

# A Theory of Political Gridlock

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## Abstract

This paper studies how electoral incentives influence the outcomes of political negotiations. It considers a game between two political parties that have to bargain over which policy to implement. While bargaining, the parties' popularity varies over time. Changes in popularity are partly exogenous and partly driven by the parties' actions. There is an election scheduled at a future date and the party with more popularity at the election date wins the vote. Electoral incentives can have substantial effects on bargaining outcomes. Periods of gridlock may arise when the election is close and parties have similar levels of popularity.

KEYWORDS: bargaining, elections, political gridlock, inefficient delay.

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# 1 Introduction

An important element of political negotiations –especially negotiations over high-profile or landmark legislation– is that agreements can have consequences that go well beyond implemented policies. As Binder and Lee (2013) put it: “Negotiation in Congress is never solely about policy; politics and policy are always intertwined.” Beyond policy considerations, political parties usually weigh whether compromising on a given issue is in their best electoral interest. Reaching an agreement can lead to changes in the parties’ level of support among voters, and these changes can be of crucial importance in determining electoral outcomes. The goal of the current paper is to study how such electoral incentives affect the outcomes of political negotiations. The results shed light on the circumstances under which electoral considerations can lead to periods of gridlock and political inaction.

I study a complete information bargaining game between two political parties that have to jointly decide which policy to implement. I consider a situation of divided government, in which neither party has enough institutional power to implement policies unilaterally. As a result, implementing a policy requires both parties to negotiate and reach an agreement.

A central element of the model is to recognize that, while bargaining, the parties’ electoral support is likely to experience changes over time. Changes in the parties’ popularity may occur due to exogenous reasons, like short-run fluctuations in the voters’ moods or preferences. In addition, the parties’ electoral support will typically also be affected by the actions that parties take; in particular, by the agreements that they reach and the policies that they implement. The model allows for both exogenous and endogenous changes in electoral support. The parties’ popularity evolves over time as an exogenous stochastic process. When parties come to an agreement, the policy that they agree to implement affects their popularity.

There is an election scheduled for some future date and the party that has more popularity at the election date wins the vote. The party that wins the election obtains a non-transferable private benefit. As a result, any agreement that parties reach has two effects on their payoffs: a direct effect, since parties have preferences over policies, and an indirect effect, since the policy that parties agree to implement affects their popularity and their electoral chances. The model is flexible, allowing implemented policies to affect the popularity of the parties in general ways. This flexibility allows me to study the dynamics of bargaining under different assumptions regarding how policies affect electoral support.

The proximity of an election can have substantial effects on bargaining outcomes. I show that the model’s unique equilibrium may involve periods of gridlock. These delays occur in spite of the fact that implementing a policy immediately is always the efficient outcome.

These periods of political inaction can only arise when the time left until the election is short enough. Intuitively, parties cannot uncouple the direct effect of a policy from its indirect effect on the election's outcome. When the election is close, this may reduce the scope of trade to the point that there is no policy that both parties are willing to accept.

The equilibrium dynamics depend on how implemented policies affect the parties' electoral support. I derive general conditions for gridlock and inefficiencies to arise. I show that electoral considerations can only lead to gridlock when the policies that are good for a party are politically costly; i.e., when good policies are bad politics. In contrast, parties are always able to compromise whenever implementing their preferred alternative weakly improves their electoral chances.

I use this general model to analyze the dynamics of bargaining under different assumptions regarding how policies affect popularity. The first setting I consider is one in which the majority party in Congress sacrifices popularity when it implements a policy that is close to her ideal point. This trade-off between ideal policies and popularity arises when voters punish the majority party if Congress implements extreme policies; i.e., policies that are far away from the median voter's ideal point. I show that gridlock will arise in this setting if the benefit parties derive from winning the election is large. Moreover, gridlock is more likely when the majority party in Congress has a small electoral advantage.

I also study a setting in which the party that obtains a better deal out of the negotiation is able to increase her popularity. This link between agreements and popularity arises when parties bargain over how to distribute discretionary spending and can use the resources they get from the negotiation to broaden their level of support among the electorate. Parties are always able to reach an agreement in this environment, but electoral incentives influence the policies that parties implement. In particular, electoral incentives lead to more egalitarian agreements relative to a setting in which implemented policies don't have electoral consequences.<sup>1</sup>

The results in this paper highlight the importance of electoral considerations in understanding the dynamics of political bargaining and provide new insights as to when gridlock is likely to arise. First, the model predicts that legislative productivity will tend to be higher immediately after an election, and will decrease as the next election approaches. This is consistent with the *honeymoon* effect, the empirical finding that presidents in the United States enjoy higher levels of legislative success during their first months in office; e.g., Dominguez

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<sup>1</sup>I also analyze a setting in which it is always politically costly for the minority party in Congress to concede to the proposals made by her opponent. I show that there will also be gridlock in this setting if parties attach a sufficiently high value to winning the election.

(2005). This result is also consistent with Mayhew (1991), who finds that the US Congress approves significantly fewer important laws in the two years prior to presidential elections compared to the two years after. Second, the model also predicts that elections will have a larger negative impact on legislative productivity in years in which the election's outcome is expected to be close. Third, my results show that the type of issue over which parties are bargaining might be an important determinant of whether the proximity of an election will lead to gridlock or not. Finally, the results in the paper suggest that elections might have different effects on bargaining dynamics under different voting rules; in particular, the results in the paper suggest that parties might find it easier to reach compromises under a proportional system than under a "first-past-the-post" system.

A technical difficulty with analyzing the current model is that it has two payoff-relevant state variables: the parties' level of relative popularity and the time left until the election. With two state variables it is difficult to obtain a tractable characterization of the equilibrium. I sidestep this difficulty by providing upper and lower bounds to the parties' equilibrium payoffs. These bounds on payoffs become tight as the election gets closer, are easy to compute, and do not depend on the specific way in which policies affect the parties' popularity. Moreover, these bounds on payoffs can be used to derive conditions for gridlock to arise, and to study how the likelihood of gridlock depends on the time until the election and on the parties' level of popularity.

Starting with Baron and Ferejohn (1989), there is a large body of literature that uses non-cooperative game theory to analyze political bargaining. Banks and Duggan (2000, 2006) generalize the model in Baron and Ferejohn (1989) by allowing legislators to bargain over a multidimensional policy space. A series of papers build on these workhorse models to study the effect of different institutional arrangements on legislative outcomes.<sup>2</sup> The current paper adds to this literature by introducing a model to study how electoral incentives affect the outcomes of political negotiations.<sup>3</sup>

This paper relates to Besley and Coate (1998) and Bai and Lagunoff (2011), who also

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<sup>2</sup>Winter (1996) and McCarty (2000) analyze models à la Baron-Ferejohn with the presence of veto players. Baron (1998) and Diermeier and Feddersen (1998) study legislatures with vote of confidence procedures. Diermeier and Myerson (1999), Ansolabehere et al. (2003) and Kalandrakis (2004a) study legislative bargaining under bicameralism. Snyder et al. (2005) analyze the effects of weighted voting within the Baron-Ferejohn framework. Cardona and Ponsati (2011) analyze the effects of supermajority rules in the model of Banks and Duggan. Diermeier and Vlaicu (2011) construct a legislative bargaining model to study the differences between parliamentarism and presidentialism in terms of their legislative success rate.

<sup>3</sup>There is also a growing literature that studies dynamic political bargaining models with an endogenous status-quo. Papers in this literature include Kalandrakis (2004b), Diermeier and Fong (2011), Duggan and Kalandrakis (2012), Dziuda and Loeper (2013), Nunnari (2012) and Bowen et al. (2013).

study models in which current policies affect future political power and electoral outcomes.<sup>4</sup> Besley and Coate (1998) and Bai and Lagunoff (2011) consider models in which the party in power chooses policies unilaterally. In contrast, the current paper considers a setting in which parties have to bargain to implement policies. This allows me to study how the link between current policies and electoral outcomes affects bargaining dynamics.

This paper also relates to Simsek and Yildiz (2014), who study a bilateral bargaining game in which the players' bargaining power evolves stochastically over time. Simsek and Yildiz (2014) study settings in which players have optimistic beliefs about their future bargaining power. They show that optimism can give rise to costly delays if players expect bargaining power to become more durable in the future. As a special application of this general insight, Simsek and Yildiz (2014) consider political negotiations in the proximity of elections. Since changes in the parties' bargaining power are likely to become durable after an election, optimism about future electoral outcomes can lead to periods of political inaction.

More broadly, this paper relates to the literature on delays and inefficiencies in bargaining.<sup>5</sup> In particular, this paper relates to the literature on conflict and bargaining failures as a result of commitment problems; e.g., Fearon (1996, 2004), Acemoglu and Robinson (2000, 2001), Powell (2004, 2006) and Schwarz and Sonin (2008). In these models, the players' inability to commit to future offers puts a limit on how much they can transfer. Inefficiencies arise when the transfers that the proposer can commit to are below what the responder is willing to accept. The inefficiencies in the current paper are also driven by a limited transferability of utility between parties. Indeed, since the benefit of winning the election is non-transferable, the only way in which parties can transfer utility among them is by choosing which policy to implement. When policies have electoral consequences, the transfers that parties can achieve might not be enough to compensate for the electoral costs of compromising, making delay inevitable.

The rest of the paper is organized as follows. Section 2 introduces the framework, establishes existence and uniqueness of equilibrium payoffs and derives some general properties of the model. Section 3 studies how the proximity of an election affects bargaining dynamics. Section 4 discusses implications of the model and presents several extensions. All proofs are collected in the Appendix.

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<sup>4</sup>Other papers featuring a link between current policies and electoral outcomes are Milesi-Ferretti and Spolaore (1994), Bourguignon and Verdier (2000) and Hassler et al. (2003).

<sup>5</sup>For bargaining models featuring delays, see Kennan and Wilson (1993), Merlo and Wilson (1995), Abreu and Gul (2000), Yildiz (2004), Compte and Jehiel (2004), Ali (2006), Acharya and Ortner (2013), Ortner (2013) or Fanning (2014).

## 2 Model

### 2.1 Framework

**Parties, policies and preferences.** Let  $[0, 1]$  be the set of alternatives or policies. Two political parties,  $i = L, R$ , bargain over which policy in  $[0, 1]$  to implement. The set of times is a continuum  $T = [0, \infty)$ , but parties can only make offers at points on the grid  $T(\Delta) = \{0, \Delta, 2\Delta, \dots\}$ . The constant  $\Delta > 0$  measures the real time between bargaining rounds. The bargaining protocol, to be described in more detail below, is a random proposer protocol: at each time  $t \in T(\Delta)$  one party is randomly selected to make an offer.

Both parties are expected utility maximizers and have a common discount factor  $e^{-r\Delta}$  across periods, where  $r > 0$  is the discount rate. Let  $z_i \in [0, 1]$  denote party  $i$ 's ideal policy. Party  $i$ 's utility from implementing policy  $z \in [0, 1]$  is  $u_i(z) = 1 - |z - z_i|$ . Throughout the paper I maintain the assumption that the parties' ideal points are at the extremes of the policy space, with  $z_R = 1$  and  $z_L = 0$ .<sup>6</sup> This implies that  $u_R(z) = z$  and  $u_L(z) = 1 - z$  for all  $z \in [0, 1]$ , so this model is equivalent to a setting in which parties bargain over how to divide a unit surplus.

Unlike models of legislative bargaining à la Baron and Ferejohn (1989) and Banks and Duggan (2000, 2006), I assume that bargaining takes place between parties, not individual legislators. This assumption reflects situations in which party leaders bargain over an issue on behalf of their respective parties. The need for parties to negotiate arises when neither party has enough institutional power to implement policies unilaterally. For instance, in the United States parties have to negotiate to implement policies when the two chambers of Congress are controlled by different parties, or when neither party has a filibuster-proof majority in the Senate. The need for parties to negotiate also arises if the president (who has veto power) is from a different party than the majority party in Congress. In sum, the model in this paper is best suited to study instances of divided government.

**Parties' popularity.** A key variable of the model is the publicly observable stochastic process  $x_t$ , which measures the parties' relative popularity. Let  $w = \{w_t, \mathcal{F}_t : 0 \leq t < \infty\}$  be a one-dimensional Brownian motion on the probability space  $(\Omega, \mathcal{F}, \mathbb{P})$ . From  $t = 0$  until the time at which parties reach an agreement,  $x_t$  evolves as a Brownian motion with constant

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<sup>6</sup>The assumption that the parties' ideal policies are at the extremes of the policy space is without loss of generality. If the policy space was  $[a, b]$  with  $a < z_R$  and  $b > z_L$ , all the alternatives in  $[a, z_L) \cup (z_R, b]$  would be strictly Pareto dominated by policies in  $[z_L, z_R]$ . It is possible to show that adding these Pareto dominated policies would not change the equilibrium outcome.

drift  $\mu$  and constant volatility  $\sigma > 0$ , with reflecting boundaries at 0 and 1. That is, from time  $t = 0$  until the agreement date

$$dx_t = \mu dt + \sigma dw_t, \tag{1}$$

if  $x_t \in (0, 1)$ . If  $x_t$  reaches either 0 or 1, it reflects back.<sup>7</sup> I use the convention that high levels of  $x$  denote situations in which party  $L$  has a high level of popularity vis-à-vis party  $R$ , while low levels of  $x$  denote situations in which party  $R$  has a high level of popularity.

