Empirical evidence on the dependence of credit default swaps and equity prices

Debbie Dupuis, Professor of Management Sciences, HEC Montreal
Eric Jacquier, Associate Professor of Finance, HEC Montreal
Nicolas Papageorgiou, Associate Professor of Finance, HEC Montreal
Bruno Rémillard, Professor of Management Sciences, HEC Montreal *

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*Corresponding author: Nicolas Papageorgiou, Finance Department, HEC Montréal, 3000 Cote Sainte-Catherine, Montreal QC H3T 2A7, Canada, tel: (514) 340-7309, fax: (514) 340-5632. All the authors are at HEC Montréal and can be reached at firstname.lastname@hec.ca. The paper has greatly benefitted from the suggestions of the referee. The work was supported in part by the FQRSC, NSERC, IFM2, and the fonds pour la formation de chercheurs et l'aide à la recherche du Québec.

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Abstract

We investigate the common practice of estimating the dependence structure between credit default swap prices on multi-name credit instruments from the dependence structure of the equity returns of the underlying firms. We find convincing evidence that the practice is inappropriate for high-yield instruments and that it may even be flawed for instruments containing only firms within a sector. To do this, we model individual credit ratings by univariate continuous time Markov chains, and their joint dynamics by copulas. The use of copulas allows us to incorporate our knowledge of the modeling of univariate processes, into a multivariate framework. However, our test and results are robust to the choice of copula.

Key Words: Copula, Markov chain, credit risk, credit rating migration, Credit Default Swaps, Equity prices, Default time.

J.E.L. classification: G10, G20, G28, C16

1 Introduction

There are two main approaches in credit default modeling: the structural approach, first studied by Merton (1974), and the reduced-form approach, introduced by Jarrow and Turnbull (1995). In the first approach, default is triggered by the market value of the borrower's assets falling below its liabilities. In the second, default is modeled directly as an unexpected arrival.

The extension to the multivariate case, namely the calibration of the parameters governing joint defaults, poses many challenges. Regardless of the approach, one must model the *joint* evolution of either the credit ratings or the asset values of the firms. These challenges are unavoidable when one seeks to price multi-name credit instruments, e.g. multi-name credit default swaps (CDS). Modeling of the joint dependence is generally done through the use of copulas.[1]

Copulas have very interesting properties. Most noteworthy is the fact that the copula linking the marginals of the default times will be the same as the one linking the marginals of any other quantity proportional to the default times. Since the value of the assets of a firm are only observable at discrete and infrequent intervals, the challenge therefore lies in finding a suitable proxy proportional to the time to default. A constant maturity CDS written on the firm's assets clearly satisfies this condition and would appear to be the obvious choice of proxy to estimate the copulas and their parameters. Although the market for single name CDS contracts has expanded dramatically over the last decade, liquidity remains a concern for many issuers. It is therefore common practice (Crouhy et al. (2000)) to use the joint dependence of equity returns as a proxy for the joint dependence of the asset returns. While this practice has been questioned by some, we have found no work presenting strong evidence that it is flawed. In fact, the results in Mashal et al. (2003) endorse the practice. The inability to produce strong evidence against the practice may be due to the lack of suitable methodology to compare dependence structures. In this paper, we use the recent results in Rémillard and Scaillet (2008) to test and reject the equality of the two dependence structures. One distinctive feature of our test is that, unlike that of Mashal et al. (2003), it is not conditioned on a specific dependence structure, i.e. copula. This is especially important given the recent evidence that the choice of copula can greatly affect pricing, see e.g., Berrada et al. (2006).

In comparing copulas of returns on CDS for several firms to those of the underlying stock returns, one faces a practical difficulty. For many firms, especially the least volatile, there are many periods with no changes in CDS prices. While this reflects the illiquidity of the market, the presence of zero-returns violates the distributional assumption of continuous returns. These zero-returns lead to upward biases in the estimates of dependencies and we remove them.[2] We create two very different portfolios of volatile securities in order to

[1] One can also make the default probabilities dependent on background variables as in Duffie and Gârleanu (2001) or use the infectious correlation approach proposed by Jarrow and Yu (2001).

[2] Note that this leads to the elimination of the least volatile as well as the most illiquid securities. One

best exploit the remaining data. First, we focus on the automobile sector where firms exhibit homogeneous risk exposures albeit with volatile assets. Our first portfolio includes the largest two US car manufacturers and three of the largest auto part producers. The different leverage of assets and equity has often been an argument against the use of (dependence structure of) equity returns as a proxy for (the dependence structure of) asset returns. While many may be willing to use equity returns as a proxy for high-grade issuers, they may object when it comes to low-grade borrowers. So, our second portfolio consists of five high-yield issuers from different sectors, i.e., with very heterogeneous credit exposure. Although the fundamental risks differ across these five companies, all are non-investment grade issuers whose equity and CDS returns exhibited high volatility in 2005.

It is important to note that many practitioners also use implied correlations from CDS indices such as the CDX.IG index as an input for pricing models. Although useful for pricing tranches on the actual indices, these implied correlations are not applicable to the pricing of credit derivatives on a specific basket of names (especially if the constituents of the basket and the index are not alike). The potential error from making such an assumption can be substantial but the topic is not pursued here.

The rest of the paper is as follows. In Section 2, we discuss dependence within a structural model. We outline the test for the equality of dependence structures in Section 3. In Section 4 we present the data in details, and give the results of the analysis. Section 5 offers concluding remarks.

2 Firm value, equity returns and default dependence

In this section, we shed some light on some model-induced links between the quantities studied in this paper.

