Prof. J. Baillieul
Prof. T. Djaferis
Electrical & Computer
  Engineering
Mechanical Engineering

(2/8/18)

ENG ME/SE 740

Exercises (Set 2)   (Due 2/15/18)

1. Consider a planar manipulator with link lengths $r_1 = r_2 = 1$ and $r_3 = 1/2$. Find θ_1, θ_2, and θ_3 such that the end effector is at (1,1) with orientation $\phi = -45°$.
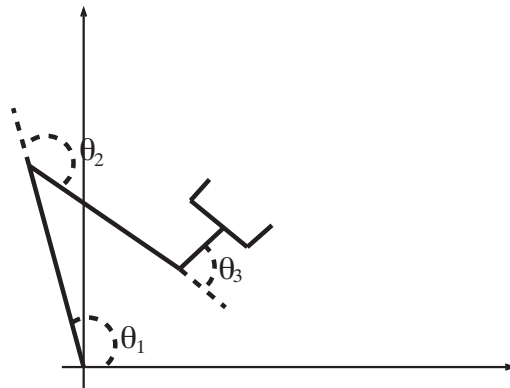

Figure 1: Mechanism of problem 1 but <u>not</u> in the configuration of the problem 1 statement.

2. We have seen a number of ways to describe the orientation of a rigid body with respect to a given coordinate frame. Consider the following rotation matrix:

$$\begin{pmatrix} 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{pmatrix}.$$

Find the corresponding axis / angle parameters, the Euler-angles and the Tait-Bryan (pitch/roll/yaw) angles.

3. Which of the following subsets of $SE(3, \mathbb{R})$ are also subgroups?

(a) $\{ \begin{pmatrix} \cos\theta & 0 & \sin\theta & x \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & z \\ 0 & 0 & 0 & 1 \end{pmatrix} : x, z \in \mathbb{R}; -\pi < \theta \leq \pi \},$

(b) $\{ \begin{pmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix} : y, z \in \mathbb{R}; -\pi < \phi \leq \pi \}$

Justify your answer.

4. Prove that for any square matrices A and B that $e^{(A+B)t} = e^{At}e^{Bt} \iff [A, B] = 0$.

5. For the following 2×2 matrices, compute e^{At}:

(a) $A = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$ (b) $A = \begin{pmatrix} -a & a \\ b & -b \end{pmatrix}$