Abstract

Cartels participating in procurement auctions frequently use bid rotation or incumbency priority to allocate market shares. However, establishing a tight link between observed allocation patterns and firm conduct has been difficult: there are cost-based competitive explanations for such behavior. We show that by focusing on the set of auctions in which the winning and losing bids are very close, it is possible to distinguish allocation patterns reflecting cost differences across firms from patterns reflecting non-competitive behavior. We apply our tests to two datasets: the sample of Ohio school milk auctions studied in Porter and Zona (1999), and a sample of municipal procurement auctions held in Japan.

KEYWORDS: procurement, collusion, backlog, incumbency, regression discontinuity, antitrust.
1 Introduction

The ability of competition authorities to proactively detect and punish collusion is crucial for achieving the goal of promoting and maintaining competition. Not only do the possibility of detection and prosecution serve as strong deterrents against collusion, they also affect the incentives of firms in existing cartels to apply for leniency programs. Successful identification of cartels thus deters collusive activity and complements enforcement programs.

In the absence of concrete leads, using data-driven screens to flag suspicious firm conduct can be useful for regulators as a first step in identifying collusion. While screens cannot substitute for direct evidence of collusion such as testimonies and records of communication, they can provide guidance on which markets or firms to focus investigation. A growing number of countries are adopting algorithm-based screens that analyze bidding data from public procurement auctions to flag suspicious behavior.\(^1\) More recently, the U.S. Department of Justice announced the formation of a procurement collusion strike force whose goal includes bolstering “data analytics employment to identify signs of potential anticompetitive, criminal collusion.”\(^2\) Imhof et al. (2018) describes an antitrust investigation initiated on the basis of statistical screens and resulting in successful cartel prosecution. The results from screens can be used in court to obtain warrants, or to support civil antitrust litigation as well as private litigation.\(^3\)

Screening cartels can also be useful to stakeholders other than antitrust authorities. For example, screening can help procurement offices counter suspected bidding rings by soliciting new bidders more aggressively or adopting auction mechanisms that are less susceptible to collusion. In large decentralized organizations, collusion may be organized by firm employees against the will of CEOs (Sonnenfeld and Lawrence, 1978).\(^4\) In that context, screening tools

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\(^{1}\) A report by the OECD (OECD, 2018) gives a brief description of the screening programs used in Brazil, Switzerland and the UK.

\(^{2}\) Announcement of the Antitrust Division’s Procurement Collusion Strike Force, November 22, 2019.

\(^{3}\) Baker and Rubinfeld (1999) give an overview on the use of statistical evidence in court for antitrust litigation.

\(^{4}\) See also Ashton and Pressey (2012), who study 56 international cartels investigated by the EU. They find that there is involvement of individuals at the most senior levels of management (CEOs, chairpersons, etc.) in about half of those cases.
can help internal auditors and compliance officers contain collusive practices initiated by employees.

Because bidding rings often adopt rotation schemes or give priority to incumbents in project allocation, bid rotation and incumbency advantage are very often suggested as indicators of collusion. However, it is well known that there are non-collusive cost-based explanations for these allocation patterns. Bid rotation can arise under competition if marginal costs increase with backlog. Incumbency advantage can be explained by cost asymmetries among competitive firms or by learning-by-doing. Hence, establishing a tight link between these bidding patterns and collusion has been difficult. As Porter (2005) describes, “An empirical challenge is to develop tests that can discriminate between collusive and non-cooperative explanations for rotation or incumbency patterns.”

We show that it is possible to discriminate between competitive and non-competitive bid rotation and incumbency patterns using the logic of regression discontinuity designs (Thistlethwaite and Campbell, 1960). We compare the backlog and incumbency status of a bidder who wins the auction by a small margin to those of a bidder who loses by a small margin. Although bids are endogenous, we show that under an appropriate notion of competition, the probability that a bidder wins or loses an auction conditional on close bids approaches 50%, regardless of the bidders’ characteristics (e.g., the size of backlog, incumbency status, etc.). Winning and losing are “as-if-random” conditional on close bids. As a result, under competition, even if backlog or incumbency status are correlated to costs, the differences in these variables between close winners and close losers should vanish as the bid difference between them approaches zero. If instead, bids are generated by collusive bidding, the differences in these variables between close winners and close losers need not disappear. For example, if the bidding ring always allocates projects to the incumbent bidder, close winners will be incumbents with significantly higher probability than close losers. Our tests of non-competitive behavior seek to detect discontinuities in the distribution of economically relevant covariates around close winners and close losers.

5See, for example, a document called “Red Flags Of Collusion” published by the U.S. DOJ listing patterns that are suggestive of collusion.
We illustrate our test using two datasets. First, we consider the sample of Ohio school milk auctions studied by Porter and Zona (1999). Firms located around Cincinnati, Ohio were charged with colluding on hundreds of school milk auctions by allocating markets according to incumbency status (State of Ohio v. Louis Trauth Dairies, Inc. et al). According to the testimony of the representatives of the colluding dairies, the firms colluded by agreeing not to undercut the bid of the incumbent firm that had served a given school district in the previous year. We test whether or not marginal winners are more likely to be incumbents than marginal losers separately for the set of collusive auctions and the set of non-collusive auctions. We find that for collusive auctions, marginal winners are significantly more likely to be incumbents than marginal losers, rejecting the null of competition. In contrast, we do not find statistically significant differences in incumbency status between marginal winners and marginal losers among non-collusive auctions despite the fact that the sample size is more than 10 times bigger.

Second, we apply our tests to a dataset of public procurement auctions held by municipalities from the Tohoku region of Japan. Firms in this dataset have not been prosecuted for collusion, but there are reasons to suspect that it is present. Kawai and Nakabayashi (2018) provide evidence that some of the firms in this dataset colluded over procurement contracts let by the Ministry of Land, Infrastructure and Transportation. Chassang et al. (2019) suggest that non-competitive behavior may have been prevalent in auctions held by a different set of Tohoku municipalities. As we discuss below, the tests presented in the current paper complement previous work by applying specifically in environments where previous tests have no bite.

To proxy for potential collusion, we split the sample of municipal auctions into high and low bid groups depending on whether the winning bid of an auction is above or below the median winning bid for the municipality holding the auction. Because the purpose of collusion is to elevate prices, we expect a higher concentration of collusive auctions in the

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6More precisely, we apply the tests separately for the set of auctions in which all of the bidders were implicated and the set of auctions in which none of the participants were implicated.

7We normalize raw bids with each auction’s reserve price to make bids more comparable.
high bid group. As in the case of collusive Ohio school milk auctions, we find that marginal winners are significantly more likely to be incumbents than marginal losers in the high bid group. We also find that backlog is significantly lower for marginal winners compared to marginal losers in this group, suggesting the presence of cartels using bid rotation. We do not find statistically significant differences in the characteristics of marginal winners and marginal losers for the low bid group.

We believe that our approach is potentially useful for practitioners. First, our test formalizes intuitive ideas often mentioned by antitrust agencies. Second, the test is easy to implement and requires no sophisticated programming. Third, our approach does not require detailed data on project or bidder characteristics because the regression discontinuity design makes it less important to control for auction and bidder heterogeneity. Fourth, our approach naturally extends to other types of auctions such as handicap auctions, scoring auctions and all-pay auctions by appropriately modifying the running variable. Finally, our approach can be easily adapted to exploit other markers of collusion. Imagine a cartel is suspected of using geographic segmentation to allocate projects. With data on the location of firms and project sites, one could assess whether or not close winners are located nearer the project site than close losers. Another possible marker of collusion is the extent of subcontracting and joint bidding. If procurement agencies require the list of subcontractors to be specified at the time of bidding, one can test whether or not marginal winners have more subcontractors than marginal losers.

**Literature.** Our work fits in the industrial organization literature interested in detecting collusion in auctions and markets. Pioneering work in this literature include Hendricks

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9For example, the Department of Justice maintains a document called “Price Fixing, Bid Rigging, and Market Allocation Schemes: What They Are and What to Look For” , in which it states “Subcontracting arrangements are often part of a bid-rigging scheme.” Similar statements are found in a report by the OECD (2013). See also Conley and Decarolis (2016) for a discussion of subcontracting and collusion.

10For example, “Subletting and Subcontracting Fair Practices Act” (Public Contract Code 4100 et seq,) of California requires that “any person making a bid or offer to perform the work, shall, in his or her bid or offer, set forth ... (T)he name, the location of the place of business, ... of each subcontractor who will perform work or labor or render service to the prime contractor.”

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and Porter (1988), Baldwin et al. (1997), as well as Porter and Zona (1993, 1999). Our contribution is particularly related to Porter and Zona (1993) who study the impact of cost shifters such as backlog and proximity to construction sites on the bids and rank order of bidders in auctions for road pavement projects. They find that the losing bids of suspected ring members do not respond to cost shifters, suggesting that those bids are likely phantom bids. Although both Porter and Zona (1993) and our paper study the relationship between the rank order of bids and possible cost shifters to screen for collusion, the underlying idea behind the proposed tests are different. Porter and Zona (1993) focus on the lack of incentives among losing cartel bidders to bid in ways that reflect their true costs. Hence, their primary focus is on losing bidders.\textsuperscript{11} The tests that we propose are based on the idea that, under collusion, winners are more likely to be incumbents (under incumbency priority) or have less backlog (under bid rotation) than losers. Hence our primary focus is on differences between winners and losers.

Recent work seeking to detect non-competitive behavior includes Bajari and Ye (2003), Ishii (2009), Athey et al. (2011), Conley and Decarolis (2016), Andreyanov (2017), Schurter (2017), and Kawai and Nakabayashi (2018).\textsuperscript{12} A complementary literature focuses on known cartels and studies the practical details of collusive arrangements: Pesendorfer (2000) studies bidding rings with and without side-payments; Asker (2010) studies knockout auctions among members of a bidding ring. Clark et al. (2018) analyze the breakdown of a cartel and its price implications. Other contributions (see for instance Ohashi (2009), or Chassang and Ortner (2019)) take a design perspective and document how changes in the auction format affect the ability of bidders to maintain collusion.

Finally, this paper complements two papers of ours, Chassang et al. (2019) and Ortner et al. (2020). In Chassang et al. (2019), we document that in a significant subset of procurement auctions held in Japan, winning bids are isolated – there are very few close winners and close losers. We show how this pattern, as well as others, can be exploited to obtain robust

\textsuperscript{11}Porter and Zona (1993) describe their tests as follows: “... our rank-based test is designed to detect differences in the ordering of higher bids, as opposed to the determinants of the probability of being the lowest bid ...”, although parts of their paper analyze the determinants of the winner.

\textsuperscript{12}For a survey of the literature up to the mid 2000s, see Porter (2005) and Harrington (2008).
lower bounds on the share of non-competitive histories. In Ortner et al. (2020), we study the equilibrium impact of screening when cartels can adapt. We show that safe tests, i.e. tests that can be passed with probability one by competitive firms, cannot be exploited by cartels to enhance collusion. In addition, safe tests can strictly reduce the scope of collusion, even when cartels adapt. The tests developed in the current paper can be applied in settings in which the isolated winning-bids anomaly studied in Chassang et al. (2019) is not present. In addition, the tests proposed in the current paper are asymptotically safe, in the sense of Ortner et al. (2020).

2 Framework

The section specifies our model of dynamic procurement. We describe our test of non-competitive behavior in Section 3, and provide theoretical foundations in Section 4. We turn to data in Section 5.

**Game form.** In each period \( t \in \mathbb{N} \), a buyer procures a single item from a finite set \( N \) of potential suppliers. The procurement contract is allocated through a sealed-bid first-price auction with a public reserve price \( r \), which we normalize to 1. Each potential bidder \( i \in N \) decides whether to participate in each auction. Bidders incur a cost \( k > 0 \) for submitting an actual bid \( b_{i,t} \in [0,1] \), and may prefer not to participate, denoted by \( b_{i,t} = \emptyset \).

We denote by \( b_t = (b_{i,t})_{i \in N} \) the profile of bids, and by \( \land b_t \) the lowest bid among participating bidders. This is the winning bid. Ties are broken with uniform probability. We denote by \( b_{-i,t} \equiv (b_{j,t})_{j \neq i} \) bids from firms other than \( i \), and by \( \land b_{-i,t} \equiv \min_{j \neq i} b_{j,t} \) the lowest bid among \( i \)'s participating competitors. Let \( \land b_{-i} > b_i \) denote the event that bidder \( i \) wins the contract, i.e. \( b_i \) is the lowest bid and possible ties are broken in favor of bidder \( i \). Bids are publicly revealed at the end of each period.

**Costs and payoffs.** In each period a profile of procurement costs \( c_t = (c_{i,t})_{i \in N} \in \mathbb{R}^N \) is realized. Firm \( i \)'s payoff at time \( t \) from submitting bid \( b_i \in [0,1] \) is \( 1_{\land b_{-i} > b_i} (b_i - c_{i,t}) - k \).
Bidders discount future payoffs using discount factor $\delta < 1$.

**Uncertainty and information.** In each period $t$, before bidding, each bidder observes a state $\theta_t \in \Theta$, with $\Theta$ finite, summarizing the state of the industry. The state follows an endogenous Markov chain: $\theta_{t+1}$ is distributed according to a c.d.f. $F(\cdot | \theta_t, b_t)$, depending only on the previous state $\theta_t$ and the profile of bids $b_t$. In addition to state $\theta_t$, each bidder $i$ privately observes a signal $z_{i,t} \in Z$, with $Z$ finite. The distribution of signal profile $z_t = (z_{i,t})_{i \in N}$ depends only on $\theta_t$ but is otherwise unrestricted. Signals may be arbitrarily correlated.