In the Merton (1974) model, the value of the firm's assets is modeled as geometric Brownian motion and is an increasing function of the equity price. The Merton approach assumes that default occurs before time horizon T if the value of the firm's assets crosses a specified barrier, an event which can only be observed at time T. The value of a CDS, which is a contingent claim that pays out when/if the default event occurs, must therefore be a decreasing function of the value of the firm's assets.

Consider the case of multiple firms. As copulas are invariant under monotone increasing transformations, it is clear that the copula associated with the values of the firms' assets is the same as the copula associated with equity prices. Similarly, it follows that the copula associated with the (negative) CDS returns is the same as the copula associated with the values of the firms' assets. The copula associated with the equity prices (a normal copula in the case of the Merton model) is thus the same as the copula associated with the (negative) CDS returns by transitivity. Note that the above reasoning can be applied to any structural

can argue that the least volatile assets are probably of very low credit risk, and therefore not a priority in the evaluation of default risk.

model.

Further note that in the Merton framework, the timing of the default is not modeled, just the joint probability of default within a given horizon. This represents the main drawback of the approach as a model for the time to default is necessary to price CDO that have, for example, Waterfall provisions.

The test that we propose can be seen as a test of an extended Merton's model (accounting for the timing of default), where we assume that default is triggered when the value of the firm, not necessarily modeled by a geometric Brownian motion, is below some barrier, and where it is also assumed that the value of the firm is an increasing function of its equity. However, even in the simple setting of joint firm values modelled by correlated geometric Brownian motions, the dependence structure of the default times is not the same as the dependence structure between the values of the firms. This can be seen in the simple case of two firms, using the results of Patras (2006).

3 Methodology

This section provides the main result used in the empirical work. Namely, we show how to compute an asymptotic p-value for the test of equality of two copulas.

Denote $S_t = (S_{1t}, \ldots, S_{dt})$, $t = 1, \ldots, n$ the equity prices of d firms over n periods, and $P_t = (P_{1t}, \ldots, P_{dt})$, $t = 1, \ldots, m$ represent the value of a credit derivatives, here a CDS of those firms, over m similar periods.[3] The goal is to compare the copula of the returns $R_t^{(S)} = \log(S_t/S_{t-1})$ of the equity prices to the copula of the returns $R_t^{(P)} = -log(P_t/P_{t-1})$ of the credit derivatives.

It is now well-known that the structure of dependence of d risk factors $X_1, \ldots, X_d$ does not depend on the marginal distribution functions $F_1, \ldots, F_d$. It only depends on the so-called copula C defined by the implicit relation between the joint distribution and its marginals. Namely,

$$C(F_1(x_1), \ldots, F_d(x_d)) = P(X_1 \leq x_1, \ldots, X_d \leq x_d),$$

as first defined by Sklar (1959).

Choosing marginals is not an easy task, but choosing a copula is even more daunting. Until lately very little quantitative assistance was available and only ad hoc guidelines existed. As shown in Berrada et al. (2006), different copulas can result in dramatically different dependence structures. Quite recently, some statistical procedures were proposed to address the problem of goodness-of-fit to copula families, e.g., Fermanian (2005), Genest et al. (2006) and Genest and Rémillard (2005), but these tests would be a roundabout way

[3] We do not assume that $n = m$. We only need the maturity of the credit instrument to be constant over time. This insures a reasonably random sample of returns and prevents maturity effects in the time series of returns.

to answer the questions raised here. This is because we do not seek to model the specific dependence structure, i.e. copula, of the CDSs or equity returns, but rather to compare them in a manner robust to the choice of copula. We now describe a procedure to do this put forward in Rémillard and Scaillet (2008).

Let C and D be two copula functions. The following procedure tests the null hypothesis $H_0 : C = D$. Suppose that $X_1, \ldots, X_{n_1}$ is a random sample from a distribution with continuous margins $F_1, \ldots, F_d$ and copula C, and $Y_1, \ldots, Y_{n_2}$ is a random sample from a distribution with continuous margins $G_1, \ldots, G_d$ and copula D.

To test the hypotheses

$$H_0: \ C = D \text{ vs } H_1: \ C \neq D,$$

Rémillard and Scaillet (2008) use the Cramér-von Mises statistic

$$S_{n_1,n_2} = \int_{[0,1]^d} \mathcal{E}^2_{n_1,n_2}(u)du,$$

where

$$\mathcal{E}_{n_1,n_2} = (C_{n_1} - D_{n_2})/\sqrt{\frac{1}{n_1} + \frac{1}{n_2}},$$

and the empirical copulas C_{n_1}, D_{n_2} are defined, for any $u = (u_1, \ldots, u_d) \in [0,1]^d$ by

$$C_{n_1}(u) = \frac{1}{n_1}\sum_{i=1}^{n_1} \mathbb{I}(U_{i,n_1} \leq u) = \frac{1}{n_1}\sum_{i=1}^{n_1}\prod_{l=1}^{d} \mathbb{I}(U_{il,n_1} \leq u_l),$$

and

$$D_{n_2}(u) = \frac{1}{n_2}\sum_{i=1}^{n_2} \mathbb{I}(V_{i,n_2} \leq u) = \frac{1}{n_2}\sum_{i=1}^{n_2}\prod_{l=1}^{d} \mathbb{I}(V_{il,n_2} \leq u_l),$$

with

$$\begin{aligned} U_{il,n_1} &= \text{rank}(X_{il})/(n_1+1), \quad 1 \leq i \leq n_1, \\ V_{il,n_2} &= \text{rank}(Y_{il})/(n_2+1), \quad 1 \leq i \leq n_2, \end{aligned}$$

and where $\text{rank}(X_{il})$ is defined to be the rank of X_{il} with respect to $X_{1l}, \ldots, X_{n_1,l}$, for any $l \in \{1, \ldots, d\}$. The value of the statistic is

$$\begin{aligned} S_{n_1,n_2} \;=\; & \left(\frac{1}{n_1} + \frac{1}{n_2}\right)^{-1} \times \left\{ \frac{1}{n_1^2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_1}\prod_{s=1}^{d}(1 - U_{is,n_1} \vee U_{js,n_1}) \right. \\ & -\frac{2}{n_1 n_2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\prod_{s=1}^{d}(1 - U_{is,n_1} \vee V_{js,n_2}) \\ & \left. +\frac{1}{n_2^2}\sum_{i=1}^{n_2}\sum_{j=1}^{n_2}\prod_{s=1}^{d}(1 - V_{is,n_2} \vee V_{js,n_2}) \right\}, \end{aligned}$$

where $a \vee b$ stands for $\max(a,b)$.

