Computable General Equilibrium Models for the Analysis of Energy and Climate Policies

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Abstract

This chapter is a simple, rigorous, practically-oriented exposition of computable general equilibrium (CGE) modeling. The general algebraic framework of a CGE model is developed from microeconomic fundamentals, and employed to illustrate (i) how a model may be calibrated using the economic data in a social accounting matrix, (ii) how the resulting system of numerical equations may be solved for the equilibrium values of economic variables, and (iii) how perturbing this equilibrium by introducing price or quantity distortions facilitates analysis of the economy-wide impacts of energy and climate policies.

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1 Introduction

Walrasian general equilibrium prevails when supply and demand are equalized across all of the interconnected markets in the economy. Computable general equilibrium (CGE) models are simulations that combine the abstract general equilibrium structure formalized by Arrow and Debreu with realistic economic data to solve numerically for the levels of supply, demand and price that support equilibrium across a specified set of markets.

CGE models have emerged as a standard pseudo-empirical tool for policy evaluation. Their strength lies in their ability to prospectively elucidate the character and magnitude of the economic impacts of energy and environmental policies. Perhaps the most important of these applications is the analysis of measures to reduce greenhouses gas (GHG) emissions—principally carbon dioxide (CO_2) from the combustion of fossil fuels. In the decade since the survey by Bhattacharyya (1996) there has been an explosion in this literature on this topic, with over 150 articles in peer-reviewed books and journals and an even greater number of working papers and technical reports. GHG mitigation policies can incorporate a range of instruments ranging from taxes and subsidies to income transfer schemes to quotas on the carbon content of energy goods. The fact that energy is an input to virtually every economic activity, coupled with the limited possibilities to substitute other commodities for fossil fuels, imply that these policies' effects will ripple through multiple markets, with far larger consequences than energy's small share of national income might suggest. This phenomenon is the central motivation for the general equilibrium approach.

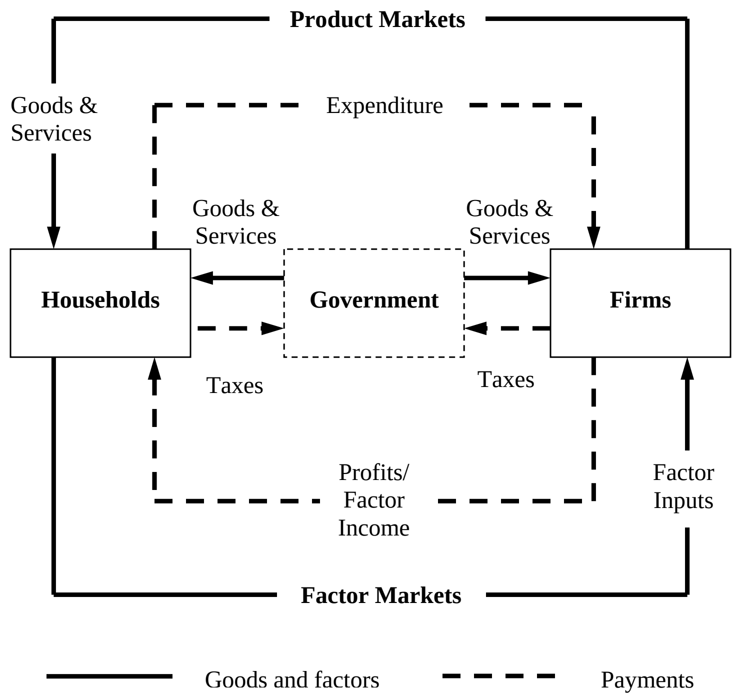
But, notwithstanding their popularity, CGE models continue to be viewed in some quarters as a "black box", whose complex internal workings obfuscate the linkages between their outputs and features of their input data, algebraic structure, or method of solution, and worse, allow questionable assumptions to be hidden within them that end up driving their results.[1] This chapter addresses this presumption by opening up the black box to scrutiny, elucidating the simple algebraic framework shared by all CGE models (regardless of their size or apparent complexity), the key features of their data base and the calibration methods used to incorporate this information into their algebraic framework, and the numerical techniques used to solve the resulting mathematical programming problem.

To accomplish all this in a single article it will be necessary to move beyond a traditional survey of the modeling literature, which is necessarily broad, and of which many examples have recently been published (e.g., Conrad, 1999, 2001; Bergman, 2005). I take the different approach of concisely synthesizing material which is usually spread across a broad cross-section of the energy economics, policy and modeling literatures.[2] Taking a cue from earlier work (Shoven and Whalley, 1984; Kehoe and Kehoe, 1995; Kehoe, 1998a) I employ the microeconomic foundations of consumer and producer maximization to develop a framework that both straightforward and sufficiently general to represent a CGE model of arbitrary size and dimension. This framework is then used to demonstrate in a practical fashion how a social accounting matrix may be used to calibrate the coefficients of the model equations, how the resulting system of numerical equations is solved, and how the equilibrium thus solved for may be perturbed and the results used to analyze the economic effects of various types of energy policies. The result is a transparent and systematic, yet also theoretically coherent and reasonably comprehensive, introduction to the subject of CGE modeling.

The plan of the chapter is as follows. Section 2 introduces the circular flow of the economy, and demonstrates how it serves as the fundamental conceptual starting point for Walrasian equilibrium theory that underlies a CGE model. Section 3 presents a social accounting matrix and illustrates how the algebra of its accounting rules reflects the conditions of general equilibrium. Section 4 develops these relationships into a workable CGE model using the device of the constant elasticity of substitution (CES) economy in which households have CES preferences and firms have CES production technology. Section 5 uses the CES economy to illustrate how models are numerically calibrated, while 6 discusses issues which arise in solving CGE models. Section 7 explains how CGE models are used to analyze energy and climate policies. An application is presented in section

8, in which the CES economy is employed to elucidate the impact of limiting CO_2 emissions in the U.S. Section 9 offers a brief summary and concluding remarks.

Figure 1: The Circular Flow



2 Foundations: The Circular Flow and Walrasian Equilibrium

The fundamental conceptual starting point for a CGE model is the circular flow of commodities in a closed economy, shown in Figure 1. The main actors in the diagram are households, who own the factors of production and are the final consumers of produced commodities, and firms, who rent the factors of production from the households for the purpose of producing goods and services that the households then consume. Many CGE models also explicitly represent the government, but its role in the circular flow is often passive: to collect taxes and disburse these revenues to firms and households as subsidies and lump-sum transfers, subject to rules of budgetary balance that are specified by the analyst. In tracing the circular flow one can start with the supply of factor inputs (e.g. labor and capital services) to the firms and continue to the supply of goods and services from the firms to the households, who in turn control the supply of factor services. One may also begin with payments, which households receive for the services of labor and capital provided to firms by their primary factor endowment, and which are then used as income to pay producing sectors for the goods and services that the households consume.

Equilibrium in the economic flows in Figure 1 results in the conservation of both product and value. Conservation of product, which holds even when the economy is not in equilibrium, reflects the physical principle of material balance that the quantity of a factor with which households are endowed, or of a commodity that is produced by firms, must be completely absorbed by the firms or households (respectively) in the rest of the economy. Conservation of value reflects the accounting principle of budgetary balance that for each activity in the economy the value of expenditures on inputs (i.e., price $\times$ quantity) must be balanced by the value of the income that it earns, and that each unit of expenditure has to purchase some amount of some type of commodity. The implication is that neither product nor value can appear out of nowhere: each activity's production or endowment must be matched by others' uses, and each activity's income must be balanced by others' expenditures. Nor can product or value disappear: a transfer of purchasing power can only be effected through an opposing transfer of some positive amount of some produced good or primary factor service, and vice versa.

These accounting rules are the cornerstones of Walrasian general equilibrium. Conservation of product, by ensuring that the flows of goods and factors must be absorbed by the production and consumption activities in the economy, is an expression of the principle of no free disposability. It implies that firms' outputs are fully consumed by households, and that households' endowment of primary factors is in turn fully employed by firms. Thus, for a given commodity the quantity produced must equal the sum of the quantities of that are demanded by the other firms and households in the economy. Analogously, for a given factor the quantities demanded by firms must completely exhaust the aggregate supply endowed to the households. This is the familiar condition of *market clearance*.