Costs $c_t = (c_{i,t})_{i \in N} \in \mathbb{R}^N$ are drawn independently conditional on state $\theta_t$, and on each private signal $z_{i,t}$. In particular, we have that

$$c_{i,t} | \theta_t, z_{i,t} \sim c_{i,t} | \theta_t, z_t, c_{-i,t}.$$ 

Bidder $i$’s cost does not provide information about the cost of other bidders beyond the information already provided in state $\theta_t$ and private signal $z_{i,t}$. We assume private values, so that each bidder observes her own costs.\(^{13}\)

Our model nests asymmetric independent private values, correlated values, and complete information. Our model also allows for settings in which a bidder’s procurement costs depend on backlog, incumbency status, or past revenue through the state variable $\theta_t$. For example, $\theta_t$ can be a vector that tracks how many auctions each bidder has recently won to capture the effect of backlog on costs. Alternatively, $\theta_t$ can be a vector that tracks whether or not a given bidder has won a particular type of auction to capture the effect of learning-by-doing. $\theta_t$ can also capture other auction characteristics such as the distance between the project site and each of the bidders or the type of work being procured.

We now discuss what is observed to the econometrician. We denote by $x_{i,t} \in X \subset \mathbb{R}^n$, with $X$ finite, the characteristics of bidder $i$ at time $t$ that the econometrician observes. The

\(^{13}\)Because the signals are allowed to be correlated, $z_{i,t}$ helps bidder $i$ predict the cost of other bidders. The main restriction is that set $Z$ is finite. This ensures that pointwise convergence results established later on hold uniformly over histories.
observables at time $t$, $x_t = (x_{i,t})_{i \in N}$, can be a subset of $\theta_t$, a coarsening of $\theta_t$, or any variable that is predetermined at the time of bidding.$^{14}$ In our application, $x_{i,t}$ corresponds to measures of bidder’s backlog or incumbency status. Given that many bidders in our data work on projects that are not in our dataset (e.g., construction work for other firms), our measures of backlog and incumbency are at best imperfect measures of backlog or incumbency status that are relevant for bidders’ costs. We emphasize that we allow for unobserved heterogeneity and measurement error by allowing the econometrician to observe only a subset of $\theta_t$ or an imperfect measure of $\theta_t$.

**Strategies and solution concepts.** A Markov strategy $\sigma_i$ is a mapping from information $h_{i,t} = (\theta_t, z_{i,t})$ and costs $c_{i,t}$ to bids $b_{i,t} \in [0, 1] \cup \{\emptyset\}$.

Throughout the paper, we focus on Markov strategies, but not necessarily on Markov Perfect Equilibria (i.e., perfect Bayesian equilibria in Markov strategies). Defining an appropriate solution concept capturing the essence of competition turns out to be tricky. While Markov perfect equilibrium (Maskin and Tirole, 2001) rules out collusive strategies when the state evolves exogenously (as in Chassang et al., 2019), this is not the case when the payoff-relevant state depends on past actions. We discuss appropriate solution concepts in Section 4 after presenting a clarifying example.

## 3 Empirical Strategy

We now delineate our test of non-competitive behavior and clarify the goal of our theoretical analysis.

Consider the problem of assessing whether firms in a given industry are engaging in collusive bid rotation. Empirically, this implies that bidders with low backlog (less likely to have won in the recent past) are more likely to win than bidders with a large backlog. The difficulty is that there may also be competitive reasons for this pattern. Suppose that firms’

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$^{14}$More generally, $x_{i,t}$ can be any garbling (in the sense of Blackwell (1953)) of bidder $i$’s information at the time of bidding in period $t$. 

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procurement costs are increasing with backlog. Even if firms are competing, on average, firms with lower backlog will have a lower cost and be more likely to win an auction than firms with higher backlog. In this environment, a test seeking to detect collusive bid rotation by comparing the backlog of winners and losers would yield false positives.

Our proposal is to compare the backlog of a selected group of firms: bidders that win or lose by a small margin. Intuitively, conditioning on close bids allows us to control for potential cost differences. The implicit hypothesis is that under competition, the identity of the winner is as-if-random conditional on close bids. As a result, marginal winners and marginal losers should be statistically similar. If instead, close winners have consistently lower backlogs than close losers, this is evidence of collusive bid rotation.

We operationalize this idea as follows. Denote by $\Delta_{i,t} \equiv b_{i,t} - \wedge b_{-i,t}$ the difference between the bid of firm $i$, and the most competitive alternative bid at time $t$. If $\Delta_{i,t} < 0$, bidder $i$ wins the auction, if $\Delta_{i,t} > 0$, bidder $i$ loses. Let $x_{i,t}$ be a measure (observed by the econometrician) of firm $i$’s backlog before bidding at time $t$ (alternatively it could be incumbency, or another relevant covariate). We define coefficient $\beta$ as the difference in average backlog between close losers and close winners:

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\beta = \lim_{\epsilon \to 0^+} \mathbb{E}[x_{i,t} | \Delta_{i,t} = \epsilon] - \lim_{\epsilon \to 0^-} \mathbb{E}[x_{i,t} | \Delta_{i,t} = \epsilon].
$$

We test the null of $\beta = 0$. When $x$ denotes backlog, we expect $\beta$ to be strictly positive under bid rotation. When $x$ denotes incumbency status, we expect $\beta$ to be strictly negative if the cartel allocates market share according to incumbency. Figure 1 foreshadows the results of Section 5 using a dataset of Japanese procurement auctions. The figure is a binned scatter plot that illustrates the relationship between bidder $i$’s 90-day backlog at time $t$ against $\Delta_{i,t}$. The null of $\beta = 0$ is rejected: the average backlog is discontinuous around $\Delta_{i,t} = 0$. Close winners have a significantly lower backlog than close losers.

A heuristic motivation. The null hypothesis of $\beta = 0$ relies on the intuition that conditional on close bids, allocation should be as-if random under competition. This argument
Note: For each firm $i$ and auction $t$, the standardized backlog of firm $i$ at $t$ is the Yen denominated amount of work it won in the 90 days prior to auction $t$, re-expressed in units of standard deviation from the firm’s time-series average. The figure is a binned scatter plot of this measure against $\Delta_{i,t}$. See section 5 for details.

Figure 1: Binned Scatter Plot of Standardized Backlog: Japanese Municipal Auctions.

can be made formal starting from the premise that under competition, a bidder’s demand conditional on information $h_{i,t} = (\theta_t, z_{i,t})$ is sufficiently smooth.

For all histories $h_{i,t} = (\theta_t, z_{i,t})$ and bids $b \in [0, 1]$, define bidder $i$’s residual demand as

$$D_i(b|h_{i,t}) \equiv \text{prob}(\land b_{-i} \succ b|h_{i,t}).$$

$D_i(b|h_{i,t})$ is the probability with which firm $i$ expects to win the auction at history $h_{i,t}$ if she places bid $b$. The probability that bidder $i$ wins conditional on submitting a close bid satisfies

$$\text{prob}(i \text{ wins | } h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| \leq \epsilon) = \frac{D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}. \quad (2)$$

It follows that whenever $D_i$ is strictly decreasing and continuously differentiable, then for $\epsilon$ small, the probability of winning conditional on close bids is approximately $1/2$, regardless
of history $h_{i,t}$. This is a straightforward consequence of the fact that the numerator on the right-hand side of (2) is approximately $\epsilon D'_i(b_{i,t}|h_{i,t})$ and the denominator is approximately $2\epsilon D'_i(b_{i,t}|h_{i,t})$.

**Lemma 1** (smooth demand). Assume that $D_i(\cdot|h_{i,t})$ is differentiable, with $D'_i(b_{i,t}|h_{i,t})$ strictly negative and continuous in bids $b_i \in [0,1]$. For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that for all histories $h_{i,t}$,

$$\left| \left| \text{prob}(i \text{ wins} \mid h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| \leq \epsilon) - \frac{1}{2} \right| \right| \leq \eta.$$  

(3)

Lemma 1 implies the following corollary.

**Corollary 1.** For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that for all $x \in X$,

$$|\text{prob}(x_i = x \mid \Delta_{i,t} \in (0, \epsilon)) - \text{prob}(x_{i,t} = x \mid \Delta_{i,t} \in (-\epsilon,0))| < \eta.$$  

In words, the corollary states that the distribution of $x_{i,t}$ has to be the same for marginal winners and marginal losers. Whenever $X$ is finite, the corollary immediately implies that the expectation of $x_{i,t}$ conditional on $\Delta$ must be continuous around $\Delta = 0$. This is not true in the data illustrated by Figure 1.

**Why equilibrium demand need not be smooth.** One difficulty with this heuristic is that there exist competitive environments in which the premise of smooth demand is false. Consider a one-shot first price auction with an incumbent $I$ and an entrant $E$. Costs to the incumbent and entrant are common knowledge, respectively satisfying $c_I < c_E$. Participation cost $k$ is equal to 0. We show in Appendix B that in any efficient equilibrium, the residual demand faced by bidders conditional on their information is not smooth. It involves either a kink or a discontinuity. In turn, the probability of winning is not independent of bidder characteristics conditional on close bids: the incumbent wins with probability 1, while the entrant wins with probability 0.

A key feature of this example is that participation cost $k$ is zero. Hence, the entrant is
willing to participate, even if she expects to make zero profits in the auction. This gives rise to discontinuities in firms’ residual demand, leading to a violation of Corollary 1. In Section 4 we show that, if participation is costly, Corollary 1 holds under a suitable notion of competition.

**Why this heuristic may be reasonable.** In the next section we show how bidders’ optimality conditions can be exploited to establish a version of Lemma 1 under an appropriate definition of competitive behavior. This being said, the premise that residual demand may be smooth under competition, but not under collusion, can be defended using partial, but perhaps more direct arguments.

As Dyer and Kagel (1996) and Ahmad and Minkarah (1988) describe, the bidding process for construction projects is affected by many seemingly random factors. In competitive environments these random factors are priced into bids and may smooth out bidders’ residual demand. In collusive settings, there is less pressure to price in random factors, since bids are not competitive: cartel members may simply be told how to bid. In fact, there are strong reasons to make bids predictable under collusion: since bids are unrelated to costs, accidentally outbidding an opponent can lead to costly misallocation, or even price wars (Athey et al., 2004). As a result, residual demand need not be smooth under collusion.

4 Theoretical Foundations

In this section we provide additional theoretical foundations for the hypothesis that assignment conditional on close bids should be as-if random. Our analysis exploits incentive compatibility constraints specific to first-price auctions to establish such a result. We begin with a discussion of an appropriate solution concept capturing competitive behavior in dynamic auctions.

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15 See Kawai and Nakabayashi (2018) for a detailed discussion of these two papers.
16 A behavioral model of Samuelson (2005) captures these ideas.
4.1 Competitive Behavior

A strategy profile $\sigma = (\sigma_i)_{i \in N}$ is a Markov perfect equilibrium (MPE) if it is a perfect Bayesian equilibrium in Markov strategies. MPEs have received much attention in the empirical industrial organization literature studying dynamic oligopolistic competition, starting with Ericson and Pakes (1995). However, depending on the richness of Markov state $\theta_t$, MPE may in fact support collusive behavior. We now describe a collusive MPE which clarifies that MPE is not the right solution concept for our analysis and suggests potential behavior restrictions capturing competition.

A collusive Markov perfect equilibrium. The following example satisfies all assumptions made so far, including positive participation costs. Three bidders $i \in \{1, 2, 3\}$ compete for contracts. The state at time $t$ takes the form $\theta_t = (\text{winner at } t-2, \text{winner at } t-1, \text{winning bid at } t-1)$. To keep the example minimal, we assume that at each time $t$, all firms have the same cost $c_t = c(\theta_t)$, with $c(\theta) \leq \bar{c} < r = 1$ for all $\theta$. Bidders incur a small cost $k > 0$ if they participate, and have access to a public randomization device. Pick a target bid $b \in (\bar{c}, 1)$ that ensures bidders have positive profits. For a discount factor $\delta$ close enough to 1, the following is a MPE:

- if the past two winners are different, and the last winning bid is equal to $b$, then the most recent losers participate with probability 1 and the most recent winner participates with probability $\alpha \in (0, 1)$; all participants bid $b$;

- if the past two winners are the same, and the last winning bid is equal to $b$, then with probability $\gamma(\theta) \in (0, 1)$ firms play the stage game Nash equilibrium, and with probability $1 - \gamma(\theta)$ they play as if the past two winners were different;\(^{17}\)

- if the last winning bid is different from $b$, then firms play the stage game Nash equilibrium.

\(^{17}\)In the stage game Nash equilibrium, firms randomize on their participation decisions, and conditional on participation, use a mixed strategy. Firm’s expected profits from participating are equal to $k$. See Sharkey and Sibley (1993) for details.
Probability $\gamma(\theta) \in (0, 1)$ is chosen to leave the winner last period indifferent between participating or not. Since all participating bidders make positive expected profits, they have incentives to bid even when they face a small participation cost.

Bidders collude using (stochastic) bid rotation. Conditional on close bids, the probability with which a past winner wins the auction is strictly smaller than the probability with which a past loser wins the auction. Indeed, a past winner always faces two competitors, while a past loser sometimes faces only one competitor. An implication is that Corollary 1 fails under this MPE: conditional on close bids, winners have on average a lower backlog than losers.

We highlight the role of continuation values in sustaining collusion. A winner’s continuation value is discontinuous in her own bid. A loser’s continuation value increases discontinuously when the winner’s bid moves from slightly below $b_i$ to $b_i$. These discontinuities in continuation values help firms internalize other firms’ profits, and dissuade bidders from undercutting.