Because of the intricate limiting distribution of the stochastic process $\mathcal{E}_{n_1,n_2}$, and also because C and D are unknown, the computation of p-values in Rémillard and Scaillet (2008) is based on the multiplier technique. To describe how it works, suppose that for any $k \in \{1,\ldots,N\}$, $\xi_1^{(k)},\ldots,\xi_{n_1}^{(k)}$, $\zeta_1^{(k)},\ldots,\zeta_{n_2}^{(k)}$ are independent standard Gaussian variables.

Set

$$\begin{aligned}
\hat{\alpha}_{n_1}^{(k)}(u) &= \frac{1}{\sqrt{n_1}}\sum_{i=1}^{n_1}\left(\xi_i^{(k)}-\bar{\xi}^{(k)}\right)\mathbb{I}(U_{i,n_1}\le u),\\
\hat{\gamma}_{n_2}^{(k)}(u) &= \frac{1}{\sqrt{n_2}}\sum_{i=1}^{n_2}\left(\zeta_i^{(k)}-\bar{\zeta}^{(k)}\right)\mathbb{I}(V_{i,n_2}\le u),
\end{aligned}$$

and for any $l \in \{1,\ldots,d\}$, define

$$\hat{\beta}_{l,n_1}^{(k)}(u_l) = \frac{1}{\sqrt{n_1}}\sum_{i=1}^{n_1}\left(\xi_i^{(k)}-\bar{\xi}^{(k)}\right)\mathbb{I}(U_{il,n_1}\le u_k),$$

$$\hat{\delta}_{l,n_2}^{(k)}(u_l) = \frac{1}{\sqrt{n_2}}\sum_{i=1}^{n_2}\left(\zeta_i^{(k)}-\bar{\zeta}^{(k)}\right)\mathbb{I}(V_{il,n_2}\le u_k).$$

Next, for any $l \in \{1,\ldots,d\}$, let

$$\widehat{\partial_{u_l}C_{n_1,h_1}}(u) = \frac{C_{n_1}(u+h_1e_l)-C_{n_1}(u-h_1e_l)}{2h_1}$$

and

$$\widehat{\partial_{u_l}D_{n_2,h_2}}(u) = \frac{D_{n_2}(u+h_2e_l)-D_{n_2}(u-h_2e_l)}{2h_2},$$

where e_l is the l-th column of the $d \times d$ identity matrix.

Finally, for all $u \in [0,1]^d$, and for all $k \in \{1\ldots,N\}$, set

$$\hat{\mathcal{C}}_{n_1,h_1}^{(k)}(u) = \hat{\alpha}_{n_1}^{(k)}(u) - \sum_{l=1}^{d}\hat{\beta}_{l,n_1}^{(k)}(u_l)\widehat{\partial_{u_l}C}_{n_1,h_1}(u),$$

$$\hat{\mathcal{D}}_{n_2,h_2}^{(k)}(u) = \hat{\gamma}_{n_2}^{(k)}(u) - \sum_{l=1}^{d}\hat{\delta}_{l,n_2}^{(k)}(u_l)\widehat{\partial_{u_l}D}_{n_2,h_2}(u),$$

$$\hat{\mathbb{E}}_{n_1,n_2}^{(k)} = \sqrt{\frac{n_2}{n_1+n_2}}\hat{\mathcal{C}}_{n,h_N}^{(k)} - \sqrt{\frac{n_1}{n_1+n_2}}\hat{\mathcal{D}}_{m,h_N}^{(k)},$$

and

$$\hat{S}^{(k)}_{n_1,n_2} = \int_{[0,1]^d} \left\{ \hat{\mathbb{E}}^{(k)}_{n_1,n_2} \right\}^2 (u) du, \qquad k \in \{1, \dots, N\}.$$

According to Theorem 2.1 of Rémillard and Scaillet (2008), if $h_i = n_i^{-1/2}$, $i = 1, 2$ and if $\min(n_1, n_2) \to \infty$ in such a way that $n_1 \big/ (n_1 + n_2) \to \lambda \in (0, 1)$, under regularity conditions on C and D, an approximate p-value for S_{n_1,n_2} is

$$\frac{1}{N} \sum_{k=1}^{N} \mathbb{I} \left(\hat{S}^{(k)}_{n_1,n_2} > S_{n_1,n_2} \right). \tag{1}$$

The closed-form expression of $\hat{S}^{(k)}_{n_1,n_2}$ used for the computations can be found in Rémillard and Scaillet (2008).

4 Data Analysis

4.1 The data

We analyze the daily prices of 5-year maturity CDS for 2005 of three different portfolios: an automobile industry portfolio, a high-yield diverse-sector portfolio and a portfolio if high grade financial institutions.

The first portfolio consists of five firms in the automobile sector: Dana, Ford, GM, Lear, and Visteon.[4] The two left plots in Figure 1 show CDS prices and equity prices for the five firms. The corresponding equity returns are shown on the right.