The policy that parties implement affects their popularity. If at time  $t \in T(\Delta)$  parties reach an agreement to implement policy  $z \in [0, 1]$ , popularity jumps at this date by  $\xi(x_t, z)$ ; that is,  $x_{t^+} = \lim_{s \downarrow t} x_s = x_t + \xi(x_t, z)$ . Then, from time  $t^+$  onwards the process  $x_t$  continues to evolve as a Brownian motion with drift  $\mu$  and volatility  $\sigma$  and with reflecting boundaries at 0 and 1. The function  $\xi(\cdot, \cdot)$  captures in a reduced-form way the effect that policies have on popularity. I impose only two conditions on  $\xi(\cdot, \cdot)$ : (i)  $x + \xi(x, z) \in [0, 1]$  for all  $x, z \in [0, 1] \times [0, 1]$ , and (ii)  $\xi(x, \cdot)$  is continuous for all  $x \in [0, 1]$ . The first condition guarantees that the parties' relative popularity always remains bounded in  $[0, 1]$  after parties reach an agreement, while the second condition guarantees that there always exists an optimal offer for the party with proposal power.

The model allows for general ways in which policies can affect popularity: not only do different policies have a different effect on the level of popularity (i.e., for a fixed  $x$ ,  $\xi(x, z)$  may vary with  $z$ ), but also the same policy may have a different effect on popularity depending on the current level of  $x$  (i.e., for a fixed  $z$ ,  $\xi(x, z)$  may vary with  $x$ ). This general model can accommodate a variety of settings. For instance, this model can accommodate environments in which the party that obtains a better deal out of the negotiation is able to increase her popularity; this is achieved by setting  $\xi(x, z)$  to be decreasing in  $z$ . The model can also accommodate settings in which the majority party in Congress loses popularity when Congress implements a policy that lies far away from the median voter's preferred alternative; for example, if party  $L$  is the majority party in Congress and the median voter's preferred policy is  $1/2$ , then this is achieved by setting  $\xi(x, z)$  to be decreasing in  $|z - 1/2|$ .

**Election.** There is an election scheduled at a future date  $t^* > 0$ , with  $t^* \in T(\Delta)$ . The outcome of this election depends on the parties' popularity at the election date. I assume a "first-past-the-post" electoral rule under which the party with more popularity at date  $t^*$  wins the election: party  $L$  wins if  $x_{t^*} \geq 1/2$  and party  $R$  wins if  $x_{t^*} < 1/2$ . The party that

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<sup>7</sup>See Harrison (1985) for a detailed description of diffusion processes with reflecting boundaries.

wins the election earns a payoff  $B > 0$ , which measures the benefit that parties derive from being in office.<sup>8</sup>

A crucial assumption of the model is that the benefit  $B$  cannot be contracted upon prior to the election.<sup>9</sup> There are two justifications for this assumption. First, some of the benefits from being in office are non-transferable (for instance, power or influence). Second, while there are other benefits from winning elections that are transferable (like resources or prestigious positions in committees), it might be impossible for parties to commit before the election to execute agreements on how to divide them.

**Bargaining protocol.** The bargaining protocol is random proposer, with the party making offers selected independently across periods. More formally, at any time  $t \in T(\Delta), t < t^*$ , party  $L$  has proposal power with probability  $p_L \in (0, 1)$  and party  $R$  has proposal power with probability  $p_R = 1 - p_L$ . The bargaining protocol after the election depends on the election's outcome: at any time  $t \geq t^*$  party  $L$  has proposal power with probability  $\hat{p}_L(x_{t^*}) \in (0, 1)$  and party  $R$  has proposal power with probability  $\hat{p}_R(x_{t^*}) = 1 - \hat{p}_L(x_{t^*})$ , where  $\hat{p}_L(\cdot)$  is weakly increasing. Note that the bargaining protocol at times  $t \geq t^*$  is allowed to depend on the outcome of the election. This assumption captures the idea that elections can lead to changes in the parties' bargaining position.

At each round  $t \in T(\Delta)$  the party with proposal power can either make an offer  $z \in [0, 1]$  to her opponent or pass. If the responder rejects the offer or if the party with proposal power passes, play moves to period  $t + \Delta$ . Party  $i$  obtains a payoff  $u_i(z)$  if the responder accepts her opponent's proposal to implement policy  $z \in [0, 1]$ . The game ends immediately after the election if parties reach an agreement before time  $t^*$ . Otherwise, if parties have not reached an agreement by time  $t^*$ , they continue bargaining according to the bargaining protocol  $\hat{p}_i(x_{t^*})$  until they reach an agreement.

**Solution concept.** Let  $\Gamma_\Delta$  denote the bargaining game with time period  $\Delta$ . I look for the subgame perfect equilibria (SPE) of  $\Gamma_\Delta$ . To guarantee uniqueness of equilibrium payoffs I focus on SPE in which the responder always accepts offers that leave her indifferent between accepting and rejecting, and in which the party with proposal power always makes an acceptable offer to her opponent whenever she is indifferent between making the acceptable

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<sup>8</sup>For simplicity, I focus on the case in which there is a single election at time  $t^*$ . Section 4.2 discusses how the results generalize to settings with multiple elections over time.

<sup>9</sup>If parties could contract before  $t^*$  on how to divide the benefits from the election, then the model would reduce to a bilateral bargaining game in which the surplus to be divided is the current policy plus the discounted value of  $B$ .



offer that maximizes her payoff or passing. The first condition rules out multiplicities arising in knife-edge cases in which all acceptable offers by the responder leave this party indifferent between accepting or rejecting, while the second condition rules out multiplicities arising in knife-edge cases in which the proposer is indifferent between making the acceptable offer that is best for her or passing. From now on I use the word equilibrium to refer to an SPE that satisfies these properties.<sup>10</sup>

**Discussion of modeling assumptions.** There are two assumptions that merit further discussion. First, I model the parties' popularity as evolving over time as a reflecting Brownian motion with drift. This type of stochastic process naturally captures the frequent fluctuations that electoral support usually exhibits. Moreover, this type of stochastic process has the property that changes in popularity have a large effect on the parties' electoral chances when the election is expected to be close, and a more muted effect when one party has a significant electoral advantage. As it will become clear below, this property of the popularity process  $x_t$  has implications regarding when the proximity of an election leads to gridlock.<sup>11</sup>

Second, I assume that the parties' popularity and electoral chances are affected by the policies that are implemented prior to the election. This assumption implies that (at least a fraction of) voters cast their vote retrospectively. There is considerable evidence supporting the assumption of retrospective voters. For instance, Canes-Wrone et al. (2002), Jones and McDermott (2004) and Jones (2010) find evidence that voters in US hold their representatives accountable for their past job performance in Congress; see also Healy and Malhotra (2013) for a recent overview of the empirical literature on retrospective voting.

## 2.2 Equilibrium

**Proposition 1**  $\Gamma_\Delta$  has unique equilibrium payoffs.

*In any SPE of  $\Gamma_\Delta$ , parties reach an agreement at any time  $t \in T(\Delta), t \geq t^*$  if they have not reached an agreement by this date. The expected payoff that party  $i \in \{L, R\}$  gets from an agreement at time  $t \geq t^*$  is  $\hat{p}_i(x_{t^*})$ .*

Proposition 1 is silent about whether parties will reach an agreement before the election or whether there will be delay. The next result shows that, if there is delay at times  $t < t^*$ , this delay will only occur when the time left until the election is short enough.

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<sup>10</sup>The restriction to SPE that satisfy these conditions is only to guarantee uniqueness of payoffs. Indeed, the results in Section 3 continue to hold if we consider the entire class of SPE.

<sup>11</sup>I stress, however, that many of the results in the paper don't rely on this particular assumption – see Section 4.2.

**Proposition 2 (uniform bound on delay)** *There exists  $s > 0$  such that, for any  $\xi(\cdot, \cdot)$ , parties always reach an agreement at times  $t \in T(\Delta)$  with  $t^* - t > s$ . The cutoff  $s$  is strictly increasing in  $B$ .*

The intuition behind Proposition 2 is as follows. The discounted benefit  $e^{-r(t^*-t)}B$  of winning the election is small when the election is far away and/or when the benefit of winning the election is small. This limits the effect that implementing a policy has on the parties' payoffs from the election, making it easier for them to reach a compromise. The cutoff  $s > 0$  in Proposition 2 is increasing in  $B$ , so gridlock may arise when the election is further away if parties attach a higher value to being in office.

## 2.3 Bounds on payoffs

In this subsection, I derive bounds to the parties's equilibrium payoffs. For any function  $f : [0, 1] \rightarrow \mathbb{R}$  and any  $s > t \geq 0$ , let  $\mathbb{E}_{NA}[f(x_s)|x_t = x]$  denote the expectation of  $f(x_s)$  conditional on  $x_t = x$  assuming that parties don't reach an agreement between times  $t$  and  $s$ ; i.e., assuming that between  $t$  and  $s$  relative popularity evolves as a Brownian motion with drift  $\mu$  and volatility  $\sigma$  and with reflecting boundaries at 0 and 1.

Let  $M_L := [1/2, 1]$  and  $M_R := [0, 1/2)$ , so that party  $i = L, R$  wins the election if  $x_{t^*} \in M_i$ . For all  $(x, t) \in [0, 1] \times [0, t^*)$  and for  $i = L, R$ , define

$$Q_i(x, t) := \mathbb{E}_{NA}[\mathbf{1}_{\{x_{t^*} \in M_i\}} | x_t = x].$$

Term  $Q_i(x, t)$  is the probability with which at time  $t < t^*$  party  $i$  is expected to win the election when  $x_t = x$  if parties don't reach an agreement between  $t$  and  $t^*$ . If parties reach an agreement to implement policy  $z$  at  $t < t^*$ , the probability that party  $i$  wins the election is  $Q_i^z(x_t, t) := Q_i(x_t + \xi(x_t, z), t)$ . Figure 1 plots  $Q_L(\cdot, t)$  for different values of  $t < t^*$ . For future reference, it is worth noting that  $Q_i(\cdot, t)$  is steep when parties have similar levels of popularity, and it becomes flatter as  $x$  approaches 0 or 1.

For  $i = L, R$  and for any  $t < t^*$ , let

$$U_i(z, x, t) := u_i(z) + e^{-r(t^*-t)}BQ_i^z(x, t)$$

be party  $i$ 's expected payoff from implementing policy  $z \in [0, 1]$  at time  $t < t^*$  when  $x_t = x$ : if parties implement policy  $z$  at time  $t < t^*$ , party  $i$  gets  $u_i(z)$  and wins the election with probability  $Q_i^z(x, t)$ .

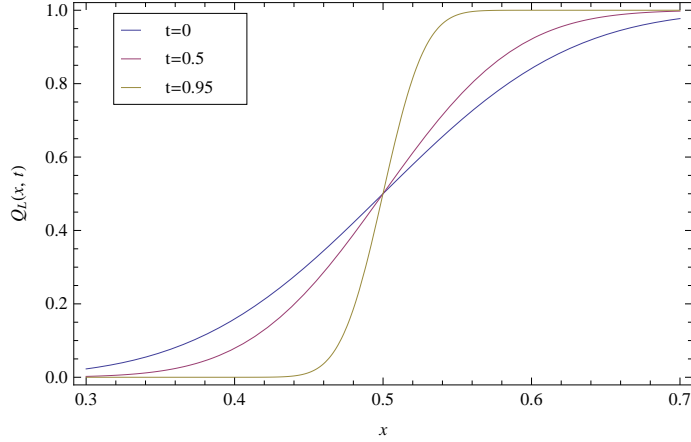


Figure 1: Probability that party  $L$  wins the election  $Q_L(x, t)$ . Parameters:  $\mu = 0$ ,  $\sigma = 0.1$ ,  $t^* = 1$ .

For all  $t < t^*$ , for all  $x \in [0, 1]$  and for  $i = L, R$ , define

$$\begin{aligned} \underline{W}_i(x, t) &:= \mathbb{E}_{NA} [e^{-r(t^*-t)} \hat{p}_i(x_{t^*}) | x_t = x] + B e^{-r(t^*-t)} Q_i(x, t) \text{ and} \\ \overline{W}_i(x, t) &:= \underline{W}_i(x, t) + 1 - e^{-r(t^*-t)}. \end{aligned}$$

Term  $\underline{W}_i(x, t)$  is the expected payoff that party  $i$  would obtain if parties delayed an agreement until the election. Note that  $\overline{W}_i(x, t) + \underline{W}_j(x, t) = 1 + B e^{-r(t^*-t)}$  for all  $t < t^*$  and  $x \in [0, 1]$ .

For all  $t \in T(\Delta)$  and all  $x \in [0, 1]$ , let  $W_i(x, t)$  denote party  $i$ 's equilibrium payoff at a subgame starting at time  $t \in T(\Delta)$  when  $x_t = x$  and parties have not reached an agreement by time  $t$  (by Proposition 1, these payoffs are unique).

**Lemma 1 (bounds on payoffs)** *For all  $t \in T(\Delta)$ ,  $t < t^*$ , for all  $x \in [0, 1]$  and for  $i = L, R$ ,  $W_i(x, t) \in [\underline{W}_i(x, t), \overline{W}_i(x, t)]$ .*

The bounds in Lemma 1 are tight as the election gets closer:  $\overline{W}_i(x, t) - \underline{W}_i(x, t) = 1 - e^{-r(t^*-t)} \rightarrow 0$  as  $t \rightarrow t^*$ . Moreover, these bounds don't depend on the way in which policies affect the parties' popularity; i.e., they don't depend on the function  $\xi(\cdot, \cdot)$ .

