1. Given that $x(0) = 1$, find $x(\cdot)$ on the interval $0 \leq t \leq T$ such that

$$J = \int_0^T \dot{x}^2 + x^2 dt$$

is minimized. (*Hint: Convert this into a control problem by setting $\dot{x} = u$.*)

2. Suppose that the partitioned system

$$
\begin{pmatrix}
\dot{w}(t) \\
\dot{y}(t)
\end{pmatrix} =
\begin{pmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{pmatrix}
\begin{pmatrix}
w(t) \\
y(t)
\end{pmatrix}
$$

with output $y(t)$ is observable. Show that $\{A_{11}, A_{21}\}$ is an observable pair.

3. Design an observer for the system shown in the figure. The observer should be of second order with both eigenvalues equal to $-3$. 

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\begin{align*}
\dot{x}_1 &= \frac{1}{s} \\
\dot{x}_2 &= \frac{1}{s+2} \\
\dot{x}_3 &= \frac{1}{s+1} + u \\
u &= \frac{1}{s+1}
\end{align*}
```