**Competition.** The previous analysis suggests that an appropriate notion of competition needs to impose constraints on the continuation values needed to support stage-game bidding behavior. We propose a criterion referred to as *competitive enforcement*.

**Definition 1 (competitive enforcement).** We say that a Markov strategy profile $\sigma = (\sigma_i)_{i \in N}$ is competitively enforceable if and only if for every bidder $i \in N$ there exists a continuation value $V_i : (\theta, 1_{b_i < \wedge b_{-i}}, \wedge b) \mapsto \mathbb{R}$ such that:

(i) For every history $h_i = (\theta, z_i)$, bid $b_i = \sigma_i(h_i, c_i)$ solves

$$\max_{b_i \in [0, 1]} \mathbb{E}_\sigma \left[ (b_i - c_i)1_{b_i < \wedge b_{-i}} + \delta V_i(\theta, 1_{b_i < \wedge b_{-i}}, \wedge b) - k1_{b_i \neq \emptyset} | \theta, z_i \right].$$

(ii) Given state $\theta$, a winner’s value $V_i(\theta, 1, b_i)$ is continuously differentiable and weakly increasing in winning bid $b_i$; a loser’s value $V_i(\theta, 0, \wedge b)$ is Lipschitz continuous and weakly decreasing in winning bid $\wedge b_{-i}$.
Point (i) of Definition 1 is without loss of generality whenever the distribution of $\theta_{t+1}$ depends on $\theta_t$, the identity of the winner at time $t$, and the value of the winning bid at $t$. Point (ii) need not always be satisfied. The collusive MPE described previously fails point (ii) and not point (i). The rationale behind point (ii) is to rule out behavior that can only be enforced using value functions that exhibit fine dependency on the winning bid, or internalize other bidders’ revenue. Point (ii) can be interpreted as an opposite interest condition: given state $\theta_t$, bidders’ continuation values express preferences for higher own revenue, and lower revenue for their competitors.\(^{18}\)

We note that condition (ii) still allows for backlog, learning-by-doing and investment to affect future costs. Changes in marginal cost from backlog and learning-by-doing are captured by the fact we allow $V_i$ to depend on whether bidder $i$ wins the auction or not, i.e., on $1_{b_i < \wedge b_{-i}}$. We allow $V_i$ to depend on the winning bid capturing the possibility that auction revenues may be invested to lower future costs.

It is helpful to relate competitive enforcement and MPE. In general, there is no inclusion relationship between competitively enforceable behavior and MPE. As the collusive MPE example shows, an MPE is not necessarily competitively enforceable. Conversely, since competitive enforcement does not require continuation values to be consistent with rational anticipations, competitively enforceable behavior need not be a MPE. However, we can show that MPE can satisfy competitive enforcement in the following environments:

- if state $\theta_t$ evolves exogenously;

- in large market settings, in which the distribution of opponents’ types is stationary and independent of the current auction’s winning bid; these are circumstances in which oblivious equilibrium (Weintraub et al., 2008) is also an MPE;

- if there are only two participating bidders in each auction, and $\theta_{t+1}$ depends on $\theta_t$ and the winner’s identity at $t$;

\(^{18}\)Our theoretical analysis also holds if we assume the opposite preferences: continuation values express preferences for lower own revenue, and higher revenue for their competitors. This reinforces the interpretation of competitive enforcement as an opposite interest condition.
• if bidders are ex-ante symmetric, and their cost distribution depends only on whether they won or lost last period’s auction.

In these environments, restrictions on state dynamics ensure that MPE rules out collusive behavior.

### 4.2 Equilibrium beliefs conditional on close bids

We now show that when play is competitively enforceable, allocation conditional on close bids is as-if random. For the results that follow, we maintain the assumption that bidders incur a strictly positive participation cost (i.e., \(k > 0\)). This rules out discontinuities in demand in competitive settings as in the example in Section 3.

**Proposition 1** (equilibrium beliefs conditional on close bids). Assume that strategy profile \(\sigma = (\sigma_i)_{i \in N}\) is competitively enforceable. For all \(\eta > 0\) there exists \(\epsilon > 0\) small enough such that, for all histories \(h_{i,t} = (\theta_{t}, z_{i,t})\) and bid \(b_{i,t} \in (\epsilon, 1 - \epsilon)\),

\[
\text{prob}_\sigma(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) \geq 1/2 - \eta.
\]

**Proof heuristic:** For simplicity, we consider the special case in which continuation values do not depend on the winning bid (Appendix B deals with the more general case):

\[
V_i(\theta, 1_{b_i < \wedge b_{-i}}, \wedge b) = V_i(\theta, 1_{b_i < \wedge b_{-i}}).
\]

Bidder \(i\)’s payoff from bid \(b \in (\epsilon, 1 - \epsilon)\) at history \(h_{i,t} = (\theta_{t}, z_{i,t})\) can be written

\[
U_i^\sigma(b|h_{i,t}) = \mathbb{E}_\sigma [(b - \kappa_{i,t})1_{b_i < \wedge b_{-i,t}}|h_{i,t}] + \delta V_i(\theta_{t}, 0) - k
\]

\[
= D_i(b|h_{i,t})(b - \kappa_{i,t}) + \delta V_i(\theta_{t}, 0) - k
\]

where \(\kappa_{i,t} \equiv c_{i,t} - \delta(V_i(\theta_{t}, 1) - V_i(\theta_{t}, 0))\) is bidder \(i\)’s cost of winning the auction, including its impact of continuation values. Note that firm \(i\) would obtain a payoff of \(\delta V_i(\theta_{t}, 0)\) if she didn’t
submit a bid. Hence, bidder $i$’s participation constraint implies that $D_i(b_{i,t}|h_{i,t})(b_{i,t} - \kappa_{i,t}) \geq k > 0$, so that $b_{i,t} - \kappa_{i,t} \geq k$.

Since bid $b_{i,t}$ is optimal, for all $\epsilon > 0$ we have that

$$U_i(b_{i,t}|h_{i,t}) \geq U_i(b_{i,t} + \epsilon|h_{i,t})$$

$$\iff D_i(b_{i,t} + \epsilon|h_{i,t})(b_{i,t} + \epsilon - \kappa_{i,t}) \leq D_i(b_{i,t}|h_{i,t})(b_{i,t} - \kappa_{i,t}), \quad (4)$$

and

$$U_i(b_{i,t}|h_{i,t}) \geq U_i(b_{i,t} - \epsilon|h_{i,t})$$

$$\iff D_i(b_{i,t} - \epsilon|h_{i,t})(b_{i,t} - \epsilon - \kappa_{i,t}) \leq D_i(b_{i,t}|h_{i,t})(b_{i,t} - \kappa_{i,t}). \quad (5)$$

Conditions (2), (4) and (5) imply that

$$\text{prob}(i \text{ wins } | h_{i,t} \text{ and } b_{i,t} - \wedge b_{-i,t} < \epsilon) = \frac{D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}$$

$$= \frac{1 - \frac{D_i(b_{i,t} - \epsilon)}{D_i(b_{i,t})}}{1 - \frac{D_i(b_{i,t} + \epsilon)}{D_i(b_{i,t})}} \geq \frac{1 - \frac{b_{i,t} - \kappa_{i,t}}{b_{i,t} + \epsilon - \kappa_{i,t}}}{1 - \frac{b_{i,t} - \kappa_{i,t}}{b_{i,t} + \epsilon - \kappa_{i,t}}} = \frac{1}{2} \frac{b_{i,t} - \kappa_{i,t} - \epsilon}{b_{i,t} - \kappa_{i,t}} \geq \frac{k - \epsilon}{2k} \to \frac{1}{2} \text{ as } \epsilon \downarrow 0.$$ 

Note that the speed of convergence of the lower bound is independent of $b_{i,t}$. Since there are finitely many histories $h_{i,t} = (\theta_t, z_{i,t})$, this concludes the proof. □

Proposition 1 provides a lower bound on firms’ winning probability at any given history, conditional on close bids. Because at most one bidder can win, and because there are at least two close bidders conditional on the existence of close bids, it cannot be that firms’ winning probability (conditional on her information) is frequently much larger than $1/2$. We now make this argument formal. For any $\epsilon > 0$, let $\epsilon$-close denote the event that the winning bid is within $\epsilon$ of the second lowest bid. For any strategy profile $\sigma$ and any $\epsilon > 0$, let $\mathbb{E}_\sigma[\cdot|\text{\epsilon-close}]$ denote the expectation over histories $h$ under $\sigma$, conditional on the event $\epsilon$-close.
Corollary 2. Assume that strategy profile $\sigma = (\sigma_i)_{i \in N}$ is competitively enforceable. For all $\eta > 0$ there exists $\epsilon > 0$ small enough such that

$$
E_\sigma \left[ \left| \text{prob}_\sigma (i \text{ wins } | h_{i,t} \text{ and } b_{i,t} - \land b_{-i,t} < \epsilon) - \frac{1}{2} \right| - \epsilon \text{-close} \right] \leq \eta.
$$

(6)

In words, winning is as-if-random conditional on close bids. An implication of Corollary 2 is that Corollary 1 (Section 3) holds under competitively enforced strategy profiles.

Sample implications. Corollary 2 holds under the joint distribution of bids and histories generated by strategy profile $\sigma$. In empirical applications, however, this distribution is not directly observed and must be replaced by a sample counterpart. In Appendix B we show that if (6) holds under the bidders’ beliefs, then it holds asymptotically under the sample joint distribution of bids $b$ and characteristic $x \in X^N$ observable to the econometrician.\footnote{We establish a similar result in Chassang et al. (2019).}

The reason such a result holds is that bidders get sufficient feedback about past states and outcomes: in our framework, bidders observe both past states $\theta$, and past bids $b$. This prevents bidders from making repeated mistakes about realized bidding profiles and characteristics.\footnote{Bidders also receive feedback from past auctions in our empirical applications. Indeed, municipalities in Japan are usually required to post auction outcomes shortly after each auction, typically within five days.}

5 Empirical Analysis

5.1 Ohio School Milk Auctions

In order to illustrate the validity of our test, we first apply it to the setting of Ohio school milk auctions analyzed by Porter and Zona (1999). Porter and Zona (1999) study bidding on school milk auctions using data collected by the state of Ohio as part of its efforts to sue the dairies for bid rigging. The dataset is an unbalanced panel of milk auctions let by Ohio school districts spanning 11 years between 1980 and 1990 with information on the bids and

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\footnote{We establish a similar result in Chassang et al. (2019).}

\footnote{Bidders also receive feedback from past auctions in our empirical applications. Indeed, municipalities in Japan are usually required to post auction outcomes shortly after each auction, typically within five days.}
the identity of the bidders.\footnote{We use the dataset constructed by Wachs and Kertész (2019).}

Several features of the setting are worth highlighting. First, the auctions are recurring. School districts hold auctions every year, typically between May and August to determine the supplier of milk for the following school year. This allows us to easily track the incumbent firm for a given auction. Second, the dataset includes bids from three bidders located around Cincinnati that were charged for collusion. According to the testimony of the individuals involved, the cartel allocated contracts according to incumbency. Aside from two years (1983 and 1989) during which the cartel broke down, conspirators respected incumbency, with non-incumbents submitting complementary bids.

Table 1 reports the summary statistics. Column (1) reports the summary statistics for all of the auctions in the sample, Column (2) reports summary statistics for the subset of auctions in which only the defendant firms participated (Non-competitive) and Column (3) reports summary statistics for those in which no cartel firm participated (Control). Because the cartel broke down in 1983 and 1989 according to the testimony of the individuals who were involved in collusion, we also report statistics for the sample that excludes years 1983 and 1989 for Columns (2) and (3). We find that, on average, the number of bidders is about 1.86 for the entire sample, and slightly higher for the non-competitive sample than for the control sample. The winning bid, reported in units of dollars per half-pint of milk, is about $0.131 for the entire sample, and slightly higher in the non-competitive sample. Table 1 also reports the average second lowest and third lowest bids.

Table 2 reports the summary statistics on incumbency. We define a bidder to be an incumbent for a given school milk auction if the bidder was the winner of the district’s auction in the previous year. Column (1) corresponds to the set of all auctions in the dataset and Column (2) and (3) correspond to auctions in which all participants were defendants and those in which none of the participants were defendants, respectively. Focusing on the row labeled 1981 in Column (1), we find that there are a total of 185 auctions in which an incumbent firm participates. Out of these auctions, the incumbent won 136 of them, or
<table>
<thead>
<tr>
<th></th>
<th>(1) All Years</th>
<th>(2) Non-Competitive Excl 83,89</th>
<th>(3) Control Excl 83,89</th>
</tr>
</thead>
<tbody>
<tr>
<td># Bidders</td>
<td>1.866</td>
<td>1.983</td>
<td>2.058</td>
</tr>
<tr>
<td></td>
<td>(0.909)</td>
<td>(0.891)</td>
<td>(0.882)</td>
</tr>
<tr>
<td>Winning Bid</td>
<td>0.131</td>
<td>0.136</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>2nd-Low Bid</td>
<td>0.135</td>
<td>0.142</td>
<td>0.144</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>3rd-Low Bid</td>
<td>0.138</td>
<td>0.147</td>
<td>0.149</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.016)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Obs.</td>
<td>3,754</td>
<td>235</td>
<td>189</td>
</tr>
</tbody>
</table>

Note: The first column corresponds to the set of all auctions, the second column corresponds to the set of auctions in which only the defendant firms bid and the last column corresponds to those in which no defendant firm bid.

Table 1: Summary Statistics of Auctions: Ohio School Milk Auctions.

about 74%. Note that we lack the necessary data to define incumbency for 1980, which is the first year of the sample. The fraction of auctions in which the incumbent wins is about 80% in Column (1), but they are generally higher in Column (2) except for 1983 and 1989 (and 1990).

