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(11/12/24)

ENG ME/EC/SE 501:

Exercises (Set 6) (Due 11/19/24)

1. The dynamics of a cd-motor are given by $J\ddot{\theta} + c\dot{\theta} = u(t)$, where J is the motor inertia, c is a coefficient representing forces opposing the motion (back EMF, viscous friction, etc.), and $u(t)$ is a control input proportional to the applied current. Find the feedback gains in a control law of the form $u = -k_v\dot{\theta} - k_p\theta$ such that the natural undamped frequency of the system (ω) is 1, and the damping ratio, ζ is also 1. (k_p and k_v are to be expressed in terms of J and c.)

2. Find a control law $u(\cdot)$ which steers the state of

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ u(t) \end{pmatrix}$$

from $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in one unit of time.

3. Identify the controllable pairs

$$(i)\ A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$(ii)\ A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(iii)\ A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$