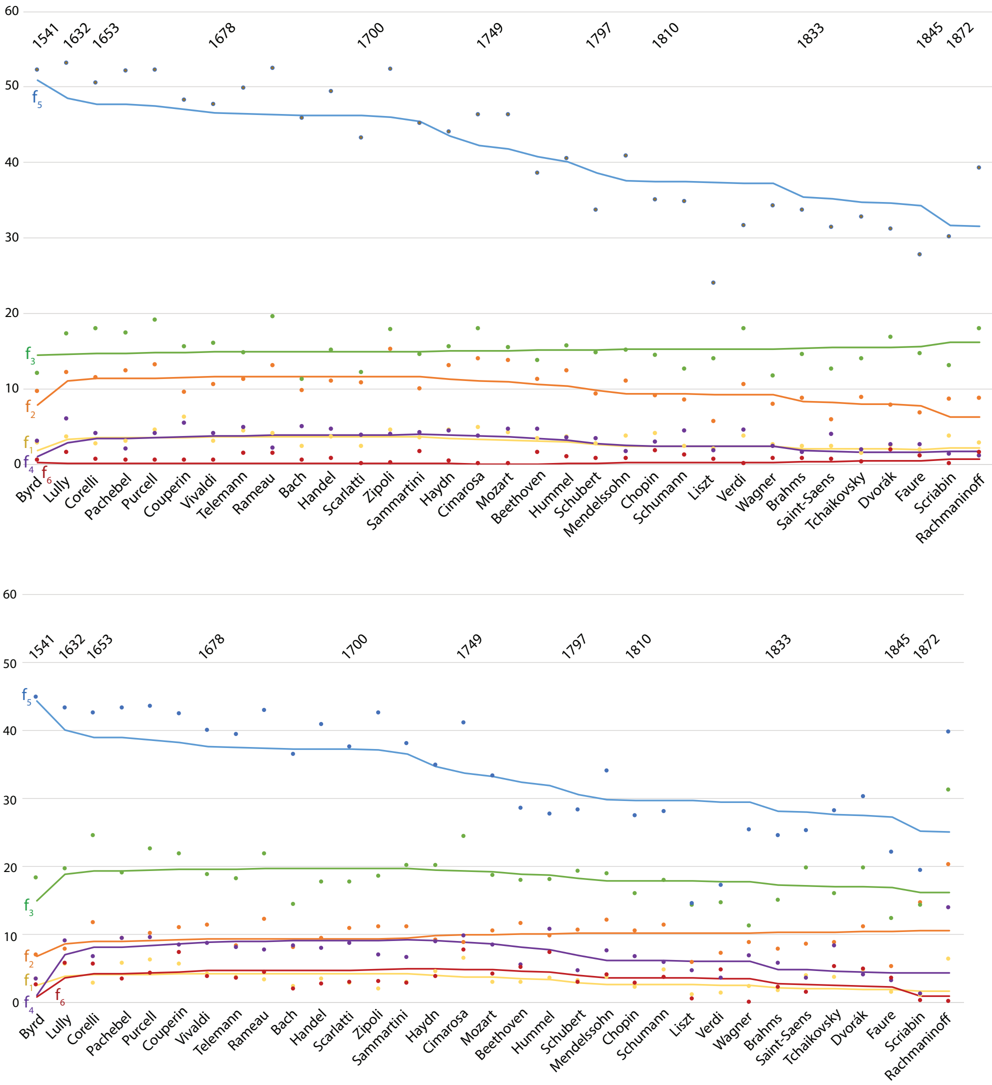



**Figure 6**. Observed and predicted magnitudes of DFT components for whole pieces, major keys, (a), and minor keys, (b).

Regressions performed on raw scale-degree values would be dominated by trends related to diatonicity. By separating this out into one of six orthogonal complex variables, other trends independent of diatonicity become observable.

One distinct historical trend is immediately evident from Fig. 6: diatonicity, though it remains the prevailing quality of tonal distributions over the full range of dates, decreases markedly over time. This component therefore appears to be an effective indicator of the gradual shift from diatonic harmonic languages to more chromatic ones. The shift is evidently not linear, however: a significant quadratic coefficient allows for a more rapid decrease in the eighteenth and nineteenth centuries. Even the quadratic coefficient appears to be insufficient to accurately capture the historical pattern, though, since most seventeenth century composers have positive residuals and nineteenth century composers have negative ones. To check this, a post-hoc cubic regression was run on just the whole-piece data, including interaction factors between all coefficients on date with mode. The resulting adjusted $R^2$s were 0.750 on reals and 0.953 on imaginaries as compared to 0.723 and 0.949 with just the quadratic factors, and the $date^3$ coefficient was significant at $p < .01$ on imaginaries (and not significant on reals).

Figure 7 shows resulting predictions of the cubic regression, which isolates the decrease in $|f_5|$ primarily to the 1700s. The diatonicity of individual composers relative to this trend intuitively matches their reputations as more or less harmonically progressive. On the progressive side are Scarlatti, Bach, Liszt, Verdi, Fauré, and Scriabin. Liszt is particularly extreme in this regard.  On the conservative side are Rameau, Zipoli, Cimarosa, Mozart, Mendelssohn, and Rachmaninoff. Most composers are similarly conservative or progressive in both modes, with the notable exceptions of Hummel (more conservative in major) and Schubert (more progressive in major).