We use the automobile sector because of its high volatility in 2005, due to severe foreign competition. The five firms include the largest two US car manufacturers and three of the largest auto part producers, all of which had a very difficult 2005. This is reflected in the increasing CDS and declining equity prices in the left plots of Figure 1. Among the many news that rocked the industry that year, two specific events stand out. First, Standard and Poor's downgraded to junk over \$290 billion of GM and Ford bonds on May 6^{th}. The downgrades were the largest ever of their kind and they had a massive impact on CDS, equity prices and their volatilities (see around observation 90 on the plots). Second, Delphi Corporation filed for Chapter 11 on October 8^{th}. Although Delphi is not in our sample - CDS and equity prices are unavailable after its Chapter 11 filing - its bankruptcy had a huge impact on the entire sector. Delphi is the largest supplier of automotive systems, components and parts to GM and therefore their insolvency fueled speculations that their competitors,

[4]The data were purchased from GFI. Recall that CDS returns are defined as minus their change in log-price such that an increase in price is seen as a negative CDS return. CDS returns defined in this way will have positive dependence with the usually defined equity returns.

Dana, Visteon and Lear, could follow suit. This uncertainty shows up in the high volatility of the CDS returns in the last few months of 2005. The market's pessimism proved well founded as Dana corporation filed for Chapter 11 early in 2006.

The second portfolio consists of five firms with very volatile CDS and equity returns as well as heterogeneous credit risk. The left plots of Figure 2 show the CDS and equity prices for Six Flags Inc, ArvinMeritor, Polyone Corp, Maytag, and Blockbuster Inc. The corresponding equity returns are on the right.

These five high-yield firms represent a heterogeneous credit exposure. Six Flags is in the General Entertainment industry, ArvinMeritor in Consumer Goods, Maytag in Consumer Staples, Blockbuster in Services and Polyone is in the Basic Materials sector. Although the fundamental risks differ across these five companies, they all share the characteristic of being non-investment grade issuers whose equity and CDS returns exhibited high volatility in 2005. It is interesting to note in Figure 2 the spike in the CDS prices in May (around observation 90) for all five companies. It appears that, although the operations of these companies are not directly related to Ford or GM, the downgrading of the latter affected the entire credit market because of its magnitude. It appears however that the equities did not respond as uniformly as the CDS to this credit event. The bottom-left plot in Figure 2 shows substantial drops only for the equity prices of Maytag and ArvinMeritor. No other specific event seems to have had a significant impact on the entire high-yield portfolio, the individual CDS and equity prices responding primarily to company or sector related events during the year.

The third portfolio consists of five financial institutions with low volatile CDS and equity returns and high credit ratings. The left plots of Figure 3 show the CDS and equity prices for American Investment Group, Citigroup, Goldman Sachs, JP Morgan, and Merril Lynch. The corresponding equity returns are on the right.

4.2 Results for the Auto Industry Portfolio

After removing missing values in the database, 133 valid CDS returns and 223 equity returns remain for the year 2005. We compute the Cramer-von Mises test and its p-value described in Section 3 as per (1) using $N = 1000$ iterations. Table 1 shows the p-values for all the bivariate copulas of firms in the portfolio. Tables 2 and 3 show the p-values for all three-, four-, and five-dimensional copulas, respectively.

Based on the p-values listed in these three tables, in many cases the null hypothesis of equality of the equity and the CDS copulas cannot be rejected. At least statistically, one cannot reject the use of the equities as a proxy for the underlying assets.[5]

The p-values listed in these three tables exhibit a pattern. The p-values tend to decrease (toward rejection) with the dimensionality of the copula. This may be because

[5]Of course, as all p-values, these are subject to the Lindley-Smith critique, and the difficulties arising from the reporting of multiple test results. For example, here a Bonferroni adjustment would not change the results since we fail to reject the null.

