ENG EC/ME/SE 501:

Exercises (Set 5)  (Due 11/7/19)

1. For which of the following systems is the origin asymptotically stable?

   (i) \( \ddot{x} + a\dot{x} + bx = 0, \) \( a > 0, b > 0, \)

   (ii) \( \ddot{x} + a\dot{x} + bx = 0, \) \( a < 0, b > 0, \)

   (iii) \( \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \) \( a < 0, \)

   (iv) \( \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -1 & -a & a \\ a & -1 & 0 \\ -a & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \)

   You should, of course, state your reasons.

2. For each of the following polynomials determine how many roots are in the right half-plane:

   (i) \( \lambda^2 - 2\lambda + 1, \)

   (ii) \( \lambda^3 + 4\lambda^2 + 5\lambda + 2, \)

   (iii) \( -2\lambda^5 - 4\lambda^4 + \lambda^3 + 2\lambda^2 + \lambda + 4. \)

3. (i) Show that reversing the order of coefficients (replacing \( a_i \) by \( a_{n-i} \) in the Routh criterion must give the same result.

   (ii) Show that this can be helpful for testing \( \lambda^6 + \lambda^5 + 3\lambda^4 + 2\lambda^3 + 4\lambda^2 + a\lambda + 8, \) where \( a \) is a parameter.

(Please turn over.)
4. You may use software for this problem. Consider the transfer function

\[ g(s) = \frac{1}{s^3 + s^2 + s + 2}. \]

(i) How many right half-plane poles are there.

(ii) By examining the Nyquist locus, determine the range of gains \( k \) (if any) such that the following closed-loop system is asymptotically stable.

(iii) Show that the closed loop poles of this system are the zeros of the polynomial \( p(s) = s^3 + s^2 + s + 2 + k \). Plot the root locus, indicating the parameter ranges (ranges of \( k \)) such that the closed loop system is stable.

(iv) Write down the Routh table for \( p(s) \).