The total sum of squared residuals from all of the regressions was also computed to see how well the result accord with musicological intuition. The composers with lower residuals (hence representative of their eras) were, in order, Mozart, Vivaldi, Telemann, Brahms, Beethoven, and Handel. Four composers stand out as having particularly high deviations from the trendlines: Verdi, Rachmaninoff, Liszt, and Wagner. Interestingly, these, plus the next two (Scriabin and Fauré) were all of the later nineteenth century, which is consistent with the intuition that harmonic languages diversified in the later nineteenth century, where

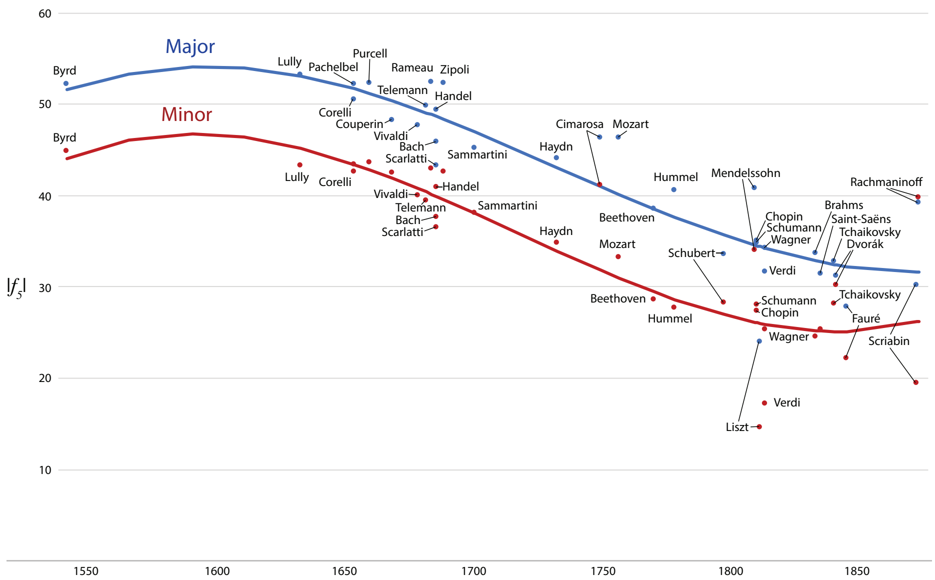



**Figure 7**. Observed $|f_5|$ and $|f_5|$ predicted from a cubic regression.

harmonically adventurous composers like Liszt and Wagner co-existed with relatively conservative ones like Verdi and Rachmaninoff.

Triadicity, $f_3$, is consistently the second most important component in Fig. 6 and $f_2$ the third, although $f_4$ seems to play a role in minor-key distributions, enough to outrank $f_2$ in a few individual instances (Lully, Pachelbel, Cimarosa, Hummel, and Tchaikovsky).

Also, while $f_5$ tends to decrease in power over time in both modes, the trendlines show little or no decrease in $f_3$ and $f_2$, so that they take a greater share of distributional weight in later styles.[1] The significant quadratic coefficients on date in $f_1$–$f_4$ and $f_6$ allow the regressions to show stable magnitudes for these components over the seventeenth and eighteenth centuries followed by decreases in the nineteenth century. For $f_2$ this pattern holds only in major, whereas in $f_3$ and $f_6$ it holds only in minor.

Figures 8a–d compare the historical trendlines in component magnitudes for beginnings and endings to those for whole pieces (duplicated from Fig. 2). These indicate one straightforward explanation for the relatively large simple effects of position: in all cases, beginnings and endings have substantially higher magnitudes than whole pieces. This reflects the unsurprising fact that whole pieces tend to have flatter pitch-class profiles (with more energy concentrated in $f_0$). The trendlines for beginnings are parallel to those for whole pieces, showing the lack of interaction between beginnings and date.[2]