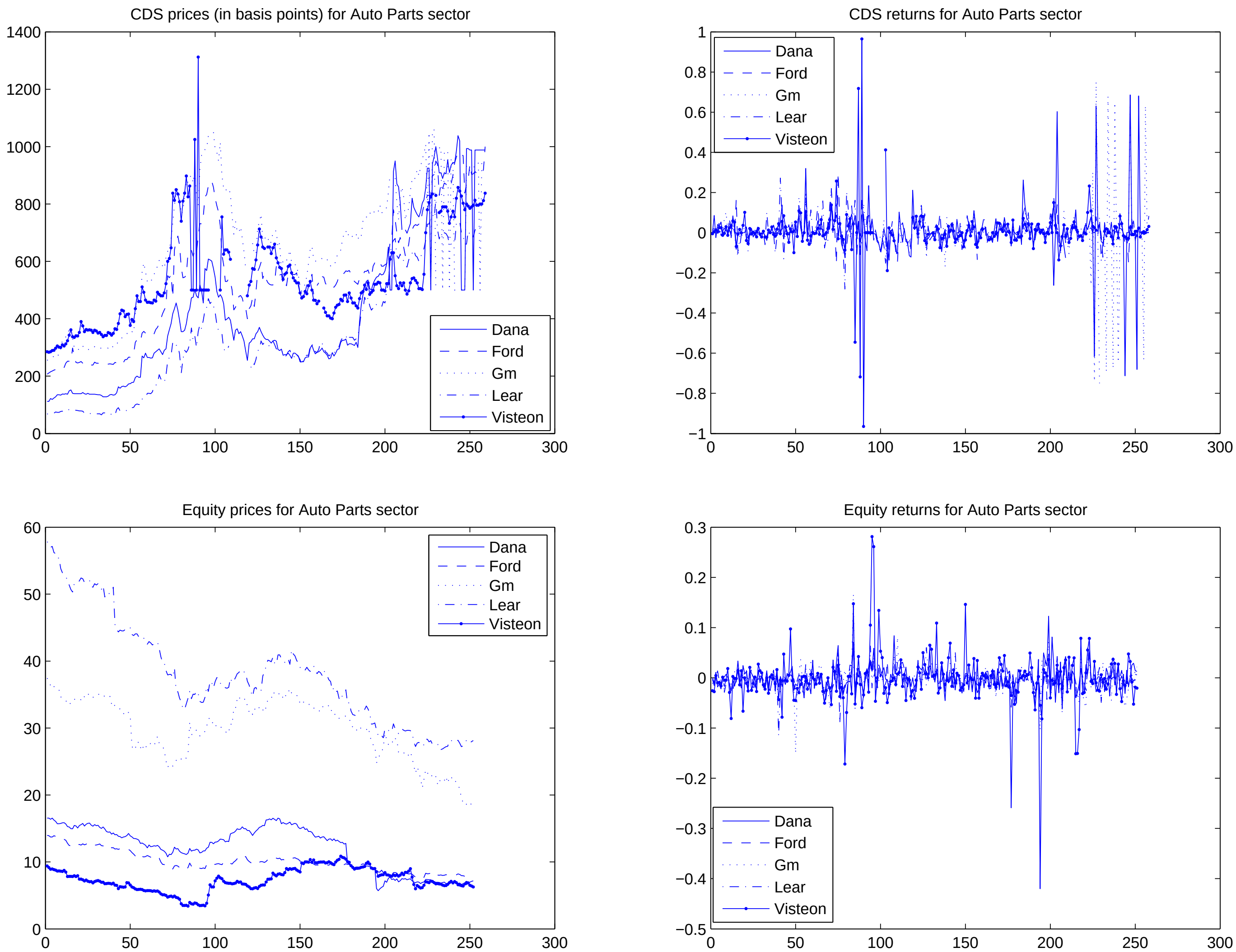

Figure 1: Daily CDS and Equity, prices (left) and returns (right), five automotive sector firms, 2005.

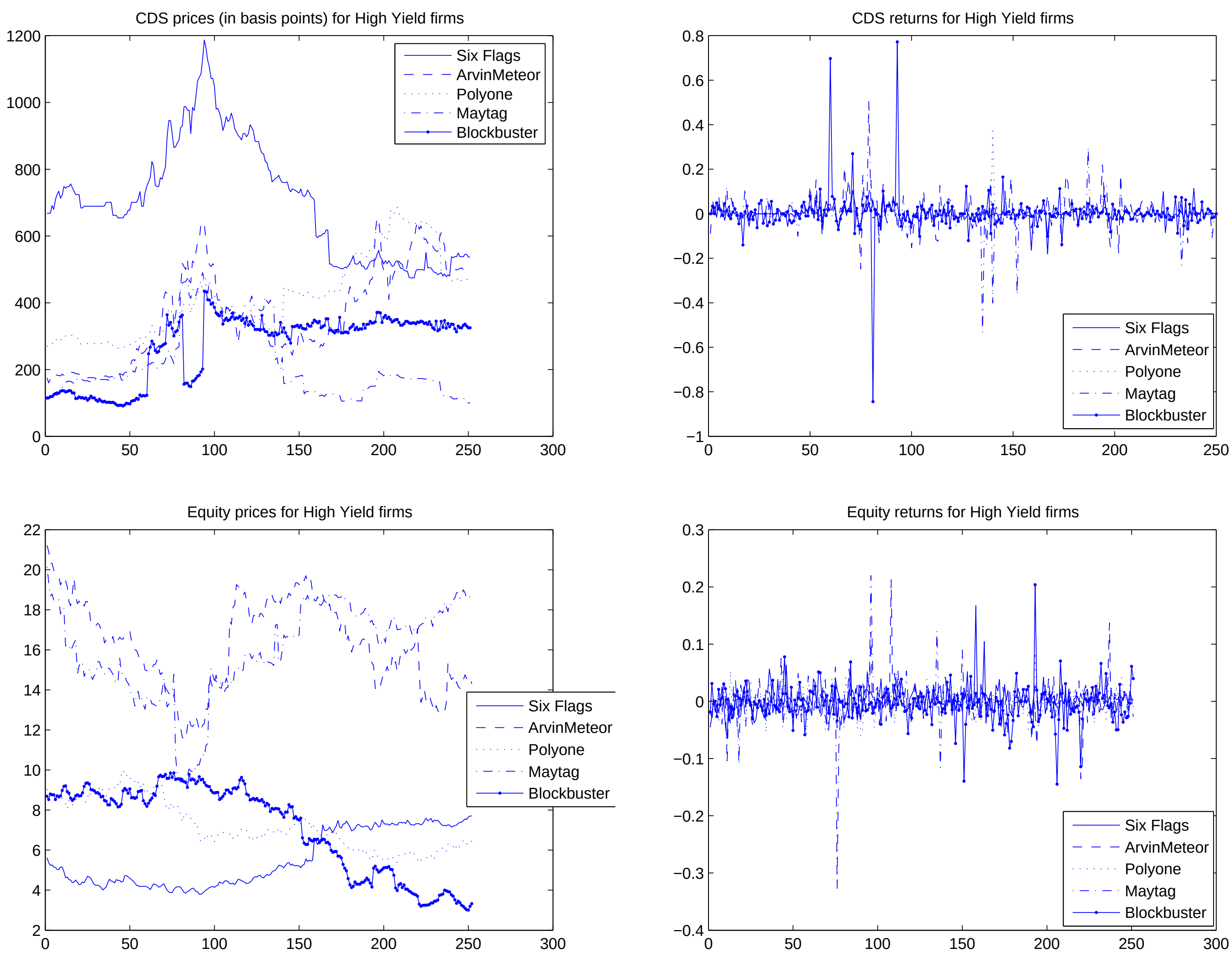

Figure 2: Daily CDS and Equity, prices (left) and returns (right), five high-yield firms, 2005.