Figure 2 plots the histogram of the running variable, $\Delta_{i,t} = b_{i,t} - b_{-i,t}$. A negative value of $\Delta_{i,t}$ implies that bidder $i$ won auction $t$, and a positive value of $\Delta_{i,t}$ implies that bidder $i$ lost auction $t$. Values of $\Delta_{i,t}$ close to zero correspond to auctions in which the winner was determined by a very small margin. The left panel of Figure 2 corresponds to the full sample, the middle panel corresponds to the sample of non-competitive auctions and the right panel corresponds to the control sample.
<table>
<thead>
<tr>
<th></th>
<th>(1) All Win/Inc Ratio Total</th>
<th>(2) Non-Competitive Win/Inc Ratio Total</th>
<th>(3) Control Win/Inc Ratio Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>.</td>
<td>249</td>
<td>.</td>
</tr>
<tr>
<td>1981</td>
<td>136/185 0.74</td>
<td>273</td>
<td>6/7 0.86</td>
</tr>
<tr>
<td>1982</td>
<td>148/188 0.79</td>
<td>287</td>
<td>9/10 0.90</td>
</tr>
<tr>
<td>1983</td>
<td>162/214 0.76</td>
<td>318</td>
<td>7/10 0.70</td>
</tr>
<tr>
<td>1984</td>
<td>199/249 0.80</td>
<td>339</td>
<td>18/20 0.90</td>
</tr>
<tr>
<td>1985</td>
<td>205/260 0.79</td>
<td>357</td>
<td>18/18 1.00</td>
</tr>
<tr>
<td>1986</td>
<td>242/293 0.83</td>
<td>378</td>
<td>16/19 0.84</td>
</tr>
<tr>
<td>1987</td>
<td>236/287 0.82</td>
<td>411</td>
<td>18/20 0.90</td>
</tr>
<tr>
<td>1988</td>
<td>253/304 0.83</td>
<td>419</td>
<td>18/20 0.90</td>
</tr>
<tr>
<td>1989</td>
<td>257/332 0.77</td>
<td>392</td>
<td>13/19 0.68</td>
</tr>
<tr>
<td>1990</td>
<td>185/247 0.75</td>
<td>331</td>
<td>17/29 0.59</td>
</tr>
<tr>
<td>Obs.</td>
<td>3,754</td>
<td>235</td>
<td>3,267</td>
</tr>
</tbody>
</table>

Note: Column (1) corresponds to the set of all auctions, Column (2) corresponds to the set of auctions in which only the defendant firms bid and the Column (3) corresponds to those in which no defendant firm bid.

Table 2: Summary Statistics on Incumbency: Ohio School Milk Auctions.

Empirical implementation. Recall the definition of coefficient $\beta$,

$$\beta = \lim_{\Delta_{i,t} \searrow 0} E[x_{i,t}|\Delta_{i,t}] - \lim_{\Delta_{i,t} \nearrow 0} E[x_{i,t}|\Delta_{i,t}]$$.
We define the variable $x_{i,t}$ as a dummy variable for incumbency status, i.e., $x_{i,t} = 1$ if firm $i$ is an incumbent in auction $t$, and 0 otherwise. If a cartel allocates contracts to incumbents, we expect $\beta$ to be strictly negative.

We estimate $\beta$ using a local linear regression as follows:

$$
\hat{\beta} = \hat{b}_0^+ - \hat{b}_0^-,
$$

$$
(\hat{b}_0^+, \hat{b}_1^+ = \arg\min \sum_{i,t} (X_{i,t} - b_0^+ - b_1^+ \Delta_{i,t})^2 K\left(\frac{\Delta_{i,t}}{h_n}\right) 1_{\Delta_{i,t}>0}
$$

$$
(\hat{b}_0^-, \hat{b}_1^-) = \arg\min \sum_{i,t} (X_{i,t} - b_0^- - b_1^- \Delta_{i,t})^2 K\left(\frac{\Delta_{i,t}}{h_n}\right) 1_{\Delta_{i,t}<0},
$$

where $h_n$ is the bandwidth and $K(\cdot)$ is the kernel. For our baseline estimates, we use a coverage error rate optimal bandwidth and a triangular kernel with a bias correction procedure as proposed in Calonico et al. (2014). Standard errors are clustered at the auction level. We test the null $H_0: \beta = 0$, against the alternative $H_1: \beta \neq 0$.

**Results.** Table 3 presents the results. Panel (A) reports results estimated on the sample of auctions in which only the defendant firms participated. In Column (1), we use all years between 1980 and 1990 while in Column (2), we exclude 1983 and 1989, the two years in which the cartel purportedly broke down. In both columns, we focus on the sample of auctions in which there is an incumbent. We find that the coefficient of $\beta$ is negative ($-0.315$) and marginally statistically significant ($p = 0.087$) for Column (1). The point estimate implies that the marginal winner is about $31.5$ percentage points more likely to be an incumbent than the marginal loser. The bandwidth used for estimation is $0.004$, or 0.4 cents. In Column (2), we find that the estimate is $-0.357$, and statistically significant at the 5 percent level.

Panel (B) reports findings for the set of control auctions. We find that the regression discontinuity estimate is $-0.056$ in Column (1), which is not statistically different from zero. Because there is no reason to expect 1983 and 1989 to be any different from other years for non-colluding firms, we do not expect any significant differences between Column (1) and Column (2) for Panel (B). Indeed, the estimate of $\beta$ in Column (2) is $-0.077$, and statistically indistinguishable from 0.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Incumbency</td>
<td></td>
</tr>
<tr>
<td></td>
<td>All Years</td>
<td>Exclude 1983 and 1989</td>
</tr>
<tr>
<td>Panel (A) :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-competitive auctions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>$-0.315^*$</td>
<td>$-0.357^{**}$</td>
</tr>
<tr>
<td></td>
<td>(0.176)</td>
<td>(0.182)</td>
</tr>
<tr>
<td>$h$</td>
<td>0.004</td>
<td>0.005</td>
</tr>
<tr>
<td>Obs.</td>
<td>309</td>
<td>266</td>
</tr>
<tr>
<td>Panel (B) :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>$-0.056$</td>
<td>$-0.077$</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>$h$</td>
<td>0.004</td>
<td>0.005</td>
</tr>
<tr>
<td>Obs.</td>
<td>3,053</td>
<td>2,455</td>
</tr>
</tbody>
</table>

Panel (A) corresponds to the sample of auctions in which only the defendant bidders bid. Panel (B) corresponds to the sample of control auctions in which none of the defendant bidders bid. Standard errors are clustered at the auction level and reported in parenthesis. The table also reports the bandwidth $h$ used for the estimation. * denotes significance at 10%, ** denotes significance at 5%, and *** denotes significance at 1%.

Table 3: Regression Discontinuity Estimates: Ohio School Milk Auctions.

Overall, the results of Table 3 suggest that our test has reasonable power and size in practice. Figure 3 illustrates the binned scatter plots that correspond to the results in Table 3. The left panel of the figure corresponds to the sample of non-competitive auctions excluding 1983 and 1989, and the right panel corresponds to the sample of all control auctions. The left panel of the figure displays a visible discontinuity in incumbency status between marginal winners and marginal losers while the right panel of the figure shows a smooth continuous relationship between $\Delta_{i,t}$ and incumbency status.
5.2 Public Procurement Auctions in Japan

Our second dataset consists of bids submitted by construction firms participating in auctions for construction projects let by 16 municipalities in the Tohoku region of Japan. Our baseline sample consists of roughly 11,000 procurement auctions taking place between 2004 and 2018. The total award amount for these auctions is about 232 billion yen, or about \$2.3 billion U.S. dollars. No firm has been charged for colluding in any of the auctions in our sample. However, as we note in the Introduction, results in Kawai and Nakabayashi (2018) and Chassang et al. (2019) suggest that at least some of these auctions might be collusive.

The baseline sample consists of auctions from municipalities in which tests of non-competitive behavior exploiting isolated winning bids (Chassang et al., 2019) do not apply.\(^{22}\)

\(^{22}\)In order to choose the set of municipalities to include in our sample, we first compute the density of the running variable for each municipality. The running variable, \(\Delta_{i,t}\), is defined as \(\Delta_{i,t} = b_{i,t} - \wedge b_{i,t}\) where bids are in percentages of the reserve price. We include the municipality in the sample if the following is satisfied:

\[
0.7 \times \max_{d \in [-3\%, -0.5\%]} f_\Delta(d) \leq \min_{d \in [-0.2\%, 0\%]} f_\Delta(d),
\]

Figure 3: Binned Scatter Plot for Incumbency: Ohio School Milk Auctions
In the Online Appendix, we show that our findings extend to the sample of all of municipalities from which we have obtained data. This is not surprising since it is likely that cartels are operating in the excluded cities.

5.2.1 Data and Empirical Implementation

Data and institutional background. Auctions are first-price sealed bid and the lowest bidder is awarded the project subject to the reserve price. Some of the municipalities use public reserve prices and others use secret reserve prices. For example, in 2012, 7 municipalities used public reserve prices, 8 municipalities used secret reserve prices, and 1 municipality used both. The lowest bid was rejected in about 12.5% of the overall sample. Online Appendix A shows that our findings are qualitatively unchanged for the subset of municipalities with a public reserve price.

We have data on all of the bids, the identity of the bidders, and a brief description of the construction project. Column (1) of Table 4 reports summary statistics of the auctions. On average, the reserve price is about 22.26 million yen, or about $222,600. The average winning bid is about 20.71 million yen. The average ratio of the winning bid to the reserve is about 92.6%. On average, 6.80 bidders participate in each auction. Column (2) reports summary statistics of the bidders in our sample. The average bidder in our sample participates in 22.56 auctions and wins about 3.32 times. The table also reports summary statistics on incumbents.

where \( f_\Delta(d) \) is the density of \( \Delta \). In municipalities with isolated bids, there will be a trough in the density of \( f_\Delta \) around 0, and the inequality is not satisfied. We also drop municipalities in which \( f_\Delta \) exhibits a mass at 0. We do so by running a McCrary test (McCrary, 2008) on the running variable and dropping municipalities with p-values less than 0.05. The auctions in these municipalities have binding price floors.

Proposition 1 and Corollary 2 extend to auctions with secret reserve price, provided we treat the auctioneer as an additional bidder and treat subsequent bidding stages as new auctions (related to past stages via state variable \( \theta \)), and provided one of the following two assumptions holds: (i) the hazard rate of the distribution of reserve prices is sufficiently low; or (ii) the residual demand of each bidder is continuous at all histories.

It is very common in auctions for buyers to retain the option of rejecting the lowest bid when the buyer believes that the price is high. The fraction of auctions in which the low bid is rejected in our sample is comparable to other settings with a secret reserve price. For example, in their study of federal offshore oil and gas drainage lease sales, Hendricks and Porter (1988) report that the most competitive bid was rejected on 7 percent of the wildcat tracts, and on 15 percent of the drainage tracts.
Table 4: Summary Statistics of Auctions and Bidders: Municipal Auctions from Japan.

<table>
<thead>
<tr>
<th></th>
<th>(1) Auctions</th>
<th>(2) Bidders</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reserve (Mil. Yen)</strong></td>
<td>22.26</td>
<td>77.14</td>
</tr>
<tr>
<td><strong>Winning Bid (Mil. Yen)</strong></td>
<td>20.71</td>
<td>71.78</td>
</tr>
<tr>
<td><strong>Win Bid/Reserve</strong></td>
<td>0.926</td>
<td>0.083</td>
</tr>
<tr>
<td><strong># of Bidders</strong></td>
<td>6.80</td>
<td>4.21</td>
</tr>
<tr>
<td><strong>Incumbent Participates (0/1)</strong></td>
<td>0.044</td>
<td>0.204</td>
</tr>
<tr>
<td><strong># of Auctions Participated</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong># of Wins</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Raw Backlog (90-Day)</strong></td>
<td>4.11</td>
<td>17.16</td>
</tr>
<tr>
<td><strong>Raw Backlog (180-Day)</strong></td>
<td>6.45</td>
<td>22.85</td>
</tr>
<tr>
<td><strong>Obs.</strong></td>
<td>11,207</td>
<td>3,377</td>
</tr>
</tbody>
</table>

Note: The reserve price, winning bid, and backlog measures are reported in units of millions of yen.

and the amount of backlog of the firms. We discuss how we define these variables next.

**Empirical implementation.** Our first covariate of interest is the firms’ backlog. We consider both raw backlog and standardized backlog. We define the raw backlog of firm $i$ at auction $t$ as either the 90-day or 180-day cumulative size (measured by the reserve price) of projects won by firm $i$. Denoting the 90-day and 180-day backlog as $x_{i,t}^{B_{90}}$ and $x_{i,t}^{B_{180}}$ respectively, they are expressed as follows:

$$x_{i,t}^{B_k} = \sum_{\tau \in T_k^t} r_{\tau} 1_{b_{\tau} > b_{i,\tau}},$$

where $r_t$ denotes the reserve price of auction $t$ and $T_k^t$ denotes the set of auctions in our sample that take place in the $k(\in \{90, 180\})$ days prior to auction $t$. We make sure not to include auction $t$ in $T_k^t$ since its outcome is not in the information set of bidders at time.
Although the raw backlog is a natural metric for capturing the amount of work recently awarded to a firm, variation in raw backlog captures both intertemporal change in backlog as well as heterogeneity in firm size. In order to construct a measure of backlog that only captures the intertemporal variation, we standardize the raw backlog at the firm level, using its within-firm mean and standard deviation. The 90-day and 180-day standardized backlogs, $x_{i,t}^{B_{90}}$ and $x_{i,t}^{B_{180}}$, are defined as follows:

$$x_{i,t}^{B_k} = \frac{x_{i,t}^{B_k} - \mu_{x_i^{B_k}}}{\sigma_{x_i^{B_k}}} ,$$

where $\mu_{x_i^{B_k}}$ is the within-firm mean of $x_{i,t}^{B_k}$ and $\sigma_{x_i^{B_k}}$ is the within-firm standard deviation of $x_{i,t}^{B_k}$. Because standardized backlog is defined relative to the firm’s own historical average, $x_{i,t}^{B_k}$ is zero if firm $i$’s raw backlog is equal to its time-series average at the time of auction $t$.