Conservation of value implies that the sum total of revenue from the production of goods must be allocated either to households as receipts for primary factors rentals, to other industries as payments for intermediate inputs, or to the government as taxes. The value of a unit of each commodity in the economy must then equal the sum of the values of all the inputs used to produce it: the cost of the inputs of intermediate materials as well as the payments to the primary factors employed in its production. The principle of conservation of value thus simultaneously reflects constancy of returns to scale in production and perfectly competitive markets for produced commodities. These conditions imply that in equilibrium producers make *zero profit*.[3]

Lastly, the returns to households' endowments of primary factors, which are the value of their factor rentals to producers, constitute income which the households exhaust on goods purchases. The fact that households' factor endowments are fully employed, so that no amount of any factor is left idle, and that households exhaust their income, purchasing some amount of commodities—even for the purpose of saving, reflects the principle of balanced-budget accounting known as *income balance*. One can also think of this principle as a zero profit condition on the production of a "utility good", whose quantity is given by the aggregate value of households' expenditures on commodities, and whose price is the marginal utility of aggregate consumption, or the unit expenditure index.

As I go on to demonstrate, CGE models employ the market clearance, zero profit and income balance conditions to solve simultaneously for the set of prices and the allocation of goods and factors that support general equilibrium. Walrasian equilibrium is defined not by the transaction processes through which this allocation comes about, but by the allocation itself, which is made up of the components of the circular flow shown by solid lines in Figure 1. General equilibrium is therefore customarily modeled in terms of barter trade in commodities and factors, without the need to explicitly keep track of (or even represent) the compensating financial transfers. Consequently, it is rare for CGE models to explicitly include money as a commodity. Nevertheless, the relative values of the different commodities and factors still need to be made denominated using some common unit of account. This is accomplished by expressing the simulated flows in terms of the value of one commodity (the so-called numeraire good) whose price is fixed. For this reason, CGE models only solve for relative prices. I expand on this point in Section 4.

3 The Algebra of Equilibrium and the Social Accounting Matrix

We now outline the algebraic expression of the circular flow, and its use in tabulating the economic data on which CGE models are calibrated. Consider a hypothetical closed economy made up of N industries, each of which produces its own type of commodity, and an unspecified number of households that jointly own an endowment of $\mathcal{F}$ different types of primary factors. Three key assumptions about this economy simplify the analysis which follows. First, there are no tax or subsidy distortions, or quantitative restrictions on transactions. Second, the households act collectively as a single representative agent who rents out the factors to the industries in exchange for income. Households then spend the latter to purchase the $\mathcal{N}$ commodities for the purpose of satisfying $\mathcal{D}$ types of demands (e.g., demands for goods for the purposes of consumption and investment). Third, each industry behaves as a representative firm that hires inputs of the F primary factors and uses quantities of the $\mathcal{N}$ commodities as intermediate inputs to produce a quantity y of its own type of output.

I use the indices $i = \{1, \dots, N\}$ to indicate the set of commodities, $j = \{1, \dots, \mathcal{N}\}$ to indicate the set of industry sectors, $f = \{1, \dots, \mathcal{F}\}$ to indicate the set of primary factors, and

$d = \{1, \ldots, \mathcal{D}\}$ to indicate the set of final demands. The circular flow of the economy can be completely characterized by three data matrices: an $\mathcal{N} \times \mathcal{N}$ input-output matrix of industries' uses of commodities as intermediate inputs, $\overline{\mathbf{X}}$, an $\mathcal{F} \times \mathcal{N}$ matrix of primary factor inputs to industries, $\overline{\mathbf{V}}$, and an $\mathcal{N} \times \mathcal{D}$ matrix of commodity uses by final demand activities, $\overline{\mathbf{G}}$.

It is straightforward to establish how the elements of the three matrices may be arranged to reflect the logic of the circular flow. First, commodity market clearance implies that the value of gross output of industry i, which is the value of the aggregate supply of the i^{th} commodity ($\overline{y}_i$) must equal the sum of the values of the j intermediate uses ($\overline{x}_{i,j}$) and the d final demands ($\overline{g}_{i,d}$) which absorb that commodity:

$$\overline{y}_i = \sum_{j=1}^{\mathcal{N}} \overline{x}_{i,j} + \sum_{d=1}^{\mathcal{D}} \overline{g}_{i,d} \tag{1}$$

Similarly, factor market clearance implies that the sum of firms' individual uses of each primary factor ($\overline{v}_{f,j}$) fully utilize the representative agent's corresponding endowment ($\overline{V}_f$):

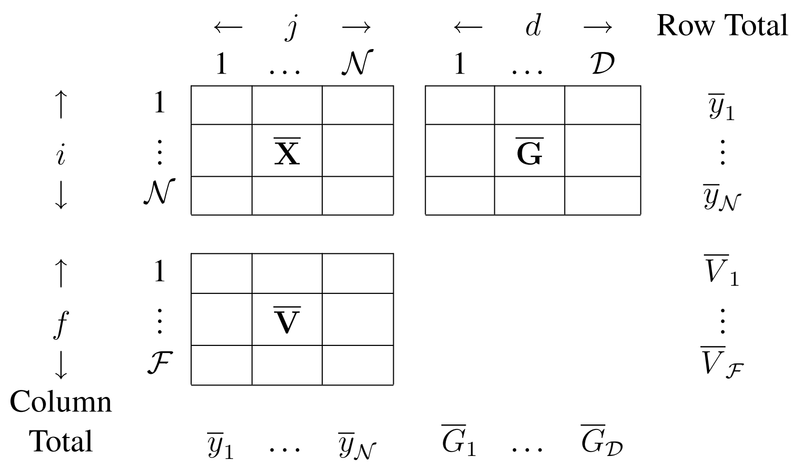
$$\overline{V}_f = \sum_{j=1}^{\mathcal{N}} \overline{v}_{f,j} \tag{2}$$

Second, the fact that industries make zero profit implies that the value of gross output of the j^{th} sector ($\overline{y}_j$) must equal the sum of the benchmark values of inputs of the i intermediate goods, $\overline{x}_{i,j}$, and f primary factors, $\overline{v}_{f,j}$, employed by that industry's production process:

$$\overline{y}_j = \sum_{i=1}^{\mathcal{N}} \overline{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} \overline{v}_{fj} \tag{3}$$

Third, the representative agent's income, $\overline{\mathcal{I}}$, is made up of the receipts from the rental of primary factors—all of which are assumed to be fully employed. The resulting income must balance the agent's gross expenditure on satisfaction of commodity demands. Together, these conditions imply that income is equivalent to the sum of the elements of $\overline{\mathbf{V}}$, which in turn must equal the sum of the

Figure 2: A Social Accounting Matrix



elements of $\overline{\mathbf{G}}$. Thus, by eq. (2),

$$\overline{\mathcal{I}} = \sum_{f=1}^{\mathcal{F}} \overline{V}_f = \sum_{i=1}^{\mathcal{N}} \sum_{d=1}^{\mathcal{D}} \overline{g}_{id} \tag{4}$$

The accounting relationships in eqs. (1)-(4) jointly imply that, in order to reflect the logic of the circular flow, the matrices $\overline{\mathbf{X}}$, $\overline{\mathbf{V}}$ and $\overline{\mathbf{G}}$ should be arranged according to Figure 2(a). This diagram is an accounting tableau known as a social accounting matrix (SAM), which is a snapshot of the inter-industry and inter-activity flows of value within an economy at equilibrium in a particular benchmark period. The SAM is an array of input-output accounts that are denominated in the units of value of the period for which the flows in the economy are recorded, typically the currency of the benchmark year. Each account is represented by a row and a column, and the cell elements record the payment from the account of a column to the account of a row. Thus, an account's components of income of (i.e., the value of receipts from the sale of a commodity) appear along its row, and the components of its expenditure (i.e., the values of the inputs to a demand activity or the production of a good) appear along its column (King 1985).

The structure the SAM reflects the principle of double-entry book-keeping, which requires that for each account, total revenue-the row total-must equal total expenditure-the column total. This is apparent from Figure fig:sam(a), where the sum across any row in the upper quadrants $\overline{\mathbf{X}}$ and $\overline{\mathbf{G}}$ is equivalent to the expression for goods market clearance from eq. (1), and the sum across any row in the south-west quadrant $\overline{\mathbf{V}}$ is equivalent to the expressions for factor market clearance from eq. (2). Likewise, the sum down any column of the left-hand quadrants $\overline{\mathbf{X}}$ and $\overline{\mathbf{V}}$ is equivalent to the expression for zero-profit in industries from eq. (3). Furthermore, once these conditions hold, the sums of the elements of the northeast and southwest quadrants ($\overline{\mathbf{G}}$ and $\overline{\mathbf{V}}$, respectively) should equal one another, which is equivalent to the income balance relationship from eq. (4). The latter simply reflects the intuition that in a closed economy GDP (the aggregate of the components of expenditure) is equal to value added (the aggregate of the components of income). These properties make the SAM an ideal data base from which to construct a CGE model.