**Proposition 3** *Consider a subgame starting at time  $t \in T(\Delta)$ ,  $t < t^*$  at which parties have not yet reached an agreement.*

- (i) *If  $\max_{z \in [0, 1]} U_i(z, x, t) < \underline{W}_i(x, t)$  for some  $i \in \{L, R\}$ , parties delay an agreement at time  $t$  if  $x_t = x$ .*

(ii) If  $\min_{z \in [0,1]} U_i(z, x, t) \leq \underline{W}_i(x, t)$  and  $\overline{W}_i(x, t) \leq \max_{z \in [0,1]} U_i(z, x, t)$  for some  $i \in \{L, R\}$ , parties reach an agreement at time  $t$  if  $x_t = x$ .

The intuition behind Proposition 3 is as follows. The range of payoffs that party  $i$  can obtain from implementing a policy at  $t < t^*$  is  $[\min_z U_i(z, x_t, t), \max_z U_i(z, x_t, t)]$ . This range is too small for parties to reach an agreement when  $\max_z U_i(z, x_t, t) < \underline{W}_i(x, t)$  for  $i \in \{L, R\}$ , since party  $i$  would never be willing to implement a policy that gives itself a payoff lower than  $\underline{W}_i(x, t)$ . On the other hand, the range of payoffs is large when  $\min_z U_i(z, x, t) \leq \underline{W}_i(x, t)$  and  $\max_z U_i(z, x, t) \geq \overline{W}_i(x, t)$ , so in this case parties are able to find a compromise policy that they are both willing to accept.

The force that reduces the range of payoffs  $[\min_z U_i(z, x_t, t), \max_z U_i(z, x_t, t)]$  is a limited transferability of utility between parties. Since the benefit from winning the election is non-transferable, the only way in which parties can achieve transfers between them is by choosing which policy  $z \in [0, 1]$  to implement. When policies have electoral consequences, the transfers that parties can achieve by choosing a policy might not be enough to compensate the electoral costs of reaching an agreement. When this happens, gridlock must arise in equilibrium; this is the content of Proposition 3(i).

Proposition 3 can be used to study the equilibrium dynamics of this model: for each  $(x, t) \in [0, 1] \times T(\Delta)$  with  $t < t^*$ , the results in Proposition 3 can be used to check whether parties will reach an agreement or not when the state of the game is  $(x, t)$ .

**Remark 1** There is a gap between the conditions in the two parts of Proposition 3: there might exist states  $(x, t)$  at which the parties' payoffs satisfy neither the conditions in part (i) of Proposition 3 nor those in part (ii). Proposition 3 is silent about whether parties will reach an agreement at those states. For each  $t \in T(\Delta)$ ,  $t < t^*$ , let  $I(t) \subset [0, 1]$  be the set of values of  $x$  such that the parties' payoffs at  $(x, t)$  satisfy neither conditions in Proposition 3. Since the bounds on payoffs become tight as the election gets closer, the (Lebesgue) measure of  $I(t)$  converges to 0 as  $t \rightarrow t^*$ .

### 3 Bargaining and gridlock in the shadow of elections

This section studies how the proximity of an election affects the dynamics of bargaining. Section 3.1 derives necessary and sufficient conditions for gridlock to arise. Section 3.2 studies bargaining dynamics under different assumptions on how policies affect popularity.

Before presenting the general results, I introduce additional notation. Recall that, for all

$(x, t)$  with  $t < t^*$  and for all  $z \in [0, 1]$ ,

$$Q_i(x, t) = \mathbb{E}_{NA}[\mathbf{1}_{\{x_{t^*} \in M_i\}} | x_t = x],$$

$$Q_i^z(x, t) = Q_i(x + \xi(x, z), t).$$

Term  $Q_i(x, t)$  is the probability with which at time  $t$  party  $i$  is expected to win the election when  $x_t = x$  if parties don't reach an agreement until time  $t^*$ . Term  $Q_i^z(x, t)$  is the probability with which party  $i$  is expected to win if parties agree to implement policy  $z$  at time  $t$ .

For any  $z \in [0, 1]$ ,  $(x, t) \in [0, 1] \times [0, t^*)$  and  $i = L, R$ , define

$$\kappa_i(z, x, t) := Q_i^z(x, t) - Q_i(x, t)$$

to be the change in the probability that party  $i$  wins the election if policy  $z$  is implemented, and

$$D_i(x, t) := \mathbb{E}_{NA}[e^{-r(t^*-t)} \hat{p}_i(x_{t^*}) | x_t = x]$$

to be the expected payoff net of the election's outcome that party  $i$  gets if parties delay an agreement until after the election.

**Definition 1** *There is gridlock if there are states  $(x, t) \in [0, 1] \times T(\Delta)$  at which parties don't reach an agreement. There is no gridlock if parties reach an agreement at all states  $(x, t) \in [0, 1] \times T(\Delta)$ .*

### 3.1 Conditions for gridlock

This section studies conditions under which gridlock and inefficiencies will arise. The first result presents general conditions under which there is no gridlock.

**Proposition 4** *If  $x \in [0, 1]$  is such that  $\xi(x, 0) \geq 0 \geq \xi(x, 1)$ , parties reach an agreement at state  $(x, t)$  with  $t < t^*$ . If  $\xi(x, 0) \geq 0 \geq \xi(x, 1)$  for all  $x \in [0, 1]$ , there is no gridlock.*

Proposition 4 shows that parties are always able to compromise whenever implementing their preferred alternative weakly improves their electoral chances. Clearly, a special case covered by Proposition 4 is one in which the agreements that parties reach don't have electoral consequences; i.e.,  $\xi(x, z) = 0$  for all  $x, z$ .

To see the why Proposition 4 holds, suppose  $\xi(x, 0) \geq 0 \geq \xi(x, 1)$  for some  $x \in [0, 1]$  and consider a subgame beginning at time  $t^* - \Delta$  with  $x_{t^* - \Delta} = x$  at which parties have not yet

reached an agreement. Suppose party  $j \neq i$  is selected to be proposer. Note that

$$\begin{aligned} U_i(z_i, x, t^* - \Delta) &= 1 + e^{-r\Delta} B Q_i^{z=z_i}(x, t^* - \Delta) \\ &\geq 1 - e^{-r\Delta} + D_i(x, t^* - \Delta) + e^{-r\Delta} B Q_i(x, t^* - \Delta) \\ &= \overline{W}_i(x, t^* - \Delta), \end{aligned}$$

where the first equality uses  $u_i(z_i) = 1$ , and the inequality uses  $D_i(x, t^* - \Delta) \leq e^{-r\Delta}$  and  $\xi(x, 0) \geq 0 \geq \xi(x, 1)$  (so that  $Q_i^{z=z_i}(x, t^* - \Delta) \geq Q_i(x, t^* - \Delta)$  for  $i = L, R$ ). Similarly,

$$\begin{aligned} U_i(z_j, x, t^* - \Delta) &= e^{-r\Delta} B Q_i^{z=z_j}(x, t^* - \Delta) \\ &\leq D_i(x, t^* - \Delta) + e^{-r\Delta} B Q_i(x, t^* - \Delta) \\ &= \underline{W}_i(x, t^* - \Delta), \end{aligned}$$

where the first equality uses  $u_i(z_j) = 0$ , and the inequality uses  $D_i(x, t^* - \Delta) \geq 0$  and  $\xi(x, 0) \geq 0 \geq \xi(x, 1)$  (so that  $Q_i^{z=z_j}(x, t^* - \Delta) \leq Q_i(x, t^* - \Delta)$  for  $i = L, R$ ). These inequalities, together with Proposition 3(ii), imply that parties reach an agreement at time  $t^* - \Delta$ . The same argument can be applied to all times  $t < t^*$ , establishing Proposition 4.

Proposition 4 presents general conditions under which there is no gridlock. The next proposition provides a counterpart to that result: if  $\xi(\cdot, \cdot)$  does not satisfy the conditions in Proposition 4, then there are parameters of the model under which there is gridlock.

**Proposition 5** *Assume that either  $\xi(x, 0) < 0$  or  $\xi(x, 1) > 0$  for some  $x \in [0, 1]$ . Then, there exists parameters  $B > 0$  and  $\hat{p}(\cdot)$  under which there is gridlock.*

Taken together, Propositions 4 and 5 show that gridlock can only arise when a party expects to lose popularity by implementing her most preferred policy. In this sense, electoral considerations can only lead to gridlock when the policies that are good for a party are politically costly; i.e., when good policies are bad politics.

While Proposition 5 shows that gridlock is possible when  $\xi(x, 0) < 0$  or  $\xi(x, 1) > 0$ , it is silent about the conditions that  $B$  and  $\hat{p}(\cdot)$  have to satisfy for delay to arise. The next result derives sufficient conditions under which there is gridlock.

Recall that  $\kappa_i(z, x, t) = Q_i^z(x, t) - Q_i(x, t)$  is the change in the probability that party  $i$  wins the election if policy  $z$  is implemented, and that  $D_i(x, t) = \mathbb{E}_{NA}[e^{-r(t^*-t)} \hat{p}_i(x_{t^*}) | x_t = x]$  is the expected payoff net of the election's outcome that party  $i$  gets if agreement is delayed until after the election. Note that  $\xi(x, 0) < 0$  iff  $\kappa_L(z_L, x, t) < 0$  and  $\xi(x, 1) > 0$  iff  $\kappa_R(z_R, x, t) < 0$ . Finally, for  $i \in \{L, R\}$ , define  $Z_i(x, t) := \{z \in [0, 1] : \kappa_i(z, x, t) < 0\}$  to be the set of policies

that reduce party  $i$ 's popularity.

**Proposition 6** *Let  $(x, t) \in [0, 1] \times [0, t^*)$  be such that  $\kappa_i(z_i, x, t) < 0$  for some  $i \in \{L, R\}$ . Assume that*

(i)  $D_i(x, t) > u_i(z)$  for all  $z$  with  $\kappa_i(z_i, x, t) \geq 0$ ; and

(ii) for all  $z' \in Z_j(x, t)$  and all  $z \in Z_i(x, t)$ ,

$$\frac{D_i(x, t) - u_i(z')}{e^{-r(t^*-t)}\kappa_i(z', x, t)} > B > \frac{D_i(x, t) - u_i(z)}{e^{-r(t^*-t)}\kappa_i(z, x, t)}. \quad (2)$$

Then, parties delay an agreement at state  $(x, t)$ .

The conditions in Proposition 6 imply that  $\max_z U_i(z, x, t) < \underline{W}_i(x, t)$ , and so by Proposition 3(i) parties delay an agreement at state  $(x, t)$ . To see why, note that for all  $z \in [0, 1]$ ,

$$U_i(z, x, t) - \underline{W}_i(x, t) = u_i(z) - D_i(x, t) + Be^{-r(t^*-t)}\kappa_i(z, x, t).$$

Term  $u_i(z) - D_i(x, t)$  is the policy payoff difference for party  $i$  between implementing  $z$  now and delaying an agreement until after the election. Term  $Be^{-r(t^*-t)}\kappa_i(z, x, t)$  measures the electoral consequences for party  $i$  from implementing policy  $z$ . The first condition in Proposition 6 implies that  $U_i(z, x, t) < \underline{W}_i(x, t)$  for all policies  $z$  with  $\kappa_i(z, x, t) = 0$ . To understand the second condition, consider first policies  $z$  that reduce party  $i$ 's popularity (i.e., such that  $\kappa_i(z, x, t) < 0$ , so that  $z \in Z_i(x, t)$ ). When the second inequality in (2) holds, the political cost  $Be^{-r(t^*-t)}\kappa_i(z, x, t)$  that party  $i$  incurs by implementing policy  $z$  outweighs the gain  $u_i(z) - D_i(x, t)$ , and so  $\underline{W}_i(x, t) > U_i(z, x, t)$ . Similarly, consider policies  $z'$  that improve party  $i$ 's popularity (i.e., such that  $\kappa_i(z', x, t) > 0$ , so that  $z' \in Z_j(x, t)$ ). The first inequality in (2) implies that party  $i$ 's electoral gain  $Be^{-r(t^*-t)}\kappa_i(z', x, t)$  from implementing policy  $z'$  is too small relative to the loss  $u_i(z') - D_i(x, t) < 0$ , and so  $\underline{W}_i(x, t) > U_i(z', x, t)$ .

The last result of this section derives sufficient conditions for there to be agreement in settings in which Proposition 4 does not apply.

**Proposition 7** *Let  $(x, t) \in [0, 1] \times [0, t^*)$  be such that  $\kappa_i(z_i, x, t) < 0$  for some  $i \in \{L, R\}$ . Assume that*

(i)  $D_i(x, t) \geq u_i(z)$  for all  $z$  with  $\kappa_i(z, x, t) \geq 0$ ; and

(ii) there exist  $z \in Z_i(x, t)$  such that

$$B \leq \frac{D_i(x, t) + (1 - e^{-r(t^*-t)}) - u_i(z)}{e^{-r(t^*-t)}\kappa_i(z, x, t)},$$

or  $z' \in Z_j(x, t)$  such that

$$B \geq \frac{D_i(x, t) + (1 - e^{-r(t^*-t)}) - u_i(z')}{e^{-r(t^*-t)}\kappa_i(z', x, t)}.$$

Then, parties reach an agreement at state  $(x, t)$ .