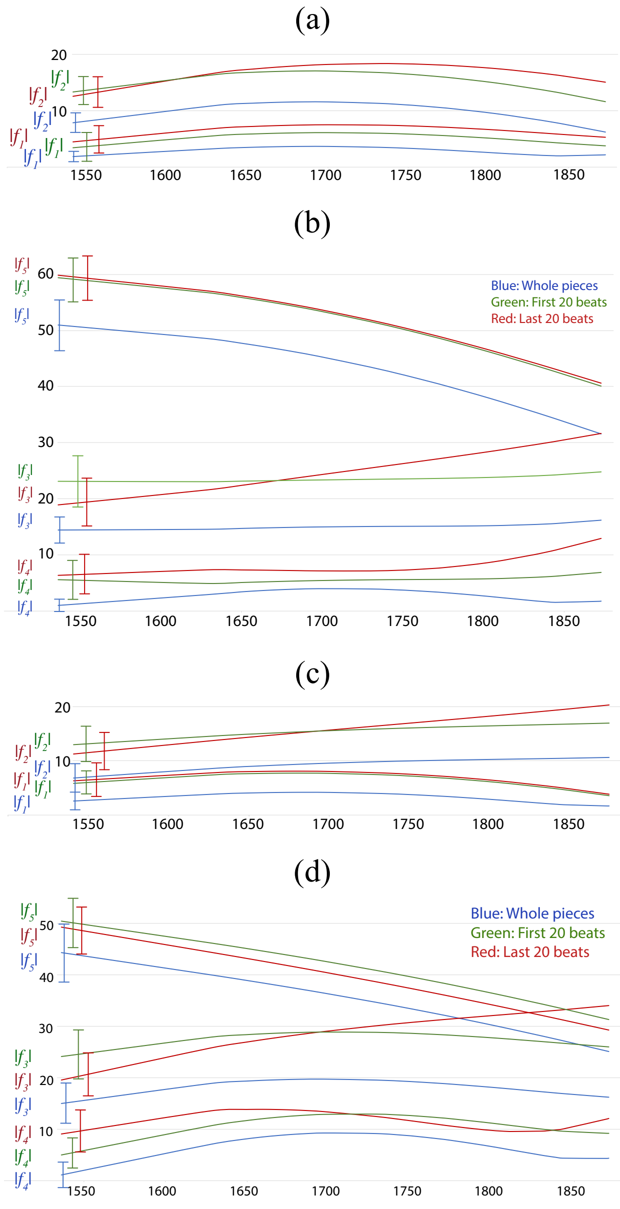



**Figure 8**. Predicted magnitudes by date and position for (a) $f_1$–$f_2$ in major, (b) $f_3$–$f_5$ in major, (c) $f_1$–$f_2$ in minor, and (d) $f_3$–$f_5$ in minor. Error bars represent ± one standard deviation of the residuals.

[1] One might wonder how *all* qualities can decrease in size, given that data are normalized. The DFT conserves total *power*, the sum of squared magnitudes. But data are normalized by cardinality ($f_0$) rather than total power. A distribution that is more evenly spread out between the pitch classes will have a greater share of its power in $f_0$, which means it will normalize to a lower-power distribution, and all components will tend to be lower. Therefore, an overall decrease of DFT magnitudes indicates a more chromatic distribution in the sense of more evenly spread between all pitch classes.

[2] This appears not to be the case for $f_4$, but this is an artifact of the conversion to polar coordinates for small-magnitude components. The trendlines by date are parallel in complex space, but because the one for beginnings passes by on a different side of, and much further from, the origin, the trend in magnitudes looks different.

Notable interactions of date and endings occur in $f_3$, $f_4$, and to a lesser extent $f_2$. Fig. 4 shows that all of these interactions occur because certain components become stronger over time in endings as compared to beginnings or whole pieces. This is evident especially in $f_3$, in both modes, and in $f_4$ in major. The trend in $f_2$ appears instead in minor. The trend in $f_3$ is so strong that the model predicts, only for endings, $f_3$s larger than $f_5$s in the later nineteenth century in minor. The interactions in $f_3$ and $f_4$ show that endings become more triadic over time: $f_3$ is the predominant component for triads, and $f_4$ is prominent in triads but not in typical key profiles.

The trendlines for $f_6$ are shown separately in Fig. 9 to allow for negative values. Positive values indicate weightings of the even whole-tone collection and negative values the odd. The trendlines are mostly flat, except on endings, which ascend markedly in the 1700s and 1800s in both modes. This may reflect increasing emphasis on the tonic note at endings.

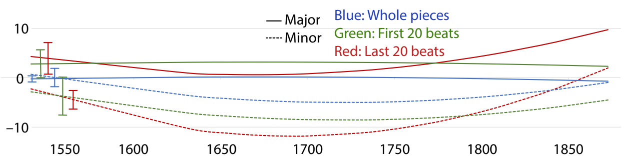



**Figure 9**. Predicted values of $f_6$ by date in all conditions.

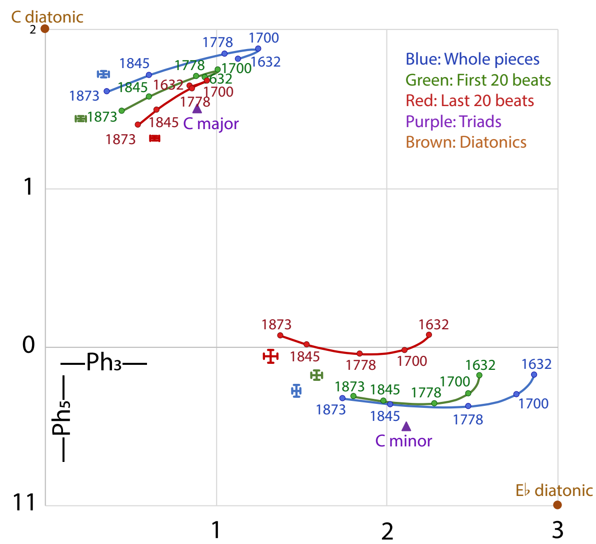
To summarize:

(1) Whole pieces have consistently flatter distributions than beginnings and endings.
(2) Endings become stronger over time in $f_3$ and $f_4$ in both modes, and $f_2$ in minor, and more positive in $f_6$.

