Using Bid Rotation and Incumbency to Detect Collusion: A Regression Discontinuity Approach

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Abstract

Cartels participating in procurement auctions frequently use bid rotation or prioritize incumbents to allocate market shares. However, establishing a link between observed allocation patterns and firm conduct has been difficult: there are cost-based competitive explanations for such patterns. We show that by focusing on auctions in which the winning and losing bids are very close, it is possible to distinguish allocation patterns reflecting cost differences across firms from patterns reflecting non-competitive environments. We apply our tests to two datasets: the sample of Ohio milk auctions studied in Porter and Zona (1999), and a sample of municipal procurement auctions from Japan.

KEYWORDS: procurement, collusion, backlog, incumbency, regression discontinuity, antitrust.

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1 Introduction

The ability of competition authorities to proactively detect and punish collusion is crucial for achieving the goal of promoting and maintaining competition. Not only do the possibility of detection and prosecution serve as strong deterrents against collusion, they also affect the incentives of firms in existing cartels to apply for leniency programs. Successful identification of cartels thus deters collusive activity and complements enforcement programs.

In the absence of concrete leads, using data-driven screens to flag suspicious firm conduct can be useful for regulators as a first step in identifying collusion. While screens cannot substitute for direct evidence of collusion such as testimonies and records of communication, they can provide guidance on which markets or firms to focus investigation. A growing number of countries are adopting algorithm-based screens that analyze bidding data from public procurement auctions to flag suspicious behavior. More recently, the U.S. Department of Justice announced the formation of a procurement collusion strike force whose goal includes bolstering “data analytics employment to identify signs of potential anticompetitive, criminal collusion.” Imhof et al. (2018) describes an antitrust investigation initiated on the basis of statistical screens and resulting in successful cartel prosecution. The results from screens can be used in court to obtain warrants, or to support civil antitrust litigation as well as private litigation.

Screening cartels can also be useful to stakeholders other than antitrust authorities. For example, screening can help procurement offices counter suspected bidding rings by soliciting new bidders more aggressively or adopting auction mechanisms that are less susceptible to collusion. In large decentralized organizations, collusion may be organized by firm employees against the will of CEOs (Sonnenfeld and Lawrence, 1978). In that context, screening tools

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1 A report by the OECD (OECD, 2018) gives a brief description of the screening programs used in Brazil, Switzerland and the UK.
2 Announcement of the Antitrust Division’s Procurement Collusion Strike Force, November 22, 2019.
4 See also Ashton and Pressey (2012), who study 56 international cartels investigated by the EU. They find that there is involvement of individuals at the most senior levels of management (CEOs, chairpersons, etc.) in about half of those cases.
can help internal auditors and compliance officers contain collusive practices initiated by employees.

Because bidding rings often adopt rotation schemes or give priority to incumbents in project allocation, bid rotation and incumbency advantage are very often suggested as indicators of collusion. However, it is well known that there are non-collusive cost-based explanations for these allocation patterns. Bid rotation can arise under competition if marginal costs increase with backlog. Incumbency advantage can be explained by cost asymmetries among competitive firms or by learning-by-doing. Hence, establishing a tight link between these bidding patterns and collusion has been difficult. As Porter (2005) describes, “An empirical challenge is to develop tests that can discriminate between collusive and non-cooperative explanations for rotation or incumbency patterns.”

We show that it is possible to discriminate between competitive and non-competitive bid rotation and incumbency patterns using the logic of regression discontinuity designs (Thistlethwaite and Campbell, 1960). We compare the backlog and incumbency status of a bidder who wins the auction by a small margin to those of a bidder who loses by a small margin. Although bids are endogenous, we show that under an appropriate notion of competition satisfied in large markets, the probability that a bidder wins or loses an auction conditional on close bids approaches 50%, regardless of the bidders’ characteristics (e.g., the size of backlog, incumbency status, etc.). Winning and losing are “as-if-random” conditional on close bids. As a result, even if backlog or incumbency status are correlated to costs, the differences in these variables between close winners and close losers should vanish as the bid difference between them approaches zero.

Conversely, we show that if covariates are not identically distributed across close winners and losers, then bidders’ continuation values are either highly sensitive to the winning bid, or increasing in the revenue of competing bidders (i.e. bidders internalize the revenues of competitors). While this does not necessarily indicate illegal collusive behavior, it suggests that the industry under examination is exhibiting unusual dynamics, and probably deserves

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5See, for example, the “Red Flags Of Collusion” report, published by the U.S. DOJ, listing patterns suggestive of collusion.
further attention.

Importantly, bids generated by natural collusive strategies generate systematic differences in characteristics across close winners and close losers. In fact, our tests can provide some insight into the type of strategies used by bidders. Specific collusive strategies suggest both a relevant covariate, and predict the sign of the difference in expected characteristics. For example, if the bidding ring allocates projects to the incumbent bidder, close winners will be incumbents with significantly higher probability than close losers. In contrast, if bids are allocated using bid rotation, bidders with lower backlog will be more likely to be close winners than close losers.

We illustrate our test using two datasets. First, we consider the sample of Ohio school milk auctions studied by Porter and Zona (1999). Firms located around Cincinnati, Ohio were charged with colluding on hundreds of school milk auctions by allocating markets according to incumbency status (State of Ohio v. Louis Trauth Dairies, Inc. et al). According to the testimony of the representatives of the colluding dairies, the firms colluded by agreeing not to undercut the bid of the incumbent firm that had served a given school district in the previous year. We test whether or not marginal winners are more likely to be incumbents than marginal losers separately for the set of collusive auctions and the set of non-collusive auctions. We find that for collusive auctions, marginal winners are significantly more likely to be incumbents than marginal losers separately for the set of collusive auctions and the set of non-collusive auctions. We do not find statistically significant differences in incumbency status between marginal winners and marginal losers among non-collusive auctions despite the fact that the sample size is more than 10 times bigger.

Second, we apply our tests to a dataset of public procurement auctions held by municipalities from the Tohoku region of Japan. Firms in this dataset have not been prosecuted for collusion, but there are reasons to suspect that it is present. To proxy for potential

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6 More precisely, we apply the tests separately for the set of auctions in which all of the bidders were implicated and the set of auctions in which none of the participants were implicated.

7 Kawai and Nakabayashi (2018) provide evidence that some of the firms in this dataset colluded over procurement contracts let by the Ministry of Land, Infrastructure and Transportation. Chassang et al. (2020) suggest that non-competitive behavior may have been prevalent in auctions held by a different set of Tohoku municipalities. As we discuss below, the tests presented in the current paper complement previous
collusion, we split the sample of municipal bids into high and low bid groups depending on whether the bid is above or below the median winning bid for the municipality holding the auction. Because the purpose of collusion is to elevate prices, we expect a higher concentration of collusive auctions in the high bid group. As in the case of collusive Ohio school milk auctions, we find that marginal winners are significantly more likely to be incumbents than marginal losers in the high bid group. We also find that backlog is significantly lower for marginal winners compared to marginal losers in this group. This is suggestive of collusive behavior using both priority to incumbents and bid rotation. We do not find statistically significant differences in the characteristics of marginal winners and marginal losers for the low bid group.

**Literature.** Our work fits in the industrial organization literature interested in detecting collusion in auctions and markets. Pioneering work in this literature include Hendricks and Porter (1988), Baldwin et al. (1997), as well as Porter and Zona (1993, 1999). Our contribution is particularly related to Porter and Zona (1993) who study the impact of cost shifters such as backlog and proximity to construction sites on the bids and rank order of bidders in auctions for road pavement projects. They find that the losing bids of suspected ring members do not respond to cost shifters, suggesting that those bids are likely phantom bids. Although both Porter and Zona (1993) and our paper study the relationship between the rank order of bids and possible cost shifters to screen for collusion, the underlying idea behind the proposed tests are different. Porter and Zona (1993) focus on the lack of incentives among losing cartel bidders to bid in ways that reflect their true costs. Hence, their primary focus is on losing bidders. Our primary focus is on differences between winners and losers. The tests we propose are based on the idea that under collusion, close winners and losers need not be statistically similar: under incumbency priority close winners are more likely to be incumbents; under bid rotation close winners are likely to have lower backlog.

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8We normalize raw bids with each auction’s reserve price to make bids more comparable.

9Porter and Zona (1993) describe their tests as follows: “... our rank-based test is designed to detect differences in the ordering of higher bids, as opposed to the determinants of the probability of being the lowest bid...”, although parts of their paper analyze the determinants of the winner.

The paper shares its emphasis on general information structures with Chassang et al. (2020), but develops a qualitatively different strategy that considerably expands the scope for applications. In this previous work, we document that in a significant subset of procurement auctions held in Japan, winning bids are isolated – there are very few close winners and close losers. This pattern, as well as others, can be exploited to obtain lower bounds on the share of non-competitive histories under general information structures. The current paper complements this previous work by focusing on settings where the missing-bids pattern does not arise, i.e. when close winning and losing bids are not rare. Moreover, it exploits the information content of arbitrary bidder characteristics. This allows us to extend the analysis to environments with intertemporal linkages such as learning by doing, or increasing marginal costs, which Chassang et al. (2020) excludes. Our framework also lets analysts use any available covariate data to test for collusion.

We believe that the tests proposed in this paper are well suited to complement standard antitrust practice, as a tool to target agency attention and effort, or to justify more invasive evidence collection. First, our test formalizes intuitive ideas often mentioned by antitrust agencies. Second, the test is easy to implement and requires no sophisticated programming. Third, our approach does not require detailed data on project or bidder characteristics because the regression discontinuity design makes it less important to control for auction and bidder heterogeneity. Fourth, our approach naturally extends to other types of auctions.

\[\text{For a survey of the literature up to the mid 2000s, see Porter (2005) and Harrington (2008).}\]
such as handicap auctions, scoring auctions and all-pay auctions by appropriately modifying the running variable. Finally, our approach can be easily adapted to exploit other markers of collusion. Imagine a cartel is suspected of using geographic segmentation to allocate projects.\footnote{Pesendorfer (2000) documents evidence of market division among school milk providers in Texas.} With data on the location of firms and project sites, one could assess whether or not close winners are located nearer the project site than close losers. Another possible marker of collusion is the extent of subcontracting and joint bidding.\footnote{For example, the Department of Justice maintains a document called “Price Fixing, Bid Rigging, and Market Allocation Schemes: What They Are and What to Look For” , in which it states “Subcontracting arrangements are often part of a bid-rigging scheme.” Similar statements are found in a report by the OECD (2013). See also Conley and Decarolis (2016) for a discussion of subcontracting and collusion.} If procurement agencies require the list of subcontractors to be specified at the time of bidding, one can test whether or not marginal winners have more subcontractors than marginal losers.\footnote{For example, “Subletting and Subcontracting Fair Practices Act” (Public Contract Code 4100 et seq.) of California requires that “any person making a bid or offer to perform the work, shall, in his or her bid or offer, set forth ... (T)he name, the location of the place of business, ... of each subcontractor who will perform work or labor or render service to the prime contractor.”}

\section{Framework}

The section specifies our model of dynamic procurement. We describe our test of non-competitive behavior in Section 3, and provide theoretical foundations in Section 4. We turn to data in Section 5.

\textbf{Game form.} In each period $t \in \mathbb{N}$, a buyer procures a single item from a finite set $N$ of potential suppliers. The procurement contract is allocated through a sealed-bid first-price auction with a public reserve price $r$, which we normalize to 1. Each potential bidder $i \in N$ decides whether to participate in each auction. Bidders incur a cost $k > 0$ for submitting an actual bid $b_{i,t} \in [0,1]$, and may prefer not to participate, denoted by $b_{i,t} = \emptyset$.

We denote by $b_t = (b_{i,t})_{i \in N}$ the profile of bids, and by $\wedge b_t$ the lowest bid among participating bidders.\footnote{If no bidder participates, i.e. $b_i = \emptyset$ for all bidders, then by convention $\wedge b_t = +\infty$.} This is the winning bid. Ties are broken with uniform probability. We denote by $b_{-i,t} \equiv (b_{j,t})_{j \neq i}$ bids from firms other than $i$, and by $\wedge b_{-i,t} \equiv \min_{j \neq i} b_{j,t}$ the lowest.
bid among i’s participating competitors. Let \( \wedge b_{-i} \succ b_i \) denote the event that bidder \( i \) wins the contract, i.e. \( b_i \) is the lowest bid and possible ties are broken in favor of bidder \( i \). Bids are publicly revealed at the end of each period.

**Costs and payoffs.** In each period a profile of procurement costs \( c_t = (c_{i,t})_{i \in N} \in \mathbb{R}^N \) is realized. Firm \( i \)'s payoff at time \( t \) from submitting bid \( b_i \in [0, 1] \) is \( 1_{\wedge b_{-i} \succ b_i}(b_i - c_{i,t}) - k \). Bidders discount future payoffs using discount factor \( \delta < 1 \).

**State transitions.** In each period \( t \), before bidding, each bidder observes a state \( \theta_t \in \Theta \), with \( \Theta \) finite, summarizing the state of the industry. The state follows an endogenous Markov chain: \( \theta_{t+1} \) is distributed according to a probability distribution \( F_\Theta(\cdot | \theta_t, w_t^*) \), depending only on the previous state \( \theta_t \), and the identity of the winning bidder \( w_t^* \in \arg \min_{i \in N} b_{i,t} \).

Our model allows for settings in which a bidder’s procurement costs depend on backlog or incumbency status through state variable \( \theta_t \). For example, \( \theta_t \) can be a vector that tracks how many auctions each bidder has recently won, as well as their reserve price, to capture the effect of backlog on costs. Alternatively, \( \theta_t \) can be a vector that tracks whether or not a given bidder has won a particular type of auction to capture the effect of learning-by-doing. State \( \theta_t \) can also capture exogenous auction characteristics such as the distance between the project site and each of the bidders, the scale of the project, or the type of work being procured.

**Information.** In addition to state \( \theta_t \), each bidder \( i \) privately observes a signal \( z_{i,t} \in Z_i \), with \( Z_i \) finite. The distribution of signal profile \( z_t = (z_{i,t})_{i \in N} \in Z = \prod_{i \in N} Z_i \) depends only on \( \theta_t \) but is otherwise unrestricted. Signals may be arbitrarily correlated. We denote by \( F_Z(\cdot | \theta) \) the distribution of signals conditional on state \( \theta \).

Costs \( c_t = (c_{i,t})_{i \in N} \in \mathbb{R}^N \) are drawn independently conditional on state \( \theta_t \), and on each private signal \( z_{i,t} \). In particular, we have that

\[
c_{i,t} | \theta_t, z_{i,t} \sim c_{i,t} | \theta_t, z_t, c_{-i,t}.
\]
Bidder $i$’s cost does not provide information about the cost of other bidders beyond the information already provided in state $\theta_t$ and private signal $z_{i,t}$. We assume private values, so that each bidder observes her own costs. This class of information structures nests asymmetric independent private values, correlated values, and complete information. We denote by $F_C(\cdot | \theta_t, z_t)$ the conditional distribution of the profile of costs $c_t$ given state $\theta_t$, and signals $z_t$.

The underlying economic environment, denoted by $\mathcal{E}$, corresponds to the tuple $\mathcal{E} = (F_\Theta, F_Z, F_C)$.

**Observables.** We now introduce variables observed by the econometrician. We denote by $x_{i,t} \in X \subset \mathbb{R}^n$, with $X$ finite, the characteristics of bidder $i$ at time $t$ that the econometrician observes. The observables at time $t$, $x_t = (x_{i,t})_{i \in N}$, can be a subset of $\theta_t$, a coarsening of $\theta_t$, or any variable that is predetermined at the time of bidding. In our application, $x_{i,t}$ corresponds to measures of a bidder’s backlog or incumbency status. Given that many bidders in our data work on projects that are not in our dataset (e.g., construction work for other firms), our measures of backlog and incumbency are at best imperfect measures of the backlog and incumbency status that are relevant for bidders’ costs. Because observables $x$ can be arbitrary noisy statistics of $\theta_t$, our framework allows for unobserved heterogeneity and measurement error.