Proposition 7 shows that the conditions for gridlock in Proposition 6 are almost tight, especially when the election is close: if  $B$  is slightly smaller or larger than the bounds in (2), then parties will reach an agreement. The conditions in part (ii) are the counterpart of those in Proposition 6(ii), and guarantee that  $\max_z U_i(z, x, t) \geq \overline{W}_i(x, t) = \underline{W}_i(x, t) + 1 - e^{-r(t^*-t)}$ . Indeed, when the first condition in part (ii) holds for some  $z \in Z_i(x, t)$ , the gain  $u_i(z) - D_i(x, t) > 0$  from implementing policy  $z$  for party  $i$  is strictly larger (by a margin of  $1 - e^{-r(t^*-t)}$ ) than the political cost  $e^{-r(t^*-t)}B\kappa_i(z, x, t)$ , and so  $U_i(z, x, t) \geq \overline{W}_i(x, t)$ . Similarly, when the second condition in part (ii) holds for some  $z' \in Z_j(x, t)$ , the political gain  $e^{-r(t^*-t)}B\kappa_i(z', x, t)$  from implementing policy  $z'$  is strictly larger (by a margin of  $1 - e^{-r(t^*-t)}$ ) than the cost  $u_i(z) - D_i(x, t) \leq 0$ , and so  $U_i(z', x, t) \geq \overline{W}_i(x, t)$ . The conditions in part (i), on the other hand, guarantee that  $\min_z U_i(z, x, t) \leq \underline{W}_i(x, t)$ . Together with Proposition 3(ii), these conditions imply that parties reach an agreement at state  $(x, t)$ .

**Remark 2** The condition that  $D_i(x, t) > u_i(z)$  for all  $z$  with  $\kappa_i(z, x, t) \geq 0$  in Proposition 6 is necessary to have  $\max_{z \in [0, 1]} U_i(z, x, t) < \underline{W}_i(x, t)$ . Indeed, if  $(x, t)$  is such that  $D_i(x, t) \leq u_i(\hat{z})$  for some  $\hat{z}$  with  $\kappa_i(\hat{z}, x, t) \geq 0$ , then  $U_i(\hat{z}, x, t) - \underline{W}_i(x, t) = u_i(\hat{z}) - D_i(x, t) + B e^{-r(t^*-t)}\kappa_i(\hat{z}, x, t) \geq 0$ . For such states, Proposition 3(i) cannot be used to establish that parties will delay an agreement.

## 3.2 Examples

This section studies how the proximity of elections affects the dynamics of bargaining under different assumptions on how policies affect popularity.



### 3.2.1 Electoral trade-off

I start by considering a setting in which the majority party in Congress faces the following trade-off: implementing a policy that is close to her ideal point lowers her level of popularity, while implementing a moderate policy allows her to maintain her level of support. This trade-off arises when voters punish the majority party if Congress implements extreme policies; i.e., policies that are far away from the median voter's ideal point.

Suppose that party  $L$  is the majority party in Congress. To model the trade-off described above, I assume that for all  $(x, z) \in [0, 1] \times [0, 1]$ ,

$$\xi(x, z) = -\min \{ \lambda |z - m(x)|, x \}, \quad (3)$$

where  $\lambda > 0$  measures the effect that implemented policies have on the majority party's popularity and  $m(x) \in (0, 1)$  is the location of the median voter's ideal point. Assume that  $m(\cdot)$  is continuous and decreasing in  $x$ , so that the median voter's ideal point is closer to  $z_i$  when party  $i$ 's popularity is high. This functional form for  $\xi(x, z)$  captures the trade-off mentioned above, since the majority party (in this case party  $L$ ) sacrifices popularity when she implements a policy that is close to her preferred alternative (and far from the median voter's ideal point). The next result follows from Proposition 6.

**Corollary 1** *Let  $\xi(\cdot, \cdot)$  be given by equation (3) with  $\lambda > 0$ . Assume that  $(x, t) \in [0, 1] \times [0, t^*]$  is such that*

(i)  $D_L(x, t) > 1 - m(x)$ , and

(ii) for all  $z \neq m(x)$ ,

$$B > \frac{D_L(x, t) - (1 - z)}{e^{-r(t^* - t)} \kappa_L(z, x, t)}.$$

*Then, parties delay an agreement at state  $(x, t)$ .*

Figure 2 illustrates the typical patterns of gridlock when  $\xi(x, z)$  satisfies equation (3) and the conditions in Corollary 1 hold. The squared areas in the figure are the values of  $(x, t)$  at which parties will delay an agreement. The shaded areas in Figure 2 are values of  $(x, t)$  at which parties will reach an agreement.

Figure 2 shows that parties will delay an agreement when the majority party has a small advantage, and that they will reach a compromise either when the majority party has a very strong position or when the minority party has more popularity. To see the intuition for this,

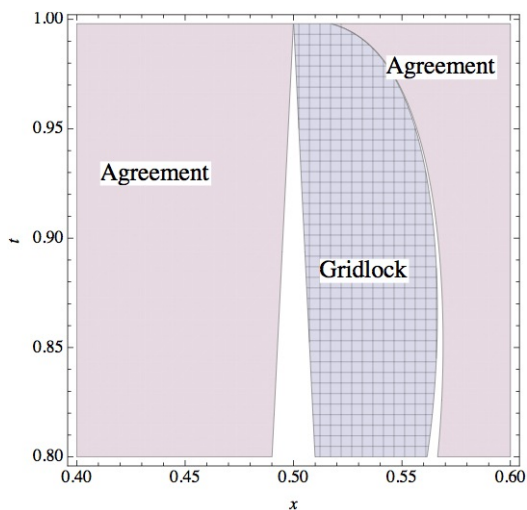


Figure 2: Agreement and gridlock regions when  $\xi$  is given by (3). Parameters:  $\mu = 0$ ,  $\sigma = 0.05$ ,  $r = 0.05$ ,  $t^* = 1$ ,  $B = 1.5$ ,  $\lambda = 0.1$ ,  $m(x) = 1/2\forall x$  and  $\hat{p}_L(x) = 1/4 + x/2$ .

consider first states at which the majority party has a small political advantage. When  $B$  is large, at such states the majority party has a lot to lose by implementing a policy close to her preferred alternative, since this would have a large negative impact on her electoral chances. Moreover, at such states the majority party doesn't want to implement a policy close to  $m(x)$  either: since she has a small political advantage, by delaying an agreement until the election date this party would likely be able to implement a policy that lies closer to her ideal point. This implies that at such states any policy  $z \in [0, 1]$  would give the majority party a lower payoff than what she could get by delaying an agreement until the election. Thus, there must be delay.

Consider next states at which the majority party has a high level of popularity. At such states, the majority party is willing to implement policies that lie relatively close to her ideal point, since she would very likely still win the election even after implementing such a policy. Moreover, at these states the minority party is also willing to implement policies that are relatively close to her opponent's ideal point, since doing this would increase (at least marginally) her chances of winning the election. Thus, at these states parties are able to find a compromise policy that they are both willing to accept.

Finally, consider states at which the minority party is leading. At such states, both parties are willing to implement policies that are only slightly favorable to the minority party. The majority party is willing to implement such a policy since it is better than what she expects to get by delaying an agreement until after the election. On the other hand, the minority

party is also willing to implement such a policy since it increases her electoral chances.

Note that, in this setting, parties are able to compromise when either party has a large political advantage. The following corollary formalizes this.

**Corollary 2** *Fix a time  $t < t^*$ , and suppose that  $\xi(\cdot, \cdot)$  is given by equation (3). Then, there exists  $\bar{\lambda} > 0$  and  $\epsilon > 0$  such that, if  $\lambda < \bar{\lambda}$ , parties reach an agreement at all states  $(x, t)$  with  $x \in [0, \epsilon] \cup [1 - \epsilon, 1]$ .*

### 3.2.2 Costly concessions

I now consider a setting in which the majority party always benefits when a policy is implemented. This specification of the model is motivated by empirical evidence showing that voters usually hold the majority party accountable for the job performance of Congress (e.g., Jones and McDermott (2004) and Jones (2010)). As journalist Ezra Klein wrote in an article for *The New Yorker*: “...it is typically not in the minority party’s interest to compromise with the majority party on big bills – elections are a zero-sum game, where the majority wins if the public thinks it has been doing a good job.”<sup>12</sup>

I model this by assuming that the majority party’s level of popularity jumps up discretely if parties reach an agreement to implement a policy. Suppose again that party  $L$  is the majority party, and that, for all  $(x, z) \in [0, 1] \times [0, 1]$ ,

$$\xi(x, z) = \min\{g, 1 - x\}, \quad (4)$$

where  $g > 0$  is a constant. Note that in this setting it is always costly for the minority party to concede to a policy put forward by her opponent: conceding to a policy lowers her popularity by  $g$  and decreases her electoral chances.<sup>13</sup>

**Corollary 3** *Let  $\xi(\cdot, \cdot)$  is given by equation (4) with  $g > 0$ . Assume that  $(x, t) \in [0, 1] \times [0, t^*)$  is such that*

$$B > \frac{D_R(x, t) - 1}{e^{-r(t^*-t)} \kappa_R(x, t, 1)}.$$

*Then, parties delay an agreement at state  $(x, t)$ .*

Figure 3 illustrates the typical patterns of gridlock when  $\xi(x, z)$  satisfies equation (4) and the conditions in Corollary 3 hold. Figure 3 shows that parties will delay an agreement

<sup>12</sup>“Unpopular Mandate. Why do politicians reverse their positions?,” *The New Yorker*, June 25, 2012.

<sup>13</sup>The results in this subsection remain qualitatively unchanged if I allow the magnitude of the jump  $g$  to depend on the implemented policy.

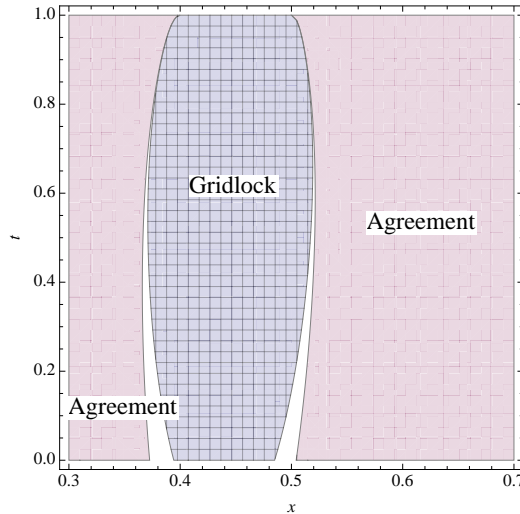


Figure 3: Agreement and gridlock regions when  $\xi$  is given by (4). Parameters:  $\mu = 0$ ,  $\sigma = 0.1$ ,  $r = 0.05$ ,  $t^* = 1$ ,  $B = 1.5$ ,  $g = 0.1$  and  $\hat{p}_L(x) = 1/4 + x/2$ .

either when they have very similar levels of popularity or when the minority party has a small advantage. Intuitively, implementing a policy has a larger negative impact on the minority party's electoral chances when popularity is either balanced or when this party only has a small advantage. When parties derive a high value from winning the election, at these states there is no policy  $z \in [0, 1]$  that compensates the minority party for her lower electoral chances, and so gridlock arises. On the other hand, the cost that the minority party incurs by accepting an offer is lower either when this party has a very large advantage in terms of popularity, or when her opponent is leading. Therefore, at such states it is easier for parties to reach a compromise.

As in Section 3.2.1, in this setting parties are also able to compromise when one of them has a large political advantage. The following corollary formalizes this.

**Corollary 4** *Fix a time  $t < t^*$ , and suppose that  $\xi(\cdot, \cdot)$  is given by equation (4). Then, there exists  $\bar{g} > 0$  and  $\varepsilon > 0$  such that, if  $g < \bar{g}$ , parties reach an agreement at all states  $(x, t)$  with  $x \in [0, \varepsilon] \cup [1 - \varepsilon, 1]$ .*

### 3.2.3 Success begets success

This subsection considers a setting in which the party that obtains a better deal out of the negotiation is able to increase her level of political support. For instance, this link between agreements and popularity arises when parties bargain over how to distribute discretionary

spending and can use the resources they get out of the negotiation to broaden their level of support among the electorate.

Suppose that  $\xi(x, \cdot)$  satisfies  $\xi(x, 0) \geq 0 \geq \xi(x, 1)$  for all  $x \in [0, 1]$ ; that is, party  $i$ 's popularity is weakly larger after the agreement if parties implement  $i$ 's preferred policy. A situation in which a better deal translates into more popularity can be modeled by further assuming that  $\xi(x, \cdot)$  is decreasing for all  $x \in [0, 1]$ . Note that, by Proposition 4, parties always reach an immediate agreement in this setting.

The next results show how the proximity of an election shapes the agreements that parties reach. Before stating the results, note that a special case of this model is one in which the agreements don't have electoral consequences; i.e.,  $\xi(x, z) = 0$  for all  $x, z$ . For all  $(x, t) \in [0, 1] \times [0, t^*)$  and for  $i = L, R$ , let  $\hat{z}_i(x, t) \in [0, 1]$  denote the agreement that parties would reach at state  $(x, t)$  when party  $i$  has proposal power and policies don't have electoral consequences.

