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ENG EC/ME/SC 501:

Exercises (Set 7) (Due 12/6/18)

1. Given that $x(0) = 1$, find $x(\cdot)$ on the interval $0 \leq t \leq T$ such that

$$J = \int_0^T \dot{x}^2 + x^2 \, dt$$

is minimized. (*Hint:* Convert this into a control problem by setting $\dot{x} = u$.)

2. Suppose that the partitioned system

$$\begin{pmatrix} \dot{w}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} w(t) \\ y(t) \end{pmatrix}$$

with output $y(t)$ is observable. Show that $\{A_{11}, A_{21}\}$ is an observable pair.

3. (a) Consider the linear system

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 0 \\ u(t) \end{pmatrix}. \tag{1}$$

For $T > 0$, find the control input that steers the state of (1) from $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ in T units of time so as to minimize the performance metric

$$\eta = \int_0^T u(t)^2 \, dt. \tag{2}$$

(b) For $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, evaluate η. Why are the values different?