Prof. J. Baillieul
Mechanical Engineering
Electrical and Computer
Engineering
Systems Engineering

(11/5/24)

ENG EC/ME/SE 501:

Exercises (Set 5) (Due 11/12/24)

1. For which of the following systems is the origin asymptotically stable?

(i) $\ddot{x} + a\dot{x} + bx = 0, \quad a > 0, b > 0,$

(ii) $\ddot{x} + a\dot{x} + bx = 0, \quad a < 0, b > 0,$

(iii) $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad a < 0,$

(iv) $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -1 & -a & a \\ a & -1 & 0 \\ -a & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$

You should, of course, state your reasons.

2. For each of the following polynomials determine how many roots are in the right half-plane:

(i) $\lambda^2 - 2\lambda + 1,$

(ii) $\lambda^3 + 4\lambda^2 + 5\lambda + 2,$

(iii) $-2\lambda^5 - 4\lambda^4 + \lambda^3 + 2\lambda^2 + \lambda + 4.$

3. (i) Show that reversing the order of coefficients (replacing a_i by a_{n-i} in the Routh criterion must give the same result.

(ii) Show that this can be helpful for testing $\lambda^6 + \lambda^5 + 3\lambda^4 + 2\lambda^3 + 4\lambda^2 + a\lambda + 8$, where a is a parameter.

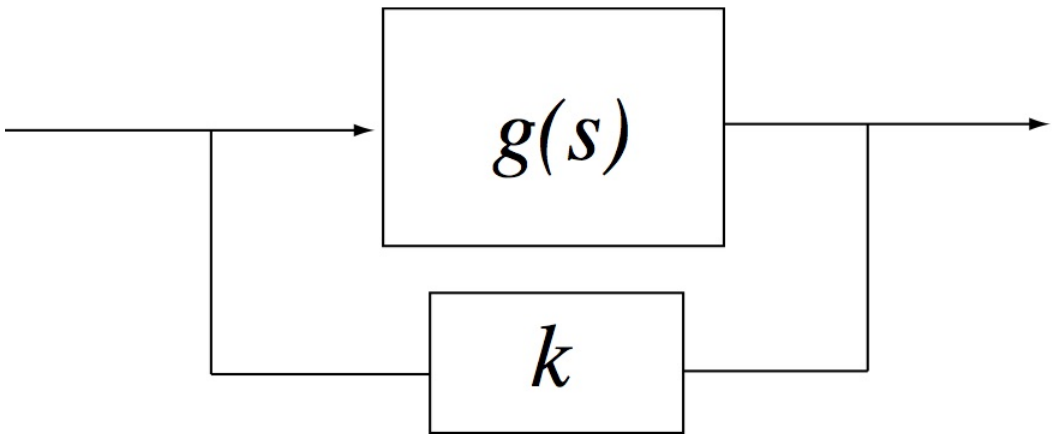
(Please turn over.)

4. You may use software for this problem. Consider the transfer function

$$g(s) = \frac{1}{s^3 + s^2 + s + 2}.$$

(i) How many right half-plane poles are there.

(ii) By examining the Nyquist locus, determine the range of gains k (if any) such that the following closed-loop system is asymptotically stable.



(iii) Show that the closed loop poles of this system are the zeros of the polynomial $p(s) = s^3 + s^2 + s + 2 + k$. Plot the root locus, indicating the parameter ranges (ranges of k) such that the closed loop system is stable.

(iv) Write down the Routh table for $p(s)$.