**Strategies and solution concepts.** A Markov strategy $\sigma_i$ is a mapping from information $h_{i,t} = (\theta_t, z_{i,t})$ and costs $c_{i,t}$ to bids $b_{i,t} \in [0, 1] \cup \{\emptyset\}$. Throughout the paper, we focus on Markov strategies, and Markov perfect equilibrium (Maskin and Tirole, 2001). A strategy profile $\sigma = (\sigma_i)_{i \in N}$ is a Markov perfect equilibrium (MPE) if it is a perfect Bayesian equilibrium in Markov strategies. MPEs have received much attention in the empirical industrial organization literature studying dynamic oligopolistic competition, starting with Ericson and

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15Because the signals are allowed to be correlated, $z_{i,t}$ helps bidder $i$ predict the cost of other bidders. The main restriction is that set $Z$ is finite. This ensures that pointwise convergence results established later hold uniformly over histories.

16More generally, $x_{i,t}$ can be any garbling (in the sense of Blackwell (1953)) of bidder $i$’s information at the time of bidding in period $t$. 

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Pakes (1995). However, as we highlight in Section 4, they are not a sufficient condition for competition: it is possible to sustain obviously collusive strategies in MPE.

3 Empirical Strategy

We now delineate our tests of non-competitive behavior and clarify the goal of our theoretical analysis.

Consider the problem of assessing whether firms in a given industry are engaging in collusive bid rotation. Empirically, this implies that bidders with a low backlog (less likely to have won in the recent past) are more likely to win than bidders with a large backlog. The difficulty is that there may also be competitive reasons for this pattern. Suppose that firms’ procurement costs are increasing with backlog. Even if firms are competing, on average, firms with lower backlog will have a lower cost and be more likely to win an auction than firms with higher backlog. In this environment, a test seeking to detect collusive bid rotation by comparing the unconditional backlog of winners and losers would yield false positives.

Our proposal is to compare the backlog of a selected group of firms: bidders that win or lose by a small margin. Intuitively, conditioning on close bids allows us to control for potential cost differences. The implicit hypothesis is that under competition, the identity of the winner is as-if-random conditional on close bids. As a result, close winners and losers should be statistically similar. If instead, close winners have consistently lower backlogs than close losers, this is evidence of collusive bid rotation.

We operationalize this idea as follows. Denote by \( \Delta_{i,t} \equiv b_{i,t} - \wedge b_{-i,t} \) the difference between the bid of firm \( i \), and the most competitive alternative bid at time \( t \). If \( \Delta_{i,t} < 0 \), bidder \( i \) wins the auction, if \( \Delta_{i,t} > 0 \), bidder \( i \) loses. Let \( x_{i,t} \) be a measure (observed by the econometrician) of firm \( i \)'s backlog before bidding at time \( t \) (alternatively it could be incumbency, or another relevant covariate). We define coefficient \( \beta \) as the difference in average backlog between close losers and close winners:

\[
\beta = \lim_{\epsilon \searrow 0^+} \mathbb{E}[x_{i,t} | \Delta_{i,t} = \epsilon] - \lim_{\epsilon \nearrow 0^-} \mathbb{E}[x_{i,t} | \Delta_{i,t} = \epsilon].
\] (1)
Note: For each firm \(i\) and auction \(t\), the standardized backlog of firm \(i\) at \(t\) is the Yen denominated amount of work it won in the 90 days prior to auction \(t\), re-expressed in units of standard deviation from the firm’s time-series average. The figure is a binned scatter plot of this measure against \(\Delta_{i,t}\). See Section 5 for details.

Figure 1: Binned Scatter Plot of Standardized Backlog, Japanese Municipal Auctions.

We test the null of \(\beta = 0\). When \(x\) denotes backlog, we expect \(\beta\) to be strictly positive under bid rotation. When \(x\) denotes incumbency status, we expect \(\beta\) to be strictly negative if the cartel allocates market share according to incumbency. Figure 1 foreshadows the results of Section 5 using a dataset of Japanese procurement auctions. The figure is a binned scatter plot that illustrates the relationship between bidder \(i\)’s 90-day backlog at time \(t\) against \(\Delta_{i,t}\). The null of \(\beta = 0\) is rejected: the average backlog is discontinuous around \(\Delta_{i,t} = 0\). Close winners have a significantly lower backlog than close losers.

A heuristic motivation. The null hypothesis that \(\beta = 0\) relies on the intuition that conditional on close bids, allocation should be as-if random under competition. This argument is easily formalized if we accept the premise that under competition, a bidder’s demand conditional on information \(h_{i,t} = (\theta_t, z_{i,t})\) is sufficiently smooth.
For all histories $h_{i,t} = (\theta_t, z_{i,t})$ and bids $b \in [0, 1]$, define bidder $i$’s residual demand as

$$D_i(b|h_{i,t}) \equiv \text{prob}(\land b > b|h_{i,t}).$$

$D_i(b|h_{i,t})$ is the probability with which firm $i$ expects to win the auction at history $h_{i,t}$ if she places bid $b$. The probability that bidder $i$ wins conditional on submitting a close bid satisfies

$$\text{prob}(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| \leq \epsilon) = \frac{D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}.$$  \hspace{1cm} (2)

It follows that whenever $D_i$ is strictly decreasing and continuously differentiable, then for a bid-difference $\epsilon$ small, the probability of winning conditional on close winning and losing bids is approximately $1/2$, regardless of history $h_{i,t}$. This is a straightforward consequence of the fact that the numerator on the right-hand side of (2) is approximately $\epsilon D'_i(b_{i,t}|h_{i,t})$ and the denominator is approximately $2\epsilon D'_i(b_{i,t}|h_{i,t})$.

**Lemma 1** (smooth demand). Assume that $D_i(\cdot|h_{i,t})$ is differentiable, with $D'_i(b_i|h_{i,t})$ strictly negative and continuous in bids $b_i \in [0, 1]$. For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that for all histories $h_{i,t}$,

$$\left| \text{prob}(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| \leq \epsilon) - \frac{1}{2} \right| \leq \eta.$$ \hspace{1cm} (3)

Lemma 1 implies the following corollary.

**Corollary 1.** For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that for all $x \in X$,

$$|\text{prob}(x_i = x \mid \Delta_{i,t} \in (0, \epsilon)) - \text{prob}(x_i = x \mid \Delta_{i,t} \in (-\epsilon, 0))| < \eta.$$  

In words, the distribution of covariates $x_{i,t}$ observable to the econometrician has to be the same for marginal winners and marginal losers.\footnote{In addition, the result must hold conditional on any information available to the bidder ahead of} Whenever $X$ is finite, Corollary 1 implies
that the expectation of $x_{i,t}$ conditional on $\Delta$ must be continuous around $\Delta = 0$. This is not true in the data illustrated by Figure 1.

**Why formal foundations are important.** A difficulty with the heuristic argument presented so far is that there exist competitive environments in which the premise of smooth demand is false. Consider a one-shot first price auction with an incumbent $I$ and an entrant $E$. Costs to the incumbent and entrant are common knowledge, respectively satisfying $c_I < c_E$. Participation cost $k$ is equal to 0. We show in Appendix C that in any efficient equilibrium, the residual demand faced by bidders conditional on their information is not smooth. It involves either a kink or a discontinuity. In turn, the probability of winning is not independent of bidder characteristics conditional on close bids: the incumbent wins with probability 1, while the entrant wins with probability 0.

A key feature of this example is that participation cost $k$ is zero. Hence, the entrant is willing to participate, even if she expects to make zero profits in the auction. This gives rise to discontinuities in firms' residual demand, leading to a violation of Corollary 1. In Section 4 we show that, if participation is costly, Corollary 1 holds under a suitable notion of competition.

**A rationale for smooth demand.** Before we turn to formal foundations for our tests in Section 4, we find it useful to clarify why the premise that residual demand is smooth under competition, but not under collusion, can be defended using partial, but perhaps more intuitive arguments.

As Dyer and Kagel (1996) and Ahmad and Minkarah (1988) describe, the bidding process for construction projects is affected by many seemingly random factors.\footnote{See Kawai and Nakabayashi (2018) for a detailed discussion of these two papers.} In competitive environments these random factors are priced into bids and may smooth out bidders’ residual demand.\footnote{A behavioral model of Samuelson (2005) captures these ideas.} In collusive settings, there is less pressure to price in random factors, since bids bidding. As a result, our tests can be applied to subsets of data adapted to bidders’ information in the sense of Chassang et al. (2020). In Section 5.2 we leverage this and apply our tests separately for bids above and below the median winning bid.
are not competitive: cartel members may simply be told how to bid. In fact, there are strong reasons to make bids predictable under collusion (Athey et al., 2004): since bids are unrelated to costs, accidentally outbidding an opponent can lead to costly misallocation, or even price wars. As a result, residual demand need not be smooth under collusion.

4 Theoretical Foundations

In this section we provide theoretical foundations for the hypothesis that assignment conditional on close bids should be as-if random under competition. Our analysis exploits incentive compatibility constraints specific to first-price auctions to establish this result. We begin with a discussion of competition in dynamic auctions where past behavior and future payoffs are linked.

4.1 Competition

Defining a solution concept capturing the essence of competition in dynamic auctions turns out to be tricky. This is because competitiveness is a joint property of the underlying economic environment $E$ and bidder conduct $\sigma$. For instance, while Markov perfect equilibrium (Maskin and Tirole, 2001) may seem intuitively competitive when the payoff-relevant state $\theta_t$ evolves exogenously, this is no longer the case when the payoff-relevant state depends on past actions. The information content of payoff-relevant states can then be used to support sophisticated collusive behavior in Markov strategies.

Ultimately, we think that competitiveness is best understood on a spectrum where some combinations of environment $E$ and conduct $\sigma$ are clearly competitive, some are clearly collusive, and some are in a gray area. Two extreme cases seem to us as fairly clear-cut: non-Markov perfect behavior is not competitive since firms are then coordinating behavior on the basis of non-payoff relevant data; in contrast, Markov perfect behavior in large markets is competitive.

In contrast Markov perfect behavior need not be competitive in small markets. Depending
on the richness of Markov state \( \theta_t \), MPE may support collusive behavior. Furthermore, restricting attention to MPEs does not necessarily imply that Corollary 1 holds. We establish these points using an explicit example described in Appendix A. We describe an environment satisfying assumptions made so far, including positive participation costs, and exhibit an MPE exploiting the information content of payoff-relevant states to sustain prices much larger than costs. Bidding data generated by this MPE fails to satisfy Corollary 1: close winners have lower average backlog than close losers.

**Exchangeable winners.** In large markets, where bidders cannot repeatedly interact with one another, MPEs are intuitively competitive. In such environments, MPEs satisfy the following form of anonymity: the identity of a recent winner does not matter to losing bidders.\(^{20}\) We formalize this property as one possible definition of unambiguously competitive auctions.

Consider an environment \( \mathcal{E} \) and an MPE \( \sigma \). For any \( i, j \in N \), and state \( \theta \in \Theta \), let \( W_i(\theta, j) \) denote bidder \( i \)'s expected continuation value conditional on state \( \theta \), and winner \( j \):

\[
W_i(\theta, j) = \mathbb{E}_{\mathcal{E},\sigma} \left[ \sum_{t=1}^{\delta^t-1} \pi_{i,t} \bigg| \theta_0 = \theta, w^*_0 = j \right]
\]

where \( \pi_{i,t} \) denotes the profits of bidder \( i \) in period \( t \) (net of participation costs).

**Definition 1** (exchangeable winners). *We say that winners are exchangeable if and only if for any bidders \( j, j' \neq i \) (including null bidder \( \emptyset \) if nobody participates), and state \( \theta \in \Theta \),

\[
W_i(\theta, j) = W_i(\theta, j').
\]

In other terms, conditional on losing, a bidder \( i \) is indifferent about the identity of the winner. It can be shown that this property holds in large markets, where the identity of future competitors is not affected by who won the current auction. Note that a bidder’s value may still depend on whether she wins or loses the auction. The bidder is only indifferent...

\(^{20}\)These are environments in which oblivious equilibrium (Weintraub et al., 2008) coincides with MPE.
over the identity of the winner conditional on losing.

4.2 Equilibrium beliefs conditional on close bids.

We now show that when winners are exchangeable, contract allocation conditional on close bids is as-if random. For the results that follow, we maintain the assumption that bidders incur a strictly positive participation cost (i.e., \( k > 0 \)). This implies that competitive bidders do not participate if they expect to lose with probability close to 1.

Take as given an environment \( \mathcal{E} \) and an MPE \( \sigma \). In order to capture bidder \( i \)'s dynamic incentives, we define the expected value \( V_i \) conditional on allocation, winning bid, and history. Let us denote by \( \zeta_i \in \{0, 1\} \) bidder \( i \)'s outcome in the auction (where \( \zeta_i = 1 \) denotes winning the auction). For any history \( h_i = (\theta, z_i) \), winning bid \( b_w \) and allocation outcome \( \zeta_i \) for bidder \( i \), let

\[
V_i(\zeta_i, b_w | h_i) \equiv \mathbb{E}_{\mathcal{E}, \sigma}[W_i(\theta, w^*) | 1_{b_i < \wedge b_{-i}} = \zeta_i, \wedge b = b_w, h_i].
\]

Value \( V_i \) is the expected continuation value of player \( i \) depending on whether she wins the auction, and the winning bid.

**Remark 1.** Conditional on winning, bidder \( i \)'s value \( V_i(1, b_i | h_i) \) does not depend on her own bid \( b_i \).

This is driven by the fact that: (i) \( V_i \) controls for the current state through history \( h_i \); (ii) state transitions depend only on the current state and the identity of the winner. In general, continuation values upon losing may depend on the winning bid, since the winning bid may be correlated with the identity of the winning bidder. This is no longer the case if winners are exchangeable.

**Remark 2.** Take as given an allocation \( \zeta_i \) to player \( i \), and a history \( h_i \). When winners are exchangeable, \( V_i(\zeta_i, b_w | h_i) \) is independent of winning bid \( b_w \).

Indeed, if bidder \( i \) loses the auction, the fact that winners are exchangeable implies that \( V_i(0, b_w | h_i) = W_i(\theta, j) \) for any fixed bidder \( j \neq i \), and all winning bids \( b_w \).
Our first main result establishes that conditional on being a close winner or loser, any bidder believes that they win with probability greater than 50%.

**Proposition 1** (equilibrium beliefs conditional on close bids). Consider an environment $\mathcal{E}$ and an MPE $\sigma$ such that winners are exchangeable. For all $\eta > 0$ there exists $\epsilon > 0$ small enough such that, for all histories $h_{i,t} = (\theta_t, z_{i,t})$ and bid $b_{i,t} \in (\epsilon, 1-\epsilon)$,

$$\text{prob}_\sigma(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| < \epsilon) \geq 1/2 - \eta.$$  

**Proof.** Since conditional value $V_i(\zeta_i, b_w|h_i)$ is independent of winning bid $b_w$ we drop it from the list of arguments. Bidder $i$’s discounted expected payoff from bid $b \in (\epsilon, 1-\epsilon)$ at history $h_{i,t} = (\theta_t, z_{i,t})$ can be written

$$U^\sigma_i(b|h_{i,t}) = \mathbb{E}_\sigma [(b - \kappa_{i,t})1_{b_{i,t} < \land b_{-i,t}}|h_{i,t}] + \delta V_i(0|h_{i,t}) - k$$

$$= D_i(b|h_{i,t})(b - \kappa_{i,t}) + \delta V_i(0|h_{i,t}) - k$$

where $\kappa_{i,t} \equiv c_{i,t} - \delta(V_i(1|h_{i,t}) - V_i(0|h_{i,t}))$ is bidder $i$’s cost of winning the auction, including its impact on continuation values. Note that firm $i$ would obtain a payoff of $\delta V_i(\theta_t, 0)$ if she didn’t submit a bid. Hence, bidder $i$’s participation constraint implies that $D_i(b_{i,t}|h_{i,t})(b_{i,t} - \kappa_{i,t}) \geq k > 0$, so that $b_{i,t} - \kappa_{i,t} \geq k$.