4 From a SAM to a CGE Model: The CES Economy

CGE models' algebraic framework results from the imposition of the axioms of producer and consumer maximization on the accounting framework of the SAM. To illustrate this point I use the pedagogic device of a constant elasticity of substitution (CES) economy in which households are treated as a representative agent with CES preferences and industry sectors are modeled as representative producers with CES production technologies. While the algebra thus far has all been developed in terms of flows of value, in the subsequent analysis it will be necessary to distinguish between the prices and quantities of goods and factors. I therefore use p_i and w_f to denote the prices of commodities and factors, respectively, and use $x_{i,j}$, $v_{f,j}$ and $g_{i,d}$ (i.e., without bars) to indicate the quantity components of the previously-defined value variables.

4.1 Households

The objective of the representative agent is to maximize utility (u) by choosing levels of goods consumption ($g_{i,C}$), subject to ruling commodity prices (p_i) and the agent's budget constraint. The agent may also demand goods and services for purposes other than consumption (C). In the present example, I assume that $d = \{C, O\}$, where O indicates other final demands (e.g., saving/investment) which are given by the exogenous vector $g_{i,O}$. Using eq. (4), the agent's disposable income is then:

$$\mu = \sum_{f=1}^{\mathcal{F}} w_f V_f - \sum_{i=1}^{\mathcal{N}} p_i g_{i,O}, \tag{5}$$

which allows us to specify the agent's problem as:

$$\max_{g_{i,C}} u[g_{1,C}, \ldots, g_{\mathcal{N},C}] \quad \text{s.t.} \quad \mu = \sum_{i=1}^{\mathcal{N}} p_i g_{i,C}. \tag{6}$$

We assume that the representative agent has CES preferences, so that her utility function is

$$u = \left(\sum_{i=1}^{\mathcal{N}} \alpha_i g_{i,C}^{(\omega-1)/\omega} \right)^{\omega/(\omega-1)},$$

where the α_is are the technical coefficients of the utility function, and ω is the elasticity of substitution.

Rather than solve (6) directly, it is advantageous to solve the dual expenditure minimization problem. The agent therefore seeks to minimize her expenditure to gain a unit of utility (θ), subject to the constraint of her utility function by choosing the levels of unit commodity demands, ($\hat{g}_{i,C}$):

$$\min_{\hat{g}_{i,C}} \theta = \sum_{i=1}^{\mathcal{N}} p_i \hat{g}_{i,C} \quad \text{s.t.} \quad 1 = \left(\sum_{i=1}^{\mathcal{N}} \alpha_i \hat{g}_{i,C}^{(\omega-1)/\omega} \right)^{\omega/(\omega-1)}. \tag{6'}$$

The variable θ is known as the unit expenditure index, and can be interpreted as the marginal utility of aggregate consumption. The solution to this problem is the vector of unit demands for the consumption of commodities ($\hat{g}_{i,C} = \alpha_i^{\omega}\theta^{\omega}p_i^{-\omega}$), which implies the conditional final demands:

$$g_{i,C} = \hat{g}_{i,C}u = \alpha_i^{\omega}\theta^{\omega}p_i^{-\omega}u, \tag{7}$$

where u indicates the representative agent's level of activity.

4.2 Producers

Each producer maximizes profit (π_j) by choosing levels of intermediate inputs ($x_{i,j}$) and primary factors ($v_{f,j}$) to produce output (y_j), subject to the ruling prices of output (p_j) intermediate inputs (p_i), factors (w_f) and the constraint of its production technology (ϑ_j). The j^{th} producer's problem is thus:

$$\max_{x_{i,j},\, v_{f,j}} \pi_j = p_j y_j - \sum_{i=1}^{\mathcal{N}} p_i x_{i,j} + \sum_{f=1}^{\mathcal{F}} w_f v_{f,j} \quad \text{s.t.} \quad y_j = \vartheta_j[x_{1,j}, \ldots, x_{\mathcal{N},j}; v_{1,j}, \ldots, v_{\mathcal{F},j}] \tag{8}$$

Producers have CES technology, so that the production function ϑ_j takes the form

$$y_j = \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j} x_{i,j}^{(\sigma_j-1)/\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j} v_{f,j}^{(\sigma_j-1)/\sigma_j} \right)^{\sigma_j/(\sigma_j-1)},$$

where, $\beta_{i,j}$ and $\gamma_{i,j}$ are the technical coefficients on intermediate commodities and primary factors respectively, while σ_j denotes each industry's elasticity of substitution.

Instead of solving (8) directly, it will prove useful to solve the dual cost minimization problem. Firm j seeks to minimize its unit cost subject to the constraint of its production technology by choosing the levels of the unit input demands for commodities ($\hat{x}_{i,j}$) and the primary factor ($\hat{v}_{f,j}$):

$$\min_{\hat{x}_{i,j},\, \hat{v}_{f,j}} p_j = \sum_{i=1}^{\mathcal{N}} p_i \hat{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} w_f \hat{v}_{f,j} \quad \text{s.t.} \quad 1 = \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j} \hat{x}_{i,j}^{(\sigma_j - 1)/\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j} \hat{v}_{f,j}^{(\sigma_j - 1)/\sigma_j} \right)^{\sigma_j/(\sigma_j - 1)} \tag{8$'$}$$

The solution to this problem yields the unit demands for inputs of intermediate commodities and primary factors ($\hat{x}_{i,j} = \beta_{i,j}^{\sigma_j} p_j^{\sigma_j} p_i^{-\sigma_j}$ and $\hat{v}_{h,j} = \gamma_{f,j}^{\sigma_j} p_j^{\sigma_j} w_f^{-\sigma_j}$), which imply the conditional input demands:

$$x_{i,j} = \hat{x}_{i,j} y_j = \beta_{i,j}^{\sigma_j} p_j^{\sigma_j} p_i^{-\sigma_j} y_j,, \tag{9}$$

$$v_{f,j} = \hat{v}_{f,j} y_j = \gamma_{f,j}^{\sigma_j} p_j^{\sigma_j} w_f^{-\sigma_j} y_j, \tag{10}$$

where y_j indicates producers' activity levels.

4.3 General Equilibrium

To formulate the algebraic structure of a CGE model it is neceesary to develop analogues of the three general equilibrium conditions developed in section 3, into which the demands derived above may be incorporated. To begin, note that for (7), (9) and (10) to be consistent with the flows in the SAM, it must be the case that $\overline{x}_{i,j} = p_i x_{i,j}$, $\overline{v}_{f,j} = w_f v_{f,j}$, $\overline{g}_{i,d} = p_i g_{i,d}$, $\overline{y}_i = p_i y_i$ and $\overline{V}_f = w_f V_f$. Using this result, eqs. (1)-(4) may be expanded to resolve prices and quantities, yielding the conditions of market clearance for goods and factors, zero profit for industries, and income balance for the representative agent:

$$p_i y_i = p_i \left(\sum_{j=1}^{\mathcal{N}} x_{i,j} + g_{i,C} + g_{i,O} \right), \tag{1$'$}$$

$$w V_f = w \sum_{j=1}^{\mathcal{N}} v_{f,j}, \tag{2$'$}$$

$$p_j y_j = \sum_{i=1}^{\mathcal{N}} p_i \hat{x}_{i,j} y_j + \sum_{f=1}^{\mathcal{F}} w_f \hat{v}_{f,j} y_j, \tag{3$'$}$$

$$\mu = \sum_{f=1}^{\mathcal{F}} w_f V_f - \sum_{i=1}^{\mathcal{N}} p_i g_{i,O} = \sum_{i=1}^{\mathcal{N}} p_i \hat{g}_{i,C} u = \theta u. \tag{4$'$}$$

A crucial insight, due to Mathiesen (1985a,b), is that eqs. (1′)-(4′) are analogous to the Karush-Kuhn-Tucker conditions for the optimal allocation of commodities and factors and the distribution of activities in the economy.[4] In particular, the variable which is the common factor in each of the foregoing equations exhibits complementary slackness with respect to the corresponding residual primal or dual constraint. Far from being a mere technical detail, this characteristic is what has revolutionized the formulation and solution of CGE models.