We emphasize that all of our backlog measures are, at best, noisy measures of the firms’ true cost-relevant backlog. The length of days we use to define our backlog measures (i.e., 90 days or 180 days) is somewhat arbitrary, and most firms are likely to work on projects that are not included in our sample, and hence, would be missed in our measures of backlog.\footnote{Many bidders who participate in auctions let by municipal governments also participate in auctions that are let by the Ministry of Land Infrastructure and Transportation and prefectural governments. Many firms also do work for other private firms.} This does not invalidate our test. As we discussed in Section 2, variables that are observed to the econometrician can be imperfect and imprecise. Corollary 1 still holds regardless.\footnote{If anything, the less our measures of backlog are related to firms’ true costs, the more plausible it is that differences in backlog between winners and losers suggests collusion.}

Column (2) of Table 4 reports summary statistics of raw backlog in millions of yen. The average 90-day backlog is around 4.11 million yen and the average 180-day backlog is around 6.45 million yen. Standardized backlog averages to zero for each firm by construction.

Another covariate of interest is whether or not a given firm is an incumbent for a given project. We define a firm to be an incumbent if it is the winner of the previous auction.
with the same project name let by the same municipality. To give an example, the city of Miyako in Iwate prefecture held procurement auctions with the project name “Restoration of Yagisawa public housing complex” on 3 occasions, November 22, 2011, September 19, 2012, and December 16, 2014. A firm named Kikuchi Painting won each time. We define this firm to be the incumbent in the second and third auctions. We define all other participants in the second and third auctions to be non-incumbents. We do not define incumbency status for any of the bidders in the first auction. Similarly, if there is only one auction for a given project name in a municipality, we do not define incumbency for any bidders. Column (1) of Table 4 reports summary statistics of incumbency status. There is an incumbent bidder in 4.4% of the auctions in our sample.

The running variable is $\Delta_{i,t} = b_{i,t} - \bar{b}_{-i,t}$, where bids are normalized by the reserve price. The left panel of Figure 4 is the histogram of $\Delta_{i,t}$. The distribution is skewed to the right of zero because the number of bidders is about 6.80 ( $\Delta_{i,t}$ is negative for only one bidder per auction, and it is positive for all of the losing bidders). Because we report our regression discontinuity results separately for the set of auctions in which the winning bid is above and below the median winning bid for the municipality letting the auction, the next two panels of Figure 4 plot the histogram of $\Delta_{i,t}$ separately for the two sets of auctions. The middle panel corresponds to the sample in which the winning bid is below the municipal median and the right panel corresponds to those in which the winning bid is above the median.

As before, we estimate discontinuities in the expectation of $x_{i,t}$ as a function of $\Delta_{i,t}$ using a local linear regression with a coverage error rate optimal bandwidth and a triangular kernel with a bias correction procedure as proposed in Calonico et al. (2014).

\footnote{We restrict our sample to auctions in which bid difference $|\Delta|$ is less than 20% of the reserve price: often, bids that are more than 20% lower than the second lowest bid are likely to be misrecorded.} Standard errors are clustered at the auction level.
Note: The left panel corresponds to the histogram of $\Delta_{i,t}$ for the entire sample. The middle panel corresponds to the sample in which the winning bid is below the median winning bid of the relevant municipality. The right panel corresponds to the sample in which the winning bid is above the median. The histogram is truncated at $\Delta_{i,t} = -0.1$ and $\Delta_{i,t} = 0.1$ for readability.

Figure 4: Histogram of $\Delta_{i,t}$: Municipal Auctions from Japan.

5.2.2 Results

Table 5 reports estimates of discontinuity $\beta$. We distinguish auctions in which the winning bid is above the municipality median (Panel (A)) and below the median (Panel (B)).

We expect the latter set of auctions to be less collusive than the former.

High winning bids. Panel (A), Column (1) of Table 5 reports estimates of discontinuity $\beta$ for the 90-day backlog (measured in millions of yen) for the sample of bids above the city-level median. We find that the 90-day backlog of marginal losers is on average 6.751 million yen higher than that of marginal winners. The estimate is statistically significant at the 5% level. The coverage error rate optimal bandwidth we use is 0.011, or about 1.1% of the reserve price. Column (2) reports estimate $\beta$ for the 90-day standardized backlog. The average standardized backlog of marginal losers is higher than that of marginal winners by 0.244 units of standard deviation. The estimate is statistically significant at the 1% level.

---

28 More precisely, we compute the median winning bid for each municipality. We then divide the auctions according to whether or not the winning bid is higher or lower than the median.

29 Note that the sample sizes for Columns (1) and (2) are slightly different. This reflects the fact that we can define the standardized backlog only for firms that win at least once in our sample. For firms that never win any contracts, the within-firm standard deviation of backlog is zero, and $x_{i,t}$ is undefined.
Columns (3) and (4) report our results for 180-day backlog. The 180-day backlog of marginal losers is on average 12.582 million yen and 0.219 standard deviations higher than that of marginal winners.\textsuperscript{30}

Column (5) reports estimates of $\beta$ using incumbency status as the outcome variable. We find that marginal losers are about 27.1 percentage points less likely to be an incumbent than marginal winners. We only use the set of auctions in which there is an incumbent for estimation. Based on these five regression results, we can confidently reject the null of competition in this sample.

**Low winning bids.** Panel (B) reports the results for auctions in which the winning bid is below the median. Estimates of $\beta$ are not statistically significant for any of the outcome variables at the 5\% level. Only for incumbency, we find that marginal winners are more likely to be incumbents than marginal losers at the 10\% level.\textsuperscript{31} Hence, we cannot reject the hypothesis that data for Panel (B) are consistent with competition.\textsuperscript{32}

Figure 5 displays binned scatter plots corresponding to the regression results reported in Columns (1) and (2) of Table 5. The top panels correspond to the results in Panel (A) and the bottom panels correspond to those in Panel (B). The left two panels plot the raw 90-day backlog against $\Delta$ and the right two panels plot the standardized 90-day backlog against $\Delta$.

\textsuperscript{30}Note that the sample size in Column (4) is larger than in Column (2). Suppose that a firm participates twice in the sample, say, January 1, 2015 and May 1, 2015. Suppose that the firm wins the first auction. According to our 90-day backlog measure, the firm’s backlog would be zero for both auctions. Hence, we cannot define the standardized backlog for this firm. However, according to our 180-day backlog measure, the firm has a positive backlog in the second auction. Hence, we can compute the within-firm standard deviation for 180-day backlog, but not for 90-day backlog.

\textsuperscript{31}The fact that the estimated coefficient for column (5) is the same for Panel (A) and Panel (B) up to the third decimal is coincidental.

\textsuperscript{32}The number of observations is smaller in Panel (A) because there are more auctions with a single bid for the sample of auctions in which the winning bid is above the median. When there is only one bid, we cannot compute $\Delta$. These auctions are used for the purpose of computing the median winning bid in each municipality, but are excluded from the sample used to estimate $\beta$. 31
<table>
<thead>
<tr>
<th></th>
<th>Panel (A): Above Median</th>
<th>Panel (B): Below Median</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw</td>
<td>Standardized</td>
</tr>
<tr>
<td>(1)</td>
<td>90-Day Backlog</td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta} )</td>
<td>6.751**</td>
<td>0.244***</td>
</tr>
<tr>
<td></td>
<td>(3.239)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>( h )</td>
<td>0.014</td>
<td>0.019</td>
</tr>
<tr>
<td>Obs.</td>
<td>30,666</td>
<td>28,650</td>
</tr>
</tbody>
</table>

Note: Panel (A) corresponds to the sample of auctions in which the winning bid is above the median. Panel (B) corresponds to the sample of auctions in which the winning bid is below the median. Standard errors are clustered at the auction level and reported in parenthesis. The table also reports the bandwidth \( h \) used for the estimation. * denotes significance at 10%, ** denotes significance at 5%, and *** denotes significance at 1%.

Table 5: Regression Discontinuity Estimates: Municipal Auctions from Japan.

There is a modest discontinuity in the binned averages at \( \Delta = 0 \) in the top left panel, corresponding to the results of Column (1) of Panel (A), Table 5. The discontinuity in the binned averages for the standardized backlog (top right panel) is more visible. The top right panel corresponds to Column (2) of Panel (A). In contrast, the graphs in the bottom panels, corresponding to Columns (1) and (2) of Panel (B), do not exhibit any clear discontinuities at \( \Delta = 0 \).

Figure 6 displays binned scatter plots corresponding to Columns (3) and (4). Similar to the case of 90-day backlog, the discontinuity is somewhat more modest in the top left panel and quite visible in the top right panel. The top left panel corresponds to the results reported in Column (3) of Panel (A) and the top right panel corresponds to that in Column
Note: Top panels correspond to Panel (A) of Table 5 and bottom panels correspond to Panel (B) of Table 5. Left panels correspond to 90-day Backlog and the right panels correspond to 90-day standardized backlog.

Figure 5: Binned Scatter Plot for 90-Day Raw Backlog and 90-Day Standardized Backlog: Municipal Auctions from Japan.

(4) of Panel (A). There are no visible discontinuities in the outcome variable for the bottom panels, corresponding to Columns (3) and (4) of Panel (B).

Figure 7 shows binned scatter plots corresponding to Column (5) of Table 5. The discontinuity in the binned averages is visible in the top panel.
5.2.3 A Placebo Test

Because the precise order of the losing bidders is unimportant for allocation by a cartel, it seems plausible that bidding rings would not have specific rules for determining which bidder should bid the second or third lowest. If this is the case, we should not expect significant differences in backlog or incumbency status between marginally second and marginally third
place bidders for both competitive and non-competitive auctions. This suggests the following placebo test.

For any non-winning bidder $i$, define $\Delta_{i,t}^2 \equiv b_{i,t} - \min\{b_{j,t} \text{ s.t. } j \neq i \text{ and } j \text{ loses}\}$ as the bid differences computed in data from which winning bids are excluded. Bid difference $\Delta_{i,t}^2$ is negative for the second lowest bidders and positive for other bidders. Even under collusion,
we do not expect that there should be systematic differences in the mean backlog and mean incumbency of close second and third (or fourth, fifth, etc.) bidders.

Table 6 reports estimates of the discontinuity in backlog and incumbency around \( \Delta^2_{i,t} = 0 \). The top and bottom panels correspond to auctions in which the winning bid is respectively above and below the municipal median. Unlike in Panel (A) of Table 5, discontinuity estimates are statistically insignificant at the 5% level in Panel (A) of Table 6. Unsurprisingly, the same holds for Panel (B). Binned scatter plots corresponding to these estimates are given in Online Appendix A.
6 Discussion

The paper proposes ways to screen for collusion based on bid rotation and incumbency patterns. While many practitioners have advocated using these patterns to screen for collusion, identifying allocation patterns that reflect agreements among cartels from those that simply reflect bidder cost heterogeneity has been difficult. Our contribution is to make this possible by conditioning on auctions that are determined by a close margin. Our approach is easy to implement, requires no sophisticated programming, and is fairly robust to model misspecification. In addition, our approach can easily be adapted to formulate tests of non-competitive behavior exploiting other covariates suspected to reflect collusive strategies, such as geographic segmentation, subcontracting, or joint bidding. Our approach can also be extended to other auction formats such as handicap auctions, scoring auctions and all-pay auctions.

We end the paper with a discussion of practical aspects of our tests: (i) the relation between the rejection of the test and collusion and (ii) firms’ responses to antitrust oversight.

Rejection of the test and collusion. Section 4 argues that the bidding patterns in our datasets cannot be rationalized by suitably competitive behavior under fairly general information structures.

While this does not necessarily imply bidder collusion, results in Section 5 suggest a correlation between the rejection of the test and non-competitive behavior. Firms bidding for school milk contracts that were charged with collusion fail our tests, while firms who were not charged with collusion pass; bids from auctions with a high winning bid fail our tests, while bids from auctions with low winning bids pass. This suggests that our test are sufficiently powered to flag cartels in practice.
Firm response to screening. Screens for collusion are perhaps most useful when firms are unaware of the details of the screening technology. When screens are known to the colluders they can potentially adapt their behavior to avoid detection. Are screens for collusion still useful if cartels adapt? Are there tests that reduce the incentives of cartels, and don’t harm competitive industries? We study these and related issues in our companion paper, Ortner et al. (2020).

We say that a test of collusive behavior is safe if the rate of false positives vanishes as the number of observations grows. The tests proposed in the current paper fall in this category. In Ortner et al. (2020), we show that antitrust oversight based on safe tests always reduces the set of enforceable collusive schemes available to cartels. Put differently, even if firms know they are being monitored and adapt their play accordingly, screens based on safe tests always make cartels worse-off.