Since bid $b_{i,t}$ is optimal, for all $\epsilon > 0$ we have that

$$U_i(b_{i,t}|h_{i,t}) \geq U_i(b_{i,t} + \epsilon|h_{i,t})$$

$$\iff D_i(b_{i,t} + \epsilon|h_{i,t})(b_{i,t} + \epsilon - \kappa_{i,t}) \leq D_i(b_{i,t}|h_{i,t})(b_{i,t} - \kappa_{i,t}),$$

and

$$U_i(b_{i,t}|h_{i,t}) \geq U_i(b_{i,t} - \epsilon|h_{i,t})$$

$$\iff D_i(b_{i,t} - \epsilon|h_{i,t})(b_{i,t} - \epsilon - \kappa_{i,t}) \leq D_i(b_{i,t}|h_{i,t})(b_{i,t} - \kappa_{i,t}).$$
Conditions (2), (4) and (5) imply that

\[
\text{prob}_\sigma(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) = \frac{D_i(b_{i,t} | h_{i,t}) - D_i(b_{i,t} + \epsilon | h_{i,t})}{D_i(b_{i,t} - \epsilon | h_{i,t}) - D_i(b_{i,t} + \epsilon | h_{i,t})}
\]

\[
= 1 - \frac{D_i(b_{i,t} + \epsilon)}{D_i(b_{i,t})} \geq 1 - \frac{b_{i,t} - \kappa_{i,t}}{b_{i,t} + \epsilon - \kappa_{i,t}}
\]

\[
\geq \frac{1}{2} \frac{k - \epsilon}{k} \rightarrow \frac{1}{2} \text{ as } \epsilon \searrow 0.
\]

Note that the speed of convergence of lower bound \(\frac{1}{2} \frac{k - \epsilon}{k}\) is independent of \(b_{i,t}\). This concludes the proof.

Proposition 1 provides a lower bound on firms’ winning probability at any given history, conditional on close bids. Because at most one bidder can win, and because there are at least two close bidders conditional on the existence of close bids, it cannot be that firms’ winning probability (conditional on her information) is frequently much larger than \(1/2\). We now make this argument formal. For any \(\epsilon > 0\), let \(\epsilon\)-close denote the event that the winning bid is within \(\epsilon\) of the second lowest bid. For any environment \(E\), MPE \(\sigma\) and threshold \(\epsilon > 0\), let \(E_{E,\sigma}[\cdot | \epsilon\text{-close}]\) denote the expectation over histories \(h\) conditional on the event \(\epsilon\text{-close} \).

**Corollary 2** (as-if random bids). Consider an environment \(E\) and MPE \(\sigma\) such that winners are exchangeable. For all \(\eta > 0\) there exists \(\epsilon > 0\) small enough such that

\[
E_{E,\sigma} \left[ \text{prob}_\sigma(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) - \frac{1}{2} \right | \epsilon\text{-close} \right] \leq \eta. \tag{6}
\]

In words, winning is as-if-random conditional on close bids. An implication of Corollary 2 is that Corollary 1 (Section 3) holds whenever winners are exchangeable.

**Sample implications.** Corollary 2 holds under the joint distribution of bids and histories generated by under an MPE \(\sigma\). In empirical applications, however, this distribution is not
directly observed and must be replaced by a sample counterpart. In Appendix C we show that if (6) holds under the bidders’ beliefs, then it holds asymptotically under the sample joint distribution of bids $b$ and characteristic $x \in X^N$ observable to the econometrician.\textsuperscript{21}

The reason such a result holds is that bidders get sufficient feedback about past states and outcomes: in our framework, bidders observe both past states $\theta$, and past bids $b$. This prevents bidders from making repeated mistakes about realized bidding profiles and characteristics.\textsuperscript{22} Expectations must match sample averages with high probability.

### 4.3 Drawing inferences from dissimilar winners and losers

We now present a converse: if the characteristics of close winners and losers are dissimilar, what inferences can we draw about the underlying environment and bidder behavior? We know that winners cannot be exchangeable, but we can in fact derive further implications regarding bidders’ continuation values conditional on losing an auction.

**Definition 2.** (i) We say that bidding behavior is sensitive if there exists $h_i$ and an assignment to $i$ such that the value of a losing bidder $V_i(0, b_w|h_i)$ is not Lipschitz continuous in $b_w$.

(ii) We say that bidding behavior internalizes competitors’ revenues if there exists a history $h_i$, and winning bids $b_w > b'_w$ such that $V_i(0, b_w|h_i) > V_i(0, b'_w|h_i)$.

When firm conduct is sensitive, a small change in bids can have a disproportionate effect on a losing bidder’s continuation value. When firm conduct internalizes competitor revenue, there are circumstances in which a bidder $i$ prefers losing to a competitor that generates higher revenues. Note that neither property holds in large markets such that winners are exchangeable, since in that case, continuation value $V_i$ is independent of the winning bid.

\textsuperscript{21}We show in Chassang et al. (2020) that this holds for any subset of histories adapted to the bidders’ information at the time of bidding.

\textsuperscript{22}Bidders do receive feedback from past auctions in our empirical applications. Indeed, municipalities in Japan are usually required to post auction outcomes shortly after each auction, typically within five days.
Proposition 2. Consider an environment $\mathcal{E}$ and an MPE $\sigma$ such that for some observable $x \in X$,

$$\limsup_{\epsilon \downarrow 0^+} |\operatorname{prob}(x_{i,t} = x | \Delta_{i,t} \in (0, \epsilon)) - \operatorname{prob}(x_{i,t} = x | \Delta_{i,t} \in (-\epsilon, 0))| > 0.$$ 

It must be that bidding behavior under $\mathcal{E}$ and $\sigma$ is either sensitive, or internalizes competitors’ revenues.

While neither sensitive behavior nor internalizing competitors’ revenue are unambiguous marker of collusion, they indicate that market dynamics are very distinct from large market dynamics, and warrant further examination. Sensitive behavior goes against the idea that “minor causes should have minor effects,” emphasized by Maskin and Tirole (2001) as a rationale for MPE. Internalizing competitors’ revenues is intuitively suspect, and arises naturally in collusive equilibria.

5 Empirical Analysis

5.1 Ohio School Milk Auctions

In order to validate our test, we first apply it to the sample of Ohio school milk auctions analyzed by Porter and Zona (1999). Porter and Zona (1999) study bidding on school milk auctions using data collected by the state of Ohio as part of its efforts to sue dairies for bid rigging. The dataset is an unbalanced panel of milk auctions let by Ohio school districts spanning 11 years between 1980 and 1990 with information on the bids and the identity of the bidders.\(^{23}\)

Several features of the setting are worth highlighting. First, the auctions are recurring. School districts hold auctions every year, typically between May and August to determine the supplier of milk for the following school year. This allows us to easily track the incumbent firm for a given auction. Second, the dataset includes bids from three bidders located around

\(^{23}\)We use the dataset constructed by Wachs and Kertész (2019).
Cincinnati that were charged for collusion. According to the testimony of the individuals involved, the cartel allocated contracts according to incumbency. Aside from two years (1983 and 1989) during which the cartel broke down, conspirators respected incumbency, with non-incumbents submitting complementary bids.

Table 1 reports summary statistics of the data. Column (1) reports summary statistics for all of the auctions in the sample, column (2) reports those for the subset of auctions in which only the defendant firms participated (Non-competitive) and column (3) reports those for the subset of auctions in which no cartel firm participated (Control). Because the cartel broke down in 1983 and 1989 according to the testimony of the individuals who were involved in collusion, we also report summary statistics for the sample that excludes years 1983 and 1989 for columns (2) and (3). We find that, on average, the number of bidders is about 1.86 for the entire sample, and slightly higher for the non-competitive sample than for the control sample. The winning bid, reported in units of dollars per half-pint of milk, is about $0.131 for the entire sample, and slightly higher in the non-competitive sample. Table 1 also reports the average second lowest and third lowest bids.

Table 2 reports summary statistics with respect to incumbency. We define a bidder to be an incumbent for a given school milk auction if the bidder was the winner of the district’s auction in the previous year. Column (1) corresponds to the set of all auctions in the dataset, while columns (2) and (3) respectively correspond to auctions in which all participants were defendants and auctions in which none of the participants were defendants. Focusing on the row labeled 1981 in column (1), we find that there are a total of 185 auctions in which an incumbent firm participates. Out of these auctions, the incumbent won 136 of them, or about 74%. Note that we lack the data needed to define incumbency for 1980, which is the first year of the sample. The fraction of auctions in which the incumbent wins is about 80% in column (1), 86% in column (2) if we exclude years 1983 and 1989 (83% if we include those two years), and 81% in column (3). While the fractions are slightly higher in column (2) than
Table 1: Summary Statistics of Auctions: Ohio School Milk Auctions.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Years</td>
<td>Non-Competitive &amp; Excl 83,89</td>
<td>Control &amp; Excl 83,89</td>
</tr>
<tr>
<td># Bidders</td>
<td>1.866</td>
<td>1.983</td>
<td>2.058</td>
</tr>
<tr>
<td></td>
<td>(0.909)</td>
<td>(0.891)</td>
<td>(0.882)</td>
</tr>
<tr>
<td>Winning Bid</td>
<td>0.131</td>
<td>0.136</td>
<td>0.138</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>2nd-Lowest Bid</td>
<td>0.135</td>
<td>0.142</td>
<td>0.144</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>3rd-Lowest Bid</td>
<td>0.138</td>
<td>0.147</td>
<td>0.149</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.016)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Obs.</td>
<td>3,754</td>
<td>235</td>
<td>189</td>
</tr>
</tbody>
</table>

Note: The first column corresponds to the set of all auctions, the second column corresponds to the set of auctions in which only the defendant firms bid and the last column corresponds to those in which no defendant firm bid.

in column (3), the differences are quite small. This highlights the general difficulty of using incumbency patterns to detect collusion since both collusive and competitive auctions are characterized by high rates of incumbency. As we will show below, the differences between the two samples become pronounced only when we condition on close auctions.

Figure 2 plots the histogram of the running variable, \( \Delta_{i,t} = b_{i,t} - \wedge b_{-i,t} \). A negative value of \( \Delta_{i,t} \) implies that bidder \( i \) won auction \( t \), and a positive value of \( \Delta_{i,t} \) implies that bidder \( i \) lost auction \( t \). Values of \( \Delta_{i,t} \) close to zero correspond to auctions in which the winner was determined by a very small margin. The left panel of Figure 2 corresponds to the full sample, the middle panel corresponds to the sample of non-competitive auctions and the right panel corresponds to the control sample. There are no obvious differences in the distribution of bid differences \( \Delta_{i,t} \).24

24This highlights the value-added of considering covariates to detect non-competitive behavior. Tests provided in Chassang et al. (2020) use only the information contained in the distribution of \( \Delta_{i,t} \), and would draw similar inference from the different datasets illustrated in Figure 2.
<table>
<thead>
<tr>
<th>Year</th>
<th>All</th>
<th>Non-Competitive</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Win/Inc Ratio</td>
<td>Total</td>
<td>Win/Inc Ratio</td>
</tr>
<tr>
<td>1980</td>
<td>.</td>
<td>249</td>
<td>0.</td>
</tr>
<tr>
<td>1981</td>
<td>136/185</td>
<td>0.74</td>
<td>6/7</td>
</tr>
<tr>
<td>1982</td>
<td>148/188</td>
<td>0.79</td>
<td>9/10</td>
</tr>
<tr>
<td>1983</td>
<td>162/214</td>
<td>0.76</td>
<td>7/10</td>
</tr>
<tr>
<td>1984</td>
<td>199/249</td>
<td>0.80</td>
<td>18/20</td>
</tr>
<tr>
<td>1985</td>
<td>205/260</td>
<td>0.79</td>
<td>18/18</td>
</tr>
<tr>
<td>1986</td>
<td>242/293</td>
<td>0.83</td>
<td>16/19</td>
</tr>
<tr>
<td>1987</td>
<td>236/287</td>
<td>0.82</td>
<td>18/20</td>
</tr>
<tr>
<td>1988</td>
<td>253/304</td>
<td>0.83</td>
<td>18/20</td>
</tr>
<tr>
<td>1989</td>
<td>257/332</td>
<td>0.77</td>
<td>13/19</td>
</tr>
<tr>
<td>1990</td>
<td>185/247</td>
<td>0.75</td>
<td>17/29</td>
</tr>
<tr>
<td>Obs.</td>
<td>3,754</td>
<td>235</td>
<td>3,267</td>
</tr>
</tbody>
</table>

Note: Column (1) corresponds to the set of all auctions, Column (2) corresponds to the set of auctions in which only the defendant firms bid and the Column (3) corresponds to those in which no defendant firm bid.

Table 2: Summary Statistics on Incumbency: Ohio School Milk Auctions.

Figure 2: Histogram of $\Delta_{i,t}$: Ohio School Milk Auctions.

**Empirical implementation.** Recall the definition of coefficient $\beta$,

$$
\beta = \lim_{\Delta_{i,t} \uparrow 0^+} E[x_{i,t} | \Delta_{i,t}] - \lim_{\Delta_{i,t} \downarrow 0^-} E[x_{i,t} | \Delta_{i,t}].
$$
We define the variable $x_{i,t}$ as a dummy variable for incumbency status, i.e., $x_{i,t} = 1$ if firm $i$ is an incumbent in auction $t$, and 0 otherwise. If a cartel allocates contracts to incumbents, we expect $\beta$ to be strictly negative.

We estimate $\beta$ using a local linear regression as follows:

$$\hat{\beta} = \hat{b}_1^+ - \hat{b}_1^-, \text{ with}$$

$$(\hat{b}_1^+, \hat{b}_1^-) = \arg \min \sum_{i,t} (X_{i,t} - b_0^+ - b_1^+ \Delta_{i,t})^2 K\left(\frac{\Delta_{i,t}}{h_n}\right) 1_{\Delta_{i,t}>0},$$

$$(\hat{b}_1^-, \hat{b}_1^-) = \arg \min \sum_{i,t} (X_{i,t} - b_0^- - b_1^- \Delta_{i,t})^2 K\left(\frac{\Delta_{i,t}}{h_n}\right) 1_{\Delta_{i,t}<0},$$

where $h_n$ is the bandwidth and $K(\cdot)$ is the kernel. For our baseline estimates, we use a coverage error rate optimal bandwidth and a triangular kernel with a bias correction procedure as proposed in Calonico et al. (2014). Standard errors are clustered at the auction level. We test the null $H_0: \beta = 0$, against the alternative $H_1: \beta \neq 0$.

**Results.** Table 3 presents the results. Panel (A) reports estimates $\hat{\beta}$ for the sample of auctions in which only the defendant firms participated. In column (1), we use all years between 1980 and 1990 while in column (2), we exclude 1983 and 1989, the two years in which the cartel purportedly broke down. In both columns, we focus on the sample of auctions in which there is an incumbent. We find that the gap $\beta$ in incumbency rates across close losers and winner is negative ($-0.300$) and marginally statistically significant ($p = 0.087$) for column (1). The point estimate implies that the marginal winner is about 30.0 percentage points more likely to be an incumbent than the marginal loser. The bandwidth used for estimation is 0.004, or 0.4 cents. In column (2), we find that the estimate is $-0.381$, and statistically significant at the 5 percent level.

