

## Energy levels of quark atoms

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We assume that free quarks exist in nature. The energy levels of atoms composed of these quarks and various nuclei are calculated. We note some important aspects of these energies.

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It is currently believed that all hadrons are composed of quarks which are confined. Whether or not quark confinement can be broken, however, is a purely experimental question. So far, experiments have shown that free, unconfined quarks, if they exist at all, must exist in concentrations of  $<1$  per  $10^{20}$  nucleons [1]. A recent proposal may push this limit by a factor of  $10^5$  [2]. Theories which accommodate this data can still allow free quarks to exist. For example, the glow model [3] predicts the existence of  $qq$  or  $\bar{q}\bar{q}$  states when the  $SU^{\text{color}}(3)$  symmetry of QCD is spontaneously broken to  $SO^{\text{glow}}(3)$  symmetry. Other models allow for free single quarks [4]. These free quarks or diquarks would most likely be remnants from the big bang [5], with the negatively charged ones existing as quark atoms, such as  $Q^4\text{He}$  (where  $Q$  is a quark or diquark with charge  $Z_Q e$ ,  $Z_Q < 0$ ), today.

The existence of free  $Q$ , in addition to its purely theoretical implications, would make catalyzed fusion feasible and useful [6]. Because the mass  $m_Q$  of a quark is most likely greater than 1 GeV, much larger than the mass of an electron, it would be easy to fuse the deuterons in a  $ddQ$  molecule. In particular, a  $\bar{u}\bar{u}Q$  with  $Z_Q = -\frac{4}{3}$  would be much more effective than a  $\mu$  in this catalysis [7]. To be able to isolate the negatively charged  $Q$  from bulk material and use the quarks for fusion catalysis, it is important to know the energy levels of quark atoms  $Q^4Z$ , which is the subject of this paper.

To calculate the ground-state energy level  $E_0$  of a quark atom with nucleus  $^AZ$  having mass  $m_A$  and charge  $+Ze$ , we numerically solve the nonrelativistic Schrödinger equation with  $l=0$  and reduced mass  $\mu = m_Q m_A / (m_Q + m_A)$ . The potential  $V(r)$  that models the quark-nucleus interaction can be separated into two parts, a Coulomb potential  $V_C(r)$  and a quark-nucleus potential,  $V_Q(r)$ , so that  $V = V_C + V_Q$ . The Coulomb potential between the  $Q$  and the nucleus is given simply by  $V_C(r) = Z_Q Z e^2 / r$  for  $r > r_A$ , where  $r_A = 1.1A^{1/3}$  fm is the radius of the nucleus. It is the attractive well from  $V_C(r)$  which allows the quark atoms to have bound states.  $V_Q(r)$ , on the other hand, is not based on the electromagnetic force but, instead, on the strong force and thus has a somewhat different contour. Even though the nucleons are colorless, when the free  $Q$  is close to a nucleon, there is a color polarization so that the  $Q$  sees the

quarks inside the nearby nucleon. Thus we approximate the quark-nucleus potential as proportional to the quark-quark potential. Now the quark-quark QCD potential begins with a "Coulomb"-like attraction and then increases linearly, providing a large constant restoring force consistent with QCD theories. For our quark-nucleus potential, we neglect the attractive, short-range forces and let the potential increase linearly starting from  $r = r_A$ . Then, at some threshold  $r = \lambda_G$ , the breaking of  $SU^{\text{color}}(3)$  symmetry occurs and there is a strong repulsion force, given by an exponentially decreasing potential. So,

$$V_Q(r) = V_0(r - r_A) \exp[-(r - r_A)/\lambda_G], \quad r > r_A. \quad (1)$$

Here,  $\lambda_G$ , the QCD-breaking distance, is related to the mass  $m_G$  of the five massive gluons resulting from the lifting of QCD degeneracy [3,8] by