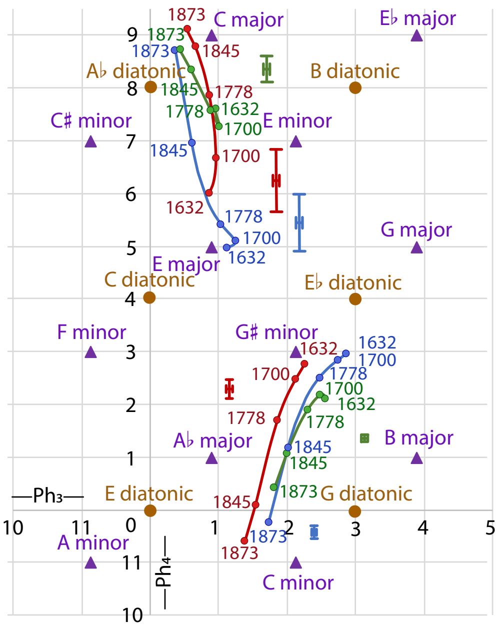
The results shown in Figs. 4–9 isolate DFT magnitudes from phases. The magnitudes indicate the strength of a given component in the profile, whereas phase gives the orientation of that component in pitch-class space. Figures 6a–b show trends in phase for components 2–5, separated into three two-dimensional phase spaces, $Ph_3/Ph_5$-, $Ph_3/Ph_4$-, and $Ph_1/Ph_2$-spaces. (See Yust 2015b, 2016). The first of these is the space used by Krumhansl (1990), and the best candidate for a two-dimensional map of tonality. The second duplicates the use of $Ph_3$ in order to compare it to $Ph_4$, which is intuitively similar as a thirds-based component and one that may relate to functional categories. The last plot ($Ph_1/Ph_2$) involves the remaining two components.

The $Ph_3/Ph_5$ plot (Fig. 10a) shows quite stable phase values, especially in major. The values are consistently very close to those of the tonic triads, but a little higher in $f_5$, indicating a slight tendency towards the sharp side. In major, the whole-piece data leans most sharpward (probably reflecting modulations to the dominant), whereas in the minor-key data the endings are more sharpward. There is more motion to the left in $Ph_3$ over time. In major, this motion is isolated roughly to the 1800s. This reflects increasing weight towards the subdominant side, especially in the nineteenth century. Endings also tend more towards the subdominant in minor (in the triadic sense—they are at the same time more sharpward in the diatonic sense).

(a)




(b)




(c)

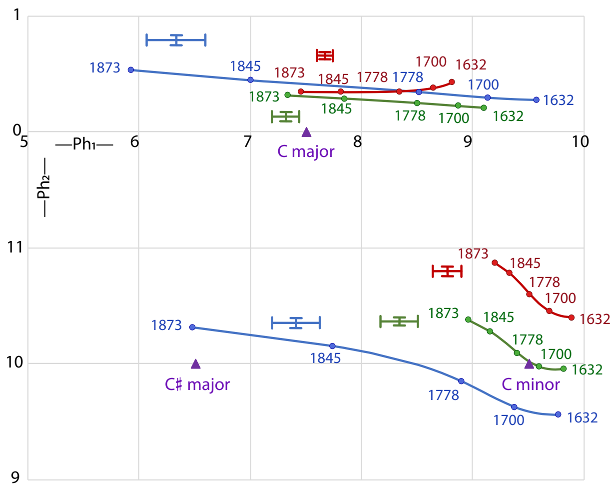



**Figure 10**. Trends by date, position, and mode in Ph$_3$/Ph$_5$-space, (a), Ph$_3$/Ph$_4$-space, (b), and Ph$_1$/Ph$_2$-space, (c). Error bars show ± one standard deviation of the residuals.

The Ph$_3$/Ph$_4$ plot (Fig. 10b) reveals trends in Ph$_4$ that are of an entirely different order of magnitude than those in Ph$_3$. Over the historical window examined, endings and whole pieces traverse about one third of a full Ph$_4$ cycle. They go in opposite directions in major and minor, but in both cases move from remote locations in the seventeenth century to values approximating the tonic triad in the late nineteenth century. The strength of this phase-based trend accounts for the relatively high predictive value of the regression on $f_4$ (Table 1) despite the generally low magnitude of this component in the profiles. The seventeenth-century Ph$_4$ positions may be understood as reflective of the diatonic scales for C major and C natural minor, which are included in Fig. 10(b) for comparison. Whole pieces are closer to the diatonics in the seventeenth century, while beginnings, especially in major, are somewhere between the Ph$_4$ position of the diatonic and that of the tonic triad. All positions move close to the tonic triads in the nineteenth century.

The other small magnitude component, $f_1$, also has large changes of phase over date, especially for whole pieces (Fig. 10c), where the predicted Ph$_1$ goes from a center of balance between D and E♭ (10–9) to between F and F♯ (7–6), a shift of melodic activity from the vicinity of $\hat{1}$ to that of $\hat{5}$. The trajectories of beginnings and endings is similar but less dramatic, stopping around E♭ (9) in minor and between E and F (8–7) in major. The significance of these trends, however, (as evident in Table 1 and Fig. 5) is weak.