Panel (B) reports findings for the set of control auctions. We find that the regression discontinuity estimate is $-0.045$ in column (1), which is not statistically different from zero.
Because there is no reason to expect 1983 and 1989 to be any different from other years for non-colluding firms, we do not expect any significant differences between column (1) and column (2) for Panel (B). Indeed, the estimate of $\beta$ in column (2) is $-0.067$, and statistically indistinguishable from 0.

Overall, the results of Table 3 suggest that our test has reasonable power and size in practice. Figure 3 illustrates the binned scatter plots that correspond to the results in Table 3. The left panel of the figure corresponds to the sample of non-competitive auctions excluding 1983 and 1989, and the right panel corresponds to the sample of all control auctions. The left panel of the figure displays a visible discontinuity in incumbency status between marginal winners and marginal losers while the right panel of the figure shows a smooth continuous relationship between $\Delta_{i,t}$ and incumbency status. As it is clear from the figure, incumbents win with high probability even among the competitive sample. It is only by looking at marginal auctions that we find differential rates of incumbency between the two samples.

5.2 Public Procurement Auctions in Japan

Our second dataset consists of bids submitted by construction firms participating in auctions for construction projects let by 16 municipalities in the Tohoku region of Japan. Our baseline sample consists of roughly 11,000 procurement auctions taking place between 2004 and 2018. The total award amount for these auctions is about 232 billion yen, or about $2.3 billion U.S. dollars. No firm has been charged for colluding in any of the auctions in our sample. However, as we note in the Introduction, results in Kawai and Nakabayashi (2018) and Chassang et al. (2020) suggest that at least some of these auctions might be collusive.

The baseline sample consists of auctions from municipalities in which tests of non-
<table>
<thead>
<tr>
<th>Panel (A) : Non-competitive auctions</th>
<th>Panel (B) : Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta} ) &amp; -0.300* &amp; -0.381**</td>
<td></td>
</tr>
<tr>
<td>( h ) &amp; 0.004 &amp; 0.005</td>
<td></td>
</tr>
<tr>
<td>Obs. &amp; 309 &amp; 266</td>
<td></td>
</tr>
</tbody>
</table>

Panel (A) corresponds to the sample of auctions in which only the defendant bidders bid. Panel (B) corresponds to the sample of control auctions in which none of the defendant bidders bid. Standard errors are clustered at the auction level and reported in parenthesis. The table also reports the bandwidth \( h \) used for the estimation. *, **, and *** respectively denote significance at the 10%, 5%, and 1% levels.

Table 3: Regression Discontinuity Estimates: Ohio School Milk Auctions.

In order to choose the set of municipalities to include in our sample, we first compute the density of the running variable for each municipality. The running variable, \( \Delta_{i,t} \), is defined as \( \Delta_{i,t} = b_{i,t} - \wedge b_{-i,t} \) where bids are in percentages of the reserve price. We include the municipality in the sample if the following is satisfied:

\[
0.7 \times \max_{d \in [-3\%, -0.5\%]} f_{\Delta}(d) \leq \min_{d \in [-0.2\%, 0\%]} f_{\Delta}(d),
\]

where \( f_{\Delta}(d) \) is the density of \( \Delta \). In municipalities with isolated bids, there will be a trough in the density of \( f_{\Delta} \) around 0, and the inequality is not satisfied. We also drop municipalities in which \( f_{\Delta} \) exhibits a mass at 0. We do so by running a McCrary test (McCrary, 2008) on the running variable and dropping municipalities with p-values less than 0.05. The auctions in these municipalities have binding price floors.

In the Online Appendix, we show that our findings extend to the sample of all of municipalities from which we have obtained data. This is not surprising since it is likely that cartels are operating in the excluded cities.
Figure 3: Binned Scatter Plot for Incumbency: Ohio School Milk Auctions

5.2.1 Data and Empirical Implementation

Data and institutional background. Auctions are first-price sealed bid and the lowest bidder is awarded the project subject to the reserve price. Some of the municipalities use public reserve prices and others use secret reserve prices. For example, in 2012, 7 municipalities used public reserve prices, 8 municipalities used secret reserve prices, and 1 municipality used both. The lowest bid was rejected in about 12.5% of the overall sample. Online Appendix B shows that our findings are qualitatively unchanged for the subset of municipalities with a public reserve price.

Our data includes all bids, the identity of bidders, and a brief description of the construction project. Column (1) of Table 4 reports summary statistics of the auctions. On average,
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th></th>
<th>(2)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>Reserve (Mil. Yen)</td>
<td>22.26</td>
<td>77.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Winning Bid (Mil. Yen)</td>
<td>20.71</td>
<td>71.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Win Bid/Reserve</td>
<td>0.926</td>
<td>0.083</td>
<td></td>
<td></td>
</tr>
<tr>
<td># of Bidders</td>
<td>6.80</td>
<td>4.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incumbent Participates (0/1)</td>
<td>0.044</td>
<td>0.204</td>
<td></td>
<td></td>
</tr>
<tr>
<td># of Auctions Participated</td>
<td></td>
<td></td>
<td>22.56</td>
<td>45.93</td>
</tr>
<tr>
<td># of Wins</td>
<td>3.32</td>
<td>6.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw Backlog (90-Day)</td>
<td>4.11</td>
<td>17.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw Backlog (180-Day)</td>
<td>6.45</td>
<td>22.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>11,207</td>
<td></td>
<td>3,377</td>
<td></td>
</tr>
</tbody>
</table>

Note: The reserve price, winning bid, and backlog measures are reported in units of millions of yen.

Table 4: Summary Statistics of Auctions and Bidders: Municipal Auctions from Japan.

The reserve price is 22.26 million yen, or about 222,000 US dollars. The average winning bid is 20.71 million yen. The average ratio of the winning bid to the reserve is about 92.6%. On average, 6.80 bidders participate in each auction. Column (2) reports summary statistics of the bidders in our sample. Bidders in our sample participate on average in 22.56 auctions and win on average 3.32 times. The table also reports summary statistics on incumbents and the amount of backlog of the firms. We discuss how we define these variables next.

**Empirical implementation.** Our first covariate of interest is the firms’ backlog. We consider both raw backlog and standardized backlog. We define the raw backlog of firm \( i \) at auction \( t \) as either the 90-day or 180-day cumulative size (measured by the reserve price) of projects won by firm \( i \). We define the 90-day and 180-day backlogs, denoted by \( B_{90}^{i,t} \) and \( B_{180}^{i,t} \) respectively.
$x_{i,t}^{B_{180}},$ as follows:

$$x_{i,t}^{B_k} \equiv \sum_{\tau \in T^k_t} r_{\tau} \mathbf{1}_{\mathbf{a}_{-i,\tau} > b_{i,\tau}},$$

where $r_t$ denotes the reserve price of auction $t$ and $T^k_t$ denotes the set of auctions in our sample that take place in the $k \in \{90, 180\}$ days prior to auction $t$. We make sure not to include auction $t$ in $T^k_t$ since its outcome is not in the information set of bidders at time $t$. Although the raw backlog is a natural metric for capturing the amount of work recently awarded to a firm, variation in raw backlog captures both intertemporal change in backlog as well as heterogeneity in firm size. In order to construct a measure of backlog that only captures the intertemporal variation, we standardize the raw backlog at the firm level, using its within-firm mean and standard deviation. The 90-day and 180-day standardized backlogs, $x_{i,t}^{\overline{B}_{90}}$ and $x_{i,t}^{\overline{B}_{180}}$, are defined as follows:

$$x_{i,t}^{\overline{B}_k} = \frac{x_{i,t}^{B_k} - \mu_{x_{i}^{B_k}}}{\sigma_{x_{i}^{B_k}}},$$

where $\mu_{x_{i}^{B_k}}$ is the within-firm mean of $x_{i,t}^{B_k}$ and $\sigma_{x_{i}^{B_k}}$ is the within-firm standard deviation of $x_{i,t}^{B_k}$. Because standardized backlog is defined relative to the firm’s own historical average, $x_{i,t}^{\overline{B}_k}$ is zero if firm $i$’s raw backlog is equal to its time-series average at the time of auction $t$. In Online Appendix B, we also consider an alternative definition of standardized backlog in which we use the moving average for $\mu_{x_{i}^{B_k}}$ in expression (7) and the moving standard error for $\sigma_{x_{i}^{B_k}}$ to compute the within-firm mean and standard deviation. The results are very similar.

We emphasize that all of our backlog measures are, at best, noisy measures of the firms’ true cost-relevant backlog. The number of days we use to define our backlog measures (90 or 180 days) is arbitrary, and most firms are likely to work on projects that are not included in
our sample, or our measures of backlog. This does not invalidate our test. As we discussed in Section 2, variables observable to the econometrician can be imperfect and imprecise. Corollary 1 holds regardless.

Column (2) of Table 4 reports summary statistics of raw backlog in millions of yen. The average 90-day backlog is around 4.11 million yen and the average 180-day backlog is around 6.45 million yen. Standardized backlog averages to zero for each firm by construction.

Another covariate of interest is whether or not a given firm is an incumbent for a given project. We define a firm to be an incumbent if it is the winner of the previous auction with the same project name let by the same municipality. To give an example, the city of Miyako in Iwate prefecture held procurement auctions with the project name “Restoration of Yagisawa public housing complex” on 3 occasions, November 22, 2011, September 19, 2012, and December 16, 2014. A firm named Kikuchi Painting won each time. We define this firm to be the incumbent in the second and third auctions. We define all other participants in the second and third auctions to be non-incumbents. We do not define incumbency status for any of the bidders in the first auction. Similarly, if there is only one auction for a given project name in a municipality, we do not define incumbency for any bidders. Column (1) of Table 4 reports summary statistics of incumbency status. There is an incumbent bidder in 4.4% of the auctions in our sample.

The running variable is \( \Delta_{i,t} = b_{i,t} - \wedge b_{-i,t} \), where bids are normalized by the reserve price. The left panel of Figure 4 is the histogram of \( \Delta_{i,t} \). The distribution is skewed to the right of zero because the average number of bidders is 6.80 (\( \Delta_{i,t} \) is negative for only one bidder per auction, and it is positive for all of the losing bidders). Because we report our regression discontinuity results separately for the set of bids above and below the median winning bid for the municipality letting the auction, the next two panels of Figure 4 plot

---

28 Many bidders who participate in auctions let by municipal governments also participate in auctions that are let by the Ministry of Land Infrastructure and Transportation and prefectural governments. Many firms also do work for other private firms.
the histogram of $\Delta_{i,t}$ separately for the two sets of auctions. The middle panel corresponds to the sample of bids below the municipal median and the right panel corresponds to bids above the median.

![Histogram of $\Delta_{i,t}$](image)

Note: The left panel corresponds to the histogram of $\Delta_{i,t}$ for the entire sample. The middle panel corresponds to the sample of bids below the median winning bid of the relevant municipality. The right panel corresponds to the sample of bids above the median. The histogram is truncated at $\Delta_{i,t} = -0.1$ and $\Delta_{i,t} = 0.1$ for readability.

Figure 4: Histogram of $\Delta_{i,t}$: Municipal Auctions from Japan.

As before, we estimate discontinuities in the expectation of $x_{i,t}$ as a function of $\Delta_{i,t}$ using a local linear regression with a coverage error rate optimal bandwidth and a triangular kernel with a bias correction procedure as proposed in Calonico et al. (2014).\textsuperscript{29} Standard errors are clustered at the auction level.

5.2.2 Results

Table 5 reports estimates of discontinuity $\beta$. We distinguish statistics computed for the subset of bids above the municipality median (Panel (A)) and statistics computed for the subset of bids below the median (Panel (B)).\textsuperscript{30} We expect the latter set of bids to be less collusive than the former.

\textsuperscript{29}We restrict our sample to auctions in which bid difference $|\Delta|$ is less than 20% of the reserve price: often, bids that are more than 20% lower than the second lowest bid are likely to be misrecorded.

\textsuperscript{30}More precisely, we compute the median winning bid for each municipality. We then categorize bids according to whether or not they are higher or lower than the median.
**High winning bids.** Panel (A), column (1) of Table 5 reports estimates of discontinuity $\beta$ for the 90-day backlog (measured in millions of yen) for the sample of bids above the municipality-level median. We find that the 90-day backlog of marginal losers is on average 5.57 million yen higher than that of marginal winners. The estimate is statistically significant at the 5% level. The coverage error rate optimal bandwidth we use is 0.016, or about 1.6% of the reserve price. Column (2) reports estimate $\beta$ for the 90-day standardized backlog. The average standardized backlog of marginal losers is higher than that of marginal winners by 0.24 units of standard deviation. The estimate is statistically significant at the 1% level.\(^{31}\) Columns (3) and (4) report our results for 180-day backlog. The 180-day backlog of marginal losers is on average 9.54 million yen and 0.22 standard deviations higher than that of marginal winners.\(^{32}\)

Column (5) reports estimates of $\beta$ using incumbency status as the outcome variable. We find that marginal losers are about 26.0 percentage points less likely to be an incumbent than marginal winners. We only use the set of auctions in which there is an incumbent for estimation. Based on these five regression results, we can confidently reject the null of competition in this sample.

**Low winning bids.** Panel (B) reports the results for the sample of bids that are below the median winning bid. Estimates of $\beta$ are not statistically significant for columns (1) - (4) at the 5% level. In the case of incumbency, we find that marginal winners are more likely to be incumbents than marginal losers. While it seems possible that the sample of bids in Panel

\(^{31}\)Note that the sample sizes for columns (1) and (2) are slightly different. This reflects the fact that we can define the standardized backlog only for firms that win at least once in our sample. For firms that never win any contracts, the within-firm standard deviation of backlog is zero, and $x^B_k$ is undefined.

\(^{32}\)Note that the sample size in column (4) is larger than in column (2). Suppose that a firm participates twice in the sample, say, January 1, 2015 and May 1, 2015. Suppose that the firm wins the first auction. According to our 90-day backlog measure, the firm’s backlog would be zero for both auctions. Hence, we cannot define the standardized backlog for this firm. However, according to our 180-day backlog measure, the firm has a positive backlog in the second auction. Hence, we can compute the within-firm standard deviation for 180-day backlog, but not for 90-day backlog.
include some non-competitive bids, the overall results in Table 5 are consistent with the notion that there is less collusion among bidders that submit low bids. We note that lack of statistical significance in Panel (B) is unlikely to be driven by the smaller sample size since Panel (A) and (B) contain the same number of bids for which $\Delta_{i,t}$ is less than zero.\textsuperscript{33} Hence, the effective sample sizes are similar. Online Appendix B considers an alternative way of partitioning the sample in which we divide auctions to high bid auctions and low bid auctions according to whether or not the winning bid is above or below the median. We find similar results as Table 5 for that case despite the fact that the sample sizes for the two partitions are roughly the same.

Figure 5 displays binned scatter plots corresponding to the regression results reported in columns (1) and (2) of Table 5. The top panels correspond to the results in the sample of high bids (Panel (A)) and the bottom panels correspond to the sample of low bids (Panel (B)). The left two panels plot the raw 90-day backlog against $\Delta$ and the right two panels plot the standardized 90-day backlog against $\Delta$.

There is a modest discontinuity in the binned averages at $\Delta = 0$ in the top left panel, corresponding to the results of column (1) of Panel (A), Table 5. The discontinuity in the binned averages for the standardized backlog (top right panel) is more visible. The top right panel corresponds to column (2) of Panel (A). In contrast, the graphs in the bottom panels, corresponding to columns (1) and (2) of Panel (B), do not exhibit any clear discontinuities at $\Delta = 0$.

