1. Write down the inverse of the matrix

\[ A = \begin{pmatrix} \cos \theta & -\sin \theta \cos \alpha & \sin \theta \sin \alpha & a \cos \theta \\ \sin \theta & \cos \theta \cos \alpha & -\cos \theta \sin \alpha & a \sin \theta \\ 0 & \sin \alpha & \cos \alpha & d \\ 0 & 0 & 0 & 1 \end{pmatrix}. \]

(Do not use Matlab, Maple, Mathematica, etc.)

2. Given a prescribed position and orientation of the planar 3-bar manipulator of the second Exercise Set, there are two possible solutions to the inverse kinematics problem. If we add one more link (in such a way that the manipulator is still planar), how many solutions are there?

3. The figure shows a 2-bar planar manipulator with rotary joints. The second link is half as long as the first \((r_1 = 2r_2)\). The joint limits are:

\[
0 < \theta_1 < 180^\circ \\
-90^\circ < \theta_2 < 180^\circ.
\]

Sketch the approximate workspace (= the set of points which can be reached by the tip of the second link).
Problem Set 4
Solutions

1. \[ A^{-1} = \begin{pmatrix} \cos \theta & \sin \theta & 0 & -a \\ -\sin \theta \cos \theta & \cos \theta & \sin \theta & -d \sin \theta \\ \sin \theta \cos \theta & -\cos \theta & \sin \theta & -d \cos \theta \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

(Hint: If \( \begin{pmatrix} u \\ v \\ 0 \end{pmatrix} \) lies in SE(3, IR), \( \begin{pmatrix} u \\ v \end{pmatrix}^{-1} = \begin{pmatrix} u^T & -u v \end{pmatrix} \).)

2. Infinitely many.

3. \[ x_1 = 2 \]
   \[ x_2 = 1 \]

```math
\text{endpointset} = ()
\text{Do}[\text{endpointset} = \text{Join}[\text{endpointset}, 
    \{(x_1 \cdot \cos((y/40) + x) + x_2 \cdot \cos((y/40) + (-x/2 + (k/80) + (-x/2 + (k/80) + x))]) \}, \{y, 1, 40\}, \{k, 1, 120\}] 
\text{ListPlot}[\text{endpointset}]
```

The workspace looks like the shaded region.