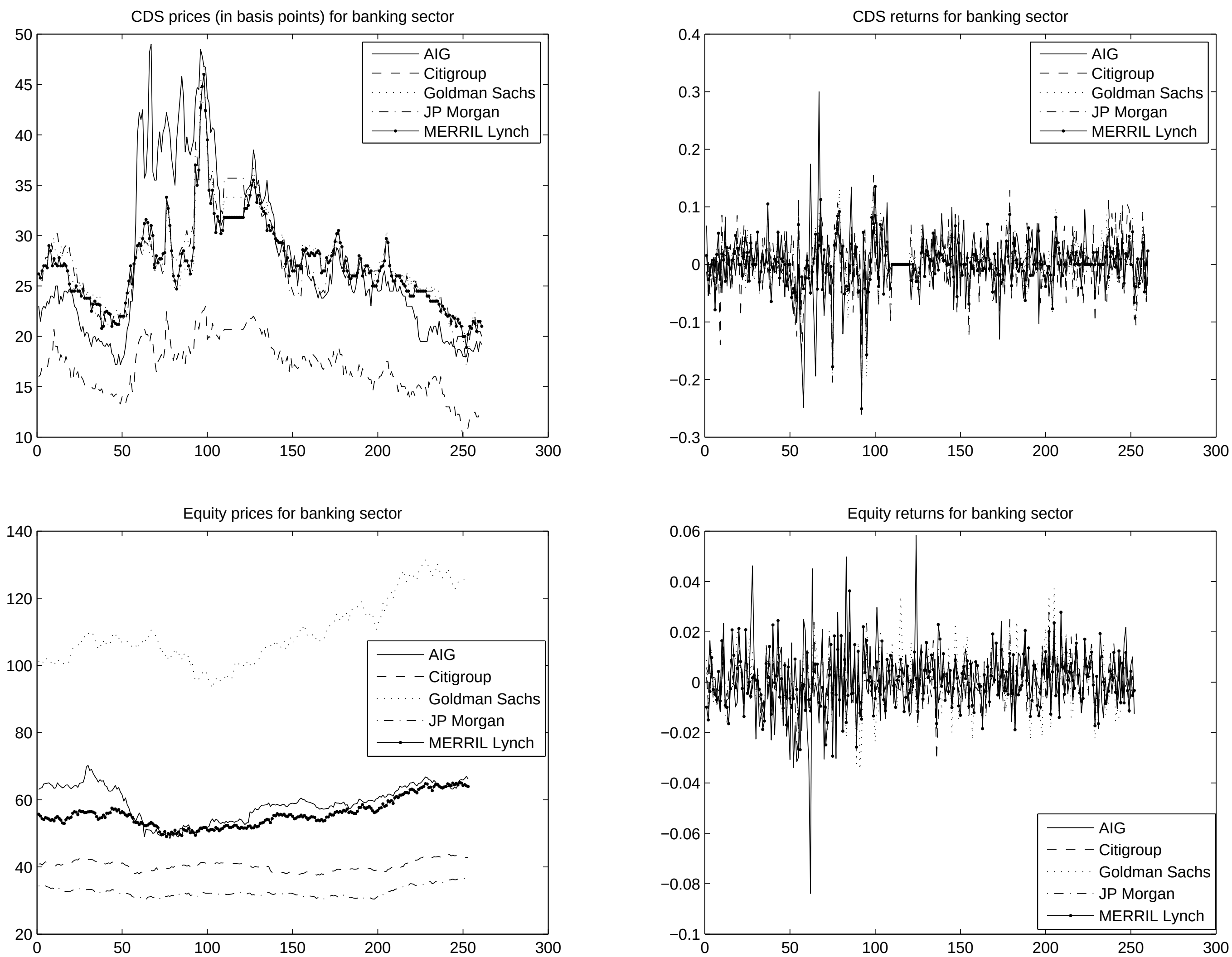

Figure 3: Daily CDS and Equity, prices (left) and returns (right), five banks, 2005.

as we increase dimensionality, a single stock, e.g. Ford, appears more often in the possible combinations. Looking at all but Ford, the "credit shock" induced by the Delphi bankruptcy was extreme. The Delphi bankruptcy had a direct effect on 1) GM its main client and 2) the other suppliers in the industry, but a lesser effect on Ford who is not a client. Since the credit shock makes up most of the equity movements of the four non-Ford stocks, the CDS and equity copulas can look similar. In contrast, for Ford, while the CDS shock was high, its equity was also driven by other non credit related factors. This could explain the equity copula being different from the CDS copula when Ford is included.

Table 1: P-values for test of equality of CDS and equity bivariate copulas, auto industry portfolio.

	Ford	GM	Lear	Visteon
Dana	0.05	0.50	0.55	0.78
Ford		0.31	0.12	0.06
GM			0.50	0.86
Lear				0.90

Table 2: P-values for test of equality of CDS and equity trivariate copulas, auto industry portfolio.

Dana	Ford	GM	0.10
Dana	Ford	Lear	0.06
Dana	Ford	Visteon	0.03
Dana	GM	Lear	0.43
Dana	GM	Visteon	0.87
Dana	Lear	Visteon	0.75
Ford	GM	Lear	0.13
Ford	GM	Visteon	0.13
Ford	Lear	Visteon	0.13
GM	Lear	Visteon	0.89

Given the profound impact of the May 2005 events on the credit market, one might think that they changed the subsequent dependence structure of the CDS returns of firms. In order to examine the relative stability of this dependence structure over time, we consider the CDS returns of the auto industry portfolio before and after the May 5th double downgrade of GM and Ford. Although there is a dramatic change in the levels of the associated CDS spreads, we find that there is no significant change in the dependence structure between the portfolio constituents before and after the downgrades. The p-value of the test is 0.64.