$$\lambda_G = \hbar c / m_G. \quad (2)$$

$V_0$  is a scaling factor determining how steep the QCD attraction will be; as we show, the energies are not sensitive to  $V_0$ . The neglect of the attractive forces for  $r < r_A$  is justified since the  $Q$  never sees this region, the probability that the  $Q$  will tunnel through the barrier being extremely small (e.g., for a  $Q^4\text{He}$  with  $m_Q = 4$  GeV,  $Z_Q = -\frac{4}{3}$ , and  $V_0 = 1.5$  GeV/f, the tunneling probability is  $\approx 10^{-200}$ , corresponding to an atom lifetime of  $10^{150}$  years). Because the potential inside the nucleus is unknown, the wave function is taken to be zero at  $r_A$  so

TABLE I. The average radii of  $Q^4\text{He}$  atoms are listed. ( $V_0 = 1$  GeV/f,  $Z_Q = -\frac{4}{3}$ ,  $m_Q = 1.5$  GeV.) Without the quark-nucleus repulsion,  $\langle r \rangle$  would be greatly reduced (e.g., to 9 f for  $m_Q = 1.5$  GeV).

$m_Q$ (GeV)	$\langle r \rangle$ (f)
1.5	86
2.0	105
4.0	188
6.0	276
8.0	368
9.5	438

that the wave function cannot penetrate into the nucleus. The potential in the region  $r > r_A$  thus becomes

$$V(r) = V_0(r - r_A) \exp[-(r - r_A)/\lambda_G] + Z_Q Z e^2 / r. \quad (3)$$

The most prominent feature of  $V$  is that the repulsion due to  $V_Q$  acts like an impenetrable barrier as seen in Figs. 1(a) and 1(b), so the binding energy of the quark atom will be much less than the corresponding energy if  $V$  con-