Figure 6 displays binned scatter plots corresponding to columns (3) and (4). Similar to the case of 90-day backlog, the discontinuity is somewhat more modest in the top left

\textsuperscript{33}Recall that we partition the sample according to whether or not a bid is above or below the median winning bid.
Table 5: Regression Discontinuity Estimates: Municipal Auctions from Japan.

<table>
<thead>
<tr>
<th></th>
<th>(1) 90-Day Backlog Raw</th>
<th>(2) 90-Day Backlog Standardized</th>
<th>(3) 180-Day Backlog Raw</th>
<th>(4) 180-Day Backlog Standardized</th>
<th>(5) Incumbent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel (A): Above Median</td>
<td>5.572**</td>
<td>0.244***</td>
<td>9.544**</td>
<td>0.223***</td>
<td>−0.260**</td>
</tr>
<tr>
<td></td>
<td>(2.793)</td>
<td>(0.049)</td>
<td>(3.906)</td>
<td>(0.049)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>h</td>
<td>0.016</td>
<td>0.017</td>
<td>0.016</td>
<td>0.021</td>
<td>0.034</td>
</tr>
<tr>
<td>Obs.</td>
<td>48,178</td>
<td>44,945</td>
<td>48,178</td>
<td>44,982</td>
<td>1,604</td>
</tr>
<tr>
<td>Panel (B): Below Median</td>
<td>0.377</td>
<td>−0.045</td>
<td>0.108</td>
<td>−0.017</td>
<td>−0.324**</td>
</tr>
<tr>
<td></td>
<td>(2.238)</td>
<td>(0.058)</td>
<td>(3.316)</td>
<td>(0.060)</td>
<td>(0.150)</td>
</tr>
<tr>
<td>h</td>
<td>0.024</td>
<td>0.030</td>
<td>0.021</td>
<td>0.027</td>
<td>0.026</td>
</tr>
<tr>
<td>Obs.</td>
<td>15,580</td>
<td>14,438</td>
<td>15,580</td>
<td>14,447</td>
<td>488</td>
</tr>
</tbody>
</table>

Note: Panel (A) corresponds to the sample of bids that are above the median winning bid. Panel (B) corresponds to the sample of bids below the median winning bid. Standard errors are clustered at the auction level and reported in parenthesis. The table also reports the bandwidth $h$ used for the estimation. *, **, and *** respectively denote significance at the 10%, 5%, and 1% levels.

panel and quite visible in the top right panel. The top left panel corresponds to the results reported in column (3) of Panel (A) and the top right panel corresponds to that in column (4) of Panel (A). There are no visible discontinuities in the outcome variable for the bottom panels, corresponding to columns (3) and (4) of Panel (B).

Figure 7 shows binned scatter plots corresponding to column (5) of Table 5. The discontinuity in the binned averages is visible in the top panel.

5.2.3 A Placebo Test

Because the precise order of the losing bidders is unimportant for allocation by a cartel, it seems plausible that bidding rings would not have specific rules for determining which bidder
Figure 5: Binned Scatter Plot for 90-Day Raw Backlog and 90-Day Standardized Backlog: Municipal Auctions from Japan.

should bid the second or third lowest. If this is the case, we should not expect significant differences in backlog or incumbency status between marginally second and marginally third place bidders for both competitive and non-competitive auctions. This suggests the following
Note: Top panels correspond to Panel (A) of Table 5 and bottom panels correspond to Panel (B) of Table 5. Left panels correspond to 180-day Backlog and the right panels correspond to 180-day standardized backlog.

Figure 6: Binned Scatter Plot for 180-Day Raw Backlog and 180-Day Standardized Backlog: Municipal Auctions from Japan.

For any non-winning bidder \(i\), define \(\Delta_{i,t}^2 \equiv b_{i,t} - \min\{b_{j,t} s.t. j \neq i \text{ and } j \text{ loses}\}\) as the bid differences computed in data from which winning bids are excluded. Bid difference \(\Delta_{i,t}^2\) is
negative for the second lowest bidders and positive for other bidders. Even under collusion, we do not expect that there should be systematic differences in the mean backlog and mean incumbency of close second and third (or fourth, fifth, etc.) bidders.

Table 6 reports estimates of the discontinuity in backlog and incumbency around $\Delta_{i,t}^2 = 0$. The top panel correspond to the sample of bids that are higher than the median winning bid.
### Table 6: Placebo: Regression Discontinuity Estimate with Respect to $\Delta^2$

<table>
<thead>
<tr>
<th></th>
<th>(1) Raw</th>
<th>(2) Standardized</th>
<th>(3) Raw</th>
<th>(4) Standardized</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90-Day Backlog</td>
<td>-0.756</td>
<td>-0.053</td>
<td>-1.847</td>
<td>-0.016</td>
<td>-0.023</td>
</tr>
<tr>
<td></td>
<td>(1.433)</td>
<td>(0.048)</td>
<td>(2.216)</td>
<td>(0.043)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>180-Day Backlog</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.494</td>
<td>0.035</td>
<td>-0.094</td>
<td>0.026</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td>(1.792)</td>
<td>(0.063)</td>
<td>(2.274)</td>
<td>(0.059)</td>
<td>(0.130)</td>
</tr>
<tr>
<td>Incumbent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.016</td>
<td>0.010</td>
<td>0.015</td>
<td>0.013</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>44,802</td>
<td>41,473</td>
<td>44,802</td>
<td>41,508</td>
<td>1,466</td>
</tr>
<tr>
<td>Below Median</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.020</td>
<td>0.025</td>
<td>0.020</td>
<td>0.026</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>10,796</td>
<td>9,795</td>
<td>10,796</td>
<td>9,805</td>
<td>317</td>
</tr>
</tbody>
</table>

Note: Panel (A) corresponds to the sample of bid that are above the median winning bid. Panel (B) corresponds to the sample of bids that are below the median winning bid. Standard errors are clustered at the auction level and reported in parenthesis. The forcing variable is $\Delta^2$. The table also reports the bandwidth $h$ used for the estimation. *, **, and *** respectively denote significance at the 10%, 5%, and 1% levels.

This paper proposes a novel method to screen for non-competitive behavior using covariates such as backlog and incumbency status. While many practitioners have advocated using

### 6 Discussion

Unlike in Panel (A) of Table 5, discontinuity estimates are statistically insignificant at the 5% level in Panel (A) of Table 6. The bottom panel corresponds to the sample of bids that are lower than the median. Unsurprisingly, the same holds for Panel (B). Binned scatter plots corresponding to these estimates are given in Online Appendix B.
these patterns to screen for collusion, identifying allocation patterns that reflect agreements among cartels from those that simply reflect bidder cost heterogeneity has been difficult. Our contribution is to make this possible by conditioning on auctions that are determined by a close margin. Our approach is easy to implement, requires no sophisticated programming, and is fairly robust to model misspecification. In addition, our approach can easily be adapted to formulate tests of non-competitive behavior exploiting any observed covariate suspected to reflect collusive strategies, such as geographic segmentation, subcontracting, or joint bidding. Our approach can also be extended to other auction formats such as handicap auctions, scoring auctions and all-pay auctions.

We end the paper with a discussion of practical aspects of our tests: (i) the relation between the rejection of the test and collusion and (ii) firms’ responses to antitrust oversight.

**Rejection of the test and collusion.** Section 4 argues that the bidding patterns in our datasets cannot be rationalized by suitably competitive behavior under fairly general information structures.

While this does not necessarily imply bidder collusion, results in Section 5 suggest a correlation between the rejection of the test and non-competitive behavior. Firms bidding for school milk contracts that were charged with collusion fail our tests, while firms who were not charged with collusion pass; the sample of bids that are relatively high fail our tests, while the sample of bids that are relatively low pass. This suggests that our test are sufficiently powered to flag potential cartels, and warrant further investigation.

**Firm response to screening.** Screens for collusion are perhaps most useful when firms are unaware of the details of the screening technology. When screens are known to the colluders they can potentially adapt their behavior to avoid detection. Are screens for collusion still useful if cartels adapt? Are there tests that reduce the incentives of cartels, and don’t harm competitive industries? We study these and related issues in our companion paper, Ortner
et al. (2020).

We say that a test of collusive behavior is *safe* if the rate of false positives vanishes as the number of observations grows. The tests proposed in the current paper satisfy this property. Ortner et al. (2020) shows that antitrust oversight based on safe tests always reduces the set of enforceable collusive schemes available to cartels. Put differently, even if firms know they are being monitored and adapt their play accordingly, screens based on safe tests always make cartels worse-off.

Moreover, as we illustrate in Ortner et al. (2020), adaptive responses by cartels may themselves lead to suspicious bidding patterns that can also be detected. Consider, for example, the test that compares the incumbency status of marginal winners and marginal losers. If the cartel has a rule of allocating projects to incumbents and wishes to maintain this rule, then the cartel needs to have the lowest bidder bid substantially lower than the second lowest bidder to avoid detection. However, this would generate isolated winning bids similar to the pattern documented in Chassang et al. (2020). Hence, avoiding one test may lead cartels to bid in ways that lead to rejection of other tests. Alternatively, the cartel can change its allocation rule so that incumbents are not always guaranteed to win. However, changing the allocation rule may reduce efficiency and increase the cost of coordination. This reduces bidders’ incentives to collude.

Online Appendix – Not for Publication

A Examples

A.1 An example of non-smooth demand.

Consider a complete information auction with an incumbent $I$ and an entrant $E$ with respective known costs $c_I < c_E$. Assume that bidding cost $k$ is zero.
Lemma A.1 (non-smooth demand). In any efficient equilibrium in weakly undominated strategies, the incumbent wins with bid $c_E$ with probability 1. The density of the entrant’s bid below $c_E$ is 0. The density of the entrant’s bids above $c_E$ is strictly positive and bounded away from 0. Specifically, for all $\epsilon > 0$, the incumbent’s demand $D_I$ satisfies $D_I(c_E + \epsilon) - 1 \leq \frac{1}{c_E + \epsilon - c_I}$.

Proof. In an efficient equilibrium in weakly undominated strategies, the incumbent cannot bid above $c_E$ with positive probability: the entrant’s optimal bid would win with positive probability.

In turn, the entrant cannot bid below $c_E$. This implies that the incumbent’s optimal bid is $c_E$. Optimality of $c_E$ implies that for any $\epsilon > 0$,

$$D_I(c_E + \epsilon)(c_E + \epsilon - c_I) \leq D_I(c_E)(c_E - c_I) = c_E - c_I \iff \frac{D_I(c_E + \epsilon) - 1}{\epsilon} \leq \frac{1}{c_E + \epsilon - c_I}.$$

A.2 A collusive Markov perfect equilibrium

We now describe an environment and an MPE which satisfy our assumptions, including positive participation costs, but nevertheless supports collusive behavior and fails to pass our tests.

Two bidders $i \in \{1, 2\}$ compete for contracts. Bidder 1 has a publicly observable cost $c_H > 0$ at even periods, and a publicly observable cost $c_L \in (0, c_H)$ at odd periods. Bidder 2 has i.i.d. costs, equal to $c_L$ with probability $q > 50\%$, and equal to $c_H$ with probability $1 - q$. Bidder 2’s cost is her private information. Auctions have reserve price $r = 1 > c_H + \kappa$. Let $\hat{c} = q c_L + (1 - q) c_H$ be bidder 2’s expected cost. We assume that $\delta < 1$ is sufficiently large, so that $c_H > \max\{r(1 - \delta) + (c_L - k)\delta, (r - k)(1 - \delta) + \delta \hat{c}\}$.

For simplicity, we expand the bidding space to deal with tied bids. For every bid $b$, we add bid $b^-$, equal in value to $b$, but such that $b^- \prec b$. We also consider a degenerate case where the impact of the state on costs is vanishingly small.$^{34}$ The state $\theta_t$ keeps track of:

- Is time period $t \in \mathbb{N}$ even or odd (i.e. $t \mod 2$)?

- Has any bidder won the auction both at times $2t$ and $2t + 1$ the past?

$^{34}$This information can be made payoff relevant in different ways, for instance by shifting costs slightly as a function of the state.
• Who has won the auction in the last period?

The collusive equilibrium we construct is as follows. If at any point in the past a bidder has won the auction in consecutive even and odd periods, or no player won an auction, players bid according to a static Nash equilibrium.\textsuperscript{35} If this is not the case, then:

• If $t \mod 2 = 0$, bidder 1 participates, and bids $r$; bidder 2 participates only if her cost is $c_L$, and bids $r^-$.

• If $t \mod 2 = 1$, and bidder 1 won in the previous period, then only bidder 2 participates, and bids $r$.

• If $t \mod 2 = 1$, and bidder 2 won in the previous period, then only bidder 1 participates, and bids $r$.

One can check that, when discount factor $\delta$ high enough, this is a Markov perfect equilibrium. Furthermore, bidder 2 is the only close winner, and conditional on being a close winner, has an expected 1-period backlog equal to $1 - q$. Bidder 1 is the only close loser, and conditional on being a close loser, bidder 1 has an expected 1-period backlog equal to $q > 1 - q$.

Note that winners are not exchangeable under this equilibrium. Indeed, each bidder’s continuation value when her opponent wins is strictly larger than her continuation value when no bidder wins. Indeed, when no bidder wins, players revert to static Nash behavior.

\section*{B Further Empirics}

In this section, we first present the binned scatter plots corresponding to the regression results in Table 6, Section 5. We next present a series of results that show robustness of the results that we report in Table 5. In particular, we present estimates of discontinuities when we partition auctions into two depending on whether or not the winning bid is above or below the median. This alternative way of partitioning equates the sample sizes across the two. We next report the results from using an alternative way of standardizing the backlog so that the it is measurable with respect to $h_{i,t}$. We also report the results when we limit our sample to the municipalities that use public reserve prices for their auctions. Finally, we report findings for the entire sample of auctions for which we have data.

\textsuperscript{35}For $t$ even or odd, the stage game has a Nash equilibrium in which bidder 1 randomizes between entering or not, and bidder 2 enters with probability 1 if her cost is $c_L$ (earning profits $c_H - c_L$), and enters with probability 0 if her cost is $c_H$. 

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Omitted binned scatter plots for Table 6. Figures B.1, B.2 and B.3 are the binned scatter plots corresponding to Table 6. In all of the panels, the horizontal axis corresponds to values $\Delta^2_{i,t} \equiv b_{i,t} - \min\{b_{j,t}, \text{s.t. } j \neq i \text{ and } j \text{ loses}\}$ for losing bidders $i$. A small negative value of $\Delta^2_{i,t}$ corresponds to a bid that is second lowest, but close to being third lowest. A small positive value of $\Delta^2_{i,t}$ corresponds to a bid that was higher than, but close to the second lowest bid.

The panels in Figure B.1 are the binned scatter plots that correspond to columns (1) and (2) of Table 6. The panels in Figure B.2 correspond to columns (3) and (4). The panels in Figure B.3 correspond to column (5). The top panels of each figure plot the outcome variable for the sample of bids that are above the municipal median. The bottom panels correspond to the sample of bids that are below the median. Unlike our results for marginal winners and marginal losers, the figures do not show any discontinuities around $\Delta^2_{i,t} = 0$.

Partitioning auctions by the winning bid. In our main analysis, we partition the sample according to whether or not they are above or below the median winning bid. This results in the sample size of the two partitions to be unequal. In order to show that our results are not driven by differences in the sample size, we consider an alternative partitioning in which we divide auctions according to whether or not the winning bid is above or below the median winning bid. This partitioning results in the same number of auctions (and hence, roughly the same number of bids) above and below the median.

Table B.1 reports the results. The top panel corresponds to the sample of bids submitted in auctions in which the winning bid is higher than the median. We find that the estimate of $\beta$ is statistically significant for all five regressions. The bottom panel corresponds to the sample of bids submitted in auctions in which the winning bid is below the median. We find that in Panel (B), none of the estimates of $\beta$ are statistically significant at the 5%. Note that the sample sizes in Panel (A) and (B) are roughly equal. The results of Table B.1 suggests that sample sizes are not driving our results in the main text.

