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Mechanical Engineering

(11/13/17)

ENG EC/ME/SE 501:

Exercises (Set 7) (Due 11/20/17)

1. Consider the system in controllability canonical form:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

Find a constant state feedback K that places the poles of the closed loop system at: $-1, -2 \pm i$.

2. a) Consider the system given below:

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(t)$$

Is the system controllable (fully explain)? If yes, find the constant state feedback K that places the poles of the closed loop system at: $-1, -1, -2$.

b) Consider now the augmented system:

$$\dot{x}(t) = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} u(t)$$

Is the open loop system asymptotically stable? Is this system controllable? Let us explore the possibility of finding a state feedback matrix K that makes the closed loop system asymptotically stable. Can one find a K that makes the closed loop eigenvalues be: $-1, -1, -2, -2, -2$? Explain all answers fully.

3. Consider the system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} u(t) \qquad y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t)$$

Show that the poles of the closed loop system can be placed arbitrarily if we use constant output feedback $u = Fy$.

4. Show by constructing a counterexample (single input single output system will suffice) that we <u>cannot</u> always asymptotically stabilize a controllable and observable linear time invariant system with constant output feedback.

5. Compute the gain matrix E such that the full-state observer for the system:

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix} x(t) \qquad y(t) = x_1(t)$$

has its poles at: $-1, -2, -3$.