Abstract

We construct a model of collective search in which players gradually approach the Pareto frontier. The players have imperfect control over which improvements to the status quo will be considered. Inefficiency takes place due to the difficulty in finding improvements acceptable to both parties. The process is path dependent, with early agreements determining long-run outcomes. It may also be cyclical, as players alternate between being more and less accommodating. Our model provides a noncooperative foundation for the “Raiffa path”.

Keywords: collective search, bargaining, path dependence, cycling, Raiffa path, delay, inefficiency.
1 Introduction

In traditional bargaining theory (e.g. Rubinstein, 1982), players are able to strike agreements that take them directly to the Pareto frontier. Yet, in many complex bargaining environments, the parties involved revise current agreements in a series of steps, slowly improving upon the existing arrangements. Real world examples of such environments featuring step-by-step negotiations include climate change agreements, international trade negotiations, nuclear disarmament, and many public policy reforms.

The complexity of the bargaining environment in many of these examples arises due to the difficulty in finding improvements to existing deals. Bargainers must search for ideas upon which to build their proposals, and it is hard to anticipate which ideas this search process yields and when. With frictions in the search process, bargainers are unlikely to find alternatives that land them directly on the Pareto frontier, and may have to content themselves with making a series of (potentially) small improvements relative to the status quo.

In this paper, we develop a tractable model that captures these frictions and in which players approach the Pareto frontier in a series of “interim agreements.” The model is a two player complete information game played over a large finite number of periods $T$. The set of feasible agreements is $X = \{ x \in \mathbb{R}^2_+ : x_1 + x_2 \leq 1 \}$. At each period $t$, player $i = 1, 2$ obtains a flow payoff equal to the coordinate $x^i_t$ of the agreement $x^t = (x^1_t, x^2_t)$ that is in place. The agreement in place at the start of the game is $(0, 0)$. In each period, a new alternative is drawn randomly from the set of feasible agreements that are Pareto improvements to the agreement last period, and players sequentially decide whether to approve or disapprove the draw. The previous period agreement is replaced if and only if both players approve the change; otherwise, it stays in place. Players share a common discount factor $\delta < 1$.

Under a key inter-temporal symmetry assumption, we are able to provide a clean characterization of the model’s equilibrium. In any period, players accept alternatives that improve their payoffs by a similar amount. Indeed, the set of alternatives that both players find acceptable is a cone defined by two lines with positive slope that pass through

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1 Even in models that feature (inefficient) delay in bargaining, once an agreement is eventually reached, the outcome typically lies on the frontier (e.g. Cho, 1990, Cramton, 1992, Abreu and Gul, 2000, Fanning, 2018).

2 The assumption, which is made on the distributions from which alternatives are drawn, implies that the game played from period $t$ onwards starting with an outstanding agreement $z \in X$ is strategically equivalent to the game played from period 0 onwards with outstanding agreement $(0, 0)$ in which the players search only for $T - t$ periods.
the last period’s agreement as its vertex. Figure 1 depicts the first two “acceptance cones” for a possible sequence of agreements \( \{x^1, x^2, x^3, x^4, \ldots \} \) that are implemented along the path of play. Alternatives that lie outside of the cone are rejected even if they are Pareto superior to the status quo. The reason for this is that players cannot commit to approve future alternatives that disproportionally benefit their opponents. As a result, a player strictly prefers to reject Pareto superior alternatives that yield a substantial improvement to her opponent, but only a mild improvement for her, since she (correctly) anticipates that approving such an alternative will “close the door” in the future to many alternatives that she finds attractive. Since players discount the future, the periods of inaction produced by the rejection of Pareto improving alternatives generate inefficiency—an inefficiency that arises due to the difficulty in finding moderate alternatives.

As Figure 1 shows, the distinctive feature of our model is that players will typically reach a sequence of interim agreements, gradually approaching the Pareto frontier. In addition, the randomness of draws and the rigidity of the status quo together imply that the process by which players approach the frontier is path dependent. In each period, the set of alternatives that players find acceptable depends on the current status quo. As a
result, at each point in time the future path of play depends crucially on the agreements that players reached at early stages.

This path dependence disappears, however, as players become infinitely patient. In the limit as $\delta \to 1$, the acceptance cone collapses to a line segment connecting the current agreement to a point on the frontier: only agreements on this line segment are implemented on the path of play. Intuitively, the cost in terms of forgone future payoff of implementing an agreement that is more beneficial to one’s opponent increases with $\delta$. In the limit, the only alternatives that both players accept are those that give a payoff vector on this line segment. When alternatives are drawn from a symmetric distribution, the long run agreement converges to an equal split of the surplus. In this case, the path that the equilibrium induces when players are arbitrarily patient coincides with the “Raiffa path”; i.e., the path of interim agreements proposed in Raiffa (1953) as a plausible outcome in settings in which the bargaining parties engage in step-by-step negotiations.

Lastly, we show that our model may give rise to equilibrium cycles, under which periods of high and low likelihood of agreements alternate. Given an existing agreement, the acceptance cone may be narrow in some periods but wide in others, following a cyclical pattern. These cycles are driven not by changes in fundamentals, but by self-fulfilling changes in the players’ expectations about future play.

Our baseline model is one in which players have no control over the offer that is generated. In this sense, our model lies at the opposite extreme of the standard approach to bargaining theory (Rubinstein, 1982) in which proposers have full control over their offers. A natural extension of our model is to the intermediate case in which proposers have partial control over the offers they put on the table. We consider such an extension in which, at each period, a randomly selected proposer chooses the distribution from which the alternative will be drawn. Our main results carry through in this environment.

Related Literature—Our paper is primarily related to the literature on collective search. Compte and Jehiel (2010a), Albrecht et al. (2010), and Moldovanu and Shi (2013) study models in which a group of agents sequentially sample alternatives from a distribution and have to choose when to stop. Closer to our model, Roberts (2007) and Penn (2009) also study settings with randomly generated alternatives and with an endogenously evolving status-quo. Roberts (2007) and Penn (2009) consider settings with supermajority rules and focus on how the dynamic nature of the problem affects players’ voting behavior when the set of available alternatives all lie on the Pareto
frontier. In contrast, we consider a setting with unanimity and focus on understanding the process by which play approaches the Pareto frontier.

The rigidity of the agreements relates our model to the growing literature on political bargaining with an endogenous status quo (e.g., Kalandrakis (2004), Duggan and Kalandrakis (2012), Dziuda and Loeper (2016)). We add to this literature by constructing a model in which players bargain over complex issues, and have imperfect control over the offers that are generated.

Because players in our model approach the Pareto frontier in incremental steps, our paper relates to prior work on incremental bargaining and partial agreements. Compte and Jehiel (2004) study a bargaining model in which each players’ outside option depends on the history of offers. In this setting players begin negotiations making incompatible offers, and make gradual concessions over time. However, there are no interim agreements in their model: the first agreement that players reach is a point on the Pareto frontier. More recently, Acharya and Ortner (2013) analyze a model in which two players bargain over two issues, one of which will only be open for negotiation at a future date. The main result is that players may reach a partial agreement on the first issue, only to complete the agreement when the second issue becomes available.

Our result on commitment and inefficiency relates our paper to the literature on bargaining failures as a result of commitment problems; e.g., Fearon (1996), Powell (2004, 2006), Acemoglu and Robinson (2000, 2001), Ortner (2017). These papers focus on understanding the conditions under which the players’ inability to commit will result in bargaining inefficiencies. Instead, we focus on how the players’ inability to commit shapes the way bargainers approach the Pareto frontier.

Our result on the Raiffa path relates our paper to others that also provide foundations for this bargaining solution. Livne (1989), Peters and Van Damme (1991), Diskin et al. (2011) and Samet (2009) provide axiomatizations for the Raiffa path. Myerson (2013), Trockel (2011) and Diskin et al. (2011) provide non-cooperative foundations by proposing bargaining models in the tradition of Rubinstein (1982). These models have the property that, in the first round, players reach an agreement at the point at which the Raiffa path intersects the Pareto frontier. In contrast to these studies, our model gives rise to interim agreements, therefore providing foundations for the path. Thus, our

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3In Myerson (2013), players reach an agreement that is “close” to the point at which the Raiffa path intersects the Pareto frontier.
paper contributes to the “Nash program” of providing non-cooperative foundations to cooperative bargaining solutions.\(^4\)

Lastly, our work is related to a set of papers in organizational economics showing how path-dependence can arise in organizations, and arguing that these dynamics may help explain why seemingly identical firms have persistent differences in performance; e.g. Chassang (2010), Li and Matouschek (2013), Halac and Prat (2016), Callander and Matouschek (2019).

## 2 Model

### 2.1 Framework

Two players, \(i = 1, 2\), play the following game. Time is discrete, with an infinite horizon, and indexed by \(t = 0, 1, 2, \ldots\). The set of feasible agreements is

\[
X := \{(y_1, y_2) \in \mathbb{R}_+^2 : y_1 + y_2 \leq 1\}.
\]

Fix \(T \in \mathbb{N}\). At each period \(t \leq T\), players jointly decide whether to change the existing agreement from the status quo \(z^t = (z^t_1, z^t_2) \in X\) to a new alternative \(x\) drawn randomly from a distribution \(F_{z^t}\) with density \(f_{z^t}\) and support over the set

\[
X(z^t) := \{x \in X : x_i \geq z^t_i \text{ for } i = 1, 2\}
\]

of alternatives that are Pareto superior to status quo \(z^t\). After alternative \(x\) is drawn, the two players sequentially decide whether or not to accept it. If both players accept it, then alternative \(x\) becomes the agreement in place in period \(t\), so \(x^t = x\). Otherwise, the status quo is implemented, so \(x^t = z^t\). The next period’s status quo is the previous period agreement, so \(z^{t+1} = x^t\) with \(z^0 = (0, 0)\). For all periods \(t \geq T + 1\) players cannot change the existing agreement, so \(x^t = x^T\). We refer to the final period \(T\) as the deadline, and we will be interested in studying the limiting case of \(T \to \infty\).