The economic intuition behind complementary slackness is straightforward. In (3′), any producer earning negative profit will shut down with an output of zero; accordingly, the expression for unit profit is complementary to the relevant producer's level of activity (y_j). The constraint qualification may therefore be written:

$$p_j < \sum_{i=1}^{\mathcal{N}} p_i \hat{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} w_f \hat{v}_{f,j}, \ y_j = 0 \quad \text{or} \quad p_j = \sum_{i=1}^{\mathcal{N}} p_i \hat{x}_{i,j} + \sum_{f=1}^{\mathcal{F}} w_f \hat{v}_{f,j}, \ y_j > 0 \tag{11}$$

An additional insight is that similar logic applies to the representative agent, whose optimal consumption decision can be thought of as zero profit in the "production" of utility: if the cost of the goods necessary to generate a unit of final consumption exceeds the latter's marginal utility, then there will be no consumption activity. The extreme right-hand equality in (4′) therefore implies:

$$\theta < \sum_{i=1}^{\mathcal{N}} p_i g_{i,C},\ u = 0 \quad \text{or} \quad \theta = \sum_{i=1}^{\mathcal{N}} p_i g_{i,C},\ u > 0 \tag{12}$$

In (1′) and (2′), any commodity or factor which is in excess supply will have a price of zero; therefore the balance between supply and demand for each of these inputs is complementary to the corresponding price level (p_j and w_f, respectively).

$$y_i > \sum_{j=1}^{\mathcal{N}} x_{i,j} + g_{i,C} + g_{i,O},\ p_i = 0 \quad \text{or} \quad y_i = \sum_{j=1}^{\mathcal{N}} x_{i,j} + g_{i,C} + g_{i,O},\ p_i > 0 \tag{13}$$

$$V_f > \sum_{j=1}^{\mathcal{N}} v_{f,j},\ w_f = 0 \quad \text{or} \quad V_f = \sum_{j=1}^{\mathcal{N}} v_{f,j},\ w_f > 0 \tag{14}$$

The incorporation of utility as a good within the equilibrium framework permits the specification of a market clearance condition for u, which states that a supply of utility in excess of that provided by consumption results in zero unit expenditure:

$$u > \mu/\theta,\ \theta = 0, \quad \text{or} \quad u = \mu/\theta,\ \theta \geq 0 \tag{15}$$

Finally, it is worth noting that the definition of disposable income, which is restated as the extreme left-hand equality in (4′), does not exhibit complementary slackness with respect to any of its constituent variables, and moreover is made redundant by (15). In the specification of general equilibrium it plays the simple role of an accounting identity. One way to make this role explicit is to designate the unit expenditure index as the numeraire price by fixing $\theta = 1$. This automatically drops eq. (15) by fixing $\mu = u$.

4.4 The CGE model in a Complementarity Format

The specification of a CGE model in a complementarity format involves pairing each of the expressions (11)-(15) with the associated complementary variable so as to make complementarity explicit (Rutherford, 1995). Using (7), (9) and (10) to make the appropriate substitutions yields the algebraic system (16a)-(16f) shown in Table 1. These equations are what is referred to as "a CGE model".

This system defines the pseudo-excess demand correspondence of the economy:

$$\Xi(\mathbf{z}) \geq 0, \quad \mathbf{z} \geq 0, \quad \mathbf{z}'\Xi(\mathbf{z}) = 0, \tag{16}$$

where $\Xi = \{\mathbf{p}, \theta, \mathbf{y}, \boldsymbol{V}, u, m\}'$ is the stacked vector of $2\mathcal{N}+\mathcal{F}+3$ equations and $\mathbf{z} = \{\mathbf{y}, u, \mathbf{p}, \mathbf{w}, \theta, m\}$ is the $2\mathcal{N} + \mathcal{F} + 3$-vector of unknowns:[5]

1. $\mathcal{N} + 1$ zero profit inequalities $\{\mathbf{p}, \theta\}$ in as many unknowns $\{\mathbf{y}, u\}$,

2. $\mathcal{N} + \mathcal{F} + 1$ market clearance inequalities $\{\mathbf{y}, \boldsymbol{V}, u\}$ as many unknowns $\{\mathbf{p}, \mathbf{w}, \theta\}$, and

3. A single income definition equation (μ) in a single unknown (μ).

Henceforth I use the shorthand notation "$\perp$" to denote the complementary slackness relationship exhibited by the model's equations and its associated variables, writing (16) compactly as:

$$\Xi(\mathbf{z}) \geq 0, \quad \perp \quad \mathbf{z}.$$

Note that in equilibrium the equations in the rightmost column of Table 1 will all be satisfied with equality, while the variables in the middle column will all be positive.

Table 1: The Equations of the CGE Model

Zero profit

$$p_j \leq \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} p_i^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} w_f^{1-\sigma_j}\right)^{1/(1-\sigma_j)}, \quad y_j \geq 0, \quad y_j \left(p_j - \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} p_i^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} w_f^{1-\sigma_j}\right)^{1/(1-\sigma_j)}\right) = 0 \quad \forall j \tag{16a}$$

$$\theta \leq \left(\sum_{i=1}^{\mathcal{N}} \alpha_i^{\omega} p_i^{1-\omega}\right)^{1/(1-\omega)}, \quad u \geq 0, \quad u\left(\theta - \left(\sum_{i=1}^{\mathcal{N}} \alpha_i^{\omega} p_i^{1-\omega}\right)^{1/(1-\omega)}\right) = 0 \tag{16b}$$

Market clearance

$$y_i \geq \sum_{j=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} p_j^{\sigma_j} p_i^{-\sigma_j} y_j + \alpha_i^{\omega} \theta^{\omega} p_i^{-\omega} u + g_{i,O}, \quad p_i \geq 0, \quad p_i\left(y_i - \sum_{j=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} p_j^{\sigma_j} p_i^{-\sigma_j} y_j + \alpha_i^{\omega} \theta^{\omega} p_i^{-\omega} u\right) = 0 \quad \forall i \tag{16c}$$

$$V_f \geq \sum_{j=1}^{\mathcal{N}} \gamma_{f,j}^{\sigma_j} p_j^{\sigma_j} w_f^{-\sigma_j} y_j, \quad w_f \geq 0, \quad w_f\left(V_f - \sum_{j=1}^{\mathcal{N}} \gamma_{f,j}^{\sigma_j} p_j^{\sigma_j} w_f^{-\sigma_j} y_j\right) = 0 \quad \forall f \tag{16d}$$

$$u \geq \mu/\theta, \quad \theta \geq 0, \quad \theta(u - \mu/\theta) = 0 \tag{16e}$$

Income balance

$$\mu = \sum_{f=1}^{\mathcal{F}} w_f V_f - \sum_{i=1}^{\mathcal{N}} p_i g_{i,O}, \quad \mu \geq 0, \quad \mu\left(\mu - \left(\sum_{f=1}^{\mathcal{F}} w_f V_f - \sum_{i=1}^{\mathcal{N}} p_i g_{i,O}\right)\right) = 0 \tag{16f}$$

5 Numerical Calibration

The problem in eq. (16) is highly non-linear, with the result that a closed-form solution for $\mathbf{z}$ does not exist. This is the reason for the "C" in CGE models: to find the general equilibrium of an economy with realistic utility and production functions, the corresponding system of equations must be calibrated on a SAM introduced in section 3 to generate a numerical problem that can be solved using optimization techniques.

To numerically calibrate our example CES economy, we need to establish equivalence between eqs. (1)-(4) and (1′)-(4′). There are different ways of doing this, depending on what kind of information is available in addition to the SAM. Most frequently however, data on benchmark prices are lacking.[6] In this situation the simplest method to "fit" eq. (16) to the benchmark equilibrium in the SAM is to treat the price variables as indices with benchmark values of unity: $p_i = w_f = \theta = 1$, and treat the activity and income variables as real values which are set equal to the row and column totals in the SAM: $x_{i,j} = \overline{x}_{i,j}$, $v_{f,j} = \overline{v}_{f,j}$, $g_{i,d} = \overline{g}_{i,d}$, $y_i = \overline{y}_i$, $V_f = \overline{V}_f$, $u = \mu = \overline{G}_C$. Then, the technical coefficients of the cost and expenditure equations may be computed by substituting these conditions into the demand functions (7), (9) and (10):