Table 3: P-values for test of equality of CDS and equity four- and five-variate copulas, auto industry portfolio.

Dana	Ford	GM	Lear		0.07
Dana	Ford	GM	Visteon		0.07
Dana	Ford	Lear	Visteon		0.02
Dana	GM	Lear	Visteon		0.68
Ford	GM	Lear	Visteon		0.11
Dana	Ford	GM	Lear	Visteon	0.04

4.3 Results for the High-yield Portfolio

After removing missing values, 148 CDS and 222 equity returns remain for the year 2005. Table 4 shows the p-values of the Cramer-von Mises test for the bivariate copulas using $N = 1000$ iterations. The results are drastically different from the auto industry portfolio. The null hypothesis is massively rejected for every bivariate combination of these five firms. So, the use of the equities as a proxy for these firms is inappropriate, at least from a statistical standpoint. Table 4 does not report results for higher dimensional copulas. As expected, if the hypothesis of equal copulas is rejected for all bivariate combinations, the hypothesis of equal copulas when considering three of more firms is also firmly rejected. All p-values are close to 0.

Table 4: P-values in % for test of equality of CDS and equity bivariate copulas, high-yield portfolio.

	ArvinMeritor	Polyone Corp	Maytag	Blockbuster
Six Flags Inc	0.000	0.000	0.000	0.000
ArvinMeritor		0.002	0.028	0.006
Polyone Corp			0.001	0.000
Maytag				0.023

The fact that copulas for equity and credit default swaps are different for our basket of high-yield firms has important implications on the pricing of multi-name credit products. Namely, the heterogeneity of the risk exposures, coupled with the diversity in financial leverage across these five firms cause the credit and equity markets to exhibit different levels of sensitivity. Given the increasing number of Collateralized Bond Obligations used to securitize high-yield debt portfolios, the choice of equity as a proxy for estimating dependence between default times seems clearly inappropriate.

4.4 Results for the High-grade Portfolio

In order to contrast the results obtained with the auto parts and High-yield portfolio, we also test for equality between CDS and equity returns on a third portfolio. This portfolio is comprised of five financial institutions that have have a credit rating of at least AA, and whose equity and CDS returns were considerably less volatile than those of the two previous portfolios.

After removing missing values, 134 CDS and 244 equity returns remain for the year 2005. The p-values of the Cramer-von Mises test for the five-dimensional copulas copulas using $N = 1000$ iterations is 0.175. Based on this p-values the null hypothesis of equality of the equity and the CDS copulas cannot be rejected.

These results contrast those found for the high-yield portfolio and at first might appear counter-intuitive. Generally, we would expect the equity returns of firms that are closer to financial distress to be more sensitive (and hence correlated) with the the CDS market. However, it is important to remember that we are not studying the dependence between CDS prices and the equity returns for each issuer; what we are investigating is whether the dependence structure between the CDS returns of the five firms is the same as the dependence structure between the equity returns of the same five firms. The results indicate that the dependence between the CDS prices of the firms in the high-grade portfolio is not statistically different than the dependence structure between the equity returns on the same portfolio. The results are also coherent with the extended Merton's model, as discussed in section 2.

As we observed in the previous section, the results for the high-yield portfolio indicate that the dependence structures were statistically different for the CDS and equity portfolios. This can in part be attributed to the high volatility of both the daily CDS and equity returns as well as the heterogeneity of the firms in high-yield portfolio both in terms of industry and financial leverage. That could also be due to the fact that for riskier assets, the extended Merton's model is not adequate.

4.5 Choice of Copula and Impact on Pricing

We have presented and used a test that shows that the dependence structures for CDS and equity returns can be statistically very different. This statistical evidence is consistent with qualitative evidence. Consider the scatter plots of normalized returns for CDS and equity, respectively, for two firms. For example, Figure 4 shows the bivariate normalized rank plots for three pairs of firms. The left and right plots show the dependence structure for CDS and equity, respectively. The top two plots show the dependence for Dana and Lear, a pair for which the p-value of the test of equality of copulas was 0.55. This lack of statistical evidence against equality is consistent with the graphical evidence: the two plots are simply not very different. In contrast, the middle and bottom plots show two pairs which exhibited strong statistical evidence against the null of equality with p-values essentially equal to zero. These plots show indeed that the CDS and equity dependence structures are qualitatively different.

The test used here does not hinge on a specific family of copulas or pricing model. One may argue that this still does not address directly the economic impact of selecting the wrong copula on the pricing of multi-name securities. To measure an economic impact requires the formulation of a pricing model. Berrada et al. (2006) for example, provide empirical evidence that the choice of the copula greatly affects the pricing of joint default risk and show the impact on the pricing of n^{th} to default credit default swaps. To do this, they model the joint dynamics of credit ratings of several firms. Namely, individual credit ratings are modeled by a univariate continuous time Markov chain, and the joint dynamics by specific copulas. Namely, they use the Normal, Student, Gumbel, Clayton and Frank copulas to price multi-name credit derivatives. A sample of the results is shown in Table 5 below. Clearly, the different copulas can produce very different prices.