Both players are expected utility maximizers and share a common discount factor \(\delta < 1\). If \(x^t = (x^t_1, x^t_2) \in X\) is the agreement in place in period \(t\), then player \(i\) earns a

\(^4\)Other contributions to the “Nash program” include Binmore et al. (1986), Gul (1989), Abreu and Pearce (2007, 2015), Compte and Jehiel (2010b) and Fanning (2016).
flow payoff $x_i^t$ at time $t$. Player $i$'s payoff from a sequence of agreements $\{x^t\}_{t=0}^\infty$ is thus

$$U_i(\{x^t\}) = (1 - \delta) \sum_{t=0}^\infty \delta^t x_i^t.$$ 

Let $\Gamma(T)$ denote the game with finite deadline $T$. We focus on the subgame perfect equilibria (SPE) of $\Gamma(T)$.

The following result is established by backward induction. Its proof, and all other proofs, appear in Appendix A.

**Proposition 1.** $\Gamma(T)$ has an SPE. Moreover, all SPE of $\Gamma(T)$ give players the same expected payoffs.

### 2.2 Recursive equilibrium characterization

For any $z \in X$ and any $x \in X(z)$, let

$$P_z(x) := \left( \frac{x_1 - z_1}{1 - z_1 - z_2}, \frac{x_2 - z_2}{1 - z_1 - z_2} \right)$$

$P_z$ is a mapping that projects points in $X(z)$ onto $X$. We make the following assumption on the distributions $F_z$ from which alternatives are drawn.

**Assumption 1.** For every $z \in X$, the density $f_z$ is such that

$$\forall x \in X(z), \quad f_z(x) = \frac{1}{(1 - z_1 - z_2)^2} f(P_z(x))$$

where $f := f_{(0,0)}$ is the density from which alternatives are drawn at the start of the game. In addition, there are constants $\underline{f}, \bar{f}$ with $\bar{f} > \underline{f} > 0$, such that $f(x) \in [\underline{f}, \bar{f}]$ for all $x \in X$.

Assumption 1 states that, for any $z \in X$, the distribution $F_z$ over $X(z)$ from which alternatives are drawn when the status quo is $z$ is “identical” to the distribution $F := F_{(0,0)}$ over $X$ from which alternatives are drawn at the start of the game. We maintain this assumption throughout the rest of the paper. Its main implication is that a subgame starting at period $t \leq T$ with status quo $z \in X$ is strategically equivalent to a game with deadline $T - t$ starting at $z^0 = (0, 0)$.

To formalize this, for any deadline $T$, time $t \leq T$ and alternative $z \in X$, let $V_i(z, t; T)$ be the continuation payoff that player $i$ obtains under an SPE of $\Gamma(T)$ at a subgame
starting at period $t$ with status quo $z^t = z$. Let $W_i(T) = V_i((0,0);T)$ be player $i$’s SPE payoff at the start of game $\Gamma(T)$. We have:

**Lemma 1.** For all $t \leq T$ and all possible values of the status quo $z^t = z = (z_1, z_2) \in X$, the players’ equilibrium payoffs satisfy

$$V_i(z, t; T) = z_i + (1 - z_1 - z_2)W_i(T - t) \text{ for } i = 1, 2. \tag{1}$$

When the status quo at time $t$ is $z$, player $i$’s equilibrium payoff is equal to the flow payoff $z_i$, that the player is guaranteed to get forever (by the persistence of the status quo), plus the payoff $(1 - z_1 - z_2)W_i(T - t)$ that the player obtains from bargaining for $T - t$ periods over the remaining surplus of size $1 - z_1 - z_2$.

We use Lemma 1 to provide a recursive characterization of the players’ equilibrium payoffs. Note first that, at the last period $T$, players accept any alternative in $X(z_T)$, where $z_T$ is the status quo at the start of period $T$. Consider next a period $t < T$ at which the status quo is $z = (z_1, z_2) \in X$. Then, player $i$ approves alternative $x = (x_1, x_2) \in X(z)$ only if

$$(1 - \delta)x_i + \delta V_i(x, t + 1; T) \geq (1 - \delta)z_i + \delta V_i(z, t + 1; T). \tag{2}$$

Let $W_i = W_i(T - t - 1)$. Then using (1) in both sides of (2) and rearranging, player $i$ accepts alternative $x$ when the status quo is $z$ only if

$$x_i \geq \ell_{i,z}(x_{-i}|W_i) := z_i + \frac{\delta W_i}{1 - \delta W_i}(x_{-i} - z_{-i}).$$

Note that $\ell_{i,z}(x_{-i}|W_i)$ is the line in $(x_i, x_{-i})$-space with slope $\delta W_i/(1 - \delta W_i)$ that passes through $z$.

For any $W = (W_1, W_2) \in X$, define

$$A_{i,z}(W_i) := \{x \in X(z) : x_i \geq \ell_{i,z}(x_{-i}|W_i)\}.$$

Then, for any pair of payoffs $W = (W_1, W_2)$ and for any $z \in X$, the set

$$A_z(W) := A_{1,z}(W_1) \cap A_{2,z}(W_2) \tag{3}$$

is the set of alternatives that are accepted by both players at period $t < T$ when the status quo is $z$ and $(W_1(T - t - 1), W_2(T - t - 1)) = (W_1, W_2)$. When $1 > \delta(W_1 + W_2)$,
the line $\ell_{1,z}(x_2|W_1)$ has steeper slope than $\ell_{2,z}(x_1|W_2)$ in $(x_1, x_2)$-space and $A_z(W)$ is a cone with vertex $z$. For any pair of values $W$ we let $A(W) := A_{(0,0)}(W)$ be the cone with vertex $(0,0)$. Such a cone is depicted in Figure 2.

For any integer $T > 0$, let $W(T) = (W_1(T), W_2(T))$ be the players’ SPE payoffs in a game with deadline $T$. By our arguments above, an alternative is accepted at the initial period if and only if it is in the set $A(W(T-1))$ with $W(T-1) = (W_1(T-1), W_2(T-1))$. Therefore, player $i$’s payoff at the start of the game is

$$W_i(T) = \text{prob}(x \in A(W(T-1)))E[(1 - \delta)x_i + \delta V_i(x, 1; T)|x \in A(W(T-1))]$$

$$+ \text{prob}(x \notin A(W(T)))[(1 - \delta)0 + \delta V_i((0,0), 1; T)]$$

$$= \text{prob}(x \in A(W(T-1)))E[x_i - (x_1 + x_2)\delta W_i(T-1)|x \in A(W(T-1))] + \delta W_i(T-1),$$

where the second line follows from equation (1).

Define the operator $\Phi : X \rightarrow X$, where for every payoff pair $W = (W_1, W_2) \in X$ and for $i = 1, 2$,

$$\Phi_i(W) := \text{prob}(x \in A(W))E[x_i - (x_1 + x_2)\delta W_i|x \in A(W)] + \delta W_i,$$

(4)

Let $\Phi^t(W)$ denote the $t$-th iteration of operator $\Phi$ over the pair $W = (W_1, W_2)$.

**Proposition 2.** For any deadline $T$,

(i) the players’ equilibrium payoffs satisfy $W(T) = \Phi^{T+1}((0,0))$, and

(ii) the set of alternatives that are accepted by both players in any period $t \leq T$ is $A_{z^t}(W(T-t-1))$ where $z^t$ is the status quo in period $t$ and $W(T-t-1)$ are the players’ equilibrium payoffs in a game with deadline $T-t-1$.

Figure 2 plots the acceptance region $A(W)$ at the initial period of the game. As the figure shows, alternatives that constitute a Pareto improvement over the initial agreement $(0,0)$ and that lie outside $A(W)$ are rejected, leading to inefficient outcomes.

The commitment problem plays an important role in exacerbating the inefficiencies that arise from search frictions. To see how, suppose that in period 0 alternative $x > (0,0)$ in Figure 2 is drawn. Alternative $x$ Pareto dominates the initial agreement, but if $x$ were to be implemented, then starting in period 1 the set of alternatives $A_{z^1}(W)$ that both players accept would be the area inside the dashed lines in Figure 2. These alternatives are significantly worse for player 2 than the alternatives that could be implemented.
in the future if the status quo \((0, 0)\) remains in place. So player 2 strictly prefers to maintain agreement \((0, 0)\) than to implement \(x\). Player 2 would approve \(x\) if player 1 could commit to accepting alternatives that are beneficial for player 2 in the future. However, player 2 rightly anticipates that player 1 would reject such alternatives in the future if \(x\) were to be implemented today. Hence, this inability to commit implies that only alternatives that improve both players’ payoffs by a similar amount (i.e., moderate alternatives) will be accepted along the path of play.

3 Infinite horizon limit

Throughout this section, we study the properties of the equilibrium in the limit as the deadline \(T\) approaches \(\infty\).

**Definition 1.** *We say that the equilibrium is convergent if the sequence \(\{W(T)\} = \{\Phi^T(0)\}\) converges as \(T \to \infty\). Otherwise, we say that the equilibrium is cycling.*

Section 3.1 studies conditions under which equilibrium is convergent. Section 3.2 discusses some properties of convergent equilibria. Section 3.3 provides conditions under which equilibrium is cycling.
3.1 Conditions for convergence

The iterative characterization of equilibrium payoffs in Proposition 2 suggests that if the sequence of payoffs \( \{W(T)\} \) converges as \( T \) goes to infinity, then the limit is a fixed point of \( \Phi \). This is confirmed by the following lemma.