In contrast, $f_2$ is a higher-magnitude component and its phase values are much more restricted, remaining close to the tonic triads. In major, there is very little systematic variation in Ph$_2$ motion, either by date or position, only a slight upward motion for whole pieces and beginnings. Minor key distributions, on the contrary, see a distinct increase in Ph$_2$ of about 0.5 over the historical window, across all positions. Furthermore, endings are about 0.5 above beginnings and beginnings about 0.5 above whole pieces in the 17$^{th}$ century, with that gap narrowing towards the 19$^{th}$ century. The higher values for endings and for later styles may reflect greater relative weight on the tonic note.

## 4. DISCUSSION

The results allow us to address two broad questions: How do pitch-class distributions in the corpus change over time? And, how do pitch-class distributions for whole pieces differ from beginnings and endings of pieces?

#### 4.1 Trends by date

The strongest trend relating to date is the decrease in the strength of diatonicity over time, which is dramatic, and as Fig. 7 shows, seems to be concentrated in the eighteenth century. Given the substantial changes in $f_5$, it is also impressive that its phase (Fig. 10a) is so stable over time. The trend by date is also nearly equivalent between the modes when stated in polar coordinates. The apparent large date × mode interaction in Fig. 5 is attributable to the substantial $Ph_5$ difference between major and minor, which puts $f_5$ in a different orientation with respect to the origin when reckoned in regular complex coordinates.

The $f_5$ trends suggest that diatonicism, or balance on the circle of fifths, remains primary for establishing tonality over the period studied, and that the circle-of-fifths positions associated with each key is stable. In other words, there is no evidence of a shift, say, from a more Dorian to a more Aeolian minor over the time period studied. What does change over time is the strength of diatonicity, which participates in an overall trend towards flatter distributions, but is more concentrated in $f_5$ than in other components. This accords with musicological intuition: not only does the music in this corpus become more chromatic over the 18th and early 19th century, in the sense of using all twelve pitch classes more frequently, but it does so largely at the expense of diatonic qualities rather than triadic ones.

This conclusion, like others here, warrants a note of caution, since it only applies to the corpus at hand, not necessarily to a "typical" sample of music from these periods, however that may be construed. Indeed, of the various ways the corpus may be biased, perhaps the strongest bias is simply towards music that has survived history and continues to be performed and valued by listeners. It is quite likely that the music that is the most unique for its time is heavily favored for inclusion in this "canon," and therefore analyzing this canon, rather than, say, a typical concert program of the *Musikverein*, probably exaggerates trends like the weakening of diatonicity, and the diversification of the later nineteenth-century harmonic palette (noted in the results section).

The $f_5$ by date trends were also not found to interact with position. The decrease in diatonicity is uniform between beginnings, endings, and whole pieces. This is not true of other subtler trends relating to changes in the corpus over time, which are

(1) increase in triadicity for endings,
(2) triadic weight shifting towards the subdominant overall,
(3) move towards the $Ph_4$ value of the tonic triad versus the home-key scale, and
(4) greater weight placed on the tonic note in minor, especially at endings.

Use of the DFT in this study was partly inspired by the work of Honingh and Bod (2010, 2011), which showed promising results distinguishing types of music on the basis of interval-type categories proposed by Quinn (2001). Quinn argued in his later work (2006–7) that the DFT improves upon and eclipses his earlier classification by interval content. In particular, their category 5 (ic5-based sets), which proves useful in, e.g., distinguishing tonal and atonal music, correlates with diatonicity ($f_5$).[1] But, as Amiot (2017) demonstrates, the DFT better reflects the intuitive sense of "diatonicity" than interval content. In addition, it is a more flexible tool, allowing us to assess not only the strength of diatonicity, $|f_5|$, but also gives a sensitive measure of the diatonic position, $Ph_5$.

The present results reinforce and refine a number of Honingh and Bod's conclusions. They

[1] This is not a perfect one-to-one correspondence, however, and the numerical correspondence of category 5 and $f_5$ is accidental. Category 2 corresponds roughly to $f_6$ and category 6 imperfectly to $f_2$, while category 3 tends to correspond to $f_4$ and category 4 to $f_3$. In general, such correspondences need not exist at all—see Quinn 2006–7.

find that tonal music has a higher proportion of category-5 sets. Although atonal music was not used in the present study, the consistently very high diatonicity values reflected in Figs. 2–4 is undoubtedly characteristic of tonality. They also found a trend of decreasing proportion of category-5 sets from Bach to Mozart to Beethoven to Brahms. This is reflective of the robust trend of decreasing diatonicity over the eighteenth and nineteenth centuries found above. One might even speculate that atonality represents the logical continuation of such a trend. Perttu's (2007) conclusion that use of non-scale tones increases over time (1700–1960) is also consistent with this finding.

The data in Figs. 2–3 also show that minor-key music has a lower diatonicity (and hence fewer category-5 sets, and Honingh and Bod find) overall. However, since diatonicity is influenced by other factors (such as date), modes are better distinguished by phase values, which, as is clear from Fig. 6, are well differentiated by mode. Indeed, as mentioned in section 2.3, this is essentially the method of distributional key finding algorithms. These typically rely upon correlation between distributions which, as Yust (2017) shows, essentially amounts to comparing positions in an appropriate two-dimensional phase space like Figs. 4a or 10a.