**Lemma 2** *Suppose  $\tilde{\xi}(\cdot, \cdot)$  is such that, for all  $x \in [0, 1]$ ,  $\tilde{\xi}(x, 0) \geq 0 \geq \tilde{\xi}(x, 1)$ . For all  $t < t^*$  and all  $x \in [0, 1]$ , let  $z_i(x, t) \in [0, 1]$  denote the agreement that parties would reach at state  $(x, t)$  when party  $i$  has proposal power and  $\xi = \tilde{\xi}$ . Then*

$$z_i(x, t) - \hat{z}_i(x, t) = e^{-r(t^*-t)} B \left[ Q_L(x + \tilde{\xi}(x, z_i(x, t)), t) - Q_L(x, t) \right]. \quad (5)$$

The next result uses equation (5) to study how electoral incentives affect the policies that parties implement in settings in which a better deal translates into more popularity.

**Proposition 8** *Suppose  $\tilde{\xi}(\cdot, \cdot)$  is such that, for all  $x \in [0, 1]$ : (i)  $\tilde{\xi}(x, \cdot)$  is decreasing, with  $\tilde{\xi}(x, 0) \geq 0 \geq \tilde{\xi}(x, 1)$ , and (ii)  $\tilde{\xi}(x, 1/2) = 0$ . For all  $t < t^*$  and all  $x \in [0, 1]$ , let  $z_i(x, t) \in [0, 1]$  denote the agreement that parties reach at state  $(x, t)$  when party  $i$  has proposal power and  $\xi = \tilde{\xi}$ .*

(i) *If  $\hat{z}_i(x, t) > 1/2$ , then  $z_i(x, t) \in (1/2, \hat{z}_i(x, t)]$ .*

(ii) *If  $\hat{z}_i(x, t) < 1/2$ , then  $z_i(x, t) \in [\hat{z}_i(x, t), 1/2)$ .*

Proposition 8 shows that electoral considerations lead to more moderate policies compared to a setting in which parties don't have electoral incentives. Intuitively, in this setting the policy that parties implement must compensate the weaker party (i.e., the party that gets a worse deal) for her lower electoral chances.

## 4 Discussion

### 4.1 Implications of the model

This paper shows how electoral considerations can affect the dynamics of inter-party negotiations, leading to long periods of political gridlock. The model predicts that there may be gridlock at times prior to an election, but that parties will always reach an agreement after the election. Importantly, this result does not depend on there being only one election; see Section 4.2 below for a discussion of how this result generalizes to settings with multiple elections. An implication of this result is that we should expect to see higher levels of legislative productivity in periods immediately after elections. This is consistent with the so-called *honeymoon* effect: the empirical finding that, in the United States, presidents enjoy higher levels of legislative success during their first months in office; see, for instance, Dominguez (2005). This result is also consistent with the empirical findings in Mayhew (1991), who shows that US Congress approves significantly fewer important laws in the two years prior to presidential elections compared to the two years after.

The model in Section 3.2.1 shows that when voters punish the majority party for implementing extreme policies, gridlock is more likely to arise when the majority party has a slight advantage in terms of popularity. On the other hand, the model of Section 3.2.2 shows that when it is electorally costly for the minority party to concede, gridlock is more likely to arise either when the minority party has an advantage in terms of popularity or when both of them have similar chances of winning the vote. Note that in both models, parties are able to reach a compromise when one of them has a high level of popularity (Corollaries 2 and 4). Taken together, these models suggest that elections will have a larger negative impact on legislative productivity in years in which the election’s outcome is expected to be close. This is a novel prediction of the model, which would be interesting to investigate empirically.

The model in the paper assumes a “first-past-the-post” electoral rule. Under this system small changes in popularity can have large effects on electoral outcomes when the election is expected to be close. This is the main reason why, in the models of Sections 3.2.1 and 3.2.2, it is more difficult for parties to reach a compromise when they have similar levels of popularity. It is worth highlighting that under different voting systems elections might have a different effect on bargaining dynamics. For instance, in parliamentary systems small changes in popularity can only have a limited effect on electoral outcomes; and hence, according to this model, parties would find it easier to reach compromises.<sup>14</sup> This might be another reason

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<sup>14</sup>Admittedly, the model in the current paper with two political parties is not well-suited to analyze most

why parliamentary systems tend to have a higher rate of legislative success than presidential systems – see Diermeier and Vlaicu (2011) for evidence on the differences in legislative success rates between parliamentary and presidential systems.

The results above show that, with an upcoming election, the type of issue over which parties are bargaining might be an important determinant of whether there will be gridlock or not. In particular, Section 3.2.3 shows that parties will be able to reach an agreement quickly when bargaining over how to distribute discretionary spending. This is another implication of the model which would be interesting to investigate empirically.

Finally, Proposition 2 shows that gridlock will only occur when the election is close enough; that is, when the time left until the election is smaller than  $s$ . The cutoff  $s$  is strictly increasing in the value  $B$  that parties attach to winning the election. This result can be used to obtain an estimate on the value that parties derive from winning an election based on observable patterns of gridlock: if Congress becomes gridlocked  $t$  days before an election, we can use the results in Proposition 2 to obtain a lower bound on  $B$ .

## 4.2 Extensions

I conclude by briefly discussing a few extensions and alternative interpretations of the model.

**Multiple elections.** The model assumes that there is a single election at time  $t^*$ . This assumption implies that any subgame starting at time  $t \geq t^*$  at which parties haven't yet reached an agreement is strategically equivalent to a game without elections. Therefore, by standard arguments in bilateral bargaining games, parties will always reach an agreement immediately after the election if they haven't done so before.

The model can be extended to allow for multiple elections over time. Indeed, suppose that there is a second election scheduled for time  $t^{**} > t^*$ . Suppose further that the time between elections is large, with  $t^{**} - t^* > s$  (where  $s$  is the threshold in Proposition 2). It then follows from Proposition 2 that parties will reach an agreement immediately after the first election if they haven't done so before. Therefore, a model with multiple elections that are sufficiently far apart in time would deliver similar equilibrium dynamics than the model with a single election. In this setting, gridlock would only arise when the next election is close, and parties would always be able to reach an agreement as soon as they pass an election.<sup>15</sup>

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parliamentary systems. This discussion is meant to be suggestive of what we should expect in an appropriate extension of the model designed to study political negotiations in parliamentary systems.

<sup>15</sup>Moreover, it can be shown that if the elections are sufficiently far apart in time, the parties' payoffs after the first election will be close to their payoffs  $\hat{p}_i(\cdot)$  at times  $t \geq t^*$  in the model with one single election.

**Other stochastic processes.** Another assumption I made throughout the paper is that the parties' relative popularity evolves over time as a Brownian motion with drift  $\mu$  and volatility  $\sigma > 0$ , and with reflecting boundaries at 0 and 1. Besides naturally capturing the frequent fluctuations that electoral support exhibits, this assumption implies that parties find it harder to reach an agreement when they have similar chances of winning the vote.<sup>16</sup>

However, many of the results in the paper don't rely on this assumption. Indeed, suppose  $x_t$  is an arbitrary stochastic process defined on some state space  $\Omega$  with the property that  $x_t(\omega) \in [0, 1]$  for all  $t$  and all  $\omega \in \Omega$ . For instance,  $x_t$  could be a mean reverting process, or a finite-state Markov chain. The election's outcome depends on the realization of  $x_{t^*}$ : party  $L$  wins the election if  $x_{t^*} \geq 1/2$  and party  $R$  wins if  $x_{t^*} < 1/2$ . As in the main text, let  $Q_i(x, t)$  be the probability with which party  $i$  is expected to win the election if there is no agreement until time  $t^*$ , and let  $Q_i^z(x, t)$  be the probability with which party  $i$  wins the election if policy  $z$  is implemented. Assume that  $Q_i^z(x, t)$  is continuous in  $z$ .

Note first that, in this setting, parties will also reach an agreement at time  $t \geq t^*$  if they haven't already done so. Indeed, any subgame that starts at a time  $t \geq t^*$  at which parties have not yet reached an agreement is equivalent to a bilateral bargaining model with random proposer. Hence, by standard arguments, parties reach an agreement at any time  $t \geq t^*$  if they haven't done so already. Moreover, party  $i$ 's expected payoff from that agreement is  $\hat{p}_i(x_{t^*})$ .

The parties' payoffs at times  $t < t^*$  are again difficult to characterize. However, by the same arguments used in the proof of Lemma 1, party  $i$ 's payoff is bounded below by  $\underline{W}_i(x, t) = \mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{p}_i(x_{t^*})|x_t = x] + Be^{-r(t^*-t)}Q_i(x, t)$ , and is bounded above by  $\overline{W}_i(x, t) = 1 - e^{-r(t^*-t)} + \underline{W}_i(x, t)$ . Using these bounds on payoffs, it can be shown that Propositions 3, 4 and 5 continue to hold in this setting. Therefore, in this more general setting it is still true that gridlock only arises when a party expects to lose popularity by implementing her most preferred policy.

**Alternative interpretations.** The model admits other interpretations beyond the political bargaining one that I emphasized throughout the text. Indeed, at an abstract level, this model can be thought of as representing a non-stationary bargaining situation in which

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The proof of this result is available upon request. Therefore, in this case the parties' equilibrium payoffs at times  $t < t^*$  would be bounded by  $\underline{W}_i(x, t) - \eta$  and  $\overline{W}_i(x, t) + \eta$ , where  $\eta$  is a positive constant that depends on the time  $t^{**} - t^*$  between elections such that  $\lim_{t^{**} - t^* \rightarrow \infty} \eta = 0$ . Applying the arguments in Section 2.3, these bounds on payoffs can be used to study the equilibrium dynamics prior to the first election.

<sup>16</sup>Another advantage of this stochastic process is that it allows for easy computations of the bounds on payoffs  $\underline{W}_i(x, t)$  and  $\overline{W}_i(x, t)$ .



the agreement that bargainers reach today affects their continuation values at a future stage. There are several real-life settings that fit this description. For example, in legal disputes, the agreement that the bargaining parties reach may set a precedent that affects the resolution of future legal conflicts between them. Similarly, in labor negotiations, the wage settlement reached today may set workers' expectations and demands at future negotiations. The results in the current paper may help shed light into how bargaining will unfold in these alternative environments. As a general principle, Propositions 4 and 5 suggest that in such strategic interactions, delays and inefficiencies may arise whenever agreements that yield high current payoffs to a party have a negative effect on her continuation value.

## A Appendix

### A.1 Proofs of Section 2.2

For any function  $f : [0, 1] \rightarrow \mathbb{R}$  and any  $s > t \geq 0$ , let  $\mathbb{E}_{NA}[f(x_s)|x_t = x]$  denote the expectation of  $f(x_s)$  conditional on  $x_t = x$  assuming that parties don't reach an agreement between times  $t$  and  $s$ ; i.e., assuming that between  $t$  and  $s$  relative popularity evolves as a Brownian motion with drift  $\mu$  and volatility  $\sigma$  and with reflecting boundaries at 0 and 1.

**Proof of Proposition 1.** Note first that any subgame starting at time  $t \geq t^*$  at which parties have not yet reached an agreement is equivalent to a standard bilateral bargaining game with a random proposer. By standard arguments, in such a game players reach an immediate agreement and the expected payoff of party  $i \in \{L, R\}$  is equal to the probability with which this party makes offers.

Next, I show that the game has unique equilibrium payoffs. Fix an equilibrium, and let  $W_i(x, t)$  denote party  $i$ 's equilibrium payoffs at time  $t \in T(\Delta)$  with  $x_t = x$  if parties have not yet reached an agreement by this date. By the arguments above,  $W_i(x, t^*) = \hat{p}_i(x_{t^*}) + B \times \mathbf{1}_{\{x \in M_i\}}$  and  $W_i(x, t) = \hat{p}_i(x_{t^*})$  for all  $t > t^*$ , where  $M_L = [1/2, 1]$  and  $M_R = [0, 1/2]$ .

For  $i = L, R$  and  $t \in T(\Delta), t < t^*$ , let  $U_i(z, x, t) = u_i(z) + e^{-r(t^*-t)} B Q_i^z(x, t)$  be the payoff that party  $i$  gets by implementing policy  $z \in [0, 1]$  at time  $t$  when  $x_t = x$ . Note that  $U_i(\cdot, x, t)$  is continuous (since  $u_i(\cdot)$ ,  $\xi(x, \cdot)$  and  $Q_i(\cdot, t)$  are continuous). Suppose that parties have not reached an agreement by time  $t^* - \Delta$ . For  $i = L, R$ , party  $i$ 's payoff if there is no agreement at time  $t^* - \Delta$  is  $e^{-r\Delta} \mathbb{E}_{NA}[W_i(x_{t^*}, t^*)|x_{t^*-\Delta} = x]$ . For  $i = L, R$ , let

$$A_i(x, t^* - \Delta) := \{z \in [0, 1] : U_i(z, x, t^* - \Delta) \geq e^{-r\Delta} \mathbb{E}_{NA}[W_i(x_{t^*}, t^*)|x_{t^*-\Delta} = x]\},$$

be the set of policies that give party  $i$  a payoff weakly larger than the payoff from delaying an agreement one round. Let  $A(x, t^* - \Delta) := A_L(x, t^* - \Delta) \cap A_R(x, t^* - \Delta)$ . If  $A(x, t^* - \Delta) = \emptyset$ , there is no policy that both parties would agree to implement. In this case there must be delay at time  $t^* - \Delta$ , so party  $i$ 's payoff is  $W_i(x, t^* - \Delta) = e^{-r\Delta} \mathbb{E}_{NA}[W_i(x_{t^*}, t^*) | x_{t^* - \Delta} = x]$ .

