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ENG EC/ME/SE 501:

Optional Exercises

1. Consider the linear difference equation

$$y(k+n) + a_{n-1}y(k+n-1) + \dots + a_0y(k) = b_{n-1}u(k+n-1) + b_{n-2}u(k+n-2) + \dots + b_0u(k).$$

Show that this equation can be put into state space form

$$x(k+1) = \mathbf{A}x(k) + \mathbf{b}u(k)$$

by writing

$$\begin{aligned} x_1(k) &= -a_0y(k-1) + b_0u(k-1) \\ x_2(k) &= -a_0y(k-2) + b_0u(k-2) \\ &\quad -a_1y(k-1) + b_1u(k-1) \\ &\quad \vdots \\ x_{n-1}(k) &= -a_0y(k-n+1) + b_0u(k-n+1) \\ &\quad -a_1y(k-n+2) + b_1u(k-n+2) \\ &\quad \vdots \\ &\quad -a_{n-2}y(k-1) + b_{n-2}u(k-1) \\ x_n(k) &= y(k). \end{aligned}$$

2. Consider the system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} s & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} u \quad (\text{S})$$

$$y = x_1 + x_2 + x_3.$$

- (i) State necessary and sufficient conditions on the system parameters for (S) to be observable.
- (ii) Design a full-state observer,

$$\dot{z} = Az + E(y - Cz) + Bu,$$

such that the error components $z_i - x_i$ approach 0 like e^{-t} .