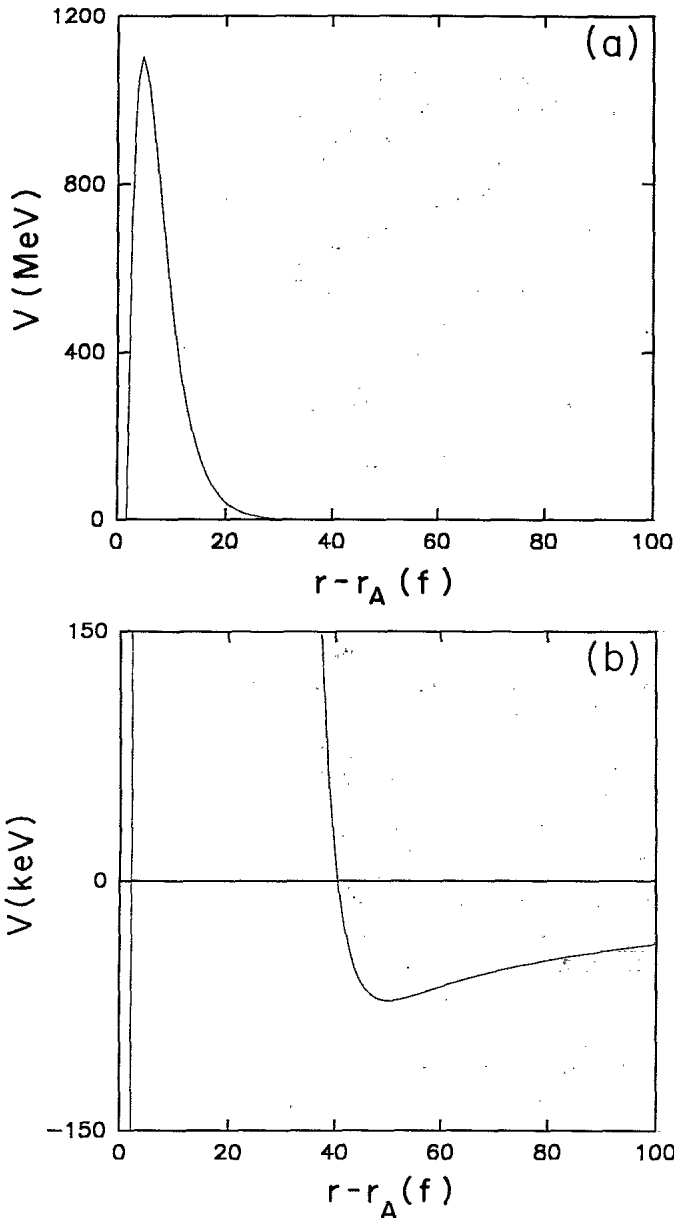


FIG. 1. (a) The potential, Eq. (3), for a  $Q^4\text{He}$  atom is graphed with  $m_Q = 1.5 \text{ GeV}$ ,  $V_0 = 1 \text{ GeV/f}$ , and  $Z_Q = -\frac{4}{3}$ . Note that since the maximum of the potential is on the order of GeV's, the potential acts as a barrier. (b) The same potential as in a with a much larger scale to show the attractive well. Note the near vertical curve at  $r \approx 40 \text{ f}$ . This allows the actual potential to be approximated by an infinite barrier.

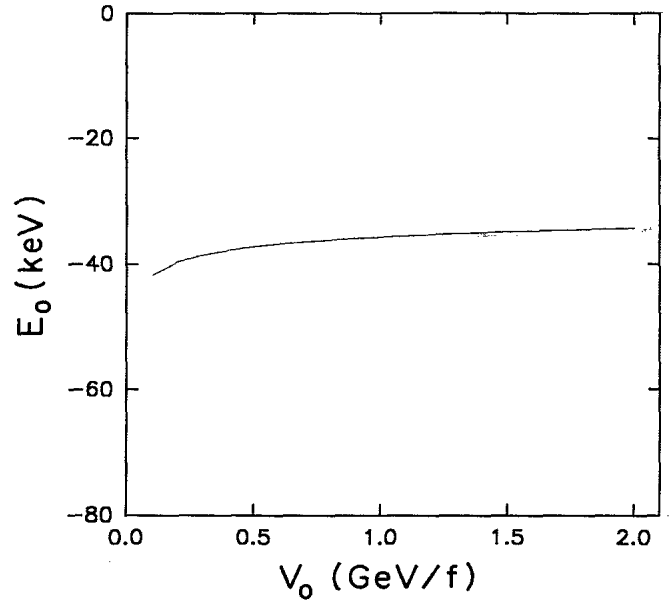


FIG. 2.  $E_0$  is plotted for a  $Q^4\text{He}$  atom while  $V_0$  is varied. ( $m_Q = 1.5 \text{ GeV}$ ,  $Z_Q = -\frac{4}{3}$ .) We see that the energy levels depend very little on large variations in  $V_0$ . (See text.)

sisted only of the attractive  $V_C$ . Because of this barrier, it is very difficult to combine two quarks which have already been separated.

The quark atoms we chose to study were  $Q^1\text{H}$ ,  $Q^4\text{He}$ ,  $Q^{12}\text{C}$ , and  $Q^{16}\text{O}$  as an illustration. (In this paper, only the energies for  $Q^4\text{He}$  and  $Q^{12}\text{C}$  atoms are shown, however, since the other energies vary with  $m_Q$  in the same

