ENG ME/EC/SE 501:

**Exercises (Set 6)**  
(Due 11/05/13)

1. The dynamics of a cd-motor are given by $J \ddot{\theta} + c \dot{\theta} = u(t)$, where $J$ is the motor inertia, $c$ is a coefficient representing forces opposing the motion (back EMF, viscous friction, etc.), and $u(t)$ is a control input proportional to the applied current. Find the feedback gains in a control law of the form $u = -k_v \dot{\theta} - k_p \theta$ such that the natural undamped frequency of the system ($\omega$) is 1, and the damping ratio, $\zeta$ is also 1. ($k_p$ and $k_v$ are to be expressed in terms of $J$ and $c$.)

2. Find a control law $u(\cdot)$ which steers the state of

$$
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} =
\begin{pmatrix}
0 & 1 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} +
\begin{pmatrix}
0 \\
u(t)
\end{pmatrix}
$$

from $\begin{pmatrix}0 \\ 0\end{pmatrix}$ to $\begin{pmatrix}1 \\ 0\end{pmatrix}$ in one unit of time.

3. Identify the controllable pairs

(i) $A = \begin{pmatrix}0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{pmatrix}$, $B = \begin{pmatrix}0 \\ 0 \\ 1\end{pmatrix}$

(ii) $A = \begin{pmatrix}0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{pmatrix}$, $B = \begin{pmatrix}0 & 0 \\ 1 & 0 \\ 0 & 1\end{pmatrix}$

(iii) $A = \begin{pmatrix}1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3\end{pmatrix}$, $B = \begin{pmatrix}1 \\ 1 \\ 1\end{pmatrix}$. 