**Lemma 2.** (i) \( \Phi \) has a fixed point, and
(ii) if the sequence of payoffs \( \{W(T)\} \) converges to \( W \), then \( W \) is a fixed point of \( \Phi \).

Our next result presents sufficient conditions for equilibrium to be convergent. In particular, it shows that equilibrium is convergent whenever players are patient enough.

**Proposition 3.** There exists a threshold \( \delta < 1 \) such that if \( \delta > \bar{\delta} \) the equilibrium is convergent.

**Symmetric distributions.** We now study conditions under which payoffs converge for the special case where \( F \) is symmetric about the 45° line, i.e. when its density \( f \) satisfies \( f(x_1, x_2) = f(x_2, x_1) \) for all \( (x_1, x_2) \in X \).

We start by noting that, when \( F \) is symmetric, both players get the same equilibrium payoffs: for all \( T \), \( W(T) = (W_1(T), W_2(T)) \) is such that \( W_1(T) = W_2(T) =: W(T) \).\(^5\)

For all \( T \) let \( \hat{W}(T) = 2W(T) \) be the sum of the players’ equilibrium payoffs in a game with deadline \( T \). With a slight abuse of notation, let \( A(\hat{W}) \) be the acceptance region when \( W = (\hat{W}/2, \hat{W}/2) \). We define the operator \( \Psi : [0, 1] \to [0, 1] \) as follows: for all \( \hat{W} \),

\[
\Psi(\hat{W}) := \Phi_1((\hat{W}/2, \hat{W}/2)) + \Phi_2((\hat{W}/2, \hat{W}/2))
\]

\[
= \text{prob}(x \in A(\hat{W})) \mathbb{E}[x_1 + x_2 | x \in A(\hat{W})](1 - \delta \hat{W}) + \delta \hat{W} \tag{5}
\]

It then follows from Proposition 2 that when \( F \) is symmetric, \( \hat{W}(T) = \Psi^{T+1}(0) \).

Our next result provides a sufficient condition for equilibrium to be convergent in the special case in which \( F \) is symmetric.

**Proposition 4.** Suppose \( F \) is symmetric. Then, if \( \Psi(\hat{W}) > -1 \) for all \( \hat{W} \in [0, 1] \), the equilibrium is convergent.

\(^5\)A formal proof of this statement is given in Lemma A.2 in the Appendix.
To get a sense as to when the condition in Proposition 4 holds, define

\[ H(\hat{W}) := \text{prob}(x \in A(\hat{W}))\mathbb{E}[x_1 + x_2|x \in A(\hat{W})], \]  

so that

\[ \Psi(\hat{W}) = H(\hat{W})(1 - \delta\hat{W}) + \delta\hat{W}, \quad \text{and} \]
\[ \Psi'(\hat{W}) = \delta(1 - H(\hat{W})) + H'(\hat{W})(1 - \delta\hat{W}). \]  

Note that \( H'(\hat{W}) < 0 \), and that the magnitude of this derivative depends on how much mass the distribution \( F \) puts on the boundary of the acceptance set: \(|H'(\hat{W})|\) is large when \( F \) puts significant mass on the boundary of \( A(\hat{W}) \). Since \( \delta(1 - H(\hat{W})) > 0 \), the condition in Proposition 4 holds whenever the distribution \( F \) is sufficiently “dispersed.”

**Example 1.** Assume \( F \) is a uniform distribution over \( X \). In this case, for any \( \hat{W} \in [0, 1] \),

\[ \Psi(\hat{W}) = \delta\hat{W} + \frac{2}{3}(1 - \delta\hat{W})^2. \]

Note that \( \Psi'(\hat{W}) = \frac{2}{3}(-1 + 4\delta\hat{W}) > -1 \), so by Proposition 4 the equilibrium is convergent. Payoffs \( W(T) = (W_1(T), W_2(T)) \) converge to \( W = (W_1, W_2) \), where for \( i = 1, 2 \),

\[ W_i = \frac{1}{8\delta^2}(3 + \delta - \sqrt{9 + 6\delta - 15\delta^2}). \]

We note that, as \( \delta \to 1 \), equilibrium payoffs \( W \) converge to \((1/2, 1/2)\).

### 3.2 Properties of convergent equilibria

In this section, we assume that the equilibrium is convergent, so \( W(T) \) converges to some \( W = \Phi(W) \). We derive several properties of the equilibrium in the limit as \( T \to \infty \).

We start by noting that, when \( W(T) \) converges to some \( W = (W_1, W_2) \in X \), each acceptance set \( A_{x^r}(W) \) is a cone with vertex \( x^r \), defined by two lines with slopes \((1 - \delta W_1)/\delta W_1 \) and \( \delta W_2/(1 - \delta W_2) \) that pass through the vertex (see Figure 2). This means that in the infinite horizon limit, the lines defining all of the acceptance cones are parallel, so the acceptance cones are nested.
Lemma 3. (nested acceptance cones) Let \( \{x^t\}_{t=0}^{\infty} \) be a realized sequence of equilibrium agreements. Then,
\[
A_{x^0}(W) \supseteq A_{x^1}(W) \supseteq A_{x^2}(W) \supseteq ... 
\]

Lemma 3 implies that there exist alternatives that are acceptable at some period \( t \), but become no longer acceptable at period \( t + 1 \) despite also being Pareto improvements relative to the \( t + 1 \) status quo \( z^{t+1} \). In words, players don’t implement alternatives that they would have both previously accepted. Figure 1 illustrates this feature of the equilibrium, by showing the acceptance cones for agreements \((0,0)\) and \(x^1 > (0,0)\).

Long-run outcomes. The game essentially ends when players reach an agreement \( x \in X \) on the Pareto frontier: if \( x \in X \) with \( x_1 + x_2 = 1 \) is implemented at time \( t \), then \( x^\tau = x \) for all periods \( \tau \geq t \). We therefore call an agreement \( x \) on the Pareto frontier a long-run outcome of the game. The game’s unique equilibrium induces a distribution \( G \) over long-run outcomes; i.e., over points on the frontier. For any subgame starting with status-quo \( z \in X \), the continuation equilibrium at that subgame induces a distribution \( G_z \) over long-run outcomes. The next result summarizes some notable features of convergent equilibria, including that the distribution over long run outcomes changes along the path of play, and exhibits path dependence. For any distribution \( \hat{G} \), we denote its support by \( \text{supp} \hat{G} \).

Proposition 5. Suppose the equilibrium is convergent. Then,

(i) (long run distribution) for any \( z \in X \), \( \text{supp} G_z = \{ y \in X : y_1 + y_2 = 1 \} \cap A_z(W) \);

(ii) (path dependence) \( G_z \neq G_{z'} \) for all \( z' \neq z \);

(iii) (gradual certainty) For every sequence of equilibrium agreements \( \{x^\tau\}_{\tau=0}^{\infty} \), \( \text{supp} G_{x^{\tau+1}} \subseteq \text{supp} G_{x^\tau} \), with strict inclusion whenever \( x^{\tau+1} \neq x^\tau \).

In the first period, any alternative \( x \) on the Pareto frontier with \( x_1 \in [\delta W_1, 1 - \delta W_2] \) lies in the support of \( G = G_{(0,0)} \). As play progresses and the players implement agreements that are closer to the Pareto frontier, the support of the long-run distribution shrinks. Figure 1 shows the support of \( G_{x^1} \) for some interim agreement \( x^1 \) on the path of play.
Patient players and the Raiffa path. We now study equilibrium behavior when players become arbitrarily patient; i.e., when $\delta \to 1$. We note that, by Proposition 3, the equilibrium is convergent whenever $\delta$ is larger than some threshold $\bar{\delta}$.

For each $\delta \in (\bar{\delta}, 1)$, we let $W^\delta = (W^1_1, W^2_2)$ denote the players’ limiting payoffs as $T \to \infty$ in a game with discount factor $\delta$. We let $G^\delta$ denote the distribution over long run outcomes in the limiting equilibrium with discount factor $\delta$.

Proposition 6. Fix a sequence $\{\delta_n\} \to 1$, and a corresponding sequence of equilibrium payoffs $\{W^\delta_n\}$. Then,

(i) (determinism) $G^{\delta_n}$ converges to a dirac measure on $(W^*_1, W^*_2) := \lim_{n \to \infty} (W^{\delta_n}_1, W^{\delta_n}_2)$;

(ii) (generalized Raiffa path) $\lim_{n \to \infty} A(W^{\delta_n}) = \{x \in X : x_1/x_2 = W^*_1/W^*_2\}$;

(iii) (efficiency) $\lim_{n \to \infty} W^{\delta_n}_1 + W^{\delta_n}_2 = 1$.

If $F$ is symmetric, $W^*_1 = W^*_2 = 1/2$.

Proposition 6(i) says that as $\delta \to 1$ the path of play approaches deterministically a particular long run outcome, namely the players’ equilibrium payoff split. Proposition 6(ii) says that, as $\delta \to 1$, the set of alternatives that both players find acceptable converges to the line segment connecting $(0, 0)$ and the point $(W^*_1, W^*_2)$. Intuitively, the cost in terms of forgone future payoff of implementing an agreement that is more beneficial to your opponent increases with $\delta$. In the limit, the only alternatives that both players accept are those that give both players a payoff on this line segment. This implies that, as players become arbitrarily patient, there is no path dependence. Lastly, Proposition 6(iii) shows that the inefficiency of delay vanishes as players become infinitely patient. This occurs in spite of the fact that, as $\delta \to 1$, the acceptance region $A(W^\delta)$ converges to a straight line, and so the probability of changing the existing agreement in any given period goes to zero.