Table 5: Default premia - basis points, for the n^{th} to default, different copula families

Model	1^{st}	2^{nd}	3^{rd}
Clayton	151	25	3
Frank	150	29	5
Gumbel	111	36	16
Gaussian	137	32	9
Student	110	39	17

5 Conclusion

We have presented empirical evidence that the dependence structures for CDS returns and equity returns can be markedly different and sensitive to the composition of the portfolio. Our results contradict previous findings, such as those in Mashal et al. (2003) who, with a likelihood-based test, found that equity returns were an efficient proxy for asset returns. A distinctive feature of our test is that, unlike that of Mashal et al. (2003), it does not hinge on a specific dependence structure, i.e. copula. The test we use does not rely on the assumption of a particular dependence structure for each return series. The test is therefore much more robust as it is not subject to model error, with respect to the choice of copula. In that respect, it is a natural complement of model-specific approaches that document the economic implications of the choice of one copula vs another.

It is important to note that given the limited scope of the empirical study, it is possible that the source of rejection/nonrejection has to do with sample selection, not necessarily the level of risk of the obligors. Let us recall that the high yield portfolio is fairly heterogeneous, while the investment-grade portfolio consists of all financial companies. With the availability of more liquid CDS prices it would definitely be interesting to conduct a more comprehensive

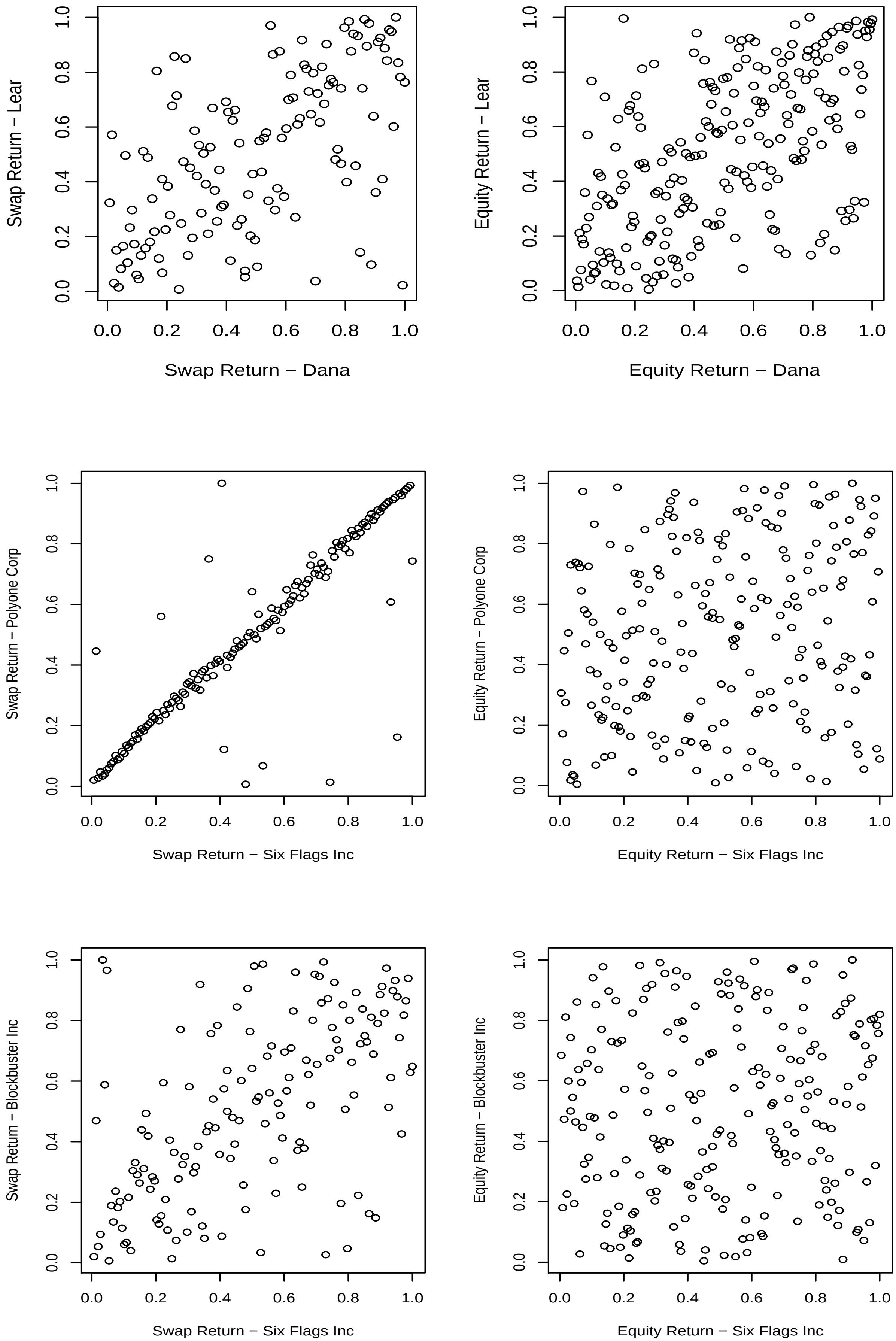

Figure 4: Bivariate normalized rank plots, three pairs of firms.

study using a larger sample (at least the scope of the 125 name CDX index) and offer a more definitive empirical evidence.

Among other directions for future research is the effect of possible lead-lag relationships between equity and CDS prices. For example, Acharya and Johnson (2007) study insider trading in the CDS market and identify the presence of advanced information revelation. However, they find no evidence that the degree of asymmetric information between the CDS and equity markets, affects prices or liquidity in either market. Consigli (2005) investigates the lead-lag relationships between the two markets with Granger causality tests. He documents a moderate leading effect of stock market volatility on CDS spreads, possibly over several days. It would be interesting to test whether our conclusions concerning the dependencies are robust to temporally aggregated data. For the moment, there is probably insufficient weekly CDS data available for a robust statistical test to give good precision.

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