4. You may use software for this problem. Consider the transfer function

\[ g(s) = \frac{1}{s^3 + s^2 + s + 2}. \]

(i) How many right half-plane poles are there.

Answer: 2 \( s = -1.35321, s = 0.176605 \pm 1.20282i \).

(ii) By examining the Nyquist locus, determine the range of gains \( k \) (if any) such that the following closed-loop system is asymptotically stable.

Answer:

For \( k \) in the interval 
\(-2 < k < -1\), the root locus encircles \(-1/k\) -2 times in the clockwise sense. 
Hence in the interval, 
there are no rhp roots.

(iii) Show that the closed loop poles of this system are the zeros of the polynomial \( p(s) = s^3 + s^2 + s + 2 + k \). Plot the root locus, indicating the parameter ranges (ranges of \( k \)) such that the closed loop system is stable.

Answer: The closed-loop poles are the zeros of \( 1 + kg(s) = 1 + k/s^3 + s^2 + s + 2 \). These coincide with the zeros of the polynomial \( p(s) = s^3 + s^2 + s + 2 + k \). See next page.

(iv) Write down the Routh table for \( p(s) \).
Problem 4 (iii) solution.

The above gives the three branches of the root locus of $s^3 + s^2 + s + 2 + k = 0$. All three roots are in the left half plane for $-2 < k < -1$. For $k > -1$, the two complex roots move onto the rhp, while for $k < -2$, the real root lies in the rhp.