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ENG EC/ME/SE 501:

Exercises (Set 1) (Due 9/20/17)

1. (a) Prove that matrix multiplication is associative: i.e. show that for any three matrices **A,B,** and **C** of compatible dimensions that **A(BC) = (AB)C**.

(b) Prove that matrix multiplication is not commutative: i.e. it is not the case that **AB = BA** for any two square matrices **A** and **B**.

2. (a) Prove that $\mathbf{(AB)}^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$.

(b) Prove that if **A** is invertible, then $\mathbf{A}^{\mathrm{T}}$ is invertible.

3. (a) Suppose $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are $m \times n$ and $n \times p$ matrices respectively. Find a formula for

$$\frac{d}{dt}[A(t)B(t)]$$

in terms of the derivatives of the individual matrices.

(b) If $\mathbf{A}(t)$ is invertible, find a formula for the derivative (with respect to t) of its inverse.

4. Using Laplace's expansion (i.e. expansion by cofactors), evaluate the determinants of the matrices

$$\begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & 3 \\ 1 & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & x & x \\ a & ax+by & ax \\ 0 & by & 1 \end{pmatrix}$$

5. Find the inverses of the matrices in the previous problem. Assume that $a, b, x,$ and y are integers. Under what further conditions on the symbolic entries a, b, x, y will the inverse of the second matrix have all its entries integers?

6. Let V be a finite dimensional vector space. A *basis* of V is any linearly independent set of vectors that *span* V. (a) Show that any maximal linearly independent set of vectors in V is a basis for V. (b) Show that any minimal spanning set of vectors is also a basis.

Definition: A square matrix $A = (a_{ij})$ is said to have *super-diagonal form* if $a_{ij} = 0$ for all $j < i$.

Definition: A matrix A is said to be *normal* if $AA^* = A^*A$, where A^* denotes the *Hermitian conjugate* of A (=transpose if A is real): $a^*_{ij} = \bar{a}_{ji}$, with the overbar denoting complex conjugate.

(Please turn over.)

7. Prove that any square super-diagonal matrix is normal if and only if it is diagonal.

Square matrices that are normal always have a diagonal Jordan normal form. This is a consequence of the following:

Theorem: A matrix A is unitarily similar to a diagonal matrix if and only if it is *normal*.