Consider next the case in which  $A(x, t^* - \Delta) \neq \emptyset$ . If party  $j$  is proposer, she offers  $z_j(x, t^* - \Delta) \in \arg \max_{z \in A(x, t^* - \Delta)} U_j(z, x, t^* - \Delta)$ , and her opponent accepts this offer.<sup>17</sup> In this case, for  $i = L, R$ , party  $i$ 's payoff is  $U_i(z_j(x, t^* - \Delta), x, t^* - \Delta)$ . Hence, when  $A(x, t^* - \Delta) \neq \emptyset$ , in any equilibrium it must be that

$$W_i(x, t^* - \Delta) = p_i U_i(z_i(x, t^* - \Delta), x, t^* - \Delta) + (1 - p_i) U_i(z_j(x, t^* - \Delta), x, t^* - \Delta).$$

The paragraphs above show that there are unique equilibrium payoffs  $W_i(x, t^* - \Delta)$  at states  $(x, t^* - \Delta)$ . Consider next time  $t^* - 2\Delta$ . Party  $i$ 's payoff in case of delay is  $e^{-r\Delta} \mathbb{E}_{NA}[W_i(x_{t^* - \Delta}, t^* - \Delta) | x_{t^* - 2\Delta} = x]$ . For  $i = L, R$ , let

$$A_i(x, t^* - 2\Delta) := \{z \in [0, 1] : U_i(z, x, t^* - 2\Delta) \geq e^{-r\Delta} \mathbb{E}_{NA}[W_i(x_{t^* - \Delta}, t^*) | x_{t^* - 2\Delta} = x]\},$$

and let  $A(x, t^* - 2\Delta) := A_L(x, t^* - 2\Delta) \cap A_R(x, t^* - 2\Delta)$ . If  $A(x, t^* - 2\Delta) = \emptyset$ , there is no policy that both parties would agree to implement. In this case there must be delay at  $t^* - 2\Delta$ , so party  $i$ 's payoff is  $W_i(x, t^* - 2\Delta) = e^{-r\Delta} \mathbb{E}_{NA}[W_i(x_{t^* - \Delta}, t^*) | x_{t^* - 2\Delta} = x]$ . If  $A(x, t^* - 2\Delta) \neq \emptyset$ , when party  $j$  has proposal power she offers  $z_j(x, t^* - 2\Delta) \in \arg \max_{z \in A(x, t^* - 2\Delta)} U_j(z, x, t^* - 2\Delta)$  and her opponent accepts this offer. In this case, in any equilibrium party  $i$ 's expected payoff at time  $t^* - 2\Delta$  is

$$\begin{aligned} W_i(x, t^* - 2\Delta) &= p_i U_i(z_i(x, t^* - 2\Delta), x, t^* - 2\Delta) \\ &\quad + (1 - p_i) U_i(z_j(x, t^* - 2\Delta), x, t^* - 2\Delta). \end{aligned}$$

Repeating these arguments for all  $t \in T(\Delta)$  establishes that this game has unique equilibrium payoffs. ■

**Proof of Proposition 2.** Note that  $W_L(x, t) + W_R(x, t) \leq 1 + B e^{-r(t^* - t)}$  for all  $t < t^*$  and all  $x \in [0, 1]$ ; that is, the sum of the parties' payoffs is bounded above by the total payoff

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<sup>17</sup>There are two things to note. First, when  $A(x, t^* - \Delta) \neq \emptyset$  the set of policies that maximize party  $j$ 's payoff is non-empty since  $A(x, t^* - \Delta)$  is compact and  $U_j(\cdot, x, t^* - \Delta)$  is continuous. Second, by our restriction on SPE, when  $A(x, t^* - \Delta) \neq \emptyset$  the party with proposal power will always make an offer in  $\arg \max_{z \in A(x, t^* - \Delta)} U_j(z, x, t^* - \Delta)$  even if she is indifferent between making this offer or delaying, and the responder will always accept such an offer even if she is indifferent between accepting and rejecting.

they would get if they implemented a policy today, which is equal to  $u_L(z) + u_R(z) = 1$ , plus the sum of the parties' discounted payoff coming from the fact that one party will win the election, which is equal to  $Be^{-r(t^*-t)}$ . Therefore, there exists  $s > 0$  such that  $e^{-r\Delta}\mathbb{E}_{NA}[W_L(x_{t+\Delta}, t + \Delta) + W_R(x_{t+\Delta}, t + \Delta)|x_t = x] \leq e^{-r\Delta}(1 + Be^{-r(t^*-t)}) < 1$  for all  $t$  with  $t^* - t > s$  and all  $x \in [0, 1]$ ; i.e.,  $s$  solves  $1 + Be^{-rs} = e^{r\Delta}$ . Note that  $s$  is strictly increasing in  $B$ . Note further that, for all  $t$  such that  $t^* - t > s$  and for all  $x \in [0, 1]$ , there exists a policy  $z(x, t) \in [0, 1]$  such that  $u_i(z(x, t)) \geq e^{-r\Delta}\mathbb{E}_{NA}[W_i(x_{t+\Delta}, t + \Delta)|x_t = x]$  for  $i = L, R$ . Since  $U_i(z(x, t), x, t) \geq u_i(z(x, t))$ , it follows that  $z(x, t) \in A_i(x, t)$  for  $i = L, R$  (recall that  $A_i(x, t) = \{z \in [0, 1] : U_i(z, x, t) \geq \mathbb{E}_{NA}[e^{-r\Delta}W_i(x_{t+\Delta}, t + \Delta)|x_t = x]\}$ ). Therefore,  $A(x, t) = A_L(x, t) \cap A_R(x, t) \neq \emptyset$ , so parties reach an agreement at  $t$ . ■

## A.2 Proofs of Section 2.3

**Proof of Lemma 1.** I first show that  $W_i(x, t) \geq \underline{W}_i(x, t)$  for all  $t < t^*$  and all  $x \in [0, 1]$ . To see this, note that party  $i$  can always unilaterally generate delay at each time  $t < t^*$ , either by rejecting offers when her opponent has proposal power or by choosing to pass on her right to make offers when making proposals. At times  $t < t^*$ , the payoff that party  $i$  gets by unilaterally delaying an agreement until time  $t^*$  is equal to  $\mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{p}_i(x_{t^*})|x_t = x] + e^{-r(t^*-t)}BQ_i(x, t) = \underline{W}_i(x, t)$ . Therefore, it must be that  $W_i(x, t) \geq \underline{W}_i(x, t)$  for all  $(x, t)$  with  $t < t^*$ .

Next, I show that  $W_i(x, t) \leq \overline{W}_i(x, t)$  for all  $t < t^*$  and all  $x \in [0, 1]$ . To see this, note first that  $W_L(x, t) + W_R(x, t) \leq 1 + Be^{-r(t^*-t)}$  for all  $t < t^*$  and all  $x \in [0, 1]$ : at any time  $t < t^*$ , the sum of the parties' payoffs cannot be larger than what they would jointly get by implementing a policy at  $t$ . From this inequality it follows that for all  $t < t^*$  and all  $x \in [0, 1]$

$$\begin{aligned} W_i(x, t) &\leq 1 + Be^{-r(t^*-t)} - W_j(x, t) \\ &\leq 1 + Be^{-r(t^*-t)} - \mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{p}_j(x_{t^*})|x_t = x] - e^{-r(t^*-t)}BQ_j(x, t) \\ &= 1 - e^{-r(t^*-t)} + \mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{p}_i(x_{t^*})|x_t = x] + e^{-r(t^*-t)}BQ_i(x, t), \end{aligned}$$

where the second inequality follows since  $W_j(x, t) \geq \underline{W}_j(x, t)$  and the equality follows since  $\hat{p}_L(x) + \hat{p}_R(x) = 1$  for all  $x$  and since  $Q_L(x, t) + Q_R(x, t) = 1$  for all  $x$  and all  $t < t^*$ . Hence,  $W_i(x, t) \leq \overline{W}_i(x, t)$  for all  $t < t^*$  and for all  $x \in [0, 1]$ . ■

**Lemma A1** Fix a time  $t \in T(\Delta)$ ,  $t < t^*$  and an  $x \in [0, 1]$ . If there exists a policy  $\hat{z} \in [0, 1]$  and a party  $j \in \{L, R\}$  such that  $U_j(\hat{z}, x, t) = \mathbb{E}_{NA}[e^{-r\Delta}W_j(x_{t+\Delta}, t + \Delta)|x_t = x]$ , then parties

reach an agreement at time  $t$  if  $x_t = x$ .

**Proof.** Suppose such a policy  $\hat{z}$  exists, and note that  $\hat{z} \in A_j(x, t)$ . Since  $W_L(x_{t+\Delta}, t + \Delta) + W_R(x_{t+\Delta}, t + \Delta) \leq 1 + Be^{-r(t^*-t-\Delta)}$  for all  $x_{t+\Delta} \in [0, 1]$ , it follows that,

$$\mathbb{E}_{NA} [e^{-r\Delta} W_i(x_{t+\Delta}, t + \Delta) | x_t] \leq e^{-r\Delta} + Be^{-r(t^*-t)} - \mathbb{E}_{NA} [e^{-r\Delta} W_j(x_{t+\Delta}, t + \Delta) | x_t]. \quad (\text{A.1})$$

Since  $U_i(z, x, t) + U_j(z, x, t) = 1 + e^{-r(t^*-t)}B$  for all  $z \in [0, 1]$ , party  $i$ 's payoff from implementing policy  $\hat{z}$  at time  $t < t^*$  with  $x_t = x$  is

$$\begin{aligned} U_i(\hat{z}, x, t) &= 1 + Be^{-r(t^*-t)} - U_j(\hat{z}, x, t) \\ &= 1 + Be^{-r(t^*-t)} - \mathbb{E}_{NA} [e^{-r\Delta} W_j(x_{t+\Delta}, t + \Delta) | x_t = x]. \end{aligned}$$

This equation together with (A.1) implies that  $U_i(\hat{z}, x, t) > \mathbb{E}_{NA} [e^{-r\Delta} W_i(x_{t+\Delta}, t + \Delta) | x_t = x]$ , so that  $\hat{z} \in A_i(x, t)$ . Hence,  $A(x, t) = A_L(x, t) \cap A_R(x, t) \neq \emptyset$ , so parties reach an agreement at time  $t$  if  $x_t = x$ . ■

**Proof of Proposition 3.** Let  $(x, t)$  be a state satisfying the conditions in part (i) of the proposition, and suppose by contradiction that parties reach an agreement at time  $t$  when  $x_t = x$ . Since  $U_i(z, x, t) < \underline{W}_i(x, t)$  for all  $z \in [0, 1]$ , this implies that party  $i$ 's equilibrium payoff at state  $(x, t)$  is strictly lower than  $\underline{W}_i(x, t)$ , a contradiction to the fact that  $\underline{W}_i(x, t)$  is a lower bound to party  $i$ 's payoff at state  $(x, t)$ . Thus, there must be delay at state  $(x, t)$ .

Next, let  $(x, t)$  be a state satisfying the conditions in Proposition 3(ii). By Lemma 1,

$$\begin{aligned} \mathbb{E}_{NA} [e^{-r\Delta} \underline{W}_i(x_{t+\Delta}, t + \Delta) | x_t] &\leq \mathbb{E}_{NA} [e^{-r\Delta} W_i(x_{t+\Delta}, t + \Delta) | x_t] \\ &\leq \mathbb{E}_{NA} [e^{-r\Delta} \overline{W}_i(x_{t+\Delta}, t + \Delta) | x_t]. \end{aligned}$$

Note that, by the law of iterated expectations,

$$\begin{aligned} &\mathbb{E}_{NA} [e^{-r\Delta} \underline{W}_i(x_{t+\Delta}, t + \Delta) | x_t = x] \\ &= \mathbb{E}_{NA} [e^{-r(t^*-t)} (\hat{p}_i(x_{t^*}) + B \mathbf{1}_{\{x_{t^*} \in M_i\}}) | x_t = x] = \underline{W}_i(x, t). \end{aligned} \quad (\text{A.2})$$

Note further that,

$$\begin{aligned} \mathbb{E}_{NA} [e^{-r\Delta} \overline{W}_i(x_{t+\Delta}, t + \Delta) | x_t = x] &= e^{-r\Delta} - e^{-r(t^*-t)} + \mathbb{E}_{NA} [e^{-r\Delta} W_i(x_{t+\Delta}, t + \Delta) | x_t] \\ &= e^{-r\Delta} - e^{-r(t^*-t)} + \underline{W}_i(x, t) < \overline{W}_i(x, t), \end{aligned} \quad (\text{A.3})$$

where the second equality follows from (A.2). Since  $U_i(\cdot, x, t)$  is continuous and since  $\min_z U_i(z, x, t) \leq \underline{W}_i(x, t)$  and  $\max_z U_i(z, x, t) \geq \overline{W}_i(x, t)$ , there exists  $\hat{z} \in [0, 1]$  such that  $U_i(\hat{z}, x, t) = \mathbb{E}_{NA}[e^{-r\Delta}W_i(x_{t+\Delta}, t + \Delta)|x_t = x]$ . By Lemma A1, there is agreement at state  $(x, t)$ . ■

### A.3 Proofs of Section 3

**Proof of Proposition 4.** Recall that  $z_R = 1$  and  $z_L = 0$  are, respectively, the ideal policies of parties  $R$  and  $L$ . When  $\xi$  satisfies the assumptions in the statement of Proposition 4, for all  $x \in [0, 1]$  and all  $t < t^*$  and for  $i, j = L, R, i \neq j$ ,

$$\begin{aligned} U_i(z_j, x, t) &= e^{-r(t^*-t)}BQ_i(x + \xi(x, z_j), t) \leq e^{-r(t^*-t)}BQ_i(x, t) \leq \underline{W}_i(x, t), \text{ and} \\ U_i(z_i, x, t) &= 1 + e^{-r(t^*-t)}BQ_i(x + \xi(x, z_i), t) \geq 1 + e^{-r(t^*-t)}BQ_i(x, t) \geq \overline{W}_i(x, t). \end{aligned}$$

Therefore, by Proposition 3(ii) parties reach an agreement at all states  $(x, t)$  with  $t \in T(\Delta), t < t^*$ . ■

**Proof of Proposition 5.** I consider the case in which  $\xi(x, 0) < 0$ ; the case in which  $\xi(x, 1) > 0$  is symmetric and omitted. Since  $\xi(x, 0) < 0$  and since  $\xi(x, \cdot)$  is continuous, there exists  $\epsilon > 0$  and  $z' \in (0, 1)$  such that  $\xi(x, z) \leq -\epsilon$  for all  $z \in [0, z']$ .

