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Mechanical Engineering
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(10/3/24)

ENG SE/ME/EC 501:

Exercises (Set 3) (Due 10/10/24)

1. Solve the following three-dimensional system:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}.$$

2. Compute e^{At} where $A = \begin{pmatrix} 5 & 5 & -4 \\ 8 & 5 & -4 \\ 12 & 6 & -5 \end{pmatrix}$.

3. Let $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$. Compute $e^{\mathbf{A}t}$ in two different ways:

(a) by exponentiating the equivalent diagonal matrix;

(b) by noting that $AB = BA \Rightarrow e^{A+B} = e^A e^B$ and further noting that $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} = A + B$ where

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$