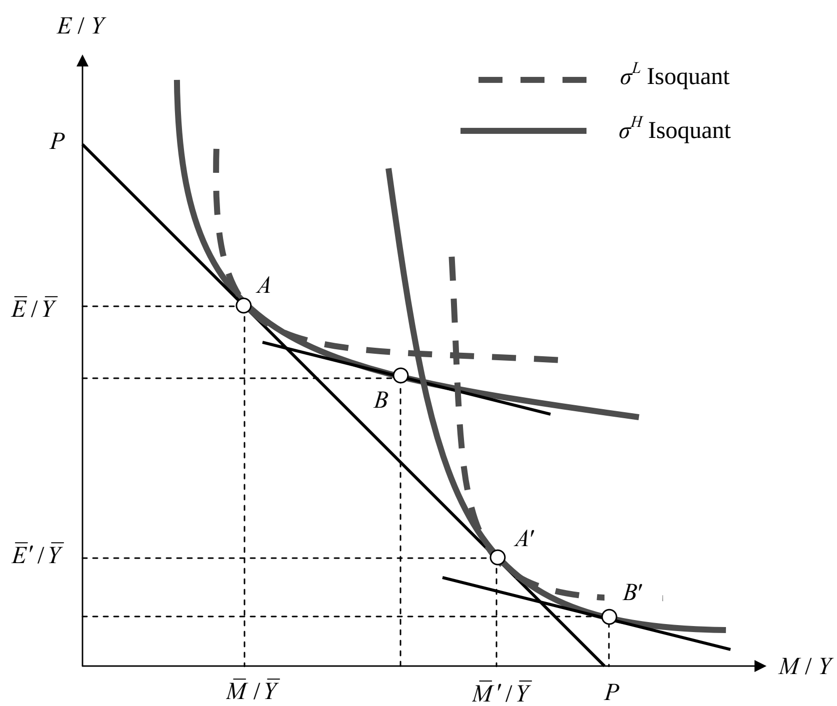
$$\alpha_{i,C} = \left(\overline{g}_{i,C}/\overline{G}_C\right)^{1/\omega}, \quad \beta_{i,j} = \left(\overline{x}_{i,j}/\overline{y}_j\right)^{1/\sigma_j} \quad \text{and} \quad \gamma_{f,j} = \left(\overline{v}_{f,j}/\overline{y}_j\right)^{1/\sigma_j}. \tag{17}$$

This result is essentially the same as the "calibrated share form" of of CES function (see, e.g., Böhringer et al., 2003).

Inserting the foregoing calibrated parameters into the expressions in Table 1, along with values for the elasticities of substitution $\boldsymbol{\sigma}$ and ω specified by the analyst, generates a system of numerical inequalities in which constitutes the actual CGE model. It is particularly important to realize that to satisfy the resulting expressions with equality, one simply has to set the price variables equal to unity and the quantity variables equal to the corresponding values in the SAM. This procedure, known as *benchmark replication*, permits the analyst to verify that the calibration is correct. The intuition is that since a balanced SAM represents the initial equilibrium of the model, plugging the values in the SAM back into the calibrated numerical pseudo-excess demand correspondence should yield an equilibrium.

Note that (17) allows us to replace the terms $\alpha^{\omega}_{i,C}$, $\beta^{\sigma_j}_{i,j}$ and $\gamma^{\sigma_j}_{f,j}$ in eq. (16) with coefficients given by the ratio of the relevant cells of the SAM and the corresponding column totals. The key implication is that the values of the substitution elasticities have no practical impact on the benchmark equilibrium, which makes intuitive sense because the model's initial equilibrium is determined by SAM, and is therefore consistent with an infinite number of potential values for $\boldsymbol{\sigma}$ and ω. The corollary is that the substitution possibilities in the economy—i.e., the degree of adjustment of economic quantities in response to changes in prices, both within and between sectors—are fundamentally determined by the SAM.

Figure 3: Calibration, the Elasticity of Substitution, and Adjustment to Price Changes



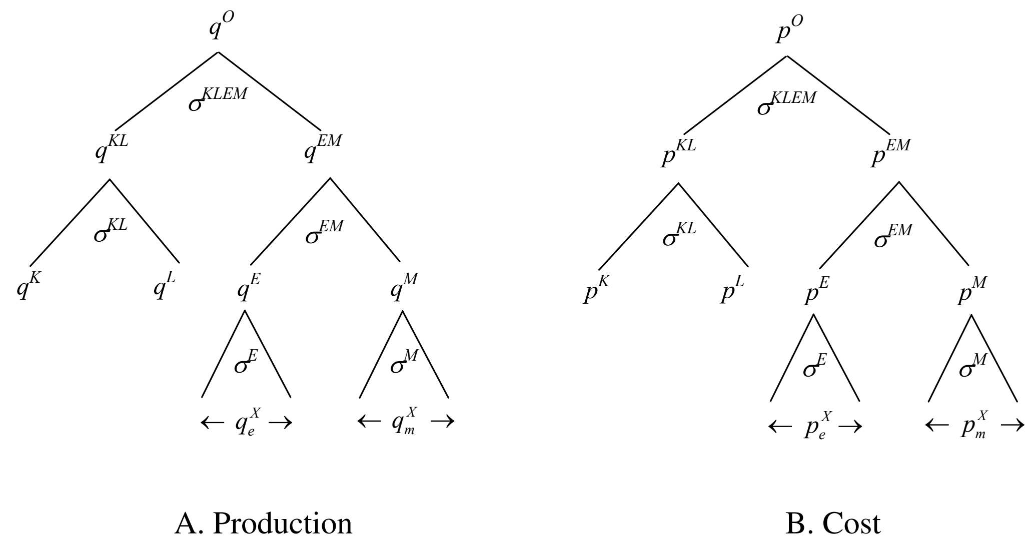
A simple example clarifies this point. Figure 3 illustrates the intra-industry margin of substitution for a hypothetical industry that produces output, Y, from inputs of energy, E and materials, M. Benchmark data on the values of the inputs ($\overline{E}$ and $\overline{M}$) and output ($\overline{Y} = \overline{E} + \overline{M}$), together with the assumption of unitary prices (given by the $-45°$ line PP), define a unique calibration point: A. An infinite number of potential isoquants pass through this point, so to pin down the industry's specific technology it is necessary to make an assumption about the elasticity of substitution (σ). A low or a high value for this parameter (σ^L or σ^H) makes the isoquant more or less highly curved, thus admitting a smaller or larger adjustment in input intensities in response to a given rotation of the relative price line, $A \rightarrow B$. The locus of the calibration point is equally important for this process: starting from another benchmark input distribution, A' (where $\overline{Y} = \overline{E}' + \overline{M}'$), the difference between the new pattern of adjustment ($A' \rightarrow B'$) and the original is easily as large as the shift induced by a change in σ.

This discussion raises the question of how precisely to determine the elasticity of substitution, which turns out to be a thorny issue. In our simple CES economy there are more free parameters than there are model equations or observations of benchmark data, which makes (17) an under-determined mathematical problem. This difficulty is magnified in real-world CGE models, in which it has become popular to specify industries' cost and consumers' expenditure functions using hierarchical CES functions, each of which has multiple elasticities of substitution.

The nested production and cost functions in the Goulder (1995) model are shown in Figure 4, in which each node of the tree denotes the output of a CES function and the branches denote the relevant inputs. In each industry, the substitution possibilities among capital (K), labor (L), energy (E) and materials (M) are controlled by five elasticity parameters: substitution between primary factors (KL) and intermediate goods (EM) by σ^O, capital-labor substitution by σ^{KL}, energy-material substitution by σ^{EM}, inter-fuel substitution by σ^E, and substitution among non-energy intermediate inputs by σ^M.

It is not possible to either estimate or compute the values of these elasticities without a host

Figure 4: Goulder (1995) KLEM Production Structure



Notes: $p^O,\ q^O$ = price and quantity of output; $\mathbf{P} = \{p^K, p^L, p^E, p^M\}$, $\mathbf{Q} = \{q^K, q^L, q^E, q^M\}$ = price and quantity of capital, labor, energy and materials; $p_e^X,\ q_e^X$ = price and quantity of intermediate energy commodities; $p_m^X,\ q_m^X$ = price and quantity of intermediate material commodities; $p^{KL},\ q^{KL}$ = price and quantity of value-added composite; $p^{EM},\ p^{EM}$ = price and quantity of energy-materials composite.

of auxiliary information.[7] Faced with this data constraint, modelers frequently resort to selecting values for these parameters from the empirical literature based on judgment and assumptions. The ad-hoc nature of this process has been criticized by mainstream empirical economists (e.g., Jorgenson, 1984; McKitrick, 1998), who advocate an econometric approach to CGE modeling in which the pseudo-excess demand correspondence is built up from statistically estimated cost and expenditure functions.

