ENG ME 740:

Exercises (Set 5)  
(Due 4/24/18)

1. Consider the mechanism depicted in Figure 1. We shall assume the base from is coincident with the first joint frame when the joint is in the home position.

![Figure 1](image)

Write down the Denavit-Hartenberg table for the mechanism.

2. In terms of these Denavit-Hartenberg parameters and the corresponding “A” matrices, the coordinate transformation from the base frame to the end effector frame is given by

\[
\begin{pmatrix}
R(\theta) & \vec{r}(\theta) \\
0 & 1
\end{pmatrix} = A_1(\theta_1)A_2(\theta_2)A_3(\theta_3) \begin{pmatrix}
1 & 0 & 0 & \epsilon \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

Write down the Jacobian of the mapping

\((\theta_1, \theta_2, \theta_3) \mapsto \vec{r}(\theta_1, \theta_2, \theta_3)\).

3. Show that all configurations with \(\theta_3 = 0\) are singular. What is the physical interpretation of this singularity.

4. The orientation of a certain manipulator is given in terms of its joint angle settings by the formula

\[
\begin{pmatrix}
n_x & o_x & a_y \\
n_y & o_y & a_y \\
n_z & o_z & a_z
\end{pmatrix} =
\begin{pmatrix}
c_1 & -s_1 & 0 \\
s_1 & c_1 & 0 \\
0 & 0 & 1
\end{pmatrix} \cdot
\begin{pmatrix}
c_2 & 0 & s_2 \\
0 & 1 & 0 \\
-s_2 & 0 & c_2
\end{pmatrix} \cdot
\begin{pmatrix}
1 & 0 & 0 \\
0 & c_\alpha & -s_\alpha \\
0 & s_\alpha & c_\alpha
\end{pmatrix} \cdot
\begin{pmatrix}
c_3 & -s_3 & 0 \\
s_3 & c_3 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

where \(c_\alpha = 1/\sqrt{2}, \ s_\alpha = 1/\sqrt{2}\) are constants, and as usual \(c_k = \cos(\theta_k)\) and \(s_k = \sin(\theta_k)\). Express the Jacobian of this transformation as a \(3 \times 3\) matrix. What configurations of the joint angles (\(\theta_i\)’s) are singular?