In general, the long-run agreement $(W^*_1, W^*_2)$ depends on the distribution $F$. Proposition 6 establishes that in the special case in which the distribution is symmetric, both players obtain the same payoff, so $(W^*_1, W^*_2) = (1/2, 1/2)$. As a result, when $F$ is symmetric, the path of play that our model induces in the limit as $\delta \to 1$ is closely related to the sequential bargaining solution proposed by Raiffa (1953). Indeed, in our frame-
The Raiffa path is the segment with slope 1 that connects the origin with the point \((1/2, 1/2)\) on the Pareto frontier.\(^6\)

### 3.3 Cycling equilibrium

We now turn to cycling equilibria. We start by providing some intuition as to why the equilibrium may be cycling.

Note that players in our model trade off implementing a Pareto improving agreement today against the benefit of waiting to see if they can change the agreement in a more preferred direction tomorrow. At the deadline \(T\), there is no benefit to waiting so the players accept every alternative in \(X(\mathbf{z}^T)\). In the second to last period, however, players are less accommodating, since they anticipate that the set of acceptable alternatives tomorrow will depend on the agreement they implement today. Graphically, the acceptance cone becomes smaller (narrower) at period \(T - 1\), and some extreme alternatives in \(X(\mathbf{z}^{T-1})\) are rejected.

Consider next period \(T - 2\). If the probability of changing the agreement next period is sufficiently small (i.e., if the distribution \(F\) places little mass on the acceptance cone tomorrow), players know that they are unlikely to enact a reform in the next period, and, in all likelihood, will have to wait until the final period to change the agreement if they don’t change it today. Since waiting for two periods is more costly than waiting only one period, players are more accommodating in period \(T - 2\) than they are in period \(T - 1\).

The arguments above suggest that, for small values of \(T\), equilibrium play may cycle, alternating between periods in which players find it relatively easy to modify existing agreements and periods in which modifying existing agreements is harder. We now show that these same cycles can occur in the limit as \(T \to \infty\).

To provide simple conditions under which such cycling occurs, we focus on the case in which the distribution \(F\) is symmetric. Recall from the discussion in Section 3.1 that when \(F\) is symmetric, players have the same equilibrium payoffs and the sum of these payoffs is the \((T + 1)\)-th iteration over 0 of the operator \(\Psi\) defined in (5).

**Proposition 7.** If \(F\) is symmetric then \(\Psi\) has a unique fixed point \(\hat{W}^*\). If, in addition,

\[(i) \ \Psi(\hat{W}) \neq \hat{W}^* \text{ for all } \hat{W} \neq \hat{W}^*, \text{ and}\]

\(^6\)When the bargaining set has a linear Pareto frontier, the Raiffa path is the segment connecting the disagreement payoff with the Pareto frontier, and passing through the utopia payoff vector; i.e., the payoff vector that would result if each player obtained her preferred outcome.
(ii) there exists $\varepsilon > 0$ such that $\Psi'(\hat{W}) \leq -1$ for all $\hat{W} \in [\hat{W}^* - \varepsilon, \hat{W}^* + \varepsilon]$,

then the equilibrium is cycling.

For some intuition as to when the conditions in Proposition 7 hold, recall that

$$\Psi(\hat{W}) = H(\hat{W})(1 - \delta \hat{W}) + \delta \hat{W},$$

$$\Psi'(\hat{W}) = \delta (1 - H(\hat{W})) + H'(\hat{W})(1 - \delta \hat{W}),$$

where $H(\hat{W}) = \text{prob}(x \in A(\hat{W}))E[x_1 + x_2|x \in A(\hat{W})]$. The magnitude of $H'(\hat{W}) < 0$ depends on how much mass the distribution $F$ puts on the boundary of the acceptance set $A(\hat{W})$. Hence, Proposition 7 holds when distribution $F$ places significant mass at the boundary of $A(\hat{W})$ for all $\hat{W}$ close to the fixed point $\hat{W}^*$.

Under the conditions in Proposition 7, the players’ equilibrium payoffs $\hat{W}(\tau)/2$ cycle around $\hat{W}^*/2$. Note that, in the symmetric case, the acceptance region $A_z(\hat{W})$ is a cone with vertex $z$ and lines with slopes $\frac{1 - \delta \hat{W}/2}{\delta \hat{W}/2}$ and $\frac{\delta \hat{W}/2}{1 - \delta \hat{W}/2}$. Therefore, the fact that payoffs $\hat{W}(\tau)/2$ cycle around $\hat{W}^*/2$ implies that there will be an alternation between periods of high and low probability of agreement; i.e., the equilibrium features cycles.

We now present an example to make the cycling result more concrete. The example also shows that the period of the cycle can vary with the model’s parameters. For expositional purposes, we consider an example in which distribution $F$ is discrete.\footnote{Two points are worth noting. First, Propositions 1 and 2 continue to hold when $F$ is discrete. Second, if we endow the space of distributions with the sup norm, operator $\Phi(W)$ is continuous in the distribution $F$. Hence, Example 2 can be approximated by a sequence of continuous distributions $\{F^n\}$ converging to the discrete distribution.}

**Example 2.** Suppose $F$ is such that

$$\text{prob}_F(x = (1/3, 1/4)) = \text{prob}_F(x = (1/4, 1/3)) = 1/2.$$  

Note that,

$$\Psi(\hat{W}) = \begin{cases} 
\delta \hat{W} & \text{if } \hat{W} > \frac{6}{73}, \\
\frac{7}{12}(1 - \delta \hat{W}) + \delta \hat{W} & \text{if } \hat{W} \leq \frac{6}{73}.
\end{cases}$$

Indeed, when $\hat{W} > \frac{6}{73}$, players’ continuation values are too high and the set of acceptable alternatives has no mass under $F$. When $\hat{W} \leq \frac{6}{73}$, the set of acceptable alternatives has probability 1. For $\delta = 0.95$, the sum of players’ equilibrium payoffs $\hat{W}(T)$ converges as $T \to \infty$ to a two-period cycle, with payoffs alternating between $\hat{W} \approx 0.93$ and $\hat{W} \approx 0.89$. 


For \( \delta = 0.98 \), equilibrium payoffs converge as \( T \to \infty \) to a five-period cycle, with payoffs alternating between \( \hat{W} \approx 0.94, \hat{W} \approx 0.92, \hat{W} \approx 0.90, \hat{W} \approx 0.88 \) and \( \hat{W} \approx 0.86 \).

4 Discussion

4.1 Strategic Search

Our model, with random proposals, is intended to capture complexities in the environment that make it difficult for players to gauge the payoff consequences of their proposals. In this section, we present a natural extension of our framework in which players have some ability to influence the direction in which they will search for new alternatives.

As we mentioned in the introduction, our model can be interpreted as a bargaining model in which the proposer has no control over the offer that is generated; and, in this sense, our model lies at the opposite extreme of the standard approach to bargaining theory in which proposers have full control over the proposals that are considered. The extension we present in this section bridges the gap between the traditional approach and our baseline model by allowing proposers to have partial control over the payoff consequences of the offers they put on the table.\(^8\) We briefly describe the model here. A formal treatment appears in Appendix B.

Two players, \( i = 1, 2 \), play the following game. Time is discrete and indexed by \( t = 0, 1, 2, \ldots \). The set of alternatives is \( X \), and players have the same preferences over alternatives as in our baseline model. At each period \( t = 0, 1, \ldots, T \), player \( i = 1, 2 \) is recognized with probability \( 1/2 \). The recognized player chooses a distribution \( F \) from a finite set of distributions \( \mathcal{F}_z \), where \( z \) is the current status quo. We assume that each distribution in \( \mathcal{F}_z \) has a density and support in \( X(z) \). The alternative \( x \) in period \( t \) is then drawn from distribution \( F \).

After the new alternative \( x \) is drawn, the two players sequentially decide whether or not to accept it. If both players accept it, then the agreement in place in period \( t \) becomes the new alternative, so \( x^t = x \). Otherwise, the status quo is implemented, so \( x^t = z^t \). The status quo at time \( t + 1 \) is the previous period agreement, so \( z^{t+1} = x^t \). For all periods \( t \geq T + 1 \) the players cannot change the agreement, so \( x^t = x^T \). As in our baseline model, for any deadline \( T \), this game can be solved by backward induction, and all equilibria generate the same expected payoffs.

\(^8\)Compte and Jehiel (2010a) consider a related model in their game in which play stops after the first agreement.
In Appendix B we show that, under a natural generalization of Assumption 1, this extended model retains all the key features of our baseline model.

4.2 Infinite Horizon Game

Throughout the paper, we considered a finite horizon game with deadline $T$. We end the paper by briefly discussing some properties of the infinite horizon version of the game; i.e., the game without a deadline.

We start by noting that the infinite horizon game always has an SPE under which players play strategies that depend on the history only through the status quo; i.e., a stationary equilibrium always exists, regardless of whether the equilibrium of the finite horizon game is convergent or cycling.

Let $W = (W_1, W_2)$ be a fixed point of operator $\Phi$ (by Lemma 2, such a fixed point always exists). It is easy to check that the following strategies constitute an SPE of the infinite horizon game: at any period $t \in \mathbb{N}$ with $z^t = z$, player $i = 1, 2$ accepts alternative $x \in X(z)$ if and only if $x \in A_{i,x}(W_i)$. Player $i$’s payoff at the start of the game under this SPE is

$$\Phi_i(W) = \text{prob}(x \in A(W))\mathbb{E}[x_i - (x_1 + x_2)\delta W_i|x \in A(W)] + \delta W_i = W_i.$$  

This implies that, when the SPE of the finite horizon game is convergent, this SPE converges as $T \to \infty$ to an SPE of the infinite horizon game.