The econometric approach remedies the problematic inconsistency between the nested CES functional forms employed in models and the flexible power series approximations of arbitrary cost or expenditure functions employed by empirical studies, which estimate the pairwise Allen-Uzawa elasticities of substitution (AUES) among the various inputs to production and consumption. For example, the Translog form of the cost function in Figure 4 might specify the logarithm of the output price as a quadratic function of the logarithms of the input prices ($\mathbf{P}$) and time (t):

$$\log p^O = \delta_0 + \boldsymbol{\delta}_P \log \mathbf{P}' + \delta_T t + \tfrac{1}{2} \log \mathbf{P} \boldsymbol{\delta}'_{PP} \log \mathbf{P} + \boldsymbol{\delta}_{PT} \log \mathbf{P}' t + \tfrac{1}{2} \delta_{TT} t^2, \qquad (18)$$

where δ_0, δ_T, δ_{TT}, $\boldsymbol{\delta}_P$, $\boldsymbol{\delta}_{PP}$, and $\boldsymbol{\delta}_{PT}$ are vectors of parameters to be estimated. By Shepard's Lemma, the derivative of this expression with respect to the logarithms of the input prices yields a vector of input cost shares, $\mathbf{s} = \{s^K, s^L, s^E, s^M\}$:

$$\frac{\partial \log p^O}{\partial \log \mathbf{P}'} = \frac{\text{diag}(\mathbf{P}\mathbf{Q}')}{p^O q^O} = \mathbf{s}' = \boldsymbol{\delta}'_P + \log \mathbf{P} \boldsymbol{\delta}_{PP} + \boldsymbol{\delta}'_{PT} t, \qquad (19)$$

where $\mathbf{Q}$ is the vector of input quantities.

Estimating (18) and (19) as a system yields a vector of numerically calibrated linear equations which may be used in our example model as an alternative to eq. (17). We would need to substitute (18) in place of the CES cost function (16a), and substitute commodity and factor demands derived from (19) into the market clearance conditions (16c) and (16d). Substitution possibilities would then be determined by the AUES between each pair of inputs k and l: $\boldsymbol{\zeta}_{kl} = 1 + \delta_{kl} / \left(\underline{s}^k \underline{s}^l \right)$, where $\underline{s}$ indicates the mean value of each input's share of total cost in the data sample. Note that our original assumption of CES technology implies that $\zeta_{KL,j} = \zeta_{KE,j} = \zeta_{KM,j} = \zeta_{LE,j} = \zeta_{LM,j} = \zeta_{EM,j} = \sigma_j$, which is a stringent restriction on the estimated parameters. Dawkins et al. (2001) provide an excellent survey of these issues.

Despite its rigor, this approach is not without drawbacks. First and foremost, it is data intensive, requiring time series observations of prices and quantities of the inputs and outputs for every industry represented in the CGE model. Oftentimes such data are simply not available, which has sharply limited its appeal.[8] A second, more subtle shortcoming involves flexible functional forms themselves. For the general equilibrium condition of no free disposability to be satisfied, the simulated cost shares must be strictly positive at all prices. But it has long been known (e.g., Lutton and LeBlanc (1984)) that large and negative estimated values of $\boldsymbol{\delta}_{PP}$ can give rise to cost shares which are negative! For this reason, Perroni and Rutherford (1998) argue that flexible functional forms lack *global regularity*, in the sense that (18) or (19) are not guaranteed to map an arbitrary vector of positive prices into $\Re^{+}$. In practice, it is not possible to predict a priori when such problems will arise, and in any case, modelers have come up with ad-hoc countermeasures.[9] Nevertheless, this remains remains an important issue for energy and climate policy simulations, as the imposition of a sufficiently high tax on energy (say) may cause p^E to increase outside of the historical range of values, with the result that $s^E < 0$ if the own- or cross-price energy elasticities are sufficiently large.

6 Computation of Equilibrium

6.1 Solution Algorithms: A Sketch

The calibration procedure transforms (16) into a square system of numerical inequalities known as a mixed complementarity problem or MCP (Ferris and Pang, 1997), which may be solved using algorithms that are now routinely embodied in modern, commercially-available software systems for optimization. Mathiesen (1985a,b) and Rutherford (1987) describe the basic approach, which is essentially a Newton-type algorithm. The algorithm iteratively solves a sequence of linear complementarity problems or LCPs (Cottle et al., 1992), each of which is a first-order Taylor series expansion of the non-linear function Ξ. The LCP solved at each iteration is thus one of finding

$$\mathbf{z} \geq 0 \quad \text{s.t.} \quad \boldsymbol{\xi}_1 + \boldsymbol{\xi}_2 \mathbf{z} \geq 0, \quad \mathbf{z}^T(\boldsymbol{\xi}_1 + \boldsymbol{\xi}_2 \mathbf{z}) = 0 \tag{20}$$

where, linearizing Ξ around $\mathbf{z}_{\{\iota\}}$, the state vector of prices, activity levels and income at iteration ι, $\boldsymbol{\xi}_1[\mathbf{z}_{\{\iota\}}] = \nabla\Xi[\mathbf{z}_{\{\iota\}}]\mathbf{z}_{\{\iota\}} - \Xi[\mathbf{z}_{\{\iota\}}]$ and $\boldsymbol{\xi}_2[\mathbf{z}_{\{\iota\}}] = \nabla\Xi[\mathbf{z}_{\{\iota\}}]$. The value of $\mathbf{z}$ that solves the subproblem (20) at the ι^{th} iteration is $\mathbf{z}^{\star}_{\{\iota\}}$. Then, starting from an initial point, $\mathbf{z}_{\{0\}}$, the algorithm generates a sequence of vectors $\mathbf{z}$ which is propagated according to the linesearch:

$$\mathbf{z}_{\{\iota+1\}} = \lambda_{\{\iota\}}\mathbf{z}^{\star}_{\{\iota\}} + (1 - \lambda_{\{\iota\}})\mathbf{z}_{\{\iota-1\}} \tag{21}$$

where the parameter $\lambda_{\{\iota\}}$ controls the length of the forward step at each iteration. The convergence criterion for the algorithm made up of eqs. (20) and (21) is $\|\Xi(\mathbf{z}^{\star})\| < \varphi$, the maximum level of excess demand, profit or income at which the economy is deemed by the analyst to have attained equilibrium. The operations research literature now contains numerous refinements to this approach, based on path-following homotopy methods described in Kehoe (1991, pp. 2061-2065) and Eaves and Schmedders (1999).[10] In modern software implementations, φ is routinely six orders of magnitude smaller than the value of aggregate income.

6.2 A Digression on the Existence and Uniqueness of Equilibrium

The foregoing exposition raises the question of how good are CGE models at finding an equilibrium. Experience with the routine use of CGE models calibrated on real-world economic data to solve for equilibria with a variety of price and quantity distortions would seem to indicate that the procedures outlined above are robust. However, an answer to this question is both involved and elusive, as it hinges on three important underlying issues which span the theoretical and empirical literatures on general equilibrium: the existence, uniqueness, and stability of equilibrium. Clearly, these are desirable attributes of a CGE model, as they imply that its solutions are both deterministic and robust to perturbations along the path to equilibrium (i.e., as $\lambda_{\{\iota\}}$ changes).

Textbook treatments of the theory of general equilibrium emphasize two properties that Ξ should satisfy. The first is the weak axiom of revealed preference (WARP), whereby an economy with multiple households exhibits a stable preference ordering over consumption bundles in the space of all possible prices and income levels, ruling out the potential for non-homothetic shifts in households' consumption vectors due to changes in the distribution of income with prices held constant. A sufficient condition for this property to hold is that households' preferences admit aggregation up to well-behaved community utility function, which is the representative agent assumption. The second property is gross substitutability (GS), where the aggregate demand for any commodity or factor is non-decreasing in the prices of all other goods and factors. Where this condition holds, a vector of equilibrium prices exists and is unique up to scalar multiple (Varian, 1992).

One can think of these properties as economic interpretations of the sufficient conditions for a unique solution to (16). From a mathematical standpoint, a (locally) unique solution for $\mathbf{z}$ can be recovered from the inverse of the pseudo-excess demand correspondence. The inverse function theorem implies that a sufficient condition for the existence of Ξ^{-1} is that the jacobian of the pseudo-excess demand correspondence is non-singular, which requires $-\nabla\Xi$ to be positive semi-definite. Loosely speaking, GS and WARP both imply that $\det(-\nabla\Xi)$ is non-negative; generally that it is positive (Kehoe, 1985). But in real-world CGE models with many sectors and agents, each specified using highly non-linear nested utility and production functions, closed-form analytical proofs of this condition impossible. For this reason, an emerging area of computational economic research is the development of algorithms to test the positive determinant property at each iteration step of the numerical sub-problem.

