1. Consider the system in controllability canonical form:

\[
\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)
\]

Find a constant state feedback $K$ that places the poles of the closed loop system at: $-1, -2 \pm i$.

2. a) Consider the system given below:

\[
\dot{x}(t) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(t)
\]

Is the system controllable (fully explain)? If yes, find the constant state feedback $K$ that places the poles of the closed loop system at: $-1, -1, -2$.

b) Consider now the augmented system:

\[
\dot{x}(t) = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} u(t)
\]

Is the open loop system asymptotically stable? Is this system controllable? Let us explore the possibility of finding a state feedback matrix $K$ that makes the closed loop system asymptotically stable. Can one find a $K$ that makes the closed loop eigenvalues be: $-1, -1, -2, -2, -2$? Explain all answers fully.
3. Consider the system:

\[
\dot{x}(t) = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u(t) \quad y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t)
\]

Show that the poles of the closed loop system can be placed arbitrarily if we use constant output feedback \( u = F y \).

4. Show by constructing a counterexample (single input single output system will suffice) that we cannot always asymptotically stabilize a controllable and observable linear time invariant system with constant output feedback.

5. Compute the gain matrix \( E \) such that the full-state observer for the system:

\[
\dot{x}(t) = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix} x(t) \quad y(t) = x_1(t)
\]

has its poles at: \(-1, -2, -3\).