Alternative standardization of backlog. In the main specification, we define the standardized backlog by subtracting the within-firm mean from the raw backlog and then dividing it by the within-firm standard error. Strictly speaking, standardized backlog defined this way is not measurable with respect to $h_{i,t}$ as required by the theory. In order to define an
Figure B.1: Binned Scatter Plot for 90-Day Backlog with Respect to $\Delta^2$: Municipal Auctions from Japan.

outcome variable that is perfectly consistent with the theory, we consider an alternative standardization of backlog in which we use the mean and standard error of the rolling backlog
Figure B.2: Binned Scatter Plot for 180-Day Backlog with Respect to $\Delta^2$: Municipal Auctions from Japan.

as follows. Let

\[
\mu'_{\text{x},i,t} = \frac{1}{N_{i,t}} \sum_{\tau < t} x_{i,\tau}^{B_k},
\]

\[
\sigma'_{\text{x},i,t} = \sqrt{\frac{1}{N_{i,t}} \sum_{\tau < t} (x_{i,\tau}^{B_k} - \mu'_{\text{x},i,t})^2},
\]
Note: The top panel corresponds to column (5), Panel (A) of Table 6. The bottom panel corresponds to column (5), Panel (B) of Table 6.

Figure B.3: Binned Scatter Plot for Incumbent with Respect to $\Delta^2$: Municipal Auctions from Japan.

where $N_{i,t}$ is the number of auctions that firm $i$ participates before auction $t$, $\mu'_{x_{i,t}}$ is the average of firm $i$'s backlog up to auction $t$, and $\sigma'_{x_{i,t}}$ is the standard deviation of firm $i$'s
Table B.1: Partitioning Sample by Auctions: Municipal Auctions from Japan.

<table>
<thead>
<tr>
<th>Panel (A) : Above Median</th>
<th>(1) 90-Day Backlog</th>
<th>(2) Standardized</th>
<th>(3) 180-Day Backlog</th>
<th>(4) Standardized</th>
<th>(5) Incumbent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>Standardized</td>
<td>Raw</td>
<td>Standardized</td>
<td>Raw</td>
<td>Standardized</td>
</tr>
<tr>
<td>Panel (A) : Above Median</td>
<td>7.155***</td>
<td>0.249***</td>
<td>13.205***</td>
<td>0.221***</td>
<td>−0.277***</td>
</tr>
<tr>
<td>h</td>
<td>0.014</td>
<td>0.019</td>
<td>0.013</td>
<td>0.025</td>
<td>0.031</td>
</tr>
<tr>
<td>Obs.</td>
<td>30,666</td>
<td>28,650</td>
<td>30,666</td>
<td>28,665</td>
<td>1,058</td>
</tr>
</tbody>
</table>

Panel (B) : Below Median

<table>
<thead>
<tr>
<th>Panel (B) : Below Median</th>
<th>(1) 90-Day Backlog</th>
<th>(2) Standardized</th>
<th>(3) 180-Day Backlog</th>
<th>(4) Standardized</th>
<th>(5) Incumbent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>Standardized</td>
<td>Raw</td>
<td>Standardized</td>
<td>Raw</td>
<td>Standardized</td>
</tr>
<tr>
<td>Panel (B) : Below Median</td>
<td>−0.290</td>
<td>−0.058</td>
<td>−1.278</td>
<td>−0.022</td>
<td>−0.268*</td>
</tr>
<tr>
<td>h</td>
<td>0.027</td>
<td>0.021</td>
<td>0.024</td>
<td>0.021</td>
<td>0.027</td>
</tr>
<tr>
<td>Obs.</td>
<td>33,100</td>
<td>30,739</td>
<td>33,100</td>
<td>30,770</td>
<td>1,032</td>
</tr>
</tbody>
</table>

Note: We partition auctions into two depending on whether or not the winning bid is above or below the median. Panel (A) corresponds to the bids of auctions that are above the median. Panel (B) corresponds to the bids of auctions that are below the median. Standard errors are clustered at the auction level and reported in parenthesis. *, **, and *** respectively denote significance at the 10%, 5%, and 1% levels.

We define standardized backlog as

\[ x_{i,t}^{B_k} = \frac{x_{i,t}^{B_k} - \mu^{B_k}}{\sigma^{B_k}}. \]

The difference between this definition and the one in the main text is that we now consider only auctions that take place before auction \( t \) in the summation \((\tau < t)\). Note that the new definition of standardized backlog is measurable with respect to \( h_{i,t} \).

We estimate \( \beta \) using a local linear regression as follows:

\[ \hat{\beta} = b_0^\dagger - b_0^\ddagger, \]

where

\[ (b_0^\dagger, b_1^\dagger) = \arg \min \sum_{x_{i,t}^{B_k} - b_0^+ - b_1^+ \Delta_{i,t}} K \left( \frac{\Delta_{i,t}}{h_n} \right) 1_{\{\Delta_{i,t}>0\}\cap\{x_{i,t}^{B_k} \neq 0\}}, \]

\[ (b_0^\ddagger, b_1^\ddagger) = \arg \min \sum_{x_{i,t}^{B_k} - b_0^- - b_1^- \Delta_{i,t}} K \left( \frac{\Delta_{i,t}}{h_n} \right) 1_{\{\Delta_{i,t}<0\}\cap\{x_{i,t}^{B_k} \neq 0\}}. \]
Note that we condition our regression discontinuity estimate on the event that \( x_{i,t}^{B_k} \neq 0 \). Because this event is measurable with respect to \( h_{i,t} \), Corollary 2 holds for this case.

<table>
<thead>
<tr>
<th></th>
<th>Panel (A) : Above Median</th>
<th>Panel (B) : Below Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta} )</td>
<td>0.591***</td>
<td>0.195*</td>
</tr>
<tr>
<td>(0.075)</td>
<td>(0.115)</td>
<td></td>
</tr>
<tr>
<td>( h )</td>
<td>0.020</td>
<td>0.025</td>
</tr>
<tr>
<td>Obs.</td>
<td>20,346</td>
<td>7,594</td>
</tr>
</tbody>
</table>

Note: Panel (A) corresponds to the sample of bids above the median winning bid. Panel (B) corresponds to the sample of bids below the median. Standard errors are clustered at the auction level and reported in parenthesis. The table also reports the bandwidth \( h \) used for the estimation. *, **, and *** respectively denote significance at the 10%, 5%, and 1% levels.

Table B.2: Alternative Standardization of Backlog: Municipal Auctions from Japan.

Table B.2 reports the results. Panel (A) corresponds to the sample of bids above the median winning bid and Panel (B) corresponds to the sample of bids below the median. The estimates for Panel (A) are statistically significant at the 5% level while the estimates in Panel (B) are not. The results of Table B.2 are similar to the results we report in column (2) and (4) of Table 5.

Results for the sample of auctions with a public reserve price. We now report the results of our analysis when we restrict the sample to auctions let by municipalities using public reserve prices. Table B.3 reports results from adapting the analysis of Table 5 to this subset. Panel (A) of Table B.3 reports estimation results for the set of bids above the
Table B.3: Restricting the Sample to Municipalities with Public Reserve Price.

Although the estimate of $\beta$ is not statistically significant for the 90-day raw backlog in column (1), we find statistically significant differences between marginal losers and marginal winners for other measures of backlog in columns (2), (3), and (4). These results are qualitatively similar to those reported in Table 5. The results overall strongly suggest that there are non-competitive auctions among the sample of public reserve auctions in which a close winner submits a high bid.

In Panel (B), we report the estimated results for the set of low bids. We find that there are no statistically significant differences between the marginal winner and the marginal loser for this subsample, implying that we cannot reject the null of competition.

---

36 As before, we compute the median winning bid for each municipality and divide the sample according to whether or not a bid is above or below the municipal median.
All municipalities. We now discuss the results of our tests when we include auctions from Japanese municipalities dropped from our main analysis. There are a total of 109 municipalities for which we have auction data. In order to construct the dataset used in Section 5, we drop municipalities for which the distribution of \( \Delta \) has a missing mass at 0 (71 municipalities) and those for which the distribution of \( \Delta \) has a point mass at exactly 0 (22 municipalities).

Figure B.4 plots the histograms of \( \Delta_{i,t} \) for auctions let by the municipalities with missing mass in the distribution of \( \Delta_{i,t} \) at 0 (first row) and for those let by municipalities with a mass in the distribution of \( \Delta_{i,t} \) at exactly zero (second row). The left two panels correspond to the histogram for all of the auctions let by each of the groups of municipalities. The middle and right panels correspond to the histogram for bids below the municipal median (middle panel) and above the municipal median (right panel).
Note: The top panels correspond to auctions from 71 municipalities with missing mass in the distribution of $\Delta_{i,t}$ at zero. The bottom panels correspond to auctions from 22 municipalities with a mass in the distribution of $\Delta_{i,t}$ at exactly zero. The left panels correspond to all auctions let by each of these groups, the middle panels condition on the winning bids to be below the municipality median and the right panels condition on the winning bids to be above the median.

Figure B.4: Histogram of $\Delta_{i,t}$: Municipal Auctions from Japan.

The missing mass in the distribution of $\Delta_{i,t}$, apparent in the top panels, has previously been documented in Chassang et al. (2020). In that paper, we show that this distinctive pattern in the distribution of $\Delta_{i,t}$ is inconsistent with competitive bidding under fairly general conditions. Because our previous paper specifically focuses on the implications of these patterns, we opted to exclude these municipalities in our baseline analysis.

The distributions of $\Delta_{i,t}$ in the bottom panels have spikes at zero which are the result of binding price floors. Price floors can result in multiple bidders bidding exactly at the price floor. Note that because the spikes are generated by price floors, and because multiple bids at the price floor typically imply that the winning bid of the auction is low, the spike is very pronounced for the middle panel, but mostly disappears in the right panel. The summary statistics of the auctions for each of the groups are reported in Table B.4. Column (1) corresponds to the sample statistics for municipalities with a missing mass at zero,
column (2) corresponds to the sample statistics for those with a mass at 0, and column (3) corresponds to the sample statistics for the baseline sample used in Section 5.

We now report the regression discontinuity results for all of the auctions in our sample. Panel (A) of Table B.5 reports the regression discontinuity estimates for bids above the median winning bid. Panel (B) of Table B.5 reports the estimates for bids below the median winning bid. Focusing on Panel (A), we find that marginal losing bidders have about 3.5 million yen more in terms of 90-day backlog (column (1)) and about 0.087 higher 90-day standardized backlog (column (2)) than marginal winners. The estimates are both statistically significant at the 1% level. Similarly, we find that marginal losing bidders have higher raw and standardized 180-day backlog (column (3), (4)) than marginal winners, and are less likely to be an incumbent (column (5)) than marginal winners. The coefficients are all

<table>
<thead>
<tr>
<th>Panel A: By Auction</th>
<th>(1) Sample with Missing Mass (71 munis)</th>
<th>(2) Sample with Mass at 0 (22 munis)</th>
<th>(3) Baseline Sample (16 munis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserve (Mil. Yen)</td>
<td>24.03 104.39</td>
<td>20.92 101.08</td>
<td>22.26 77.14</td>
</tr>
<tr>
<td>Winning Bid (Mil. Yen)</td>
<td>22.60 97.64</td>
<td>19.09 95.63</td>
<td>20.71 71.78</td>
</tr>
<tr>
<td>Win Bid/Reserve</td>
<td>0.940 0.073</td>
<td>0.911 0.078</td>
<td>0.926 0.083</td>
</tr>
<tr>
<td># of Bids</td>
<td>7.44 3.78</td>
<td>8.00 4.64</td>
<td>6.80 4.21</td>
</tr>
<tr>
<td>Incumbent</td>
<td>0.064 0.244</td>
<td>0.043 0.202</td>
<td>0.044 0.204</td>
</tr>
<tr>
<td>Obs.</td>
<td>44,993</td>
<td>54,153</td>
<td>11,207</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: By Bidder</th>
<th>(1) Sample with Missing Mass (71 munis)</th>
<th>(2) Sample with Mass at 0 (22 munis)</th>
<th>(3) Baseline Sample (16 munis)</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Participation</td>
<td>24.13 63.23</td>
<td>37.31 85.84</td>
<td>24.75 49.31</td>
</tr>
<tr>
<td># of Wins</td>
<td>2.87 7.88</td>
<td>4.06 10.10</td>
<td>2.80 6.52</td>
</tr>
<tr>
<td>Raw Backlog (90-Day)</td>
<td>3.60 20.06</td>
<td>4.55 20.66</td>
<td>3.47 15.83</td>
</tr>
<tr>
<td>Raw Backlog (180-Day)</td>
<td>5.89 33.27</td>
<td>6.81 26.86</td>
<td>5.44 21.10</td>
</tr>
<tr>
<td>Obs.</td>
<td>15,694</td>
<td>13,350</td>
<td>4,005</td>
</tr>
</tbody>
</table>

Note: Column (1) reports summary statistics for the sample of auctions with missing mass in the distribution of $\Delta_{i,t}$ at zero (71 municipalities). Column (2) reports summary statistics for the sample with mass at exactly zero (22 municipalities). Column (3) reports sample statistics for the sample used in Section 5.

Table B.4: Summary Statistics by Auctions and Bidders: All Municipalities.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90-Day Backlog</td>
<td>180-Day Backlog</td>
<td>Incumbent</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Raw</td>
<td>Standardized</td>
<td>Raw</td>
<td>Standardized</td>
<td></td>
</tr>
<tr>
<td>Panel (A) :</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Above Median</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>3.218**</td>
<td>0.086***</td>
<td>6.506***</td>
<td>0.117***</td>
<td>−0.341***</td>
</tr>
<tr>
<td></td>
<td>(1.279)</td>
<td>(0.019)</td>
<td>(2.175)</td>
<td>(0.019)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>$h$</td>
<td>0.013</td>
<td>0.012</td>
<td>0.009</td>
<td>0.013</td>
<td>0.011</td>
</tr>
<tr>
<td>Obs.</td>
<td>558,152</td>
<td>530,966</td>
<td>558,152</td>
<td>531,357</td>
<td>19,929</td>
</tr>
<tr>
<td>Panel (B) :</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Below Median</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>1.484**</td>
<td>−0.026</td>
<td>2.366**</td>
<td>−0.012</td>
<td>−0.091***</td>
</tr>
<tr>
<td></td>
<td>(0.701)</td>
<td>(0.016)</td>
<td>(1.168)</td>
<td>(0.018)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>$h$</td>
<td>0.015</td>
<td>0.013</td>
<td>0.014</td>
<td>0.011</td>
<td>0.012</td>
</tr>
<tr>
<td>Obs.</td>
<td>188,032</td>
<td>177,304</td>
<td>188,032</td>
<td>177,457</td>
<td>6,864</td>
</tr>
</tbody>
</table>

In addition to the auctions used in the baseline analysis, we include auctions from 70 municipalities with missing mass in the distribution of $\Delta_{i,t}$ at zero and those from 18 municipalities with mass in the distribution of $\Delta_{i,t}$ at exactly zero. Panel (A) corresponds to the sample of bids above the median. Panel (B) corresponds to the sample of bids below the median. Standard errors are clustered at the auction level and reported in parenthesis. The forcing variable is $\Delta^1$. The table also reports the bandwidth used for the estimation. *, **, and *** respectively denote significance at the 10%, 5%, and 1% levels.

Table B.5: Regression Discontinuity Estimates: All Municipalities.

statistically significant at the 1% level. These findings lead us to reject the null hypothesis of competition for this sample.

