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ENG ME/EC 501:

Exercises (Set 1) (Due 9/16/08)

1. (a) Prove that matrix multiplication is associative: i.e. show that for any three matrices $\mathbf{A}$,$\mathbf{B}$, and $\mathbf{C}$ of compatible dimensions that $\mathbf{A(BC) = (AB)C}$.

(b) Prove that matrix multiplication is not commutative: i.e. it is not the case that $\mathbf{AB = BA}$ for any two square matrices $\mathbf{A}$ and $\mathbf{B}$.

2. (a) Prove that $\mathbf{(AB)}^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$.

(b) Prove that if $\mathbf{A}$ is invertible, then $\mathbf{A}^{\mathrm{T}}$ is invertible.

3. (a) Suppose $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are $m \times n$ and $n \times p$ matrices respectively. Find a formula for

$$\frac{d}{dt}[A(t)B(t)]$$

in terms of the derivatives of the individual matrices.

(b) If $\mathbf{A}(t)$ is invertible, find a formula for the derivative (with respect to t) of its inverse.

4. Using Laplace's expansion (i.e. expansion by cofactors), evaluate the determinants of the matrices

$$\begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & 3 \\ 1 & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix}.$$

5. Find the inverses of the matrices in the previous problem.

6. Let $\mathbf{p}_1,\mathbf{p}_2,\ldots,\mathbf{p}_n$ be a basis for $\mathbb{R}^n$, and let $\mathbf{x}$ be a given vector in $\mathbb{R}^n$. Show that the representation of $\mathbf{x}$ in terms of this basis is unique.