Moreover, as we illustrate in Ortner et al. (2020), adaptive responses by cartels may themselves lead to suspicious bidding patterns that can also be detected. Consider, for example, the test that compares the incumbency status of marginal winners and marginal losers. If the cartel has a rule of allocating projects to incumbents and wishes to maintain this rule, then the cartel needs to have the lowest bidder bid substantially lower than the second lowest bidder to avoid detection. However, this would generate an unnatural pattern in the distribution of $\Delta_{i,t}$ similar to that documented in Chassang et al. (2019). Hence, avoiding one test may lead cartels to bid in ways that lead to rejection of other tests. Alternatively, the cartel can change its allocation rule so that incumbents are not always guaranteed to win. However, changing the allocation rule may reduce efficiency and increase the cost of coordination. These costs are likely to reduce the incentives of firms to collude.
Online Appendix – Not for Publication

A Further Empirics

In this section, we first present the binned scatter plots corresponding to the regression results in Table 6, Section 5. We then present estimates of discontinuities when we limit our sample to the municipalities that use public reserve prices for their auctions. Finally, we report findings for the entire sample of auctions for which we have data.

Omitted binned scatter plots for Table 6. Figures A.1, A.2 and A.3 are the binned scatter plots corresponding to Table 6. In all of the panels, the horizontal axis corresponds to values $\Delta^2_{i,t} \equiv b_{i,t} - \min\{b_{j,t}, s.t. j \neq i \text{ and } j \text{ loses}\}$ for losing bidders $i$. A small negative value of $\Delta^2_{i,t}$ corresponds to a bid that is second lowest, but close to being third lowest. A small positive value of $\Delta^2_{i,t}$ corresponds to a bid that was higher than, but close to the second lowest bid.

The panels in Figure A.1 are the binned scatter plots that correspond to Columns (1) and (2) of Table 6. The panels in Figure A.2 correspond to Columns (3) and (4). The panels in Figure A.3 correspond to Column (5). The top panels of each figure plot the outcome variable for the sample of auctions with winning bids above the municipal median. The bottom panels correspond to the sample with winning bids below the median. Unlike our results for marginal winners and marginal losers, the figures do not show any discontinuities around $\Delta^2_{i,t} = 0$.

Results for the sample of auctions with public reserve price. We now report the results of our analysis when we restrict the sample to auctions let by municipalities using public reserve prices. Table A.1 reports results from adapting the analysis of Table 5 to this subset. Panel (A) of Table A.1 reports estimation results for the set of auctions with high winning bids. We find that the difference in the backlog of marginal losers and marginal winners is higher than zero by a margin that is statistically significant at the 5% level in Columns (2), (3), and (4). We find that the difference is statistically significant at the 10% level in Column (1). These results are qualitatively similar to those reported in Table

---

33 As before, we compute the median winning bid for each municipality and divide the sample according to whether or not the winning bid is above or below the municipal median.
Figure A.1: Binned Scatter Plot for 90-Day Backlog with Respect to $\Delta^2$: Municipal Auctions from Japan.

5. While the estimated difference for incumbency status (Column (5)) is not statistically significant in Panel (A), the overall results strongly suggest that there are non-competitive auctions among the sample of public reserve auctions with high winning bids.

   In Panel (B), we report the estimated results for the set of auctions with low winning bids. We find that there are no statistically significant differences between the marginal winner and the marginal loser for this subsample, implying that we cannot reject the null of competition.
The similarity of the results between those reported in Table 5 and Table A.1 suggests that the power and size of our tests are robust to whether or not we include municipalities that use secret reserve prices.

**All municipalities.** We now discuss the results of our tests when we include auctions from municipalities that we drop in our main analysis. There are a total of 109 municipalities for
Note: Top panels correspond to Panel (A) of Table 6 and bottom panels correspond to Panel (B) of Table 6.

Figure A.3: Binned Scatter Plot for Incumbent with Respect to $\Delta^2$: Municipal Auctions from Japan.

which we have auction data. In order to construct the dataset used in Section 5, we drop municipalities for which the distribution of $\Delta$ has a missing mass at 0 (71 municipalities) and those for which the distribution of $\Delta$ has a point mass at exactly 0 (22 municipalities).

Figure A.4 plots the histograms of $\Delta_{i,t}$ for auctions let by the municipalities with missing mass in the distribution of $\Delta_{i,t}$ at 0 (first row) and for those let by municipalities with a mass
Table A.1: Regression Discontinuity Estimates: Municipal Auctions with Public Reserve Price.

in the distribution of $\Delta_{i,t}$ at exactly zero (second row). The left two panels correspond to the histogram for all of the auctions let by each of the groups of municipalities. The middle and right panels correspond to the histogram for auctions in which the winning bid is below the municipal median (middle panel) and above the municipal median (right panel).

The missing mass in the distribution of $\Delta_{i,t}$ that is apparent in the top panels has previously been documented in Chassang et al. (2019). In that paper, we show that this distinctive pattern in the distribution of $\Delta_{i,t}$ is inconsistent with competitive bidding under fairly general conditions. Because our previous paper specifically focuses on the implications of these patterns, we opted to exclude these municipalities in our baseline analysis.

The distributions of $\Delta_{i,t}$ in the bottom panels have spikes at zero which are the result of binding price floors. Price floors can result in multiple bidders bidding exactly at the price floor. Note that because the spikes are generated by price floors, and because multiple
Note: The top panels correspond to auctions from 71 municipalities with missing mass in the distribution of $\Delta_{i,t}$ at zero. The bottom panels correspond to auctions from 22 municipalities with a mass in the distribution of $\Delta_{i,t}$ at exactly zero. The left panels correspond to all auctions let by each of these groups, the middle panels condition on the winning bids to be below the municipality median and the right panels condition on the winning bids to be above the median.

Figure A.4: Histogram of $\Delta_{i,t}$: Municipal Auctions from Japan.

bids at the price floor typically imply that the winning bid of the auction is low, the spike is very pronounced for the middle panel, but mostly disappears in the right panel. The summary statistics of the auctions for each of the groups are reported in Table A.2. Column (1) corresponds to the sample statistics for municipalities with a missing mass at zero, Column (2) corresponds to the sample statistics for those with a mass at 0, and Column (3) corresponds to the sample statistics for our baseline sample.

We now report the regression discontinuity results for all of the auctions in our sample. Panel (A) of Table A.3 reports the regression discontinuity estimates for auctions with above-median winning bids. Panel (B) of Table A.3 reports the estimates for auctions with below-median winning bids. Focusing on Panel (A), we find that marginal losing bidders have about 3.5 million yen more in terms of 90-day backlog (Column (1)) and about 0.087 higher
### Table A.2: Summary Statistics by Auctions and Bidders: All Municipalities.

<table>
<thead>
<tr>
<th></th>
<th>(1) Sample with Missing Mass (71 munis)</th>
<th>(2) Sample with Mass at 0 (22 munis)</th>
<th>(3) Baseline Sample (16 munis)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Std.</td>
<td>Mean Std.</td>
<td>Mean Std.</td>
</tr>
<tr>
<td>Reserve (Mil. Yen)</td>
<td>24.03 104.39</td>
<td>20.92 101.08</td>
<td>22.26 77.14</td>
</tr>
<tr>
<td>Winning Bid (Mil. Yen)</td>
<td>22.60 97.64</td>
<td>19.09 95.63</td>
<td>20.71 71.78</td>
</tr>
<tr>
<td>Win Bid/Reserve</td>
<td>0.940 0.073</td>
<td>0.911 0.078</td>
<td>0.926 0.083</td>
</tr>
<tr>
<td># of Bids</td>
<td>7.44 3.78</td>
<td>8.00 4.64</td>
<td>6.80 4.21</td>
</tr>
<tr>
<td>Incumbent</td>
<td>0.064 0.244</td>
<td>0.043 0.202</td>
<td>0.044 0.204</td>
</tr>
<tr>
<td>Obs.</td>
<td>44,993 54,153</td>
<td>11,207</td>
<td></td>
</tr>
</tbody>
</table>

#### Panel A: By Auction

- The estimates are both statistically significant at the 1% level. Similarly, we find that marginal losing bidders have higher raw and standardized 180-day backlog (Column (3), (4)) than marginal winners, and are less likely to be an incumbent (Column (5)) than marginal winners. The coefficients are all statistically significant at the 1% level. These findings lead us to reject the null hypothesis of competition for this sample.

#### Panel B: By Bidder

- The bottom panel of Table A.3 reports the results for the set of auctions with winning bids below the median. While the regression discontinuity estimate is statistically significant at the 5% level in Column (5) and significant at the 10% level in Column (3), the estimated differences between marginal winners and losers are much smaller than in Panel (A). The results suggest the existence of some collusive bidding among this ample, but the tests are

Note: Column (1) reports summary statistics for the sample of auctions with missing mass in the distribution of $\Delta_{i,t}$ at zero (71 municipalities). Column (2) reports summary statistics for the sample with mass at exactly zero (22 municipalities). Column (3) reports sample statistics for the sample that we use in our baseline analysis.
In addition to the auctions used in the baseline analysis, we include auctions from 70 municipalities with missing mass in the distribution of $\Delta_{i,t}$ at zero and those from 18 municipalities with mass in the distribution of $\Delta_{i,t}$ at exactly zero. Panel (A) corresponds to the sample of auctions in which the winning bid is above the median. Panel (B) corresponds to the sample of auctions in which the winning bid is below the median. Standard errors are clustered at the auction level and reported in parenthesis. The forcing variable is $\Delta^1$. The table also reports the bandwidth used for the estimation. * denotes significance at 10%, ** denotes significance at 5%, and *** denotes significance at 1%.

Table A.3: Regression Discontinuity Estimates: All Municipalities.

much less conclusive. Overall, the results of Table A.3 suggest that the null of competitive bidding is strongly rejected for auctions with relative high winning bids, but that the evidence is more mixed for auctions with relatively low winning bids. This is consistent with the expectation that there would be more collusion among auctions with high winning bids than among those with low winning bids.
B Proofs

B.1 Proofs for Section 3

Proof of Lemma 1. We show that for all \( \eta > 0 \), there exists \( \epsilon > 0 \) small enough such that for all histories \( h_{i,t} \),

\[
\left| \text{prob}(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| \leq \epsilon) - \frac{1}{2} \right| \leq \eta.
\]

By assumption, \( D'_i(b_i|h_i) \) is continuous in \( b_i \in [0,1] \) and strictly negative for all histories \( h_i = (\theta, z_i) \). Since there are finitely many histories \( (\theta, z_i) \), it follows that there exists \( \nu > 0 \) such that \( D'_i(b_i|h_{i,t}) \leq -\nu \) for all \( b_i \) and all histories \( h_{i,t} \). In addition, for all \( \hat{\eta} > 0 \), there exists \( \epsilon \) small enough that for all \( \hat{b}_i \in [b_i - \epsilon, b_i + \epsilon] \), \( |D'_i(\hat{b}_i|h_{i,t}) - D'_i(b_i|h_{i,t})| \leq \hat{\eta} \).

This implies that for \( \epsilon \) small

\[
\left| \text{prob}(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| \leq \epsilon) - \frac{1}{2} \right| \leq \frac{|D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t}) - D_i(b_{i,t} - \epsilon|h_{i,t}) + \epsilon h_{i,t})| - \frac{1}{2}|}{\epsilon} \leq \frac{|D'_i(b_{i,t}|h_{i,t}) - \epsilon \hat{\eta}|}{\epsilon^2 (\epsilon) + 2\epsilon \hat{\eta} - \frac{1}{2}}.
\]

Lemma 1 follows by taking \( \hat{\eta} \) small enough. ■

An example of non-smooth demand. Consider a complete information auction with an incumbent \( I \) and an entrant \( E \) with respective known costs \( c_I < c_E \). Suppose that bidding cost \( k \) is zero.

Lemma B.1 (non-smooth demand). In any efficient equilibrium in weakly undominated strategies, the incumbent wins with bid \( c_E \) with probability 1. The density of the entrant’s bid below \( c_E \) is 0. The density of the entrant’s bids above \( c_E \) is strictly positive and bounded away from 0. Specifically, for all \( \epsilon > 0 \), the incumbent’s demand \( D_I \) satisfies \( \frac{D_i(c_E+\epsilon)-1}{\epsilon} \leq -\frac{1}{c_E+c_I} \).

Proof. In an efficient equilibrium in weakly undominated strategies, the incumbent cannot bid above \( c_E \) with positive probability: the entrant’s optimal bid would win with positive probability.
In turn, the entrant cannot bid below $c_E$. This implies that the incumbent’s optimal bid is $c_E$.

Optimality of $c_E$ implies that for any $\epsilon > 0$,

$$D_I(c_E + \epsilon)\left(c_E + \epsilon - c_I\right) \leq D_I(c_E)(c_E - c_I) = c_E - c_I \iff \frac{D_I(c_E + \epsilon) - 1}{\epsilon} \leq -\frac{1}{c_E + \epsilon - c_I}.$$ 

B.2 Proofs for Section 4

We begin with the proof of Proposition 1. We first establish two preliminary lemmas.