Consider next the case in which the unique equilibrium of the finite horizon game is cycling. Suppose further that payoffs $W(T) = \Phi^{T+1}(0)$ converge to a cycle of length $k \geq 2$, with payoffs $W^0, ..., W^{k-1}$. In this case, the following strategies constitute an SPE of the infinite horizon game. At every period $t$ with $z^t = z$, player $i = 1, 2$ accepts alternative $x \in X(z)$ if and only if $x \in A_{i,x}(W_i^t \mod k)$. Under this SPE, player $i$’s expected payoff at the start of the game is

$$\Phi_i(W^0) = \text{prob}(x \in A(W^0))\mathbb{E}[x_i - (x_1 + x_2)\delta W_i^0|x \in A(W^0)] + \delta W_i^0 = W_i^1.$$  

Therefore, when payoffs in the finite horizon game converge to a cycle, the infinite horizon game also has an SPE with the same cycle.

\footnote{Recall that in Example 2, payoffs converge to a cycle of length 2 when $\delta = 0.95$, and a cycle of length 5 when $\delta = 0.98$.}
5 Conclusion

The paper constructs a bargaining model based on the assumption that players have imperfect control over the proposals that are considered.

Our model suggests a new source of inefficiency in bargaining, namely the difficulty in finding moderate agreements that are acceptable to all players involved. In our model, the bargaining process is path dependent and may be cyclical as the players alternate between being more and less accommodating. Inefficiency is exacerbated by the commitment problem, and cycling is driven by an alternating pattern of changes in the players’ self-fulfilling expectations about the likelihood of making an improvement to existing agreements.

Our model provides an answer to the question of how two bargainers approach the Pareto frontier. They do so in steps, while ensuring that these steps fit within the set of trajectories that ensure long-run moderation. In the symmetric case, as the players become fully forward looking, the only acceptable trajectory is the one hypothesized by Raiffa under which the players are guaranteed to achieve an equal split of the surplus.

Appendix

A Proofs

A.1 Proofs for Section 2

Proof of Proposition 1. In the subgame starting in period \( T \), it is optimal for both players to accept any alternative in \( X(z^T) \). Moreover, the alternative that is drawn will remain in place in all future periods so the payoff to each player \( i \) at this subgame is

\[
V_i(z^T, T; T) = \mathbb{E}_{z^T}[x_i]
\]

where \( \mathbb{E}_{z^T}[\cdot] \) is the expectation operator under the distribution \( F_{z^T} \).

At any subgame starting in period \( T - 1 \) with status quo \( z^{T-1} \), it is optimal for player \( i \) to accept alternative \( x \in X(z^{T-1}) \) if

\[
(1 - \delta)x_i + \delta V_i(x, T; T) \geq (1 - \delta)z_i^{T-1} + \delta V_i(z^{T-1}, T; T)
\]
So the set of alternatives that are acceptable to both players is

\[ A_{z^{T-1}} := \{ x \in X(z^{T-1}) : (9) \text{ holds for both } i = 1, 2 \} \]

This defines the payoff that each player \( i \) gets at such a subgame, which is

\[
V_i(z^{T-1}, T - 1; T) = \text{prob}(x \in A_{z^{T-1}}) E_{z^{T-1}} [(1 - \delta)x_i + \delta V_i(x, T; T) | x \in A_{z^{T-1}}] \\
+ \text{prob}(x \notin A_{z^{T-1}}) [(1 - \delta)z_i^{T-1} + \delta V_i(x^{T-1}, T; T)]
\]

Repeating these arguments for all \( t < T \) establishes existence of a SPE, and uniqueness of SPE payoffs.\(^{10}\) \qed

**Proof of Lemma 1.** Recall that for all \( z \in X \), \( E_z[\cdot] \) is the expectation operator under distribution \( F_z \). Let \( E[\cdot] \) be the expectation operator under distribution \( F_{(0,0)} = F \). We prove the result by induction.

Consider first a subgame starting at period \( t = T \) with status quo \( z^T = z \in X \). Note that

\[
V_i(z, T; T) = E_z[x_i] = z_i + (1 - z_1 - z_2)E[x_i],
\]

where the first equality follows from equation (8) and the second equality follows from Assumption 1.

Now, consider the game with deadline \( T = 0 \). By equation (8), player \( i \)'s equilibrium payoffs satisfy \( W_i(0) = E[x_i] \). Hence,

\[
V_i(z, T; T) = z_i + (1 - z_i - z_j)W_i(0)
\]

which establishes the basis case.

For the induction step, suppose that (1) holds for all \( t \) such that \( T - t = 0, 1, ..., n - 1 \) and for all \( z \in X \). Fix a subgame starting at period \( \tilde{t} \) with \( T - \tilde{t} = n \) and with status quo \( z^{\tilde{t}} = z \in X \). Let \( A_{z^{\tilde{t}}} \) be the set of alternatives that both players accept at period

\(^{10}\)Multiplicity of SPE arises for two reasons. First, for alternatives \( x \notin A_{z^{t}} \), there are SPE in which only one player rejects and the other accepts, and other SPE in which both players reject. Second, players are indifferent between accepting or rejecting when \( x \) is such that (9) holds with equality. Since the set of such alternatives have measure zero, players obtain the same expected payoffs in all SPE.
\( \bar{t} \) when \( z^\bar{t} = z \); that is,

\[
A_z(\bar{t}) = \left\{ x \in X(z) : (1 - \delta)x_i + \delta V_i(x, \bar{t} + 1; T) \geq (1 - \delta)z_i + \delta V_i(z, \bar{t} + 1; T) \text{ for } i = 1, 2 \right\}
\]

\[
= \left\{ x \in X(z) : (x_i - z_i) \geq (x_1 + x_2 - z_1 + z_2)\delta W_i(T - \bar{t} - 1) \text{ for } i = 1, 2 \right\},
\]

where the second line follows since, by the induction hypothesis, (1) holds for \( t = \bar{t} + 1 \).

Note then that

\[
V_i(z, \bar{t}; T) = \text{prob}(x \in A_z(\bar{t})) \mathbb{E}_z \left[ (1 - \delta)x_i + \delta V_i(x, \bar{t} + 1; T) \mid x \in A_z(\bar{t}) \right] 
+ \text{prob}(x \notin A_z(\bar{t})) \left( (1 - \delta)z_i + \delta V_i(z, \bar{t} + 1; T) \right)
\]

\[
= \text{prob}(x \in A_z(\bar{t})) \mathbb{E}_z \left[ x_i + (1 - x_1 - x_2)\delta W_i(T - \bar{t} - 1) \mid z \in A_z(\bar{t}) \right] 
+ \text{prob}(x \notin A_z(\bar{t})) \left( z_i + (1 - z_1 - z_2)\delta W_i(T - \bar{t} - 1) \right)
\]

\[
= \text{prob}(x \in A_z(\bar{t})) \mathbb{E}_z \left[ (x_i - z_i) + (z_1 + z_2 - x_1 - x_2)\delta W_i(T - \bar{t} - 1) \mid x \in A_z(\bar{t}) \right] 
+ z_i + (1 - z_1 - z_2)\delta W_i(T - \bar{t} - 1)
\]

(10)

where the second equality follows since, by the induction hypothesis, (1) holds for \( t = \bar{t} + 1 \), and the last inequality follows since \( \text{prob}(x \notin A_z(\bar{t})) = 1 - \text{prob}(x \in A_z(\bar{t})) \).

Consider next a game with deadline \( T - \bar{t} \). Let \( \tilde{A} \) be the set of alternatives that both players accept at the first period of the game:

\[
\tilde{A} = \left\{ x \in X : (1 - \delta)x_i + \delta V_i(x, 1; T - \bar{t}) \geq \delta V_i((0, 0), 1; T - \bar{t}) \text{ for } i = 1, 2 \right\}
\]

\[
= \left\{ x \in X : x_i \geq (x_1 + x_2)\delta W_i(T - \bar{t} - 1) \text{ for } i = 1, 2 \right\},
\]

where the second line follows since, by the induction hypothesis, for all \( V_i(x, 1; T - \bar{t}) = x_i + (1 - x_i - x_j)W_i(T - \bar{t}) \) for all \( x \). Player \( i \)'s payoff in this game is equal to

\[
W_i(T - \bar{t}) = \text{prob}(x \in \tilde{A}) \mathbb{E} \left[ (1 - \delta)x_i + \delta V_i(x, 1; T - \bar{t}) \mid x \in \tilde{A} \right] + \text{prob}(x \notin \tilde{A}) \delta V_i((0, 0), 1; T - \bar{t})
\]

\[
= \text{prob}(x \in \tilde{A}) \mathbb{E} \left[ x_i - (x_1 + x_2)\delta W_i(T - \bar{t} - 1) \mid x \in \tilde{A} \right] + \delta W_i(T - \bar{t} - 1)
\]

(11)

Assumption 1 implies that

\[
\text{prob}(x \in A_z(\bar{t})) \mathbb{E}_z \left[ x_i - z_i + (z_1 + z_2 - x_1 - x_2)\delta W_i(T - \bar{t} - 1) \mid x \in A_z(\bar{t}) \right] 
= (1 - z_1 - z_2)\text{prob}(x \in \tilde{A}) \mathbb{E} \left[ x_i - (x_1 + x_2)\delta W_i(T - \bar{t} - 1) \mid x \in \tilde{A} \right].
\]
Combining this with (10) and (11),

\[ V_i(z, \tilde{t}; T) = z_i + (1 - z_1 - z_2)W_i(T - \tilde{t}). \]

which establishes the result. ■

**Proof of Proposition 2.** (i) The proof is by induction. Consider the game with deadline \( T = 0 \). Since it is optimal for both players to accept any alternative \( x \in X \) that is drawn, player \( i \)'s payoff in this game satisfies \( W_i(T) = \mathbb{E}[x_i] = \Phi_i((0, 0)) \).