The bottom panel of Table B.5 reports the results for bids below the median. While the regression discontinuity estimate is statistically significant at the 5% level in columns (1), (3), and (5), the estimated differences between marginal winners and losers are smaller than in Panel (A). The results suggest the existence of some collusive bidding among this sample, but likely to a lesser extent than the sample in Panel (A). Overall, the results of Table B.5 suggest that the null of competitive bidding is strongly rejected for the sample of high bids, but that the evidence is less strong for the sample of low winning bids. This is consistent with the expectation that there would be more collusion among auctions with high winning bids than among those with low winning bids.

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C Proofs

C.1 Proofs for Section 3

Proof of Lemma 1. We show that for all $\eta > 0$, there exists $\epsilon > 0$ small enough such that for all histories $h_{i,t}$,

$$\left| \text{prob}(i \text{ wins} | h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| \leq \epsilon) - \frac{1}{2} \right| \leq \eta.$$ 

By assumption, $D'_i(b_i|h_i)$ is continuous in $b_i \in [0, 1]$ and strictly negative for all histories $h_i = (\theta, z_i)$. Since there are finitely many histories $(\theta, z_i)$, it follows that there exists $\nu > 0$ such that $D'_i(b_i|h_i) \leq -\nu$ for all $b_i$ and all histories $h_{i,t}$. In addition, for all $\hat{\eta} > 0$, there exists $\epsilon$ small enough that for all $\hat{b}_i \in [b_i - \epsilon, b_i + \epsilon]$, $|D'_i(\hat{b}_i|h_{i,t}) - D'_i(b_i|h_{i,t})| \leq \hat{\eta}$.

This implies that for $\epsilon$ small

$$\left| \text{prob}(i \text{ wins} | h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| \leq \epsilon) - \frac{1}{2} \right| = \left| \frac{D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t}) - \frac{1}{2}}{D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})} \right| \leq \frac{\nu - \hat{\eta}}{2\nu + 2\hat{\eta}} - \frac{1}{2}.$$

Lemma 1 follows by taking $\hat{\eta}$ small enough. ■

Proof of Corrolary 1. Note that, for each $x \in X$,

$$\text{prob}(x_i = x | \Delta_{i,t} \in (-\epsilon, 0)) = \text{prob}(x_{i,t} = x | i \text{ wins and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon)$$

$$= \text{prob}(x_{i,t} = x | |b_{i,t} - \wedge b_{-i,t}| < \epsilon) \frac{\text{prob}(i \text{ wins} | x_{i,t} = x \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon)}{\text{prob}(i \text{ wins} | |b_{i,t} - \wedge b_{-i,t}| < \epsilon)}$$

Similarly,

$$\text{prob}(x_i = x | \Delta_{i,t} \in (0, \epsilon)) = \text{prob}(x_{i,t} = x | i \text{ loses and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon)$$

$$= \text{prob}(x_{i,t} = x | |b_{i,t} - \wedge b_{-i,t}| < \epsilon) \frac{\text{prob}(i \text{ loses} | x_{i,t} = x \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon)}{\text{prob}(i \text{ loses} | |b_{i,t} - \wedge b_{-i,t}| < \epsilon)}$$
By Lemma 1, we have that
\[
\lim_{\epsilon \searrow 0} \Pr(i \text{ wins } | x_{i,t} = x \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) = \lim_{\epsilon \searrow 0} \Pr(i \text{ wins } | |b_{i,t} - \wedge b_{-i,t}| < \epsilon) = \frac{1}{2},
\]
\[
\lim_{\epsilon \searrow 0} \Pr(i \text{ loses } | x_{i,t} = x \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) = \lim_{\epsilon \searrow 0} \Pr(i \text{ loses } | |b_{i,t} - \wedge b_{-i,t}| < \epsilon) = \frac{1}{2}.
\]
Hence, for each \( x \in X \),
\[
\lim_{\epsilon \searrow 0} \left| \Pr(x_i = x | \Delta_{i,t} \in (-\epsilon, 0)) - \Pr(x_i = x | \Delta_{i,t} \in (0, \epsilon)) \right| = 0.
\]
Since \( X \) is finite, for all \( \eta > 0 \) there exists \( \epsilon > 0 \) small enough such that for all \( x \in X \),
\[
\left| \Pr(x_i = x | \Delta_{i,t} \in (0, \epsilon)) - \Pr(x_i = x | \Delta_{i,t} \in (-\epsilon, 0)) \right| < \eta.
\]
This completes the proof. \( \blacksquare \)

### C.2 Proofs for Section 4

#### C.2.1 Observables under exchangeable winners

**Proof of Corollary 2.** For each \( \epsilon > 0 \), let \( \Pr_{\sigma}(. | \epsilon\text{-close}) \) denote the distribution over histories conditional on event \( \epsilon\text{-close} \). Then, for each \( i \in N \) and each \( \epsilon > 0 \), the probability with which firm \( i \) wins an auction under \( \sigma \) conditional on event \( \epsilon\text{-close} \) satisfies
\[
\Pr_{\sigma}(i \text{ wins} | \epsilon\text{-close}) = \mathbb{E}_{\mathcal{E},\sigma} \left[ \Pr_{\sigma}(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) | \epsilon\text{-close} \right] \times \Pr_{\sigma}(|b_{i,t} - \wedge b_{-i,t}| < \epsilon | \epsilon\text{-close}).
\]
By Proposition 1, it follows that
\[
\forall i \in N, \quad \liminf_{\epsilon \searrow 0} \mathbb{E}_{\mathcal{E},\sigma} \left[ \Pr_{\sigma}(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) | \epsilon\text{-close} \right] \geq \frac{1}{2}, \quad (9)
\]
Towards a contradiction, suppose that the result is not true. Hence, there exists a player \( j \) and a number \( \eta > 0 \) such that
\[
\limsup_{\epsilon \searrow 0} \mathbb{E}_{\mathcal{E},\sigma} \left[ \Pr(j \text{ wins } | h_{j,t} \text{ and } |b_{j,t} - \wedge b_{-j,t}| < \epsilon) | \epsilon\text{-close} \right] \geq \frac{1}{2} + \eta. \quad (10)
\]
Note that, for each $\epsilon > 0$, we have that
\[
\sum_{i \in N} \text{prob}_\sigma(i \text{ wins } | \epsilon\text{-close}) = 1 \text{ and }
\sum_{i \in N} \text{prob}_\sigma(|b_{i,t} - \wedge b_{-i,t}| < \epsilon | \epsilon\text{-close}) = \mathbb{E}_{\mathcal{E},\sigma}[^{\{i \text{ s.t. } |b_{i,t} - \wedge b_{-i,t}| < \epsilon\} } | \epsilon\text{-close}] \geq 2.
\]

Using (8), (9) and (10), we obtain that
\[
1 = \limsup_{\epsilon \downarrow 0} \sum_{i \in N} \text{prob}_\sigma(i \text{ wins } | \epsilon\text{-close}) \geq \frac{1}{2} \limsup_{\epsilon \downarrow 0} \sum_{i \in N} \text{prob}_\sigma(|b_{i,t} - \wedge b_{-i,t}| < \epsilon | \epsilon\text{-close})
+ \eta \text{prob}_\sigma(|b_{j,t} - \wedge b_{-j,t}| < \epsilon | \epsilon\text{-close})
\geq 1 + \eta \limsup_{\epsilon \downarrow 0} \text{prob}_\sigma(|b_{j,t} - \wedge b_{-j,t}| < \epsilon | \epsilon\text{-close}) > 1,
\]
a contradiction. ■

**Sample implications of Corollary 2.** We now show that when the sample size is large, Corollary 2 must hold approximately under the sample distribution of bids and characteristics $b, x$.

Data consists of bids and observable characteristics $(b_t, x_t)_{t \in \{0, \cdots, T\}}$ for auctions happening at times $t \in \{0, \cdots, T\}$. We denote by $\hat{\text{prob}}$ the sample joint distribution of bids and characteristics in the data.

Given $\epsilon > 0$ and $x \in X$, we define $B_{x,\epsilon} \equiv \{(i, t) \text{ s.t. } x_{i,t} = x, \ |b_{i,t} - \wedge b_{-i,t}| \leq \epsilon\}$ the subsample of close bids such that the bidders characteristics $x_i$ are equal to $x$. We denote by $B_x \equiv \{(i, t) \text{ s.t. } |b_{i,t} - \wedge b_{-i,t}| \leq \epsilon\}$ the sample of close bids. A bidder’s sample probability of winning conditional on close bids and type $x$ is denoted by $\hat{P}_{x,\epsilon}$. Formally, we have,
\[
\hat{P}_{x,\epsilon} \equiv \hat{\text{prob}}(i \text{ wins } | x_i = x, \ |b_i - \wedge b_{-i}| \leq \epsilon)
= \frac{|\{(i, t) \in B_{x,\epsilon} \text{ s.t. } b_{i,t} < \wedge b_{-i,t}\}|}{|B_{x,\epsilon}|}
\]
(11)

We make the following assumption about data.

**Assumption C.1.** There exists $\lambda > 0$ such that for all datasets of interest $B$, and all $x \in X$,
\[
\frac{\sum_{x' \in X \setminus \{x\}} |B_{x',\epsilon}|}{|B_{x,\epsilon}|} \leq \lambda
\]

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The following result holds:

**Proposition C.1** (winning is independent of bidder characteristics). For all \( \eta > 0 \), there exists \( \epsilon > 0 \) small enough such that with probability approaching 1 as \( |B| \) goes to infinity,

\[
\forall x \in X, \quad \left| \hat{P}_{x,\epsilon} - \frac{1}{2} \right| \leq \eta.
\]

**Proof.** Take \( \eta' > 0 \) as given. We know from Proposition 1 that for epsilon small enough, for all histories \( h_{i,t} \), \( \text{prob}(i \text{ wins} | h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) \geq 1/2 - \eta' \).

Fix \( x \in X \). We show that with probability approaching 1 as \( |B| \) goes to infinity, \( \hat{P}_{x,\epsilon} \geq \frac{1}{2} - 2\eta' \). Observe first that, by Assumption C.1, when \( |B| \) grows large, \( |B_{x,\epsilon}| \) grows proportionally large:

\[
\frac{|B_{x,\epsilon}|}{|B|} = 1 - \frac{\sum_{x' \in X \setminus x} |B_{x',\epsilon}|}{|B|} + \sum_{x' \neq x} |B_{x',\epsilon}| \geq 1 - \frac{\lambda}{1 + \lambda}.
\]

We denote by \( \{t_1, \cdots, t_n\} \) auctions occurring at times \( t \) such that \( (i, t) \in B_{x,\epsilon} \), ordered according to the timing of the auction. Since the number \( N \) of bidders is finite, \( n \) grows large proportionally with \( |B_{x,\epsilon}| \). We define \( C_k = \{i \in N \text{ s.t. } (i, t_k) \in B_{x,\epsilon}\} \). In equilibrium,

\[
H_K = \sum_{k=1}^{K} \sum_{i \in C_k} 1_{b_{i,t_k} \prec \wedge b_{-i,t_k}} - \text{prob}_i(b_{i,t_k} \prec \wedge b_{-i,t_k} | i \in C_k)
\]

is a martingale. Indeed note that given the information \( I_K \) available at the time of bidding in auction \( K \),

\[
\text{E} \left[ \sum_{i \in C_K} 1_{b_{i,t_K} \prec \wedge b_{-i,t_K}} \bigg| I_K \right] = \text{E} \left[ \sum_{i \in C_K} 1_{1_{i \in C_K} b_{i,t_K} \prec \wedge b_{-i,t_K}} \bigg| I_K \right]
\]

\[
= \text{E} \left[ \text{E}_{C_K} \left[ \sum_{i \in N} 1_{i \in C_K} 1_{b_{i,t_K} \prec \wedge b_{-i,t_K}} \bigg| I_K \right] \right]
\]

\[
= \text{E} \left[ \sum_{i \in N} 1_{i \in C_K} \text{prob}_i(1_{b_{i,t_K} \prec \wedge b_{-i,t_K}} | i \in C_K) | I_K \right]
\]

\[
= \text{E} \left[ \sum_{i \in C_K} \text{prob}_i(1_{b_{i,t_K} \prec \wedge b_{-i,t_K}} | i \in C_K) | I_K \right].
\]
Using Proposition 1, this implies that

\[ G_K \equiv \sum_{k=1}^{K} \sum_{i \in C_k} 1_{b_i \prec b_{i-1}} - \frac{1}{2} + \eta' \]

is a submartingale with increments bounded by \(|N|\) (the maximum number of bidders in an auction). It follows for the Azuma-Hoeffding Theorem that as \(n\) grows large, with probability approaching 1, \(G_n \geq -\eta'n\). Since \(n \leq |B_{x,\epsilon}|\), this implies that with probability approaching 1,

\[ \hat{P}_{x,\epsilon} \equiv \frac{1}{|B_{x,\epsilon}|} \sum_{k=1}^{n} \sum_{i \in C_k} 1_{b_i \prec b_{i-1}} \geq \frac{1}{2} - 2\eta'. \]

Since \(X\) is finite, with probability approaching 1 as \(|B_{\epsilon}|\) becomes large, we have that for all \(x \in X\), \(\hat{P}_{x,\epsilon} \geq \frac{1}{2} - 2\eta'\). In addition, since \(\sum_{x' \in X} |B_{x',\epsilon}| \hat{P}_{x',\epsilon} = |\{(i, a) \in B_{\epsilon} \text{ s.t. } i \text{ wins }\}|\), it follows that

\[ \frac{\sum_{x' \in X} |B_{x',\epsilon}| \hat{P}_{x',\epsilon}}{\sum_{x' \in X} |B_{x',\epsilon}|} \leq \frac{1}{2}. \]

Hence, with probability approaching 1, we have that

\[ |B_{x,\epsilon}| \hat{P}_{x,\epsilon} = \frac{1}{2} |B_{x,\epsilon}| + \sum_{x' \in X \backslash x} |B_{x',\epsilon}| \left( \frac{1}{2} - \hat{P}_{x',\epsilon} \right) \]

\[ \Rightarrow \hat{P}_{x,\epsilon} \leq \frac{1}{2} + 2\eta' \frac{\sum_{x' \in X \backslash x} |B_{x',\epsilon}|}{|B_{x,\epsilon}|} \leq \frac{1}{2} + 2\eta' \lambda. \]

Hence by selecting \(\eta'\) sufficiently small in the first place, it follows that for any \(\eta > 0\), there exists \(\epsilon\) such that as \(|B_{\epsilon}|\) grows large, \(\hat{P}_{x,\epsilon} - \frac{1}{2} \leq \eta\) with probability 1.

A corollary of Proposition C.1 is that our regression discontinuity design remains valid: conditional on close bids, the sample distribution of covariates is independent of whether the bidder wins or loses the auction.