Lemma B.2 (minimum demand). Consider $\sigma$ a competitively enforceable strategy profile. There exists $\nu > 0$ such that for every history $h_i = (\theta, z_i)$ and bid $b_i \in [0, 1]$ in the support of $\sigma_i|h_i$, $D_i(b_i|h_i) \geq \nu$. In addition,

$$b_i - c_i + \delta \mathbb{E}_\sigma [V_i(\theta, 1, b_i) - V_i(\theta, 0, \land b_{-i}) | h_i, b_i \lt \land b_{-i}] \geq k.$$ 

Proof. Since firm $i$ chooses to participate, it must be that

$$\mathbb{E}_\sigma \left[ \mathbf{1}_{b_i \lt \land b_{-i}}(b_i - c_i + \delta V_i(\theta, 1, b_i)) + \mathbf{1}_{b_i \gt \land b_{-i}}\delta V_i(\theta, 0, \land b_{-i}) | h_i \right] - k \geq \mathbb{E}_\sigma [\delta V_i(\theta, 0, \land b_{-i}) | h_i]$$

$$\iff \mathbb{E}_\sigma \left[ \mathbf{1}_{b_i \lt \land b_{-i}}(b_i - c_i + \delta V_i(\theta, 1, b_i) - \delta V_i(\theta, 0, \land b_{-i})) | h_i \right] \geq k$$

$$\iff D_i(b_i|h_i)(b_i - c_i + \delta \mathbb{E}_\sigma [V_i(\theta, 1, b_i) - V_i(\theta, 0, \land b_{-i}) | h_i, b_i \lt \land b_{-i}] \geq k.$$ 

Since $D_i \geq 0$, it must be that both left-hand side factors are strictly positive. In addition, since continuation values are bounded by some constant $\overline{V}$, it follows that $D_i(b_i|h_i) \geq k/(1 + 2\overline{V})$. Similarly, since demand is bounded above by 1, we have that

$$b_i - c_i + \delta \mathbb{E}_\sigma [V_i(\theta, 1, b_i) - V_i(\theta, 0, \land b_{-i}) | h_i, b_i \lt \land b_{-i}] \geq k.$$ 

This concludes the proof. \qed

Lemma B.3 (continuous demand). Consider $\sigma$ a competitively enforceable strategy profile. For every history $h_i = (\theta, z_i)$, residual demand $D_i(b_i|h_i)$ is continuous in $b_i$ over $(0, 1)$.

Proof. The proof is by contradiction. Assume that demand $D_i(\cdot|h_i)$ is discontinuous at bid $b_0$. There must exist a bidder $j$ and a history $(\theta, z_j)$ such that firm $j$ bids $b_j = b_0$ with
probability \( q > 0 \). By Lemma B.2, bidder \( j \) must win with probability at least \( \nu > 0 \) when bidding \( b_0 \).

Consider a bidder \( l \) and a history \( h_l = (\theta, z_l) \) such that history \( h_j \) has positive probability, and bidder \( l \) loses with positive probability against bidder \( j \) when bidder \( j \) bids \( b_0 \). Since the number of histories is finite, there exists \( \nu_1 > 0 \) such that at any such history \( h_l \) bidder \( j \) bids \( b_0 \) with positive probability \( \nu_1 \).

Pick \( \epsilon > 0 \) and consider the payoff of bidder \( l \) bidding \( b_l \in [b_0, b_0 + \epsilon) \). Bidder \( l \) gets payoff (excluding participation costs and payoffs upon non-participation)

\[
U_l(b_l|h_l, c_l) = D(b_l|h_l) (b_l - c_l + \delta \mathbb{E}_\sigma[V_i(\theta, 1, b_l) - V_i(\theta, 0, \land b_{-l}) | h_l, b_l < \land b_{-l}]).
\]

We know from Lemma B.2 that

\[
b_l - c_l + \delta \mathbb{E}_\sigma[V_i(\theta, 1, b_l) - V_i(\theta, 0, \land b_{-l}) | h_l, b_l < \land b_{-l}] \geq k.
\]

By assumption, \( V_i(\theta, 1, b_l) \) is continuous in \( b_l \). Hence, for all \( \eta > 0 \), there exists \( \epsilon \) small enough so that \( V_i(\theta, 1, b_l - \epsilon) \geq V_i(\theta, 1, b_l) - \eta \).

The assumption that the continuation value of losers, \( V_i(\theta, 0, \land b_{-l}) \) is decreasing in \( \land b_{-l} \) implies that

\[
\mathbb{E}_\sigma[V_i(\theta, 0, \land b_{-l}) | h_l, b_l - \epsilon < \land b_{-l}] \leq \mathbb{E}_\sigma[V_i(\theta, 0, \land b_{-l}) | h_l, b_l < \land b_{-l}].
\]

Altogether, it follows that for every \( \eta > 0 \), there exists \( \epsilon > 0 \) small enough that

\[
b_l - \epsilon - c_l + \delta \mathbb{E}_\sigma[V_i(\theta, 1, b_l - \epsilon) - V_i(\theta, 0, \land b_{-l}) | h_l, b_l - \epsilon < \land b_{-l}] \\
\geq b_l - c_l + \delta \mathbb{E}_\sigma[V_i(\theta, 1, b_l) - V_i(\theta, 0, \land b_{-l}) | h_l, b_l < \land b_{-l}] - \eta \geq k - \eta.
\]

Hence, it follows that by bidding \( b_l - \epsilon \), bidder \( l \) gets a payoff

\[
U_l(b_l - \epsilon|h_l, c_l) = D(b_l - \epsilon|h_l) (b_l - \epsilon - c_l + \delta \mathbb{E}_\sigma[V_i(\theta, 1, b_l - \epsilon) - V_i(\theta, 0, \land b_{-l}) | h_l, b_l - \epsilon < \land b_{-l}]) \\
\geq U_l(b_l|h_l, c_l) - \eta + \nu_1(k - \eta)
\]

Since \( \nu_1 \) is fixed, it follows that for \( \epsilon \) small enough \( U_l(b_l - \epsilon|h_l, c_l) > U_l(b_l|h_l, c_l) \). Hence, there exists \( \epsilon \) small such that bidder \( l \) does not bid in \([b_0, b_0 + \epsilon]\). Since there are only finite histories, this implies that there exists \( \epsilon > 0 \) such that no bidder \( l \) that loses against bidder \( j \) bidding \( b_0 \) bids in the range \([b_0, b_0 + \epsilon]\). Hence, bidder \( j \) would benefit from bidding \( b_0 + \epsilon/2 \) rather
than $b_0$. This contradicts the assumption that $\sigma$ is competitively enforced and concludes the proof.

\[\square\]

**Proof of Proposition 1.** Fix a Markov strategy $\sigma$ that is competitively enforceable, and a history $h_{i,t} = (\theta_t, z_{i,t})$ of firm $i$. Let $b_{i,t} < r = 1$ denote firm $i$'s bid at this history when her cost is $c_{i,t}$. For any bid $b$, let $U_i(b|h_{i,t}, c_{i,t})$ denote $i$’s payoff from bidding $b$ at history $h_{i,t}$ when her cost is $c_{i,t}$:

\[U_i(b|h_{i,t}, c_{i,t}) = \mathbb{E}_\sigma \left[ 1_{A_{b_{-i,t}>b}}(b - c_{i,t} + \delta V_i(\theta_t, 1, b)) + (1 - 1_{A_{b_{-i,t}>b}})\delta V_i(\theta_t, 0, A_{b_{-i,t}}) \mid h_{i,t} \right] - k.\]

Since bid $b_{i,t}$ is optimal, for all $\epsilon > 0$ it must be that,

\[U_i(b_{i,t}|h_{i,t}, c_{i,t}) \geq U_i(b_{i,t} + \epsilon|h_{i,t}, c_{i,t}) \iff (D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t}))(b_{i,t} - \kappa_{i,t}^+) \geq D_i(b_{i,t} + \epsilon|h_{i,t})(\epsilon + \Delta V_{i,t}^+) \quad (7)\]

where $\kappa_{i,t}^+ \equiv c_{i,t} - \mathbb{E}_\sigma[\delta(V_i(\theta, 1, b_{i,t}) - V_i(\theta, 0, A_{b_{-i,t}}))|h_{i,t}, b_{i,t} + \epsilon > A_{b_{-i,t}} > b_{i,t}]$ and $\Delta V_{i,t}^+ \equiv \delta(V_i(\theta, 1, b_{i,t} + \epsilon) - V_i(\theta, 1, b_{i,t}))$. Since $D_i(\cdot|h_{i,t})$ is continuous at $b_{i,t}$ (Lemma B.3), since $D_i(b_{i,t}|h_{i,t}) > 0$ (Lemma B.2) and since $V_i(\theta, 1, b)$ is increasing, $b_{i,t} - \kappa_{i,t}^+ > 0$ for $\epsilon > 0$ small.

Similarly, for all $\epsilon > 0$ it must be that

\[U_i(b_{i,t}|h_{i,t}, c_{i,t}) \geq U_i(b_{i,t} - \epsilon|h_{i,t}, c_{i,t}) \iff (D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t}|h_{i,t}))(b_{i,t} - \kappa_{i,t}^-) \leq D_i(b_{i,t} - \epsilon|h_{i,t})(\epsilon + \Delta V_{i,t}^-) - (D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t}|h_{i,t}))(\kappa_{i,t}^- - \kappa_{i,t}^+) \quad (8)\]

where $\kappa_{i,t}^- \equiv c_{i,t} - \mathbb{E}_\sigma[\delta(V_i(\theta, 1, b_{i,t}) - V_i(\theta, 0, A_{b_{-i,t}}))|h_{i,t}, b_{i,t} > A_{b_{-i,t}} > b_{i,t} - \epsilon]$ and $\Delta V_{i,t}^- \equiv \delta(V_i(\theta, 1, b_{i,t}) - V_i(\theta, 1, b_{i,t} - \epsilon))$.

Using (7) and (8), together with $b_{i,t} - \kappa_{i,t}^+ > 0$, we have that

\[\text{prob}_\sigma(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - A_{b_{-i,t}}| < \epsilon) = \frac{D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})} \geq \frac{D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) + \epsilon + \Delta V_{i,t}} \left( \frac{\kappa_{i,t}^- - \kappa_{i,t}^+}{\epsilon + \Delta V_{i,t}} + D_i(b_{i,t} + \epsilon|h_{i,t}) \right) \quad (9)\]

Since $D_i(\cdot|\theta, z_{i,t})$ is continuous on $[0, 1]$, it is uniformly continuous. Since there are finitely
many \((\theta, z_i)\), for every \(\gamma_D > 0\) there exists \(\bar{\epsilon} > 0\) such that, for all \(i, \theta, z_i\) and for all \(b, b'\) with \(|b - b'| \leq 2\bar{\epsilon}\), \(D_i(b|\theta, z_i) - D_i(b'|\theta, z_i) < \gamma_D\).

Moreover, since \(V_i(0, \theta, \wedge \bm{b}_{-i})\) is Lipschitz continuous and decreasing in \(\wedge \bm{b}_{-i}\), and since there are finitely many states \(\theta\), there exists a Lipschitz constant \(L > 0\) such that, for all \(i, \theta, z_i, c_{i,t}, \kappa_{i,t}^+ - \kappa_{i,t}^- \geq -2\epsilon L\).

Lastly, since \(V_i(\theta_t, 1, \cdot)\) is continuously differentiable on \([0, 1]\), there exists \(b^{+}\in [b_{i,t} - \epsilon, b_{i,t}]\) such that \(\Delta V_{i,t}^{+\epsilon} = \delta \epsilon V_i'(\theta_t, 1, b^{+})\) and \(\Delta V_{i,t}^{-\epsilon} = \delta \epsilon V_i'(\theta_t, 1, b^{-})\). Since \(V_i'(\theta_t, 1, \cdot)\) is continuous on \([0, 1]\), it is uniformly continuous on this range. Since there are finitely many states, for every \(\gamma_V > 0\) there exists \(\bar{\epsilon} > 0\) such that, for all \(i, \theta, z_i, c_{i,t}, |V_i'(\theta_t, 1, b) - V_i'(\theta_t, 1, b')| < \gamma_V\) for all \(b, b'\) with \(|b - b'| \leq 2\bar{\epsilon}\). Hence, \(\Delta V_{i,t}^{+\epsilon} - \Delta V_{i,t}^{-\epsilon} \leq \delta \epsilon \gamma_V\) for all \(\epsilon \leq \bar{\epsilon}\).

Using (9), for every \(\gamma_D > 0\) and \(\gamma_V > 0\), there exists \(\bar{\epsilon} > 0\) such that, for all \(\epsilon < \bar{\epsilon}\),

\[
\text{prob}_{\sigma}(i \text{ wins } | h_{i,t}\text{ and } |b_{i,t} - \wedge \bm{b}_{-i,t}| < \epsilon) \geq \frac{D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} + \epsilon|h_{i,t}) + \gamma_D D_i(b_{i,t} + \epsilon|h_{i,t}) + \gamma_D D_i(b_{i,t} + \epsilon|h_{i,t}) + \gamma_D D_i(b_{i,t} + \epsilon|h_{i,t}) + \gamma_D D_i(b_{i,t} + \epsilon|h_{i,t})} \geq \frac{\nu - \gamma_D}{\nu(1 + \delta \gamma_V) + \gamma_D 2L + \nu - \gamma_D},
\]

where the second inequality uses \(V_i'(\theta_t, 1, b^{+\epsilon}) \geq 0\) and \(D_i(b_{i,t} + \epsilon|h_{i,t}) \geq D_i(b_{i,t}|h_{i,t}) - \gamma_D \geq \nu - \gamma_D\) (Lemma B.2). Picking \(\gamma_D, \gamma_V\) small, we obtain that \(\text{prob}_{\sigma}(i \text{ wins } | h_{i,t}\text{ and } |b_{i,t} - \wedge \bm{b}_{-i,t}| < \epsilon) \geq 1/2 - \eta\).