Suppose next that \( W_i(\tau) = \Phi^{\tau+1}_i((0, 0)) \) for all \( \tau = 0, ..., T - 1 \), and consider game with deadline \( T \). The set of alternatives that both players accept in the initial period are given by

\[
\tilde{A} = \{ x \in X : (1 - \delta)x_i + \delta V_i(x, 1; T) \geq \delta V_i((0, 0), 1; T) \text{ for } i = 1, 2 \}
\]

\[ = \{ x \in X : x_i \geq (x_1 + x_2)\delta W_i(T - 1) \text{ for } i = 1, 2 \}, \]

where the second line follows from Lemma 1. Player \( i \)'s payoff \( W_i(T) \) satisfies

\[
W_i(T) = \text{prob}(x \in \tilde{A})\mathbb{E}\left[(1 - \delta)x_i + \delta V_i(x, 1; T) \mid x \in \tilde{A}\right] + \text{prob}(x \notin \tilde{A})\delta V_i((0, 0), 1; T)
\]

\[ = \text{prob}(x \in \tilde{A})\mathbb{E}\left[x_i - (x_1 + x_2)\delta W_i(T - 1) \mid x \in \tilde{A}\right] + \delta W_i(T - 1) \quad (12) \]

where the equality follows after using Lemma 1. By the induction hypothesis, \( W(T - 1) = \Phi(T)((0, 0)) \), and so \( \tilde{A} = A(\Phi(T)((0, 0))) \). Using this in (12), it follows that \( W_i(T) = \Phi(\Phi(T)((0, 0))) = \Phi^{T+1}(0, 0) \).

(ii) Fix a period \( t \leq T \) and an alternative \( z \in X \), and consider a subgame starting at period \( t \) with status quo \( z' = z \). At such a subgame, player \( i \) finds it optimal to accept alternatives \( x \in X(z) \) satisfying

\[
(1 - \delta)x_i + \delta V_i(x, t + 1; T) \geq (1 - \delta)z_i + \delta V_i(z, t + 1; T)
\]

or, using equation (1) in Lemma 1, alternatives that satisfy

\[
x_i - z_i \geq (x_1 + x_2 - z_1 - z_2)\delta W_i(T - t - 1). \quad (13) \]

The set of alternatives that both players accept at period \( t \) when the status quo is \( z' = z \) is therefore the set of alternatives \( x \in X(z) \) for which (13) is satisfied for both \( i = 1, 2 \).
This is precisely the set $A_z(W(T-t-1))$ defined in (3). ■

**Proof of Lemma 2.**
(i) $\Phi$ is continuous and maps $X$ onto itself, so by Brouwer’s fixed point theorem, it has a fixed point.

(ii) If $\{W(T)\}$ converges to $W$, then Proposition 2(i) implies that

$$W = \lim_{T \to \infty} \Phi^T((0,0)) = \Phi\left(\lim_{T \to \infty} \Phi^{T-1}((0,0))\right) = \Phi(W),$$

so $W$ is a fixed-point of $\Phi$. ■

For every $\delta < 1$, let $A^\delta(W)$ and $\Phi^\delta$ be, respectively, the acceptance sets and the operator defined in equation (4) when the discount factor is $\delta$. Let $W^\delta = (W_1^\delta, W_2^\delta)$ be a fixed point of $\Phi^\delta$: for $i, j = 1, 2, i \neq j$,

$$W_i^\delta = \delta W_i^\delta + \text{prob}(x \in A^\delta(W^\delta))E[x_i - (x_i + x_j)\delta W_i^\delta|x \in A^\delta(W^\delta)]$$

$$\iff W_i^\delta = \frac{\text{prob}(x \in A^\delta(W^\delta))E[x_i|x \in A(W^\delta)]}{1 - \delta + \delta \text{prob}(x \in A^\delta(W^\delta))E[x_i + x_j|x \in A^\delta(W^\delta)]}.$$\hspace{1cm} (14)

**Lemma A.1.** Fix a sequence of discount factors $\{\delta_n\} \to 1$, and let $W^{\delta_n} = (W_1^{\delta_n}, W_2^{\delta_n}) \in X$ be a sequence such that $W^{\delta_n} = \Phi^{\delta_n}(W^{\delta_n})$ for all $n$. Then, $\lim_{n \to \infty} (W_1^{\delta_n} + W_2^{\delta_n}) = 1$.

**Proof.** Towards a contradiction, suppose this is not true. Hence, there exists a sequence $\{\delta_n\} \to 1$ and a positive number $\eta > 0$ such that $W_1^{\delta_n} + W_2^{\delta_n} < 1 - \eta$ for all $n$. Note that this implies that there is a set $B$ with nonempty interior such that $B \subseteq A^{\delta_n}(W^{\delta_n})$ for all $n$ large enough. Therefore, $\text{prob}(x \in A^{\delta_n}(W^{\delta_n})) > \text{prob}(x \in B) > 0$ for all $n$ large enough. It follows that

$$\lim_{n \to \infty} W_1^{\delta_n} + W_2^{\delta_n} = \lim_{n \to \infty} \frac{\text{prob}(x \in A^{\delta_n}(W^{\delta_n}))E[x_1 + x_2|x \in A^{\delta_n}(W^{\delta_n})]}{1 - \delta_n + \delta_n \text{prob}(x \in A^{\delta_n}(W^{\delta_n}))E[x_1 + x_2|x \in A^{\delta_n}(W^{\delta_n})]} = 1,$$

a contradiction. Hence, it must be that $W_1^{\delta_n} + W_2^{\delta_n} \to 1$ as $\delta_n \to 1$. ■
Proof of Proposition 3. We start by showing that, for any \( \delta < 1 \), there exists \( V^\delta < 1 \) with \( \delta V^\delta \rightarrow 1 \) as \( \delta \rightarrow 1 \) such that, for all \( W = (W_1, W_2) \) with \( W_1 + W_2 < V^\delta \), \( \Phi_1(W) + \Phi_2(W) > W_1 + W_2 \). Note that this property implies that, for any fixed point \( W^\delta = (W_1^\delta, W_2^\delta) \) of \( \Phi^\delta \), it must be that \( V^\delta \leq W_1^\delta + W_2^\delta \).\(^{11}\)

To see why such a \( V^\delta \) exists, pick \( g \in (0, f) \) with \( g < 1 \) and note that for any \( W \in X \),

\[
\Phi_1^\delta(W) + \Phi_2^\delta(W) = \delta(W_1 + W_2) + \text{prob}(x \in A^\delta(W)) \mathbb{E}[x_1 + x_2 | x \in A^\delta(W)](1 - \delta(W_1 + W_2)) \\
\geq \delta(W_1 + W_2) + \frac{1}{3}f(1 - \delta(W_1 + W_2))^2 \\
> \delta(W_1 + W_2) + \frac{1}{3}g(1 - \delta(W_1 + W_2))^2,
\]

where the first inequality follows since \( f(x) \geq f > 0 \) for all \( x \).\(^{12}\) Equation (15) implies that \( \Phi_1^\delta(W) + \Phi_2^\delta(W) > W_1 + W_2 \) for all \( W \) when

\[
\frac{1}{3}g \left( \frac{1 - \delta(W_1 + W_2)}{1 - \delta} \right)^2 > W_1 + W_2.
\]

Let \( V^\delta \) be the smallest positive solution to \( \frac{1}{3}g \left( \frac{1 - \delta}{1 - \delta} \right)^2 = V^\delta \); i.e.,

\[
V^\delta = \frac{3(1 - \delta)}{2g^2} \left( 1 + \frac{2g\delta}{3(1 - \delta)} - \sqrt{1 + \frac{4g\delta}{3(1 - \delta)}} \right).
\]

It follows that \( \Phi_1^\delta(W) + \Phi_2^\delta(W) > W_1 + W_2 \) for all \( W \) with \( W_1 + W_2 < V^\delta \). Note that \( V^\delta < 1 \) for all \( \delta < 1 \), and that \( \delta V^\delta \rightarrow 1 \) as \( \delta \rightarrow 1 \).

We show next that there exists \( \delta < 1 \) such that, for all \( \delta > \delta \) and for all \( W = (W_1, W_2) \in X \) with \( W_1 + W_2 \geq V^\delta \), \( (\Phi^\delta)^T(W) \) converges to a fixed point of \( \Phi^\delta \) as

\[\text{prob}(x \in A^\delta(W)) \mathbb{E}[x_1 + x_2 | x \in A^\delta(W)] = \int_{x \in A^\delta(W)} (x_1 + x_2) f(x) dx \geq f \int_{x \in A^\delta(W)} (x_1 + x_2) dx = \frac{1}{3}f(1 - \delta(W_1 + W_2)).\]
\( T \to \infty \). Towards establishing this, note that for \( i, j = 1, 2, i \neq j \),

\[
\frac{\partial \Phi^\delta_i(W)}{\partial W_i} = \delta - \delta \int_{x \in A^\delta_i(W)} (x_1 + x_2) f(x) dx \in \left[ \delta - \frac{T}{3} (1 - \delta(W_1 + W_2)), \delta \right]
\]

\[
\frac{\partial \Phi^\delta_i(W)}{\partial W_j} = -\int_0^{1-\delta W_j} \frac{\delta x_i^2 f \left( x, \frac{\delta W_j x_i}{1 - \delta W_j} \right)}{1 - \delta W_j} \cdot \frac{1 - \delta(W_1 + W_2)}{(1 - \delta W_j)^3} \in \left[ -\frac{T}{3} (1 - \delta(W_1 + W_2)), 0 \right],
\]

where we used the assumption that \( f(x) \leq \overline{f} \) for all \( x \in X \). Since \( \delta V^\delta \to 1 \) as \( \delta \to 1 \), there exists \( \underline{\delta} < 1 \) such that, for all \( \delta > \underline{\delta} \) and all \( W \in X \) with \( W_1 + W_2 \geq \delta V^\delta \), \( \delta - \frac{2T}{3} (1 - \delta(W_1 + W_2)) \geq 0 \). Note that, for all \( \delta > \underline{\delta} \) and all \( W \in X \) with \( W_1 + W_2 \geq \delta V^\delta \)

\[
\frac{\partial \Phi^\delta_i(W)}{\partial W_i} + \frac{\partial \Phi^\delta_i(W)}{\partial W_j} \in (-\delta, \delta).
\]  

