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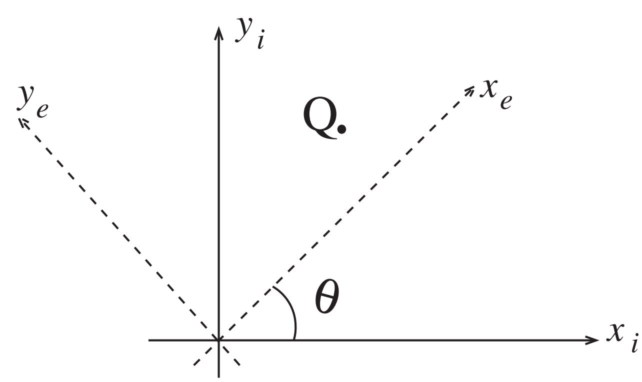
(2/1/18)

ENG MN 740:

Exercises (Set 1) (Due 2/8/18)

1. Referring to the figure below, the point Q may be expressed in terms of either coordinate system. Show that relation between the coordinates for Q in the two systems is given by

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_e \\ y_e \end{pmatrix}.$$



2. Verify by a direct calculation that the group of rigid planar motions satisfies an associative law. (I.e. that for any three rigid motions $T_1,\ T_2,\ T_3$, we have $T_1 \circ (T_2 \circ T_3) = (T_1 \circ T_2) \circ T_3$.)

3. Verify that the group of rigid planar motions is not Abelian. (Recall that a group is said to be Abelian if and only if $a \cdot b = b \cdot a$ for all a and b in the group.)

4. A vector ${}^{A}P$ is rotated about $\hat{Z}_A$ by θ degrees and is subsequently rotated about $\hat{X}_A$ by ϕ degrees. Give the rotation matrix which will accomplish these rotations in the given order.

5. A vector ${}^{A}P$ is rotated about $\hat{Y}_A$ by 30 degrees and is subsequently rotated about $\hat{X}_A$ by 45 degrees. Give the rotation matrix which will accomplish these rotations in the given order.

6. A frame $\{B\}$ is located as follows: initially coincident with a frame $\{A\}$ we rotate $\{B\}$ about $\hat{Z}_B$ by θ degrees and then we rotate the resulting frame about $\hat{X}_B$ by ϕ degrees. Give the rotation matrix which will change the description of vectors from ${}^{B}P$ to ${}^{A}P$.

7. A frame $\{B\}$ is located as follows: initially coincident with a frame $\{A\}$ we rotate $\{B\}$ about $\hat{Z}_B$ by 30 degrees and then we rotate the resulting frame about $\hat{X}_B$ by 45 degrees. Give the rotation matrix which will change the description of vectors from ${}^{B}P$ to ${}^{A}P$.

8. ${}^{A}_{B}R$ is a 3×3 matrix with eigenvalues 1, $e^{+\alpha i}$, and $e^{-\alpha i}$, where $i = \sqrt{-1}$. What is the physical meaning of the eigenvector of ${}^{A}_{B}R$ associated with the eigenvalue 1?