#### 4.2 Functional categories

Two of the DFT components, $f_3$ and $f_4$, relate to theories of functional classification because of the thirds-based organization of their spaces. The present study cannot, of course, say anything about harmonic function *per se*, in the sense of a syntax of progressions (as the term is used, e.g., by White [2013], White and Quinn [2018], and others) since all data points average over multiple measures of music and will typically contain multiple chords. Yet both components separate notes typical of tonic ($\hat{1}$, $\hat{3}$), dominant ($\hat{7}$), and subdominant ($\hat{4}$, $\hat{6}$) functional categories, so they will reflect the overall frequency of chords of one type versus another. The two components are mathematically independent of one another, and hence reflect different concepts of "dominantness" and "subdominantness." Cohn's (1999, 2012) idea of hexatonic function is represented by $f_3$, whereas $f_4$ represents the kind of "axial" or Riemannian function advanced by DeJong and Noll (2008, 2011).

Two stylistic trends found above have opposite implications according to the two functional classifications. Distributions move toward the subdominant side in $Ph_3$ in the nineteenth century, while moving towards the tonic in $Ph_4$ (Figs 10a–b).

Overall the data show that hexatonic function ($f_3$) is stable, strong, and tonic-centered, whereas axial function ($f_4$) is weak in the overall distribution. This may reflect the fact that dominant and subdominant cancel one another out in $f_3$, whereas they reinforce one another, opposite tonic, in $f_4$, particularly in minor.

The pronounced $Ph_4$ trend by date therefore reflects a greater presence of tonic versus dominant and subdominant in the nineteenth century pieces, which effects the phase of $f_4$ (which is highly sensitive due to low magnitude) but effects $f_3$ primarily in magnitude. The trend in $Ph_3$ is then able to show a concurrent slight shift in from dominant to subdominant.

The trend in $Ph_3$ lends some support to claims made by theorists such as Harrison (1994) that subdominant-type functions gain in importance over the nineteenth century.

The relatively high $R^2$ reported here for $f_4$ may suggest revising one of the conclusions of Yust 2017, who implies that $f_4$ is negligible for tonality because of its suppression in pitch-class distributions for whole-piece data. This finding is replicated here, but at the same time $f_4$ helps distinguish between beginnings, endings, and whole-piece data, and reveals historical trends. The suppression of $f_4$ when averaging over many measures may instead reflect a relative balance between harmonic functions being averaged together. There is therefore a strong possibility that $f_4$ may be important at the level of local harmonic motion. Whether this is so, and whether local $f_3$ or $f_4$ motion may reflect syntactic norms of harmonic progression, can only be determined by further research, analyzing time-dependent distributions taken over smaller windows.

### 4.3 Beginnings and endings

The inclusion of a position factor allows us to address a question posed in the literature on distributional studies of key and mode: Are short passages from the beginning and ending of pieces more representative of a given key than entire pieces? Albrecht and Shanahan (2013) and Albrecht and Huron (2014) argue that beginnings and endings will tend to be better representatives of a key's distributional properties, since they are less likely to include modulations. The regressions presented here, as Fig. 5 indicates, did indeed find a number of differences between beginnings and endings and entire pieces, differences that interact with mode and, only for endings, with date.

One strong difference between beginnings and endings versus whole pieces is found here, but it is an obvious one, that entire pieces have flatter overall distributions than beginnings and endings. The DFT isolates this difference in $f_0$, allowing us to make other comparisons that would have meaningful implications for, e.g., algorithmic key finding.

It is difficult to directly litigate the question of whether beginnings and endings or whole pieces better represent a key because there is no ground truth for what a "better" representation of a key is. Nonetheless, the present results provide some significant qualifications and cautions regarding such claims.

The most important qualification is that lumping together of beginnings and endings is not appropriate, in that endings behave differently than beginnings in a number of ways. The data do, on the other hand, show a remarkable correspondence between beginnings and entire pieces. This correspondence suggests that beginnings and whole pieces constitute a fairly robust common distributional representation of a key, and the typical choices of contrasting key areas tend to reinforce the distributional properties of the home key in some respects and cancel one another out in others.

The two kinds of effects involving beginnings in the regressions, a main effect and an interaction with mode, may be accounted for by the fact that beginnings are more concentrated in their pitch-class content and therefore have higher magnitude coefficients. This can be seen in Figs. 4–5; note also that the magnitude differences are approximately the same in major and minor. The mode × beginnings interactions, like the date × mode interaction, are an artifact of the conversion to polar coordinates. Because major and minor have substantially different phase values, motion to or from the origin is accomplished differently in complex coordinates.