(16)

Fix \( \delta > \underline{\delta} \), and let \( Y^\delta := \{ W \in X : W_1 + W_2 \geq \delta V^\delta \} \). Let \( \| \cdot \| \) be the sup-norm on \( \mathbb{R}^2 \). By equation (16), for all \( W, W' \in Y^\delta \) and for \( i = 1, 2 \), \( |\Phi^\delta_i(W) - \Phi^\delta_i(W')| \leq \delta \times \|W - W'\| \). Hence, for all \( W, W' \in Y^\delta \), \( \|\Phi^\delta_i(W) - \Phi^\delta_i(W')\| \leq \delta \times \|W - W'\| \). Moreover, \( \Phi^\delta(W) \in Y^\delta \) for all \( W \in Y^\delta \).\(^\text{13}\) Note that this implies that, for all \( \delta > \underline{\delta} \) and for all \( W \in Y^\delta \), \((\Phi^\delta)^T(W)\) converges to a fixed point of \( \Phi^\delta \) as \( T \to \infty \).

Lastly, we show that the equilibrium is convergent whenever \( \delta > \underline{\delta} \). Fix \( \delta > \underline{\delta} \). There are two cases to consider: (i) \( \Phi^\delta(0) \in Y^\delta \), and (ii) \( \Phi^\delta(0) \notin Y^\delta \). In case (i), for any deadline \( T \geq 0 \), \((\Phi^\delta)^T(\Phi^\delta(0))\) converges to a fixed point of \( \Phi^\delta \) as \( T \to \infty \).

Consider next case (ii). Since \( \Phi^\delta_i(W) + \Phi^\delta_j(W) \geq W_1 + W_2 \) for all \( W = (W_1, W_2) \) with \( W_1 + W_2 < V^\delta \), there exists \( t \geq 1 \) such that \( \Phi^\delta_t((\Phi^\delta)^t(0)) + \Phi^\delta_t((\Phi^\delta)^t(0)) \geq V^\delta \). Hence, by our arguments above, \((\Phi^\delta)^{t+s}(0)\) converges to a fixed point of \( \Phi^\delta \) as \( s \to \infty \), and so the equilibrium is convergent. \( \blacksquare \)

Lemma A.2. If \( F \) is symmetric then the players have the same equilibrium payoffs for all deadlines, i.e. \( W_1(T) = W_2(T) =: W(T) \) for all \( T \geq 0 \).

\(^\text{13}\)Proof: Note that the function \( G(V) = \delta V + \frac{1}{3} g(1 - \delta V)^2 \) is increasing in \( V \) whenever \( \delta - \frac{1}{3} g(1 - \delta V) \geq 0 \). Then, since \( \delta - \frac{1}{3} g(1 - (\delta(W_1 + W_2))) \geq \delta - \frac{2T}{3} (1 - \delta(W_1 + W_2)) \geq 0 \) for all \( \delta \geq \underline{\delta} \) and all \( W \in Y^\delta \), it follows that,

\[
\Phi^\delta_i(W) + \Phi^\delta_j(W) \geq \delta(W_1 + W_2) + \frac{1}{3} g(1 - \delta(W_1 + W_2))^2
\]

\[
\geq \delta V^\delta + \frac{1}{3} g(1 - \delta V^\delta)^2 = V^\delta.
\]

for all \( \delta \geq \underline{\delta} \) and all \( W \in Y^\delta \). Hence, for all \( \delta \geq \underline{\delta} \) and all \( W \in Y^\delta \), \( \Phi^\delta(W) \in Y^\delta \).
Proof. If $F$ is symmetric, then

$$W_1(0) = \Phi_1((0,0)) = \mathbb{E}[x_1] = \mathbb{E}[x_2] = \Phi_2((0,0)) = W_2(0)$$

Now suppose that $W_1(t) = W_2(t)$ for all $t = 0, ..., T - 1$. Then, $W_1(T - 1) = W_2(T - 1)$ implies that the set $A((W_1(T - 1), W_2(T - 1))$ is symmetric, i.e. if $x = (x_1, x_2) \in A((W_1(T - 1), W_2(T - 1))$ then $(x_2, x_1) \in A((W_1(T - 1), W_2(T - 1))$. Then, we have

$$W_1(T) = \Phi_1(W(T - 1))$$

$$= \text{prob}(x \in A(W(T - 1)) \mathbb{E}[x_1 - (x_1 + x_2)W_1(T - 1)|x \in A(W(T - 1))]} + \delta W_1(T - 1)$$

$$= \text{prob}(x \in A(W(T - 1)) \mathbb{E}[x_2 - (x_1 + x_2)W_2(T - 1)|x \in A(W(T - 1))]} + \delta W_2(T - 1)$$

$$= \Phi_2(W(T - 1)) = W_2(T),$$

where the third equality follows since $W_1(T - 1) = W_2(T - 1)$ and since $F$ is symmetric.

\[\blacksquare\]

**Proof of Proposition 4.** For any $\hat{W} \in [0,1]$, define

$$H(\hat{W}) := \text{prob}(x \in A(\hat{W})) \mathbb{E}[x_1 + x_2|x \in A(\hat{W})],$$

so that $\Psi(\hat{W}) = \delta \hat{W} + H(\hat{W})(1 - \delta \hat{W})$. Note that $H'(\hat{W}) \leq 0$: indeed, $\hat{W}'' > \hat{W}'$ implies that $A(\hat{W}'') \subset A(\hat{W}')$, so for any $\hat{W}'' > \hat{W}'$,

$$\text{prob}(x \in A(\hat{W}'')) \mathbb{E}[x_1 + x_2|x \in A(\hat{W}'')] \leq \text{prob}(x \in A(\hat{W}')) \mathbb{E}[x_1 + x_2|x \in A(\hat{W}')]$$.

It then follows that $\Psi'(\hat{W}) = \delta (1 - H(\hat{W})) + H'(\hat{W})(1 - \delta \hat{W}) \leq \delta < 1$ for all $\hat{W} \in [0,1]$. When $\Psi'(\hat{W}) > -1$ for all $\hat{W} \in [0,1]$, $|\Psi'(\hat{W})| < 1$ for all $\hat{W} \in [0,1]$. This implies that $\Psi$ is a contraction, and the sequence \{$\hat{W}(T)$\} converges to its unique fixed point. Hence, the equilibrium is convergent.

\[\blacksquare\]

**A.2 Proofs for Section 3.2**

**Proof of Lemma 3.** Fix any $\tau \geq t$. Since $x_{\tau + 1} \in A_{x_{\tau}}(W)$ we have

$$x_{i_{\tau + 1}} \geq \ell_{i,x_{\tau}}(x_{i_{\tau + 1}}|W_i) = x_{i_{\tau}} + \frac{\delta W_i}{1 - \delta W_i}(x_{i_{\tau + 1}}^\tau - x_{i_{\tau}}^\tau)$$
for both $i = 1, 2$. For any $y = (y_1, y_2) \in A_{x^{\tau+1}}(W)$, add $y_i \delta W_i/(1 - \delta W_i)$ to both sides of the above inequality and rearrange to get

$$x_i^{\tau+1} + \frac{\delta W_i}{1 - \delta W_i} (y - x_i^{\tau+1}) \geq x_i^{\tau} + \frac{\delta W_i}{1 - \delta W_i} (y - x_i^{\tau+1})$$

This means that

$$\ell_{i,x^{\tau+1}}(y_{-i}|W_i) \geq \ell_{i,x^{\tau}}(y_{-i}|W_i), \quad i = 1, 2. \quad (17)$$

Thus if $y \in A_{x^{\tau+1}}(W)$ then $y_i \geq \ell_{i,x^{\tau+1}}(y_{-i}|W_{-i})$ for $i = 1, 2$, and by (17), $y_i \geq \ell_{i,x^{\tau}}(y_{-i}|W_{-i})$ for $i = 1, 2$. This means that $y \in A_{x^{\tau}}(W)$, and thus $A_{x^{\tau+1}}(W) \subseteq A_{x^{\tau}}(W)$. \blacksquare

**Proof of Proposition 5.** For each $z \in X$, define $LR_z := A_z(W) \cap \{y \in X : y_1 + y_2 = 1\}$. Since distribution $F_z$ has full support and since $LR_z \subseteq A_z(W)$, any point in $LR_z$ can arise as a long term outcome; i.e., $LR_z \subseteq \text{supp} \ G_z$.

Consider next a subgame starting at period $t$ with $z^t = z$. By Lemma 3, $x^\tau \in A_z(W)$ for all $\tau \geq t$. Since $LR_z = A_z(W) \cap \{z \in X : y_1 + y_2 = 1\}$, any point on the frontier that is not in $LR_z$ cannot arise as a long term outcome when $z^t = z$. Hence, $\text{supp} \ G_z \subseteq LR_z$.