Recall that  $D_L(x, t) := \mathbb{E}_{NA}[e^{-r(t^*-t)}\hat{p}_L(x_{t^*})|x_t = x]$ . Note that, for any  $d \in (0, 1)$ , there exists a bargaining protocol after the election  $\hat{p}(\cdot)$  and a date  $t < t^*$  such that  $D_L(x, t) = d$ . From now on, I fix a bargaining protocol  $\hat{p}(\cdot)$  and a time  $t < t^*$  such that  $D_L(x, t) \geq 1 - z'$ . I show that, for such a bargaining protocol, there exists a benefit from the election  $B > 0$  such that  $U_L(z, x, t) < \underline{W}_L(x, t)$  for all  $z \in [0, 1]$ . By Proposition 3, this implies that parties delay an agreement at time  $t < t^*$  if  $x_t = x$ .

I first show that there exists  $\hat{B} > 0$  such that  $U_L(z, x, t) < \underline{W}_L(x, t)$  for all  $z \in [0, z']$  whenever  $B > \hat{B}$ . Let  $\eta_1 := \min_{z \in [0, z']} Q_L(x, t) - Q_L^z(x, t) \geq Q_L(x, t) - Q_L(x - \epsilon, t) > 0$ , and note that for  $z \in [0, z']$ ,

$$\begin{aligned} U_L(z, x, t) - \underline{W}_L(x, t) &= 1 - z - D_L(x, t) + e^{-r(t^*-t)}B[Q_L^z(x, t) - Q_L(x, t)] \\ &\leq 1 - D_L(x, t) - e^{-r(t^*-t)}B\eta_1. \end{aligned}$$

Therefore, a sufficient to have  $U_L(z, x, t) < \underline{W}_L(x, t)$  for all  $z \in [0, z']$  is that

$$B > \hat{B} := \frac{1 - D_L(x, t)}{e^{-r(t^*-t)}\eta_1}. \quad (\text{A.4})$$

Next, I define  $z'' \in (z', 1]$  as follows. If there exists  $z$  such that  $\xi(z, x) > 0$ , then  $z'' := \inf\{z \in [0, 1] : \xi(z, x) > 0\}$  (since  $\xi(\cdot, x)$  is continuous,  $\xi(z'', x) = 0$ ). Otherwise,  $z'' = 1$ . Note that in either case,  $z'' > z'$ . Since  $D_L(x, t) \geq 1 - z'$  and since  $\xi(z, x) \leq 0$  for all  $z \in (z', z'']$ , it follows that

$$U_L(z, x, t) - \underline{W}_L(x, t) = 1 - z - D_L(x, t) + e^{-r(t^*-t)}B[Q_L^z(x, t) - Q_L(x, t)] < 0$$

for all  $z \in (z', z'']$ . Therefore, if  $z'' = 1$ , there is delay at state  $(x, t)$  whenever  $B > \hat{B}$ .

Consider next the case in which  $z'' < 1$ , so there exists  $z > z''$  such that  $\xi(z, x) > 0$ . Let  $\eta_2 := \max_{z \in [z'', 1]} Q_L^z(x, t) - Q_L(x, t) > 0$ , and note that for all  $z \in [z'', 1]$ ,

$$\begin{aligned} U(z, x, t) - \underline{W}_L(x, t) &= 1 - z - D_L(x, t) + Be^{-r(t^*-t)}[Q_L^z(x, t) - Q_L(x, t)] \\ &\leq 1 - z'' - D_L(x, t) + Be^{-r(t^*-t)}\eta_2. \end{aligned}$$

Therefore,  $U(z, x, t) < \underline{W}_L(x, t)$  for all  $z \in [z'', 1]$  whenever

$$B < \tilde{B} := \frac{D_L(x, t) - (1 - z'')}{e^{-r(t^*-t)}\eta_2} \quad (\text{A.5})$$

This, together with arguments above, implies that a sufficient condition for there to be delay at state  $(x, t)$  is that  $B \in (\hat{B}, \tilde{B})$ . Using equations (A.4) and (A.5),

$$\tilde{B} > \hat{B} \iff D_L(x, t) > d := \frac{\eta_2 + (1 - z'')\eta_1}{\eta_2 + \eta_1} \in (0, 1).$$

Thus, for  $\hat{p}$  such that  $D_L(x, t) > d$  and for  $B \in (\hat{B}, \tilde{B})$ , there is delay at state  $(x, t)$ . ■

**Proof of Proposition 6.** Let  $(x, t)$  be such that the conditions in the statement of the proposition hold. I now show that, in this case,  $U_i(z, x, t) < \underline{W}_i(x, t)$  for all  $z \in [0, 1]$ . This, together with Proposition 3(i), implies that parties delay an agreement at state  $(x, t)$ . Note that for all  $z \in [0, 1]$  and  $(x, t) \in [0, 1] \times [0, t^*)$ ,

$$U_i(z, x, t) - \underline{W}_i(x, t) = u_i(z) + e^{-r(t^*-t)}B\kappa_i(z, x, t) - D_i(x, t).$$

Consider first policies  $z \in Z_i(x, t) = \{z \in [0, 1] : \kappa_i(z, x, t) < 0\}$ . Note that, for such policies,

$$U_i(z, x, t) < \underline{W}_i(x, t) \iff B > \frac{D_i(x, t) - u_i(z)}{e^{-r(t^*-t)}\kappa_i(z, x, t)}.$$

Consider next policies  $z$  with  $\kappa_i(z, x, t) = 0$ . For such policies,  $U_i(z, x, t) - \underline{W}_L(x, t) = u_i(z) - D_i(x, t) < 0$ , where I used the assumption that  $D_i(x, t) > u_i(z)$  for all  $z$  with  $\kappa_i(z, x, t) \geq 0$ .

Finally, consider policies  $z \in Z_j(x, t) = \{z \in [0, 1] : \kappa_j(z, x, t) < 0\} = \{z \in [0, 1] : \kappa_i(z, x, t) > 0\}$ . For such policies,

$$U_i(z, x, t) < \underline{W}_i(x, t) \iff B < \frac{D_i(x, t) - u_i(z)}{e^{-r(t^*-t)}\kappa_i(z, x, t)}.$$

The arguments above imply that, for  $(x, t)$  such that the conditions in the statement of the proposition hold,  $U_i(z, x, t) < \underline{W}_i(x, t)$  for all  $z \in [0, 1]$ . Hence, by Proposition 3(i) parties delay an agreement at state  $(x, t)$ . ■

**Proof of Proposition 7.** Let  $(x, t)$  be such that the conditions in the statement of the proposition hold. I now show that, in this case,  $\min_z U_i(z, x, t) \leq \underline{W}_i(x, t)$  and  $\max_z U_i(z, x, t) \geq \overline{W}_i(x, t)$ . This, together with Proposition 3(ii), implies that parties reach an agreement at state  $(x, t)$ .

I start by showing that  $\min_z U_i(z, x, t) \leq \underline{W}_i(x, t)$ . Suppose first that there exist  $z$  such that  $\kappa_i(z, x, t) \geq 0$ . Note that, for all such  $z$ ,

$$U_i(z, x, t) - \underline{W}_i(x, t) = u_i(z) + e^{-r(t^*-t)}B\kappa_i(z, x, t) - D_i(x, t) \leq 0,$$

where I used  $D_i(x, t) \geq u_i(z)$  for all  $z$  such that  $\kappa_i(z, x, t) \geq 0$ .

Suppose next that  $\kappa_i(z, x, t) < 0$  for all  $z \in [0, 1]$ . Note then that

$$U_i(z_j, x, t) - \underline{W}_i(x, t) = e^{-r(t^*-t)}B\kappa_i(z_j, x, t) - D_i(x, t) < 0,$$

since  $u_i(z_j) = 0$  and since  $\kappa_i(z_j, x, t) < 0$ . Therefore, in either case,  $\min_z U_i(z, x, t) \leq \underline{W}_i(x, t)$ .

Next I show that  $\max_z U_i(z, x, t) \geq \overline{W}_i(x, t)$ . Note that, for all  $z \in Z_i(x, t)$ ,

$$\begin{aligned} U_i(z, x, t) - \overline{W}_i(x, t) &= u_i(z) - (1 - e^{-r(t^*-t)}) - D_i(x, t) + Be^{-r(t^*-t)}\kappa_i(z, x, t) \geq 0 \\ \iff B &\leq \frac{D_i(x, t) + (1 - e^{-r(t^*-t)}) - u_i(z)}{e^{-r(t^*-t)}\kappa_i(z, x, t)} \end{aligned} \quad (\text{A.6})$$

Hence, when (A.6) holds for some  $z \in Z_i(x, t)$ ,  $\max_z U_i(z, x, t) \geq \overline{W}_i(x, t)$ .

Similarly, for all  $z \in Z_j(x, t)$ ,

$$\begin{aligned} U_i(z, x, t) - \overline{W}_i(x, t) &= u_i(z) - (1 - e^{-r(t^*-t)}) - D_i(x, t) + Be^{-r(t^*-t)}\kappa_i(z, x, t) \geq 0 \\ \iff B &\geq \frac{D_i(x, t) + (1 - e^{-r(t^*-t)}) - u_i(z)}{e^{-r(t^*-t)}\kappa_i(z, x, t)} \end{aligned} \quad (\text{A.7})$$

Hence, when (A.7) holds for some  $z \in Z_j(x, t)$ ,  $\max_z U_i(z, x, t) \geq \overline{W}_i(x, t)$ .

The arguments above imply that  $\min_z U_i(z, x, t) \leq \underline{W}_i(x, t)$  and  $\max_z U_i(z, x, t) \geq \overline{W}_i(x, t)$  whenever  $(x, t)$  is such that the conditions in the statement of the proposition hold. Hence, by Proposition 3(ii) parties reach an agreement at state  $(x, t)$ . ■

**Proof of Corollary 1.** Suppose  $\xi(\cdot, \cdot)$  is given by (3), and note that  $\kappa_L(z, x, t) \leq 0$  for all  $(x, t) \in [0, 1] \times [0, t^*)$  and all  $z \in [0, 1]$ , with strict inequality for all  $z \neq m(x)$ . By Proposition 6, parties delay an agreement at any state  $(x, t)$  such that: (i)  $D_L(x, t) > 1 - m(x)$ , and (ii)  $B > (D_L(x, t) - (1 - z))/e^{-r(t^*-t)}\kappa_L(z, x, t)$  for all  $z \neq m(x)$ . ■

**Proof of Corollary 2.** Before proving the corollary, I establish some properties of  $Q_i(x, t)$ . For all  $s > 0$  and all  $x, y \in [0, 1]$ , let  $p(x, y, s) = \text{Prob}(x_s = y | x_0 = x)$  be the transition density function of the process  $x_t$ . It is well-known that  $p(x, y, s)$  solves Kolmogorov's backward equation (i.e., Bhattacharya and Waymire (2009), chapter V.6),

$$\frac{\partial}{\partial s} p(x, y, s) = \mu \frac{\partial}{\partial x} p(x, y, s) + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial x^2} p(x, y, s), \quad (\text{A.8})$$

with  $\lim_{s \rightarrow 0} p(x, y, s) = \mathbf{1}_{\{y=x\}}$  and  $\partial p(x, y, s)/\partial x|_{x=0} = \partial p(x, y, s)/\partial x|_{x=1} = 0$  for all  $s > 0$ . Note that for all  $t < t^*$  and for  $i = L, R$ ,  $Q_i(x, t) = \mathbb{E}_{NA}[\mathbf{1}_{\{x_s \in M_i\}} | x_0 = x] = \int_{M_i} p(x, y, t^* - t) dy$ . Since  $p(x, y, s)$  solves (A.8) with  $\partial p(x, y, s)/\partial x|_{x=0} = \partial p(x, y, s)/\partial x|_{x=1} = 0$ , it follows that  $Q_i(x, t) \in C^{2,1}$  for all  $(x, t) \in [0, 1] \times [0, t^*)$ , with  $\partial Q_i(x, t)/\partial x|_{x=0} = \partial Q_i(x, t)/\partial x|_{x=1} = 0$ .