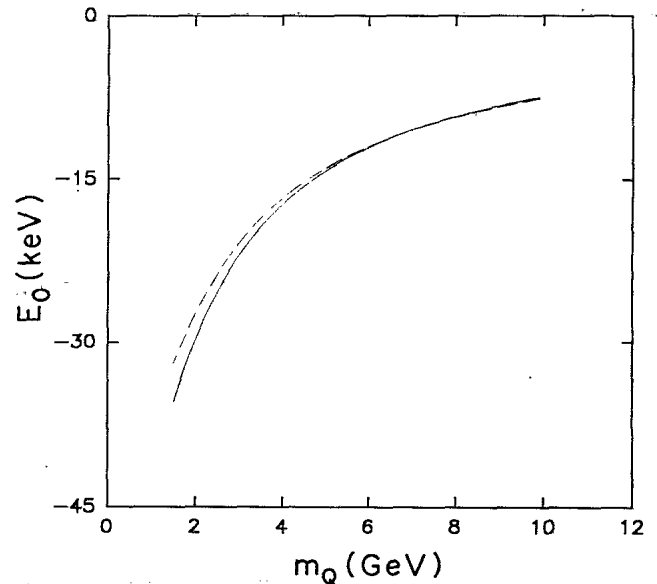


FIG. 3.  $E_0$  for a  $Q^4\text{He}$  atom is plotted against the mass of the  $Q$ . ( $V_0 = 1 \text{ GeV/f}$ ,  $Z_Q = -\frac{4}{3}$ .) The dashed points are the energies if the quark-nucleus repulsion is replaced by an infinite barrier. (See text.) If there was no repulsive quark-nucleus potential  $V_Q$ , the binding energy would increase (e.g., to about 200 keV for  $m_Q = 1.5 \text{ GeV}$ ).

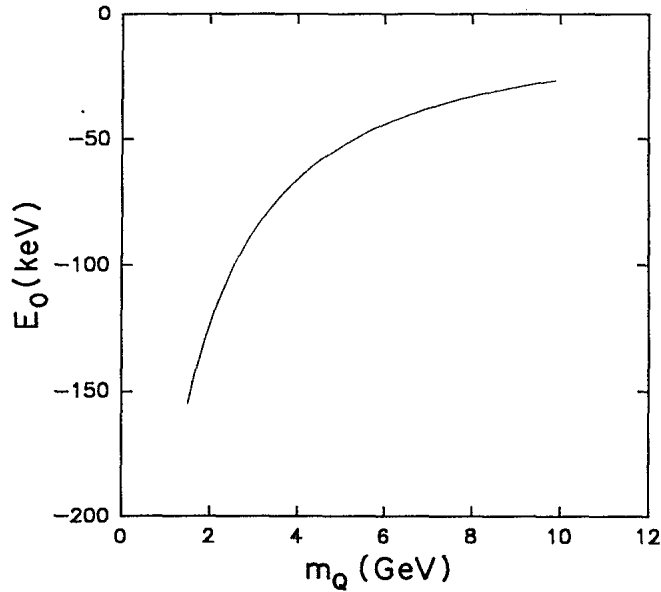


FIG. 4.  $E_0$  for  $Q^{12}\text{C}$  atom is plotted against  $m_Q$ . ( $V_0=1$  GeV/ $f$ ,  $Z_Q=-\frac{4}{3}$ .)

manner.) These calculations could be extended similarly for higher  $Z$  nuclei. We calculate the lowest energies  $E_0$  of the quark atoms while varying  $m_Q$  from 1.5 GeV to 10 GeV. Since we expect  $m_Q$  to depend on the size  $1/m_G$  of the broken  $\text{SU}^{\text{color}}(3)$  field, we take  $m_Q$  proportional to  $1/m_G$ , with the constant of proportionality chosen as follows: A typical meson has a radius of 1 fm and a mass of 500 MeV. Thus, we want  $\lambda_G=1$  fm to correspond to  $m_Q=500$  MeV. But by (2),  $\lambda_G=1$  fm when  $m_G=200$  MeV. So, we set

$$m_Q = (500 \text{ MeV})(200 \text{ MeV}/m_G). \quad (4)$$

In order not to contradict experimental limits of  $\lambda_G > 3$  fm [2,8], we must take  $m_Q > 1.5$  GeV. Given  $m_Q$ ,  $\lambda_G$  is uniquely determined (for example,  $m_Q=1.5$  GeV yields  $\lambda_G=3$  fm), and the energy levels are then calculated with a standard search pattern. First, we vary  $V_0$  from 0.1 GeV/ $f$  to 2 GeV/ $f$ . Then  $Z_Q$  was taken to be  $-\frac{1}{3}$ ,  $-\frac{2}{3}$ , or  $-\frac{4}{3}$  (corresponding to  $Q$  being a  $d/\bar{u}\bar{d}$ ,  $\bar{u}$ , or  $\bar{u}\bar{u}$ , respectively).

