ENG EC/ME/SE 501:

**Mini Quiz 4 Study Questions**  (Quiz date 12/10/20)

1. Consider the system \( \ddot{x} = u \), \( y = x \).

   (a) Write the system in (first order) state-space form.

   (b) Is the system you have written controllable? Observable?

   (c) Design a full-state observer with poles at \(-1, -2, -3\).

2. Consider the linear time invariant system

\[
\dot{x} = Ax + bu, \quad y = cx,
\]

where \( A = \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix}, \ b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \) and \( c = (1, 1) \).

   (a) State necessary and sufficient conditions for the system to be both controllable and observable.

   (b) Consider the system

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u
\]

\[
y = x_1 + x_2,
\]

and let \( \lambda_1 \) and \( \lambda_2 \) be any two points in the complex plane. Design a state feedback control law such that the closed loop system has its poles at \( \lambda_1 \) and \( \lambda_2 \).

3. Consider the system

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}.
\]

Write down a feedback law \( u = Kx \) such that the closed loop system has all its eigenvalues at \( \lambda = -1 \).

Also study all problems in Homework 7.