Besides this difference in magnitudes, beginnings are very similar to whole pieces. With a few small qualifications, the predicted phases for beginnings and whole pieces are quite close in Fig. 10. In the small magnitude components, $f_1$ and $f_4$, the less dramatic changes of phase for beginnings are a consequence of the magnitude difference, since a similar motion in complex space further from the origin amounts to a smaller change of phase. This leaves two other small differences that can be identified. First, in Fig. 10a, beginnings have a slightly lower $\mathrm{Ph}_5$ than whole pieces in major, which probably reflects the tendency to choose sharp-side secondary key areas in major-mode pieces. Second, in Fig. 10c, beginnings in minor tend to have a slightly higher $\mathrm{Ph}_2$. This may reflect a greater weight on the tonic note, C, in beginnings, and/or a greater representation of the third, E♭—the tonic of the relative major—in whole pieces. Thus, there do seem to be some reliable differences in whole pieces that may be attributed to the inclusion of passages in common secondary keys. However, these differences are surprisingly slight.

Endings, on the other hand, are different from whole-piece distributions in a number of respects. Like beginnings, they also have higher magnitude coefficients overall. But unlike beginnings, the differences interact with date. In particular, the magnitudes of $f_3$, $f_4$, and $f_6$—and $f_2$ in minor—increase over time for endings but not for beginnings or whole pieces. The difference is especially strong for $f_3$. This indicates that endings become more triadic over the time period examined. In addition, endings have more differences of phase from whole pieces (Fig. 10), particularly in minor. In major, they have lower $Ph_5$ (flatter distributions). In minor, they have substantially higher $Ph_5$ (sharper), lower $Ph_3$ (more subdominant), and higher $Ph_2$.

## 5. CONCLUSION

The results reported here validate the premise that using the DFT to interpret pitch-class distributions helps to separate out trends that would otherwise be overwhelmed by diatonicity, which is consistently orders of magnitude larger than other components. Yust's (2017) main result, that tonal distributions averaged over multiple measure are primarily determined by three components, $f_5$, $f_3$, and $f_2$, in that order, is replicated on a larger body of data. However, the results also suggest that the low value of $f_4$ for distributions taken over large spans of music may not necessarily mean that $f_4$ is irrelevant to tonality, and its role may become more clear in a study of more local distributions, where it might have a role in something like functional syntax.

Consistent with other studies, this one drives home the obvious fact that music becomes more chromatic over the eighteenth and nineteenth centuries. However, it also refines this observation in important ways, showing that increased chromaticism comes largely at the expense of the diatonic rather than the triadic aspect of tonal profiles, and that the circle-of-fifths locations of keys remain remarkably stable as diatonicity decreases in overall weight. Furthermore, other more subtle trends, such as an increased weight towards the subdominant in the nineteenth century, become observable after factoring out the much larger diatonic trend. Also, individual composers may be compared to the overall trend, substantiating and supplementing musicological evaluations of these composer's relationship to their historical era. Future studies might also consider how trends within the output of individual composers (such as Beethoven) compare to the overall historical trends found here.

The results also show that beginnings have tonal profiles more similar to whole pieces than endings. The question of what kind of profile is a better representative of a key is harder to definitively answer.

Overall, the results here suggest that future research should consider using the DFT when analyzing trends in pitch-class distributions. One worthwhile goal would be to validate conclusions arrived at here in other corpora that may not have the same biases as the YCA data. Perhaps the most pressing question for further study, however, is whether there are significant regularities in more local distributions and how they change over time in tonal music, and whether the DFT can aid in discovering such regularities. Further differentiating such temporal dependencies of local distributions over historical periods may significantly fill out the picture of stylistic change drawn here.

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## 7. REFERENCES

Aarden, B. J. (2003). *Dynamic Melodic Expectation* (Unpublished doctoral dissertation). Ohio State University.

Albrecht, J. D., & Huron, D. (2014). A statistical approach to the historical development of major and minor pitch distributions, 1400–1750. *Music Perception 31*, 223–243. doi: MP.2014.31.3.223

Albrecht, J. D., & Shanahan, D. (2013). The use of large corpora to train a new type of key-finding algorithm: An improved treatment of the minor mode. *Music Perception 31*, 59–67. doi: 10.1525/MP.2013.31.1.59

Amiot, E. (2016). *Music through Fourier space: Discrete Fourier transform in music theory*. Heidelberg: Springer.

———. (2017). Interval content vs. DFT. In O. A. Agustín-Aquino, E. Luis-Puebla, M. Monteil (Eds.) *Mathematics and computation in music: Sixth international conference, MCM 2017.* Heidelberg: Springer.

Bernardes, G., Cocharro, D., Caetano, M., Guedes, C., & Davies, M. E. P. (2016). A multi-level tonal interval space for modelling pitch relatedness and musical consonance. *Journal of New Music Research* 45: 281–94.

Cohn, R. L. (1999). "As wonderful as star clusters": Instruments for gazing at tonality in Schubert. *19th-Century Music 22*, 213–232. doi: 10.2307/746799

———. (2012). *Audacious euphony: The triad's second nature*. New York: Oxford University Press.

Cuddy, L. L. & Badertscher, B. (1987). Recovery of the tonal hierarchy: Some comparisons across age and levels of musical experience. *Perception & Psychophysics 41*: 609–620. doi: 10.3758/BF03210493

deClerq, T. (2016) Big data, big questions: A Closer look at the Yale-Classical Archives corpus. *Empirical Musicology Review* 11: 59–67.