Suppose  $\xi(\cdot, \cdot)$  is given by (3). Fix  $t < t^*$  and let  $\eta > 0$  be such that  $\max_{x \in [0, 1]} u_L(z_L) - D_L(x, t) - (1 - e^{-r(t^*-t)}) = \max_{x \in [0, 1]} e^{-r(t^*-t)} - D_L(x, t) \geq \eta B e^{-r(t^*-t)}$ .<sup>18</sup> Since  $Q_L(x, t) \in C^{2,1}$  and since  $\partial Q_i(x, t)/\partial x|_{x=0} = \partial Q_i(x, t)/\partial x|_{x=1} = 0$ , there exists  $\bar{\lambda} > 0$  and  $\epsilon > 0$  such that, for all  $z$ ,  $\max_{x \in [0, \epsilon] \cup [1 - \epsilon, 1]} \kappa_L(z, x, t) > -\eta$  whenever  $\lambda < \bar{\lambda}$ . Therefore, for  $\lambda < \bar{\lambda}$ ,

$$\begin{aligned} u_L(z_L) - D_L(x, t) - (1 - e^{-r(t^*-t)}) + Be^{-r(t^*-t)}\kappa_L(z_L, x, t) &\geq 0 \forall x \in [0, \epsilon] \cup [1 - \epsilon, 1] \\ \iff U_L(z_L, x, t) &\geq \overline{W}_L(x, t) \forall x \in [0, \epsilon] \cup [1 - \epsilon, 1]. \end{aligned}$$

<sup>18</sup>Such an  $\eta > 0$  exists since  $D_i(x, t) < e^{-r(t^*-t)}$  for all  $x$ .



Moreover, for all  $x$ ,

$$u_L(z_R) - D_L(x, t) + Be^{-r(t^*-t)}\kappa_L(z_R, x, t) \leq 0 \iff U_L(z_R, x, t) \leq \underline{W}_L(x, t),$$

where the first inequality uses  $u_L(z_R) = 0$  and  $\kappa_L(z_R, x, t) \leq 0$ . Therefore, by Proposition 3(ii), parties reach an agreement at state  $(x, t)$  with  $x \in [0, \epsilon] \cup [1 - \epsilon, 1]$ . ■

**Proof of Corollary 3.** Suppose  $\xi(\cdot, \cdot)$  is given by (4), and note that  $\kappa_R(z, x, t) < 0$  for all states  $(x, t) \in [0, 1] \times [0, t^*)$  and all  $z \in [0, 1]$ . Note further that, for all  $z \in [0, 1]$ ,

$$\frac{D_R(x, t) - 1}{e^{-r(t^*-t)}\kappa_R(z = 1, x, t)} \geq \frac{D_R(x, t) - u_R(z)}{e^{-r(t^*-t)}\kappa_R(z, x, t)}.$$

Therefore, by Proposition 6, parties delay an agreement at state  $(x, t)$  when

$$B > \frac{D_R(x, t) - 1}{e^{-r(t^*-t)}\kappa_R(z = 1, x, t)}.$$

■

**Proof of Corollary 4.** Suppose  $\xi(\cdot, \cdot)$  is given by (4), and fix  $t < t^*$ . Let  $\eta > 0$  be such that  $\max_{x \in [0, 1]} u_R(z_R) - D_R(x, t) - (1 - e^{-r(t^*-t)}) = \max_{x \in [0, 1]} e^{-r(t^*-t)} - D_R(x, t) \geq \eta Be^{-r(t^*-t)}$ . Since  $Q_R(x, t) \in C^{2,1}$  and since  $\partial Q_i(x, t)/\partial x|_{x=0} = \partial Q_i(x, t)/\partial x|_{x=1} = 0$  (see proof of Corollary 2), there exists  $\bar{g} > 0$  and  $\varepsilon > 0$  such that, for all  $z$ ,  $\max_{x \in [0, \varepsilon] \cup [1 - \varepsilon, 1]} \kappa_R(z, x, t) > -\eta$  whenever  $g < \bar{g}$ . Therefore, when  $g < \bar{g}$ ,

$$\begin{aligned} u_R(z_R) - D_R(x, t) - (1 - e^{-r(t^*-t)}) + Be^{-r(t^*-t)}\kappa_R(z_R, x, t) &\geq 0 \forall x \in [0, \varepsilon] \cup [1 - \varepsilon, 1] \\ \iff U_R(z_R, x, t) &\geq \overline{W}_R(x, t) \forall x \in [0, \varepsilon] \cup [1 - \varepsilon, 1]. \end{aligned}$$

Moreover, for all  $x$ ,

$$u_R(z_L) - D_R(x, t) + Be^{-r(t^*-t)}\kappa_R(z_L, x, t) \leq 0 \iff U_R(z_L, x, t) \leq \underline{W}_R(x, t),$$

where I used  $u_R(z_L) = 0$  and  $\kappa_R(z_L, x, t) \leq 0$ . Therefore, by Proposition 3(ii), parties reach an agreement at state  $(x, t)$  with  $x \in [0, \varepsilon] \cup [1 - \varepsilon, 1]$ . ■

The following Lemma is used in the proof of Lemma 2.

**Lemma A2** *Let  $\tilde{\xi}(\cdot, \cdot)$  and  $\hat{\xi}(\cdot, \cdot)$  be such that, for all  $x \in [0, 1]$ ,  $\tilde{\xi}(x, 0) \geq 0 \geq \tilde{\xi}(x, 1)$  and  $\hat{\xi}(x, 0) \geq 0 \geq \hat{\xi}(x, 1)$ . For all  $x \in [0, 1]$  and all  $t < t^*$ , let  $\tilde{W}_i(x, t)$  and  $\hat{W}_i(x, t)$  be party  $i$ 's*

payoff at state  $(x, t)$  when  $\xi = \tilde{\xi}$  and  $\xi = \hat{\xi}$ , respectively. Then, for all  $x \in [0, 1]$  and all  $t < t^*$ ,  $\tilde{W}_i(x, t) = \hat{W}_i(x, t)$ .

**Proof.** Note that, by Proposition 4, there is no gridlock when  $\xi = \tilde{\xi}$  or when  $\xi = \hat{\xi}$ .

As a first step, I show that when  $\xi$  is such that  $\xi(x, 0) \geq 0 \geq \xi(x, 1)$  for all  $x \in [0, 1]$ , then for all  $(x, t) \in [0, 1] \times [0, t^*)$  and for  $j = L, R$ , there exists an offer  $z \in [0, 1]$  such that  $U_j(z, x, t) = \mathbb{E}_{NA}[e^{-r\Delta}W_j(x_{t+\Delta}, t + \Delta)|x_t = x]$  (i.e., such that party  $j$  is indifferent between accepting and rejecting). To see this, note that by the proof of Proposition 4,  $\min_z U_j(z, x, t) \leq \underline{W}_j(x, t)$  and  $\max_z U_j(z, x, t) \geq \overline{W}_j(x, t)$ . Note further that  $\underline{W}_j(x, t) = \mathbb{E}_{NA}[e^{-r\Delta}\underline{W}_j(x_{t+\Delta}, t + \Delta)|x_t = x]$  and that  $\overline{W}_j(x, t) > \mathbb{E}_{NA}[e^{-r\Delta}\overline{W}_j(x_{t+\Delta}, t + \Delta)|x_t = x]$  (see equations (A.2) and (A.3)). Then, by continuity of  $U_j(\cdot, x, t)$  and by the fact that  $W_j(x, t + \Delta) \in [\underline{W}_j(x, t + \Delta), \overline{W}_j(x, t + \Delta)]$  for all  $x$  (Lemma 1), there must exist an offer  $z \in [0, 1]$  such that  $U_j(z, x, t) = \mathbb{E}_{NA}[e^{-r\Delta}W_j(x_{t+\Delta}, t + \Delta)|x_t = x]$ . Note that this offer maximizes party  $i$ 's payoff among the offers that party  $j \neq i$  finds acceptable at state  $(x, t)$  when  $i$  has proposal power, and hence is the offer that party  $i$  makes in equilibrium.

I now use the observation in the previous paragraph to show that parties obtain the same payoffs when  $\xi = \tilde{\xi}$  than when  $\xi = \hat{\xi}$ . The proof is by induction. Consider time  $t = t^* - \Delta$ . By the previous paragraph, if  $x_t = x$  and party  $i$  has proposal power, she will make an offer  $z$  such that  $U_j(z, x, t) = \mathbb{E}_{NA}[e^{-r\Delta}\hat{p}_j(x_{t^*})|x_{t^*-\Delta} = x] + e^{-r\Delta}BQ_j(x, t^* - \Delta)$ , and party  $j$  will accept such an offer. Since  $\hat{p}_j(\cdot)$  and  $Q_j(\cdot, \cdot)$  do not depend on  $\xi$ , player  $j$  gets the same payoff at time  $t = t^* - \Delta$  if party  $i$  is proposer regardless of whether  $\xi = \tilde{\xi}$  or  $\xi = \hat{\xi}$ . Since parties reach an agreement at  $t^* - \Delta$ , the sum of their payoffs is  $1 + Be^{-r\Delta}$ . Hence, if party  $i$  is proposer she must also get the same payoff at time  $t^* - \Delta$  regardless of whether  $\xi = \tilde{\xi}$  or  $\xi = \hat{\xi}$ . Therefore, for all  $x \in [0, 1]$  and  $k = L, R$ ,  $\tilde{W}_k(x, t^* - \Delta) = \hat{W}_k(x, t^* - \Delta) = W_k(x, t^* - \Delta)$ .

Suppose next that, for all  $x \in [0, 1]$  and  $k = L, R$ ,  $\tilde{W}_k(x, t) = \hat{W}_k(x, t) = W_k(x, t)$  for all  $t = t^* - \Delta, t^* - 2\Delta, \dots, t^* - n\Delta$ . Let  $s = t^* - n\Delta$ . At state  $(x, s - \Delta)$ , if party  $i$  has proposal power she will make an offer  $z$  such that  $U_j(z, x, s - \Delta) = \mathbb{E}_{NA}[e^{-r\Delta}W_j(x_s, s)|x_{s-\Delta} = x]$ , and party  $j$  accepts such an offer. By the induction hypothesis,  $W_j(x_s, s)$  is the same regardless of whether  $\xi = \tilde{\xi}$  or  $\xi = \hat{\xi}$ . It follows that, regardless of whether  $\xi = \tilde{\xi}$  or  $\xi = \hat{\xi}$ , party  $j$  gets the same payoff at time  $s - \Delta$  if party  $i$  is proposer. Since parties reach an agreement at  $s - \Delta$ , the sum of their payoffs is  $1 + Be^{-r(t^* - s + \Delta)}$ . Hence, if party  $i$  is proposer she must also get the same payoff at time  $s - \Delta$  regardless of whether  $\xi = \tilde{\xi}$  or  $\xi = \hat{\xi}$ . Therefore, for all  $x \in [0, 1]$  and  $k = L, R$ ,  $\tilde{W}_k(x, s - \Delta) = \hat{W}_k(x, s - \Delta) = W_k(x, s - \Delta)$ . ■

**Proof of Lemma 2.** Let  $\hat{z}_i(x, t)$  be the offer that party  $i$  makes at time  $t$  if  $x_t = x$  when  $\xi(x, z) = 0$  for all  $(x, z)$  (i.e., when policies don't affect popularity). Note that the payoff

that party  $L$  gets from this offer is  $U_L(\hat{z}_i(x, t), x, t) = 1 - \hat{z}_i(x, t) + e^{-r(t^* - t)} BQ_L(x, t)$ . On the other hand, if  $\xi(x, z) = \tilde{\xi}(x, z)$ , party  $i$  makes offer  $z_i(x, t)$ , and party  $L$  gets a payoff equal to  $U_L(z_i(x, t), x, t) = 1 - z_i(x, t) + e^{-r(t^* - t)} BQ_L(x + \tilde{\xi}(x, z_i(x, t)), t)$ . Since by Lemma A2 parties get the same payoff regardless of whether  $\xi(x, z) = 0$  or  $\xi(x, z) = \tilde{\xi}(x, z)$ , it must be that  $U_L(\hat{z}_i(x, t), x, t) = U_L(z_i(x, t), x, t)$ , which implies equation (5). ■

**Proof of Proposition 8.** Suppose that  $x$  and  $t$  are such that  $\hat{z}_i(x, t) > 1/2$ . Note that the left-hand side of (5) would be strictly positive if  $z_i(x, t) > \hat{z}_i(x, t)$ , while the right-hand side would be weakly negative (since  $Q_L(\cdot, t)$  is strictly increasing and  $\tilde{\xi}(x, \cdot)$  is decreasing and satisfies  $\tilde{\xi}(x, 1/2) = 0$ ). On the other hand, if  $z_i(x, t) \leq 1/2$  then the left-hand side of (5) would be strictly negative and the right-hand side would be weakly positive. Hence, it must be that  $z_i(x, t) \in (1/2, \hat{z}_i(x, t)]$ . A symmetric argument establishes that  $z_i(x, t) \in [\hat{z}_i(x, t), 1/2)$  for all  $(x, t)$  such that  $\hat{z}_i(x, t) < 1/2$ . ■

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