**Corollary C.1** (close winners and losers have similar characteristics). For all \(\eta > 0\), there exists \(\epsilon > 0\) small enough such that with probability approaching 1 as \(|B_{\epsilon}|\) goes to infinity,

\[ \forall x \in X, \quad \left| \hat{\text{prob}}(x_i = x \mid i \text{ wins }, |b_i - \wedge b_{-i}| \leq \epsilon) - \hat{\text{prob}}(x_i = x \mid i \text{ loses }, |b_i - \wedge b_{-i}| \leq \epsilon) \right| \leq \eta. \]
Proof. Observe that

\[ \widehat{\text{prob}}(x_i = x \mid i \text{ wins }, |b_i - \land_{-i}| \leq \epsilon) = \frac{\text{prob}(i \text{ wins } | x_i = x, |b_i - \land_{-i}| \leq \epsilon)}{\text{prob}(i \text{ wins } | |b_i - \land_{-i}| \leq \epsilon)} \]

\[ \widehat{\text{prob}}(x_i = x \mid i \text{ loses }, |b_i - \land_{-i}| \leq \epsilon) = \frac{\text{prob}(i \text{ loses } | x_i = x, |b_i - \land_{-i}| \leq \epsilon)}{\text{prob}(i \text{ loses } | |b_i - \land_{-i}| \leq \epsilon)}. \]

Therefore,

\[
\begin{align*}
\left| \widehat{\text{prob}}(x_i = x \mid i \text{ wins }, |b_i - \land_{-i}| \leq \epsilon) - \widehat{\text{prob}}(x_i = x \mid i \text{ loses } |b_i - \land_{-i}| \leq \epsilon) \right|
\leq & \left| \frac{\text{prob}(i \text{ wins } | x_i = x, |b_i - \land_{-i}| \leq \epsilon)}{\text{prob}(i \text{ wins } | |b_i - \land_{-i}| \leq \epsilon)} - \frac{\text{prob}(i \text{ loses } | x_i = x, |b_i - \land_{-i}| \leq \epsilon)}{\text{prob}(i \text{ loses } | |b_i - \land_{-i}| \leq \epsilon)} \right| \\
= & \left| \frac{\sum_{x' \in X} \frac{|B_{x',\epsilon}|}{|B_{x'|}} \widehat{P}_{x',\epsilon}}{1 - \sum_{x' \in X} \frac{|B_{x',\epsilon}|}{|B_{x'|}} \widehat{P}_{x',\epsilon}} - \frac{1 - \widehat{P}_{x,\epsilon}}{1 - \sum_{x' \in X} \frac{|B_{x',\epsilon}|}{|B_{x'|}} \widehat{P}_{x',\epsilon}} \right|.
\end{align*}
\]

It follows from Proposition C.1 that for any \( \eta' > 0 \), there exists \( \epsilon \) such that with probability 1 as \( |B_{\epsilon}| \) grows large,

\[
\frac{\widehat{P}_{x,\epsilon}}{\sum_{x' \in X} \frac{|B_{x',\epsilon}|}{|B_{x'|}} \widehat{P}_{x',\epsilon}} - \frac{1 - \widehat{P}_{x,\epsilon}}{1 - \sum_{x' \in X} \frac{|B_{x',\epsilon}|}{|B_{x'|}} \widehat{P}_{x',\epsilon}} \in \left[ \frac{1}{2} - \eta', \frac{1}{2} - \frac{1}{2} + \eta' \right] \cup \left[ \frac{1}{2} + \eta', \frac{1}{2} + \frac{1}{2} + \eta' \right].
\]

By picking \( \eta' \) small enough, this implies that with probability approaching 1,

\[
\left| \widehat{\text{prob}}(x_i = x \mid i \text{ wins }, |b_i - \land_{-i}| \leq \epsilon) - \widehat{\text{prob}}(x_i = x \mid i \text{ loses } |b_i - \land_{-i}| \leq \epsilon) \right| \leq \eta.
\]

\[\square\]

C.2.2 Inference from dissimilar winners and losers

We now establish Proposition 2 by establishing its contraposite. Throughout this section we consider an environment \( \mathcal{E} \) and an MPE \( \sigma \) such that \( V_i(0, b_w|\zeta_i) \) is Lipschitz continuous in \( b_w \), and \( V_i(0, b_w|\zeta_i) \) is weakly decreasing in \( b_w \). We then show that bids must be as if random conditional on close bids.

We begin by establishing two intermediary lemmas. Recall that continuation value \( V_i(\zeta, b_w|\zeta_i) \) does not depend on winning bid \( b_w \) when bidder \( i \) wins. Hence, we suppress the dependency of \( V_i \) on \( b_w \) when \( \zeta_i = 1 \).
Lemma C.1 (minimum demand). There exists $\nu > 0$ such that for every history $h_i = (\theta, z_i)$ and bid $b_i \in [0, 1]$ in the support of $\sigma_i|h_i$, $D_i(b_i|h_i) \geq \nu$. In addition,

$$b_i - c_i + \delta \mathbb{E}_\sigma [V_i(1|h_i) - V_i(0, \wedge b_{-i}|h_i) | h_i, b_i, b_i < \wedge b_{-i}] \geq k.$$  

Proof. Since firm $i$ chooses to participate, it must be that

$$\mathbb{E}_\sigma \left[ 1_{b_i < \wedge b_{-i}} (b_i - c_i + \delta V_i(1|h_i)) + 1_{b_i > \wedge b_{-i}} \delta V_i(0, \wedge b_{-i}|h_i) | h_i \right] - k \geq \mathbb{E}_\sigma [\delta V_i(0, \wedge b_{-i}|h_i) | h_i]$$

$$\iff \mathbb{E}_\sigma \left[ 1_{b_i < \wedge b_{-i}} (b_i - c_i + \delta V_i(1|h_i) - \delta V_i(0, \wedge b_{-i}|h_i)) | h_i \right] \geq k$$

$$\iff D_i(b_i|h_i) (b_i - c_i + \delta \mathbb{E}_\sigma [V_i(1|h_i) - V_i(0, \wedge b_{-i}|h_i) | h_i, b_i < \wedge b_{-i}] \geq k.$$  

Since $D_i \geq 0$, it must be that both left-hand side factors are strictly positive. In addition, since continuation values are bounded by some constant $\overline{V}$, it follows that $D_i(b_i|h_i) \geq k/(1 + 2\overline{V})$. Similarly, since demand is bounded above by 1, we have that

$$b_i - c_i + \delta \mathbb{E}_\sigma [V_i(1|h_i) - V_i(0, \wedge b_{-i}|h_i) | h_i, b_i < \wedge b_{-i}] \geq k.$$  

This concludes the proof. \hfill $\square$

Lemma C.2 (continuous demand). For every history $h_i = (\theta, z_i)$, residual demand $D_i(b_i|h_i)$ is continuous in $b_i$ over $(0, 1)$.

Proof. The proof is by contradiction. Assume that demand $D_i(\cdot|h_i)$ is discontinuous at bid $b_0$. There must exist a bidder $j$ and a history $h_j = (\theta, z_j)$ such that firm $j$ bids $b_j = b_0$ with probability $q > 0$. By Lemma C.1, bidder $j$ must win with probability at least $\nu > 0$ when bidding $b_0$.

Consider a bidder $l$ and a history $h_l = (\theta, z_l)$ such that history $h_j$ has positive probability, and bidder $l$ loses with positive probability against bidder $j$ when bidder $j$ bids $b_0$. Since the number of histories is finite, there exists $\nu_1 > 0$ such that at any such history $h_l$ bidder $j$ bids $b_0$ with positive probability $\nu_1$.

Pick $\epsilon > 0$ and consider the payoff of bidder $l$ bidding $b_l \in [b_0, b_0 + \epsilon)$. Bidder $l$ gets payoff (excluding participation costs and payoffs upon non-participation)

$$U_l(b_l|h_l, c_l) = D(b_l|h_l) (b_l - c_l + \delta \mathbb{E}_\sigma [V_i(1|h_l) - V_i(0, \wedge b_{-i}|h_l) | h_l, b_l < \wedge b_{-i}]).$$

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We know from Lemma C.1 that
\[ b_i - c_i + \delta \mathbb{E}_\sigma [V_i(1|h_i) - V_i(0, \land b_{-i}|h_i) \mid h_i, b_i < \land b_{-i}] \geq k. \]

The assumption that the continuation value of losers, \( V_i(0, \land b_{-i}|h_i) \) is decreasing in \( \land b_{-i} \) implies that
\[ \mathbb{E}_\sigma [V_i(0, \land b_{-i}|h_i) \mid h_i, b_i - \epsilon < \land b_{-i}] \leq \mathbb{E}_\sigma [V_i(0, \land b_{-i}|h_i) \mid h_i, b_i < \land b_{-i}] . \]

Altogether, it follows that for every \( \eta > 0 \), there exists \( \epsilon > 0 \) small enough that
\[ b_i - \epsilon - c_i + \delta \mathbb{E}_\sigma [V_i(1|h_i) - V_i(0, \land b_{-i}|h_i) \mid h_i, b_i - \epsilon < \land b_{-i}] \geq b_i - c_i + \delta \mathbb{E}_\sigma [V_i(1|h_i) - V_i(0, \land b_{-i}|h_i) \mid h_i, b_i - \epsilon < \land b_{-i}] - \eta \geq k - \eta. \]

Hence, it follows that by bidding \( b_i - \epsilon \), bidder \( l \) gets a payoff
\[ U_i(b_i - \epsilon|h_i, c_i) = D(b_i - \epsilon|h_i) (b_i - \epsilon - c_i + \delta \mathbb{E}_\sigma [V_i(1|h_i) - V_i(0, \land b_{-i}|h_i) \mid h_i, b_i - \epsilon < \land b_{-i}]) \geq U_i(b_i|h_i, c_i) - \eta + \nu_1(k - \eta). \]

Since \( \nu_1 \) is fixed, it follows that for \( \epsilon \) small enough \( U_i(b_i - \epsilon|h_i, c_i) > U_i(b_i|h_i, c_i) \). Hence, there exists \( \epsilon \) small such that bidder \( l \) does not bid in \([b_0, b_0 + \epsilon]\). Since there are only finite histories, this implies that there exists \( \epsilon > 0 \) such that no bidder \( l \) that loses against bidder \( j \) bidding \( b_0 \) bids in the range \([b_0, b_0 + \epsilon]\). Hence, bidder \( j \) would benefit from bidding \( b_0 + \epsilon/2 \) rather than \( b_0 \). This contradicts the assumption that \( \sigma \) is an MPE and concludes the proof. \( \square \)

Finally we establish that Proposition 1 continues to hold when value functions \( V_i(\zeta_i, b_w|h_i) \) are not sensitive and do not internalize competitors’ profits.

**Lemma C.3.** Consider an environment \( \mathcal{E} \) and an MPE \( \sigma \) such that bidding behavior is not sensitive and does not internalize competitors’ revenues. For all \( \eta > 0 \) there exists \( \epsilon > 0 \) small enough such that, for all histories \( h_{i,t} = (\theta_t, z_{i,t}) \) and bid \( b_{i,t} \in (\epsilon, 1 - \epsilon) \),
\[ \text{prob}_\sigma(i \text{ wins} \mid h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| < \epsilon) \geq 1/2 - \eta. \]

**Proof.** Consider an environment \( \mathcal{E} \) and an MPE \( \sigma \) such that bidding behavior is not sensitive and does not internalize competitors’ revenues. Fix a history \( h_{i,t} = (\theta_t, z_{i,t}) \) of firm \( i \). Let \( b_{i,t} < r = 1 \) denote firm \( i \)’s bid at this history when her cost are \( c_{i,t} \). For any bid \( b \), let
$U_i(b|h_{i,t}, c_{i,t})$ denote $i$'s payoff from bidding $b$ at history $h_{i,t}$ when her cost is $c_{i,t}$:

$$U_i(b|h_{i,t}, c_{i,t}) = \mathbb{E}_\sigma \left[ 1_{\mathbb{A}_{-i,t} \succ b}(b - c_{i,t} + \delta V_i(1|h_{i,t}) + (1 - 1_{\mathbb{A}_{-i,t} \succ b})\delta V_i(0, \mathbb{A}_{-i,t}|h_{i,t}) \mid h_{i,t} \right] - k.$$ 

Since bid $b_{i,t}$ is optimal, for all $\epsilon > 0$ it must be that,

$$U_i(b_{i,t}|h_{i,t}, c_{i,t}) \geq U_i(b_{i,t} + \epsilon|h_{i,t}, c_{i,t})$$

$$\iff (D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t}))(b_{i,t} - \kappa_{i,t}^+ \epsilon) \geq D_i(b_{i,t} + \epsilon|h_{i,t}) \times \epsilon$$  \hspace{1cm} (12)

where $\kappa_{i,t}^+ \equiv c_{i,t} + \delta \mathbb{E}_\sigma[V_i(1|h_{i,t}) - V_i(0, \mathbb{A}_{-i,t}|h_{i,t})|h_{i,t}, b_{i,t} + \epsilon > \mathbb{A}_{-i,t} > b_{i,t}]$. Since $D_i(\cdot|h_{i,t})$ is continuous at $b_{i,t}$ (Lemma C.2), and since $D_i(b_{i,t}|h_{i,t}) > 0$ (Lemma C.1) it must be that $b_{i,t} - \kappa_{i,t}^+ > 0$ for $\epsilon > 0$ small.

Similarly, for all $\epsilon > 0$ it must be that

$$U_i(b_{i,t}|h_{i,t}, c_{i,t}) \geq U_i(b_{i,t} - \epsilon|h_{i,t}, c_{i,t})$$

$$\iff (D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t}|h_{i,t}))(b_{i,t} - \kappa_{i,t}^- \epsilon) \leq D_i(b_{i,t} - \epsilon|h_{i,t}) \times \epsilon$$

$$- (D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t}|h_{i,t}))(\kappa_{i,t}^+ \epsilon - \kappa_{i,t}^- \epsilon)$$  \hspace{1cm} (13)

where $\kappa_{i,t}^- \equiv c_{i,t} - \delta \mathbb{E}_\sigma[V_i(1|h_{i,t}) - V_i(0, \mathbb{A}_{-i,t}|h_{i,t})|h_{i,t}, b_{i,t} > \mathbb{A}_{-i,t} > b_{i,t} - \epsilon]$.

Using (12) and (13), together with $b_{i,t} - \kappa_{i,t}^+ > 0$, we have that

$$\text{prob}_\sigma(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \mathbb{A}_{-i,t}| < \epsilon)$$

$$= \frac{D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}$$

$$\geq \frac{D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) - (D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t}|h_{i,t}))(\kappa_{i,t}^+ \epsilon - \kappa_{i,t}^- \epsilon) + D_i(b_{i,t} + \epsilon|h_{i,t})}.$$  \hspace{1cm} (14)

Since $D_i(\cdot|\theta, z_{i})$ is continuous on $[0, 1]$, it is uniformly continuous. Since there are finitely many $(\theta, z_{i})$, for every $\gamma_D > 0$ there exists $\tau > 0$ such that, for all $i, \theta, z_{i}$ and for all $b, b'$ with $|b - b'| \leq 2\tau$, $D_i(b|\theta, z_{i}) - D_i(b'|\theta, z_{i}) < \gamma_D$.

Moreover, since $V_i(0, \mathbb{A}_{-i})$ is Lipschitz continuous and decreasing in $\mathbb{A}_{-i}$, and since there are finitely many states $\theta$, there exists a Lipschitz constant $L > 0$ such that, for all $i, \theta, z_{i}, c_{i,t}, \kappa_{i,t}^+ - \kappa_{i,t}^- \geq -2\epsilon L$.  

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Using (14), for every $\gamma_D > 0$, there exists $\epsilon > 0$ such that, for all $\epsilon < \epsilon$,\[
\text{prob}_s(i \text{ wins } \mid h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| < \epsilon) \geq \frac{D_i(b_{i,t} + \epsilon|h_{i,t})}{\nu + \gamma_D \nu - \gamma_D},
\]

where the second inequality uses the inequality $D_i(b_{i,t} + \epsilon|h_{i,t}) \geq D_i(b_{i,t}|h_{i,t}) - \gamma_D \geq \nu - \gamma_D$ (Lemma C.1). Picking $\gamma_D$ small, we obtain that $\text{prob}_s(i \text{ wins } \mid h_{i,t} \text{ and } |b_{i,t} - \land b_{-i,t}| < \epsilon) \geq 1/2 - \eta$.

Proof of Proposition 2. Assume that continuation values are not sensitive, and do not internalize competitors’ revenue. Together, Lemmas C.3, and Corollary 2 imply that Corollary 1 must hold. Hence, if Corollary 1 does not hold, it must be that bidding behavior is either sensitive, or internalizes competitors’ revenue. □

References


OECD (2013): “Ex officio cartel investigations and the use of screens to detect cartels,”.