Theoretical studies of general equilibrium have focused on finding the least restrictive conditions on Ξ that enable WARP and/or GS to ensure uniqueness, and have largely circumvented the details of algebraic functional forms employed in applied models. The signal exception is Mas-Colell's (1991) proof that so-called "super Cobb-Douglas economies"—i.e., those with CES utility and production functions and greater-than-unitary substitution elasticities—are guaranteed to have a unique equilibrium in the absence of taxes and other distortions. Although this result is both directly relevant and encouraging, it is tempered by concern that the introduction of tax distortions can induce multiple equilibria, even in models with a representative consumer and convex production technologies.[11] The potential for distortions to introduce instability is worrying because, as the next section will elaborate, CGE models are the workhorse of the empirical analysis of the incidence of price and quantity policies' distortionary effects.

Empirical tests of these problem have focused on the construction, diagnosis and analysis of multiple equilibria in simple, highly stylized CGE models. Kehoe (1998b) analyzes a model that has two consumers, each with Cobb-Douglas preferences, and four commodities produced with an activity analysis technology. The model's pseudo-excess demand correspondence satisfies the GS property, yet it exhibits three equilibria, indicating the minor role played by the GS condition in determining the equilibrium of economies with production. However, changing the model's production functions to Cobb-Douglas technologies collapses the number of equilibria to one, corroborating Mas-Colell's result. Kehoe (1998b) concludes that the only guarantees of uniqueness are the very restrictive conditions of a representative consumer and complete reversibility of production. The latter condition implies that the supply side of the economy is an input-output system where there is no joint production and consumers possess no initial holdings of produced goods, but do hold initial endowments of at least one non-reproducible commodity or factor.

Whether these conditions ensure uniqueness in the presence of energy policy distortions remains a question. The point of contention is the complex feedback effects on commodity demands and producers' activity levels of the rents generated by these policies, which redound to the representative agent. Whalley and Zhang (2002) illustrate pure exchange economies that have either a unique equilibrium without taxes and multiple equilibria with taxes, or multiple equilibria without taxes and a unique equilibrium with the introduction of a small tax. Kehoe (1998b) shows that sufficient condition for uniqueness in the presence of a tax distortion is that the weighted sum of the income effects, in which the weights are given by the "efficiency" or net-of-tax prices, must be positive. In the presence of pre-existing distortions in the benchmark SAM, the fact that calibration of the model will set all prices to unity makes this condition easy to verify. However, if taxes are specified as algebraic functions of variables within the model, this condition may be difficult to check prior to actually running the model and inspecting the equilibrium to which it converges.

The intuition is that, with a specified revenue requirement and endogenous taxes, even models that satisfy all of the other prerequisites for uniqueness will have a Laffer curve that yields two equilibria, one in which the tax rate is high and the other in which it is low.[12] All this suggests the lurking possibility that multiplicity may be induced by changes in tax parameters, and may be difficult to predict ex ante, or detect.

It is therefore unsurprising that tests of multiplicity of equilibria in real-world CGE models are few and far between.[13] Research in this area is ongoing, focusing on translating theoretical results into numerical diagnostic tools (e.g., Dakhlia, 1999). But without the ability to test for—or remedy—the problem of multiple equilibria, most applied work proceeds on the assumption that the solutions generated by their simulations are unique and stable. As Dakhlia (1999) points out, whether this is in fact the case, or whether multiplicity usually just goes undetected, is still an open question. The remainder of the chapter deals with the effects of exogenous distortions in more detail.

7 Modeling Energy and Climate Policies

Policy variables in CGE models most often take the form of parameters that are exogenously specified by the analyst, and are either price-based—i.e., taxes and subsidies, or quantity-based—i.e., constraints on demand and/or supply. Beginning with the initial equilibrium represented in the SAM, a change one or more of these parameters perturbs the vector of prices and activity levels, causing the economy to converge to a new equilibrium. To evaluate the effect of the policy represented by this change, the analyst compares the pre- and post-change equilibrium vectors of prices, activity levels, and income levels, subject to the caveats of the accuracy and realism of the model's assumptions.

This approach has the advantage of measuring policies' ultimate impact on consumers' aggregate well-being in a theoretically consistent way, by quantifying the change in the income and consumption of the representative agent that result from the myriad supply-demand interactions among the markets in the economy. Ironically, this functionality is at the root of the "black box" criticism articulated in the introduction, as policymakers may be tempted to treat CGE models as a sort of economic crystal ball. By contrast, CGE models' usefulness as tool for policy analysis owes less to their predictive accuracy, and more to their ability to shed light on the mechanisms responsible for the transmission of price and quantity adjustments among markets. Therefore, CGE models should properly be regarded as computational laboratories within which to analyze the dynamics of the economic interactions from which policies derive their impacts.

The remainder of the chapter focuses its attention on production in the energy sectors and the uses of energy commodities in the economy. It bears emphasizing that energy commodities are reproducible, created by combining inputs of natural resources (i.e., primary energy reserves) with labor, capital and intermediate goods. Accordingly, I assume that the vectors of commodities and industries are partitioned into subsets of $\mathcal{E}$ energy goods/sectors, indexed by e, and $\mathcal{M}$ non-energy material goods/sectors, indexed by m.

7.1 Price Instruments

It is easiest to illustrate the impact on equilibrium of price instruments such as taxes and subsidies. Within CGE models, taxes are typically specified in an ad-valorem fashion, whereby a tax at a given rate determines the fractional increase in the price level of the taxed commodity. For example, an ad-valorem tax at rate τ on the output of industry e drives a wedge between the producer price of output, p_e, and the consumer price, $(1+\tau)p_e$, in the process generating revenue from the y_e units of output in the amount of $\tau p_e y_e$. A subsidy which lowers the price may be also incorporated in this way, by specifying $\tau < 0$.

Conceptually, there are three types of markets in the economy in which basic energy taxes or subsidies can be levied: the markets for the output of energy sectors (indicated by the superscript

Y), the market for consumption of energy (indicated by the superscript C), and the markets for energy inputs to production in each industry (indicated by the superscript X). Let the tax or subsidy rates that correspond to each of these markets be denoted by τ_e^Y, τ_e^C and $\tau_{e,j}^X$, respectively. These ad-valorem rates are easily integrated into our CES economy by treating them as exogenous policy parameters. The representative agent's problem becomes

$$\min_{\hat{g}_{i,C}} \theta = \sum_{e=1}^{\mathcal{E}} (1+\tau_e^C)(1+\tau_e^Y) p_e \hat{g}_{e,C} + \sum_{m=1}^{\mathcal{M}} p_m \hat{g}_{m,C} \quad \text{s.t.} \quad 1 = \left(\sum_{i=1}^{\mathcal{N}} \alpha_i \hat{g}_{i,C}^{(\omega-1)/\omega} \right)^{\omega/(\omega-1)}, \tag{6''}$$

while the producer's problem becomes

$$\min_{\hat{x}_{i,j},\, \hat{v}_{f,j}} p_j = \sum_{e=1}^{\mathcal{E}} (1+\tau_{e,j}^X)(1+\tau_e^Y) p_e \hat{x}_{e,j} + \sum_{m=1}^{\mathcal{M}} p_m \hat{x}_{m,j} + \sum_{f=1}^{\mathcal{F}} w_f \hat{v}_{f,j}$$

$$\text{s.t.} \quad 1 = \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j} \hat{x}_{i,j}^{(\sigma_j-1)/\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j} \hat{v}_{f,j}^{(\sigma_j-1)/\sigma_j} \right)^{\sigma_j/(\sigma_j-1)}, \tag{8''}$$

giving rise to new final and intermediate demands for energy commodities:

$$g_{e,C} = \alpha_e^{\omega} \theta^{\omega} \left((1+\tau_e^C)(1+\tau_e^Y) p_e \right)^{-\omega} u, \tag{22}$$

$$x_{e,j} = \beta_{e,j}^{\sigma_j} p_j^{\sigma_j} \left((1+\tau_{e,j}^X)(1+\tau_e^Y) p_e \right)^{-\sigma_j} y_j. \tag{23}$$