De Jong, K., & Noll, T. (2008). Contiguous fundamental bass progressions. *Dutch Journal of Music Theory 13*, 84–97.

———. (2011). Fundamental passacaglia: Harmonic function and the modes of the musical tetractys. In C. Agon, M. Andreatta, G. Assayag, E. Amiot, J. Bresson, and J. Mandreau (Eds.), *Mathematics and computation in music: Third international conference, MCM 2011*, 98–114. Heidelberg: Springer. doi: 10.1007/978-3-642-21590-2

Harte, C., Sandler, M., & Gasser, M. (2006). Detecting harmonic change in musical audio. In Amatriain, X., Chew, E., and Foote, J. (Eds.) *Proceedings of the 1st ACM workshop on audio and music computing multimedia*, 21–6.

Harrison, D. (1994) *Harmonic function in chromatic music: A renewed dualist theory and an account of its precedents*. University of Chicago Press.

Honingh, A. K., and Bod, R. (2010). Pitch class set categories as analysis tools for degrees of tonality. In J.S. Downie & R.C. Veltkamp (Eds.), *Proceedings of the 11th International Society of Music Information Retrieval Conference (ISMIR 2010),* 459–464.

———. (2011). Clustering and classification of music by interval categories. In C. Agon, M. Andreatta, G Assayag, E. Amiot, J. Bresson, and J. Mandreau (Eds.), *Mathematics and computation in music: Third international conference, MCM 2011*, 346–349. Heidelberg: Springer. doi: 10.1007/978-3-642-21590-2

Krumhansl, C. L. (1990). *Cognitive foundations of musical pitch*. New York: Oxford University Press.

Krumhansl, C. L., & Cuddy, L. L. (2010). A theory of tonal hierarchies in music. In M.R. Jones, A.N. Popper, & R.R. Fay (Eds.), *Music Perception* (55–99). New York: Springer. doi: 10.1007/978-1-4419-6114-3_3

Lewin, D. (1959). Re: Intervallic relations between two collections of notes. *Journal of Music Theory 3*, 298–301.

———. (2001). Special cases of the interval function between pitch-class sets *X* and *Y*. *Journal of Music Theory 45*, 1–29. doi: 10.1215/00222909-45-1-1

Meyer, L. B. (1957). Meaning in music and information theory. *Journal of Aesthetics and Art Criticism* 15, 412–24.

Perttu, D. (2007). A quantitative study of chromaticism: Changes observed in historical eras and individual composers. *Empirical Musicology Review 2*, 47–54. doi: 10.18061/1811/24822

Prince, J. B., & Schmuckler, M. A. (2014). The tonal-metrical hierarchy: A corpus analysis. *Music Perception 31*, 254–270. doi: 10.1525/mp.2014.31.3.254

Quinn, I. (2001). Listening to similarity relations. *Perspectives of New Music 39*, 108–158.

———. (2006–7). General equal-tempered harmony. *Perspectives of New Music 44*, 114–159 and *45*, 151–163.

Quinn, I. & Mavromatis, P. (2011). Voice-leading prototypes and harmonic function in two chorale corpora. In C. Agon, M. Andreatta, G. Assayag, E. Amiot, J. Bresson, and J. Mandreau (Eds.), *Mathematics and computation in music: Third international conference, MCM 2011,* 230–40.

Sapp, C. S. (2011). *Computational methods for the analysis of musical structure* (Unpublished doctoral dissertation, Stanford University).

Temperley, D. (2007). *Music and probability*. Cambridge, Mass.: MIT Press.

Tompkins, D. C. (2017). Early seventeenth-century harmonic practice: A corpus study of tonality, modality, and harmonic function in Italian secular song with Baroque guitar accompaniment in Alfabeto tablature. PhD Diss. Florida State University.

White, C. W. (2013). *Some statistical properties of tonality, 1650–1900.* (Unpublished doctoral dissertation, Yale University).

White, C. W. (2014). Changing styles, changing corpora, changing tonal models. *Music Perception 31*, 244–53.

White, C. W. and Quinn, I. (2016) The Yale-Classical Archives corpus. *Empirical Musicology Review 11*, 50–8

———. (2018) Chord context and harmonic function in tonal music. *Music Theory Spectrum* 40, 314–37.

Youngblood, J.E. (1958) Style as information. *Journal of Music Theory* 2: 24–35.

Yust, J. (2015a). Applications of DFT to the theory of twentieth-century harmony. In T. Collins, D. Meredith, & A. Volk (Eds.), *Mathematics and computation in music: Fifth international conference, MCM 2015*, 207–218. Heidelberg: Springer. doi: 10.1007/978-3-319-20603-5

———. (2015b). Schubert's harmonic language and Fourier phase space. *Journal of Music Theory 59*: 121–181. doi: 10.1215/00222909-2863409

———. (2016). Special collections: Renewing set theory. *Journal of Music Theory 60*, 213–262. doi: 10.1215/00222909-3651886.

———. (2017). Probing questions about keys: Tonal distributions through the DFT. In O. A. Agustín-Aquino, E. Luis-Puebla, M. Monteil (Eds.), *Mathematics and computation in music: Sixth international conference, MCM 2017.* Heidelberg: Springer.