

ENG ME 740:

Exercises (Set 5) (Due 4/24/18)

1. Consider the mechanism depicted in Figure 1. We shall assume the base from is coincident with the first joint frame when the joint is in the home position.

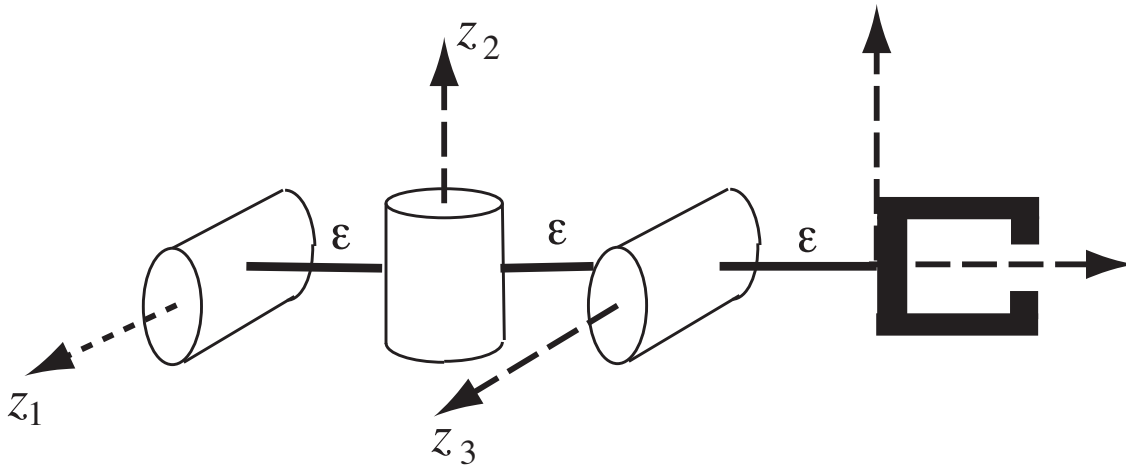


Figure 1

Write down the Denavit-Hartenberg table for the mechanism.

2. In terms of these Denavit-Hartenberg parameters and the corresponding “A” matrices, the coordinate transformation from the base frame to the end effector frame is given by

$$\begin{pmatrix} R(\theta) & \vec{r}(\theta) \\ 0 & 1 \end{pmatrix} = A_1(\theta_1)A_2(\theta_2)A_3(\theta_3) \begin{pmatrix} 1 & 0 & 0 & \epsilon \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Write down the Jacobian of the mapping

$$(\theta_1, \theta_2, \theta_3) \mapsto \vec{r}(\theta_1, \theta_2, \theta_3).$$

3. Show that all configurations with $\theta_3 = 0$ are singular. What is the physical interpretation of this singularity.

4. The orientation of a certain manipulator is given in terms of its joint angle settings by the formula

$$\begin{pmatrix} n_x & o_x & a_y \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{pmatrix} = \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & -s_\alpha \\ 0 & s_\alpha & c_\alpha \end{pmatrix} \cdot \begin{pmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_\alpha = 1/\sqrt{2}$, $s_\alpha = 1/\sqrt{2}$ are constants, and as usual $c_k = \cos(\theta_k)$ and $s_k = \sin(\theta_k)$. Express the Jacobian of this transformation as a 3×3 matrix. What configurations of the joint angles (θ_i 's) are singular?