This establishes that $\text{supp} \ G_z = LR_z$, and it follows that $G_z \neq G_{z'}$ for $z \neq z'$. Lemma 3 then implies that along a realized equilibrium path $\{x^{\tau}\}_{\tau=t}^\infty$, we have $\text{supp} \ G_{x^{\tau+1}} \subseteq \text{supp} \ G_{x^{\tau}}$. The inclusion is strict when $x^{\tau+1} \neq x^\tau$ since $LR_{x^{\tau+1}} \neq LR_{x^\tau}$ in this case. \blacksquare

**Proof of Proposition 6.** Fix a sequence $\{\delta_n\}$ with $\delta_n \rightarrow 1$. For each $n$, let $W^{\delta_n} = (W_1^{\delta_n}, W_2^{\delta_n})$ be the players’ equilibrium payoffs in the limit as $T \rightarrow \infty$ in a game with discount factor $\delta_n$. By Lemma 2, for each $n$, $W^{\delta_n}$ is a fixed point of $\Phi^{\delta_n}$. By Lemma A.1, $\{W^{\delta_n}\}$ is such that $\lim_{n \rightarrow \infty} W_1^{\delta_n} + W_2^{\delta_n} = 1$. This establishes part (iii).

Consider next part (i). By Proposition 5, for each $n$ the support of the long-run distribution $G^{\delta_n}$ is

$$A(W) \cap \{y \in X : y_1 + y_2 = 1\} = \{x \in X : x_1 + x_2 = 1 \text{ and } x_1 \in [\delta W_1^{\delta_n}, 1 - \delta W_2^{\delta_n}]\}.$$ 

By part (iii), $\delta_n(W_1^{\delta_n} + W_2^{\delta_n})$ converges to 1 as $n \rightarrow \infty$. Hence, $[\delta_n W_1^{\delta_n}, 1 - \delta_n W_2^{\delta_n}]$ converges to a point $W_1^*$, and so $G^{\delta_n}$ converges to a dirac measure on $(W_1^*, W_2^*)$. 

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Finally, recall that
\[
A^{\delta_n}(W^{\delta_n}) = \bigg\{ x \in X : x_i \geq \frac{\delta_n W^{\delta_n}_i}{1 - \delta_n W^{\delta_n}_i} x_{-i}, \text{ for } i = 1, 2 \bigg\}.
\]

Using part (iii), \( A^{\delta_n}(W^{\delta_n}) \) converges to \( \{ x \in X : x_1/x_2 = W^*_1/W^*_2 \} \). ■

**Proof of Proposition 7.** First we prove that if \( F \) is symmetric then the fixed point of \( \Psi \) is unique. Operator \( \Psi \) is continuous and maps \([0, 1]\) onto itself, so by Brouwer’s fixed point theorem, it has a fixed point.

Let \( \hat{W} \) be a fixed point of \( \Psi \). Then, \( \hat{W} \) satisfies
\[
\hat{W} = \frac{\text{prob}(x \in A(\hat{W}))E[x_1 + x_2|x \in A(\hat{W})]}{1 - \delta + \delta \text{prob}(x \in A(\hat{W}))E[x_1 + x_2|x \in A(\hat{W})]}.
\]

(18)

Note that \( A(\hat{W}'') \subset A(\hat{W}') \) for any \( \hat{W}'' > \hat{W}' \). Therefore, for any \( \hat{W}'' > \hat{W}' \),
\[
\text{prob}(x \in A(\hat{W}''))E[x_1 + x_2|x \in A(\hat{W}'')] \leq \text{prob}(x \in A(\hat{W}'))E[x_1 + x_2|x \in A(\hat{W}')].
\]

Thus, the right side of (18) is decreasing in \( \hat{W} \), and so \( \Psi \) has a unique fixed point.

Next, the sum of the players’ equilibrium payoff in a game with deadline \( T \) is \( \hat{W}(T) = \Psi^{T+1}(0) \). By standard results in dynamical systems (e.g., Theorem 4.2 in De la Fuente (2000)), under conditions (i) and (ii) in the statement of the proposition the sequence \{\( \hat{W}(T) \)\} does not converge. So the equilibrium must be cycling. ■

**B Strategic search**

In this appendix we study the extension described in Section 4.1. We start by noting that this game has an essentially unique SPE – this can be established using the same arguments as in the proof of Proposition 1.

Fix a deadline \( T \) and a SPE \( \sigma^* \). For every time \( t \leq T \) and any \( z \in X \), let \( V_i(z, t; T) \) by player \( i \)'s SPE continuation payoff at period \( t \) in a game with deadline \( T \) when the status quo at time \( t \) is \( z \). Let \( W_i(T) \) denote player \( i \)'s equilibrium payoff at the start of the game.
We make the following assumptions on the sets of distributions $F_x$. First, for all $x, y \in X$, $\text{card}(F_x) = \text{card}(F_y)$; i.e., all the sets $F_x$ have the same cardinality. Second, for all $x \in X$ and all $F_x \in F_x$ with density $f_x$, there exists $F \in \mathcal{F} = \mathcal{F}_{(0,0)}$ with density $f$ such that $f_x(y) = \frac{1}{(1 - z_1 - z_2)^2} f(F_x(y))$ for all $y \in X(x)$. We further assume that there exists $\bar{f} > f > 0$ such that $f(x) \in [\bar{f}, f]$ for all $x \in X$. Note that these assumptions are a generalization of Assumption 1 to the current environment.

The following result generalizes Lemma 1 to the current environment. The proof is identical to the proof of Lemma 1, and hence omitted.

**Lemma B.1.** For all $t \leq T$ and all $z' = z = (z_1, z_2) \in X$,

$$V_i(z, t; T) = z_i + (1 - z_1 - z_2)W_i(T - t). \tag{19}$$

Lemma B.1 can be used to obtain a recursive characterization of equilibrium payoffs. Consider a period $t \leq T$ at which the status quo is $z = (z_1, z_2) \in X$. As in our baseline model, player $i$ approves a alternative $x = (x_1, x_2) \in X(z)$ only if

$$(1 - \delta)x_i + \delta V_i(x, t + 1; T) \geq (1 - \delta)z_i + \delta V_i(z, t + 1; T)$$

$$x_i + (1 - x_1 - x_2)\delta W_i(T - t - 1) \geq z_i - (1 - x_1 - x_2)\delta W_i(T - t - 1),$$

where we used Lemma B.1. Let $W_i = W_i(T - t - 1)$. Then, at period $t$ player $i$ accepts alternative $x$ when the status quo is $z$ only if $x_i \in A_{i, z}(W_i) = \{x \in X(z) : x_i \geq \ell_i, x_{-i}(x_{-i}|W_i)\}$, where $\ell_{i, x}(x_{-i}|W_i)$ is defined as in the main text. For any pair of payoffs $W = (W_1, W_2)$ and for any $z \in X$, the set $A_z(W)$ defined in the main text is the set of alternatives that are accepted by both players at period $t < T$ when the status quo is $z$ and $(W_1(T - t - 1), W_2(T - t - 1)) = (W_1, W_2)$.

Consider a game with deadline $T$. Suppose player $i = 1, 2$ is recognized to choose the distribution from which the alternative will be drawn at the initial period. If player $i$ chooses distribution $F \in \mathcal{F}$, she obtains payoffs equal to

$$\text{prob}_F(x \in A(W(T - 1))) \text{E}_F[x_i - (x_1 + x_2)\delta W_i(T - 1)|x \in A(W(T - 1))] + \delta W_i(T - 1).$$

For any $W \in X$ and for $i = 1, 2$, let

$$F^*_{W, i} \in \arg\max_{F \in \mathcal{F}} \text{prob}_F(x \in A(W)) \text{E}_F[x_i - (x_1 + x_2)W_i|x \in A(W)],$$
and let $F^*_W := \frac{1}{2}F^*_{W,1} + \frac{1}{2}F^*_{W,2}$. Note that, when $W(T - 1) = W$, the initial period alternative is drawn from distribution $F^*_W$.

Define the operator $\hat{\Phi} : X \rightarrow X$ as follows: for $i = 1, 2$ and for all $W \in X$,

$$\hat{\Phi}_i(W) = \text{prob}_{F^*_W}(x \in A(W)) E_{F^*_W}[x_i - (x_1 + x_2)\delta W_i | x \in A(W)] + \delta W_i.$$ 

For any integer $t$, let $\hat{\Phi}_t$ denote the $t$-th iteration of operator $\hat{\Phi}$.

For any integer $T$, let $W(T)$ denote the players’ SPE payoffs in a game with deadline $T$. The following result extends Proposition 2 to the current environment – the proof uses the same arguments as the proof of Proposition 2, and hence we omit it.

**Proposition B.1.** For any deadline $T$,

(i) the players’ equilibrium values satisfy $W(T) = \hat{\Phi}^{T+1}((0,0))$, and

(ii) the set of alternatives that are accepted by both players in any period $t \leq T$ is $A_{z^t}(W(T-t-1))$ where $z^t$ is the status quo in period $t$ and $W(T-t-1)$ are the players’ equilibrium payoffs in a game with deadline $T-t-1$.

This characterization of equilibrium payoffs can be used to generalize the main results in the main text to the current environment. First, the equilibrium features inefficient delays. Second, when the equilibrium is convergent, the acceptance regions are nested, and the distribution over long-run outcomes that the equilibrium induces at a subgame starting with status quo payoff $z$ has support equal to $\{y \in X : y_1 + y_2 = 1\} \cap A_z(W)$. Therefore, the equilibrium also displays path-dependence. It can be shown that Proposition 6 continues to hold in this setting, so the equilibrium outcome also becomes deterministic in the limit as $\delta \rightarrow 1$. Finally, the game with strategic proposals can also give rise to cycling equilibria.

**References**


14The proofs of all of these results are available upon request.