**Proof of Corollary 2.** Fix a Markov strategy \(\sigma\) that is competitively enforced. For each \(\epsilon > 0\), let \(\text{prob}_{\sigma,h}(\cdot|\epsilon\text{-close})\) denote the distribution over histories under \(\sigma\) conditional on event \(\epsilon\text{-close}\). Then, for each \(i \in \mathcal{N}\) and each \(\epsilon > 0\), the probability with which firm \(i\) wins an auction under \(\sigma\) conditional on event \(\epsilon\text{-close}\) satisfies

\[
\text{prob}_{\sigma,h}(i \text{ wins }|\epsilon\text{-close}) = \mathbb{E}_\sigma \left[ \text{prob}_{\sigma}(i \text{ wins } | h_{i,t}\text{ and } |b_{i,t} - \wedge \bm{b}_{-i,t}| < \epsilon) | \epsilon\text{-close} \right] \times \text{prob}_{\sigma,h}(|b_{i,t} - \wedge \bm{b}_{-i,t}| < \epsilon|\epsilon\text{-close}).
\]

By Proposition 1, it follows that

\[
\forall i \in \mathcal{N}, \liminf_{\epsilon \to 0} \mathbb{E}_\sigma \left[ \text{prob}_{\sigma}(i \text{ wins } | h_{i,t}\text{ and } |b_{i,t} - \wedge \bm{b}_{-i,t}| < \epsilon) | \epsilon\text{-close} \right] \geq \frac{1}{2}. \tag{12}
\]
Towards a contradiction, suppose that the result is not true. Hence, there exists a player \( j \) and an \( \eta > 0 \) such that

\[
\limsup_{\epsilon \downarrow 0} \mathbb{E}_{\sigma} \left[ \mathbf{prob}_{\sigma}(j \text{ wins} \mid h_{j,t} \text{ and } |b_{j,t} - \wedge b_{-j,t}| < \epsilon) \mid \epsilon\text{-close} \right] \geq \frac{1}{2} + \eta. \tag{13}
\]

Note that, for each \( \epsilon > 0 \), we have that

\[
\sum_{i \in N} \mathbf{prob}_{\sigma,h}(i \text{ wins} \mid \epsilon\text{-close}) = 1 \quad \text{and} \quad \sum_{i \in N} \mathbf{prob}_{\sigma,h}(|b_{i,t} - \wedge b_{-i,t}| < \epsilon \mid \epsilon\text{-close}) = \mathbb{E}_{\sigma} \mathbb{E}[\{i \text{ s.t. } |b_{i,t} - \wedge b_{-i,t}| < \epsilon \mid \epsilon\text{-close}] \geq 2.
\]

Using (11), (12) and (13), we obtain that

\[
1 = \limsup_{\epsilon \downarrow 0} \sum_{i \in N} \mathbf{prob}_{\sigma,h}(i \text{ wins} \mid \epsilon\text{-close}) \geq \frac{1}{2} \limsup_{\epsilon \downarrow 0} \sum_{i \in N} \mathbf{prob}_{\sigma,h}(|b_{i,t} - \wedge b_{-i,t}| < \epsilon \mid \epsilon\text{-close})
\]

\[
+ \eta \mathbf{prob}_{\sigma,h}(|b_{j,t} - \wedge b_{-j,t}| < \epsilon \mid \epsilon\text{-close})
\]

\[
\geq 1 + \eta \limsup_{\epsilon \downarrow 0} \mathbf{prob}_{\sigma,h}(|b_{j,t} - \wedge b_{-j,t}| < \epsilon \mid \epsilon\text{-close}) > 1,
\]

a contradiction. \( \blacksquare \)

**B.3 Sample Implications**

In this section we show that Corollary 2 extends under the sample distribution of bids \( b \), and characteristics \( x \).

Data consists of bids and observable characteristics \((b_t, x_t)_{t \in \{0, \ldots, T\}}\) for auctions happening at times \( t \in \{0, \ldots, T\} \). We denote by \( \hat{\mathbf{prob}} \) the sample joint distribution of bids and characteristics in the data.

Given \( \epsilon > 0 \) and \( x \in X \), we define \( B_{x,\epsilon} \equiv \{(i, t) \text{ s.t. } x_{i,t} = x, \ |b_{i,t} - \wedge b_{-i,t}| \leq \epsilon\} \) the subsample of close bids such that the bidders characteristics \( x_i \) are equal to \( x \). We denote by \( B_{\epsilon} \equiv \{(i, t) \text{ s.t. } |b_{i,t} - \wedge b_{-i,t}| \leq \epsilon\} \) the sample of close bids. A bidder’s sample probability
of winning conditional on close bids and type $x$ is denoted by $\hat{P}_{x,\epsilon}$. Formally, we have,

$$\hat{P}_{x,\epsilon} \equiv \widehat{\text{prob}}(i \text{ wins} \mid x_i = x, \ |b_i - \land b_{-i}| \leq \epsilon) = \frac{|\{(i, t) \in B_{x,\epsilon} \text{ s.t. } b_{i,t} \prec \land b_{-i,t}\}|}{|B_{x,\epsilon}|} \quad (14)$$

We make the following assumption about data.

**Assumption B.1.** There exists $\lambda > 0$ such that for all datasets of interest $B$, and all $x \in X$,

$$\frac{\sum_{x' \in X \setminus x} |B_{x',\epsilon}|}{|B_{x,\epsilon}|} \leq \lambda$$

The following result holds:

**Proposition B.1** (winning is independent of bidder characteristics). For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that with probability approaching 1 as $|B_\epsilon|$ goes to infinity,

$$\forall x \in X, \quad \left| \frac{\hat{P}_{x,\epsilon}}{2} - 1 \right| \leq \eta.$$

**Proof of Proposition B.1.** Take $\eta' > 0$ as given. We know from Proposition 1 that for epsilon small enough, for all histories $h_{i,t}$, $\text{prob}(i \text{ wins} \mid h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| < \epsilon) \geq 1/2 - \eta'$.

Fix $x \in X$. We show that with probability approaching 1 as $|B_\epsilon|$ goes to infinity, $\hat{P}_{x,\epsilon} \geq \frac{1}{2} - 2\eta'$. Observe first that, by Assumption B.1, when $|B_\epsilon|$ grows large, $|B_{x,\epsilon}|$ grows proportionally large:

$$\frac{|B_{x,\epsilon}|}{|B_\epsilon|} = 1 - \frac{\sum_{x' \in X \setminus x} |B_{x',\epsilon}|}{|B_{x,\epsilon}| + \sum_{x' \neq x} |B_{x',\epsilon}|} \geq 1 - \frac{\lambda}{1 + \lambda}.$$  

We denote by $\{t_1, \cdots, t_n\}$ auctions occurring at times $t$ such that $(i, t) \in B_{x,\epsilon}$, ordered according to the timing of the auction. Since the number $N$ of bidders is finite, $n$ grows large proportionally with $|B_{x,\epsilon}|$. We define $C_k = \{i \in N \text{ s.t. } (i, t_k) \in B_{x,\epsilon}\}$. In equilibrium,

$$H_K \equiv \sum_{k=1}^{K} \sum_{i \in C_k} 1_{b_{i,t_k} \prec \land b_{-i,t_k}} - \text{prob}_i(b_{i,t_k} \prec \land b_{-i,t_k} \mid i \in C_k)$$

is a martingale. Indeed note that given the information $I_K$ available at the time of bidding.
in auction $K$,

$$
E \left[ \sum_{i \in C_K} 1_{b_{i,t_K} \prec \land b_{-i,t_K}} | I_K \right] = E \left[ \sum_{i \in N} 1_{i \in C_K} 1_{b_{i,t_K} \prec \land b_{-i,t_K}} | I_K \right]
$$

$$
= E \left[ E_{C_K} \left[ \sum_{i \in N} 1_{i \in C_K} 1_{b_{i,t_K} \prec \land b_{-i,t_K}} | I_K \right] \right]
$$

$$
= E \sum_{i \in C_K} 1_{i \in C_K} \text{prob}_i \left( 1_{b_{i,t_K} \prec \land b_{-i,t_K}} | i \in C_K \right) | I_K \right]
$$

$$
= E \sum_{i \in C_K} \text{prob}_i \left( 1_{b_{i,t_K} \prec \land b_{-i,t_K}} | i \in C_K \right) | I_K \right].
$$

Using Proposition 1, this implies that

$$
G_K \equiv \sum_{k=1}^{K} \sum_{i \in C_k} 1_{b_{i,t_k} \prec \land b_{-i,t_k}} - \frac{1}{2} + \eta'
$$

is a submartingale with increments bounded by $|N|$ (the maximum number of bidders in an auction). It follows for the Azuma-Hoeffding Theorem that as $n$ grows large, with probability approaching 1, $G_n \geq -\eta' n$. Since $n \leq |B_{x,\epsilon}|$, this implies that with probability approaching 1,

$$
\hat{P}_{x,\epsilon} \equiv \frac{1}{|B_{x,\epsilon}|} \sum_{k=1}^{n} \sum_{i \in C_k} 1_{b_{i,t_k} \prec \land b_{-i,t_k}} \geq \frac{1}{2} - 2\eta'.
$$

Since $X$ is finite, with probability approaching 1 as $|B_{\epsilon}|$ becomes large, we have that for all $x \in X$, $\hat{P}_{x,\epsilon} \geq \frac{1}{2} - 2\eta'$. In addition, since $\sum_{x' \in X} |B_{x',\epsilon}| \hat{P}_{x',\epsilon} = \left| \{(i, a) \in B_{\epsilon} \text{ s.t. } i \text{ wins } \} \right|$, it follows that

$$
\sum_{x' \in X} |B_{x',\epsilon}| \hat{P}_{x',\epsilon} \leq \frac{1}{2}.
$$

Hence, with probability approaching 1, we have that

$$
|B_{x,\epsilon}| \hat{P}_{x,\epsilon} = \frac{1}{2} |B_{x,\epsilon}| + \sum_{x' \in X \setminus x} |B_{x',\epsilon}| \left( \frac{1}{2} - \hat{P}_{x',\epsilon} \right)
$$

$$
\Rightarrow \hat{P}_{x,\epsilon} \leq \frac{1}{2} + 2\eta' \sum_{x' \in X \setminus x} \frac{|B_{x',\epsilon}|}{|B_{x,\epsilon}|} \leq \frac{1}{2} + 2\eta' \lambda.
$$

Hence by selecting $\eta'$ sufficiently small in the first place, it follows that for any $\eta > 0$, there
exists $\epsilon$ such that as $|B_i|$ grows large, $|\hat{P}_{x,\epsilon} - \frac{1}{2}| \leq \eta$ with probability 1.

A corollary of Proposition B.1 is that our regression discontinuity design remains valid: conditional on close bids, the distribution of covariates is independent of whether the bidder wins or loses the auction.

**Corollary B.1** (close winners and losers have similar characteristics). For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that with probability approaching 1 as $|B_i|$ goes to infinity,

$$\forall x \in X, \quad \left| \hat{\text{prob}}(x_i = x \mid i \text{ wins }, |b_i - \wedge b_{-i}| \leq \epsilon) - \hat{\text{prob}}(x_i = x \mid i \text{ loses }, |b_i - \wedge b_{-i}| \leq \epsilon) \right| \leq \eta.$$

**Proof of Corollary B.1.** Observe that

$$\hat{\text{prob}}(x_i = x \mid i \text{ wins }, |b_i - \wedge b_{-i}| \leq \epsilon) = \frac{\hat{\text{prob}}(i \text{ wins } \mid x_i = x, |b_i - \wedge b_{-i}| \leq \epsilon)}{\hat{\text{prob}}(i \text{ wins } \mid |b_i - \wedge b_{-i}| \leq \epsilon)}$$

$$\hat{\text{prob}}(x_i = x \mid i \text{ loses }, |b_i - \wedge b_{-i}| \leq \epsilon) = \frac{\hat{\text{prob}}(i \text{ loses } \mid x_i = x, |b_i - \wedge b_{-i}| \leq \epsilon)}{\hat{\text{prob}}(i \text{ loses } \mid |b_i - \wedge b_{-i}| \leq \epsilon)}.$$

Therefore,

$$\left| \frac{\hat{\text{prob}}(i \text{ wins } \mid x_i = x, |b_i - \wedge b_{-i}| \leq \epsilon)}{\hat{\text{prob}}(i \text{ wins } \mid |b_i - \wedge b_{-i}| \leq \epsilon)} - \frac{\hat{\text{prob}}(i \text{ loses } \mid x_i = x, |b_i - \wedge b_{-i}| \leq \epsilon)}{\hat{\text{prob}}(i \text{ loses } \mid |b_i - \wedge b_{-i}| \leq \epsilon)} \right|$$

$$\leq \left| \sum_{x' \in X} \frac{|B_{x'\epsilon}|}{|B_i|} \frac{\hat{P}_{x',\epsilon}}{\hat{P}_{x,\epsilon}} - \frac{1 - \hat{P}_{x,\epsilon}}{1 - \sum_{x' \in X} \frac{|B_{x'\epsilon}|}{|B_i|} \hat{P}_{x',\epsilon}} \right|.$$

It follows from Proposition B.1 that for any $\eta' > 0$, there exists $\epsilon$ such that with probability 1 as $|B_i|$ grows large,

$$\sum_{x' \in X} \frac{|B_{x'\epsilon}|}{|B_i|} \frac{1 - \hat{P}_{x',\epsilon}}{1 - \sum_{x' \in X} \frac{|B_{x'\epsilon}|}{|B_i|} \hat{P}_{x',\epsilon}} \in \left[ \frac{1/2 - \eta'}{1/2 + \eta'}, \frac{1/2 + \eta'}{1/2 - \eta'} \right].$$

By picking $\eta'$ small enough, this implies that with probability approaching 1,

$$\left| \hat{\text{prob}}(x_i = x \mid i \text{ wins }, |b_i - \wedge b_{-i}| \leq \epsilon) - \hat{\text{prob}}(x_i = x \mid i \text{ loses }, |b_i - \wedge b_{-i}| \leq \epsilon) \right| \leq \eta.$$
References


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OECD (2013): “Ex officio cartel investigations and the use of screens to detect cartels.”


