ENG EC/ME/SC 501:

Exercises (Set 8)  (Due 12/5/19)

1. Given that \( x(0) = 1 \), find \( x(\cdot) \) on the interval \( 0 \leq t \leq T \) such that

\[
J = \int_0^T \dot{x}^2 + x^2 \, dt
\]

is minimized. (Hint: Convert this into a control problem by setting \( \dot{x} = u \).)

2. Suppose that the partitioned system

\[
\begin{pmatrix}
\dot{w}(t) \\
\dot{y}(t)
\end{pmatrix} =
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
\begin{pmatrix}
w(t) \\
y(t)
\end{pmatrix}
\]

with output \( y(t) \) is observable. Show that \( \{A_{11}, A_{21}\} \) is an observable pair.

3. (a) Consider the linear system

\[
\begin{pmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
x_1(t) \\
x_2(t)
\end{pmatrix} +
\begin{pmatrix}
0 \\
u(t)
\end{pmatrix}.
\]

For \( T > 0 \), find the control input that steers the state of (1) from \( \begin{pmatrix} 0 \\ 0 \end{pmatrix} \) to \( \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \) in \( T \) units of time so as to minimize the performance metric

\[
\eta = \int_0^T u(t)^2 \, dt.
\]

(b) For \( \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \), evaluate \( \eta \). Why are the values different?