The results are given in Figs. 2–5. First, notice that the energies  $E_0$  do not depend significantly on  $V_0$  as seen in Fig. 2. Even if  $V_0$  is doubled or tripled, the energy levels change by only a few percent. This shows that although  $V_0$  is not well known, we can still calculate the energies to high accuracy. To understand the reason for this result, we constructed a “hard core” potential  $V_{\text{HC}}(r)$  where we replace the quark-nucleus repulsion

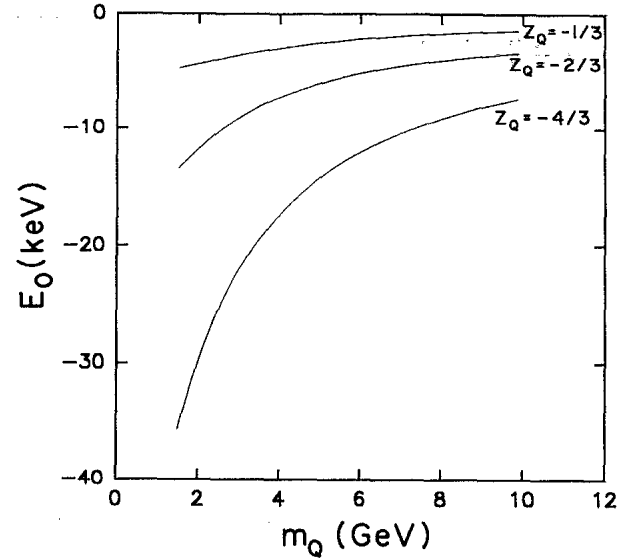


FIG. 5.  $E_0$  is plotted versus  $m_Q$  for a  $Q^4\text{He}$  atom while  $Z_Q$  is varied from  $-\frac{1}{3}$  to  $-\frac{2}{3}$  to  $-\frac{4}{3}$ . ( $V_0=1$  GeV/ $f$ ,  $m_Q=1.5$  GeV.)

with an infinite barrier. Energies using this potential were calculated and closely approximate the actual energies as seen in Fig. 3. Now, if we vary  $V_0$  in Eq. (1) by a large amount, then, due to the exponential factor in the quark-nucleus repulsion, the width of the approximating barrier changes very little. But the energies for  $V_{\text{HC}}(r)$  depend solely on the width of the barrier, so the energies will only vary slightly, too.

The most important aspect of the energies (using the actual potential) is their magnitude as shown in Figs. 3 and 4. Typical values of  $E_0$  for quark atoms range from a few keV to hundreds of keV. If there were no QCD potential  $V_Q$ , though, these energies would be the usual Coulomb result  $E_0 = -(Z_Q Z)^2 \alpha^2 \mu c^2 / 2$ , which is on the order of MeV. So, we demonstrate that the binding energies of the atoms are reduced by more than an order of magnitude due to the repulsion potential  $V_Q$ . Also, the  $E_0$  are inversely proportional to  $m_Q$  as expected. If the charge  $Z_Q$  of the quark is changed, the energy levels change proportionally as seen in Fig. 5.

In addition to the energy levels, we also calculated  $\langle r \rangle$ , the expectation value of the radius. The radii are on the order of hundreds of fm as seen in Table I. If the potential consisted of only the Coulomb attraction,  $\langle r \rangle$  would shrink by about an order of magnitude.

Our findings show that the quark-nucleus repulsion potential dramatically reduces the quark atom binding energy. This may be significant in fusion catalysis schemes involving free quarks and in the process of isolating the free charge from bulk material.

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