Every tax (subsidy) generates a positive (negative) revenue stream that increments (decrements) the income of some consumer while negatively (positively) affecting the production and absorption of the commodity in question. In representative-agent models, the simplest way to represent this phenomenon is to treat the government as a passive entity that collects tax revenue and immediately recycles it to the single household as a lump-sum supplement to the income from factor returns. This approach circumvents the need to represent the government as an explicit sector within the model; taxes and subsidies may be specified simply as transfers of purchasing power from to and from the representative agent. In this situation, the demand functions (22) and (23), as well as the necessary adjustments to income lead to the transformation of (16) into the new pseudo-excess demand correspondence (24):

$$p_j \leq \left(\sum_{e=1}^{\mathcal{E}} \beta_{e,j}^{\sigma_j} \left((1+\tau_{e,j}^X)(1+\tau_e^Y)p_e\right)^{1-\sigma_j} + \sum_{m=1}^{\mathcal{M}} \beta_{m,j}^{\sigma_j} p_m^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} w_f^{1-\sigma_j} \right)^{1/(1-\sigma_j)} \quad \perp y_j \tag{24a}$$

$$\theta \leq \left(\sum_{e=1}^{\mathcal{E}} \alpha_e^{\omega} \left((1+\tau_e^C)(1+\tau_e^Y)p_e\right)^{1-\omega} + \sum_{m=1}^{\mathcal{M}} \alpha_m^{\omega} p_m^{1-\omega} \right)^{1/(1-\omega)} \quad \perp u \tag{24b}$$

$$y_e \geq \sum_{j=1}^{\mathcal{N}} \beta_{e,j}^{\sigma_j} p_j^{\sigma_j} \left((1+\tau_{e,j}^X)(1+\tau_e^Y)p_e\right)^{-\sigma_j} y_j + \alpha_e^{\omega} \theta^{\omega} \left((1+\tau_e^C)(1+\tau_e^Y)p_e\right)^{-\omega} u + g_{e,O} \quad \perp p_e$$

$$y_m \geq \sum_{j=1}^{\mathcal{N}} \beta_{m,j}^{\sigma_j} p_j^{\sigma_j} p_m^{-\sigma_j} y_j + \alpha_m^{\omega} \theta^{\omega} p_m^{-\omega} u + g_{m,O} \quad \perp p_m \tag{24c}$$

$$V_f \geq \sum_{j=1}^{\mathcal{N}} \gamma_{f,j}^{\sigma_j} p_j^{\sigma_j} w_f^{-\sigma_j} y_j \quad \perp w_f \tag{24d}$$

$$u \geq \mu/\theta, \quad \perp \theta \tag{24e}$$

$$\mu = \sum_{f=1}^{\mathcal{F}} w_f V_f - \sum_{i=1}^{\mathcal{N}} p_i g_{i,O} + \sum_{e=1}^{\mathcal{E}} \tau_e^Y p_e y_e + \sum_{e=1}^{\mathcal{E}} \tau_e^C (1+\tau_e^C)^{-\omega} \left((1+\tau_e^Y)p_e\right)^{1-\omega} \alpha_e^{\omega} \theta^{\omega} u + \sum_{e=1}^{\mathcal{E}} \sum_{j=1}^{\mathcal{N}} \tau_{e,j}^X (1+\tau_{e,j}^X)^{-\sigma_j} \left((1+\tau_e^Y)p_e\right)^{1-\sigma_j} \beta_{e,j}^{\sigma_j} p_j^{\sigma_j} y_j \quad \perp \mu \tag{24f}$$

The foregoing system of equations may be solved for a new, tariff-ridden equilibrium, whose price and quantity allocation may be compared with that of the original benchmark equilibrium without taxes.[14] The measure of the taxes' aggregate impact on economic well-being is equivalent variation. This is approximated by the change in the representative agent's consumption (u) with respect to the initial equilibrium, which is the loss of household's real purchasing power induced by the distortion in relative prices. It is noteworthy that the most significant adjustments to the original pseudo-excess demand correspondence are the additional terms in the income definition equation (24f). The implication is that the welfare effect of a single tax or subsidy depends on the interactions among a myriad of factors: the level of the tax and the distribution of other taxes and subsidies across all markets in the economy, the characteristics of the particular market in which the tax is levied, the linkages between this market and the others in the economy, and the values of the vectors of calibrated parameters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

The ability to rigorously account for the income consequences of inter-market price and quantity adjustments is what sets the current approach apart from partial equilibrium analysis. But it also highlights a kernel of truth to the black box criticism. The non-linearity and dimensionality of the pseudo-excess demand correspondence make it difficult to intuit the net impact of adding or removing a single distortion, even in models with only a modest number of sectors and/or households. Moreover, to sort through and understand the web of interactions that give rise to the post-tax equilibrium often requires the analyst to undertake a significant amount ex post analysis and testing.

7.2 Quantity Instruments

In comparison with taxes, quantity instruments vary widely in their characteristics and methods of application. It is useful to draw a distinction between the instrument itself, which is represented by one or more exogenous quantity parameters, and its effect on supply or demand in a particular market or set of markets, which must be expressed using one or more auxiliary equations. Although quantity instruments may be simple to parameterize, capturing the subtle characteristics of their economic effects through proper formulation of the auxiliary equations can sometimes be

Figure 5: Quantity Instruments: Taxonomy and Examples

	Relative	**Absolute**
Direct	Renewable Portfolio Standard	Rationing/Supply Curtailment
Indirect	GHG Intensity Cap	GHG Emission Cap

a challenge. Modeling quantity constraints within the complementarity framework necessitates the introduction of an additional (dual) variable with which the (primal) auxiliary equation can be paired. Intuitively, the quantity distortion defined by this equation generates a complementary price distortion which has the same effect as a tax or a subsidy. Thus, while in the previous section the price distortion was an exogenous parameter, here it is a shadow price—an endogenous variable that exhibits complementary slackness with respect to the quantity instrument. Furthermore, as with taxes, quantity distortions generate a stream of rents that must be allocated somewhere in the economy.

The auxiliary equation is often specified as a rationing constraint in which the quantity instrument sets an upper or lower bound on the supply and/or use of one or more energy commodities. Such constraints may be direct, where the energy good in question itself is the subject of restriction, or indirect, where some attribute of the good (e.g., its CO_2 content) is being limited. They may also be expressed in absolute or relative terms, with the former corresponding to an exogenous limit on energy or its attributes, and the latter tying these quantities to other variables in the economy.

Figure 5 summarizes these considerations and provides examples. Production and/or consumption of an energy commodity may be rationed directly, a situation which corresponds to a curtailment of energy supply, or the sorts of direct government intervention in markets seen in times of crisis. As well, policies such as the renewable portfolio standard (RPS), which has emerged as a popular means to promote alternative sources of electricity supply, act as a relative rationing constraint by imposing a lower bound on the production of renewable energy that is indexed to the sales of conventional energy. In contrast to such direct measures, policies such as climate change mitigation limit the emissions from a portfolio of fossil fuels, which ends up indirectly and endogenously curtailing demand for the most CO_2-intensive fuels. Finally, emission caps may be posed in a relative form such as the intensity target discussed by Ellerman and Sue Wing (2003). By judiciously choosing the level of such a target, the ex-ante impact on GHG emissions and the supply and demand for energy can be the same as its absolute counterpart under certainty. However, the introduction of ex-post uncertainty (e.g., by simulating the CGE model with different elasticity parameters) will lead to the targets denominated in absolute and intensity terms having different economic effects.[15]

The case of pure rationing is straightforward. Returning to the no-tariff world of eq. (16), assume there is a particular energy commodity (say, e') whose supply faces a binding quantity limit $q_{e'}$. The endogenous ad-valorem equivalent tax for the rationing instrument is the output tariff $\tau_{e'}^{Y}$, which is complementary to the following additional equation:

$$y_{e'} \leq q_{e'} \qquad \perp \tau_{e'}^{Y}, \tag{25g}$$

and the income definition equation (16f) becomes:

$$\mu = \sum_{f=1}^{\mathcal{F}} w_f V_f - \sum_{i=1}^{\mathcal{N}} p_i g_{i,O} + \tau_{e'}^{Y} q_{e'} \qquad \perp \mu, \tag{25f}$$

while the other equations in the pseudo-excess demand correspondence remain unchanged. The last term on the right-hand side is the pure rent from constraining supply, which are assumed to redound to the representative agent. It is possible to make alternative assumptions about where to allocate this stream of revenue. For example, we could model the rents as accruing to a particular industry (say, j'), by defining an endogenous ad-valorem subsidy to that sector's output ($\tau_{j'}^{Y} < 0$) in which the value of the subsidy revenue was constrained to equal the value of the rent: $\tau_{e'}^{Y} q_{e'} =$

$\tau^Y_{j'} p_{j'} y_{j'}$. This constraint would constitute an additional auxiliary equation, to which $\tau^Y_{j'}$ would be the complementary variable. Moreover, it would be necessary to re-specify the zero profit and market clearance conditions in a manner similar to (24a) and (24c) to account for the distortionary effects of the subsidy on relative prices.

The second example is an RPS policy in which the government mandates that a proportion of the aggregate energy supply ($\rho \in (0, 1)$) must come from renewable sources. Let the set of energy industries be partitioned into conventional and renewable sources, indicated by $\mathcal{E}^C$ and $\mathcal{E}^R$, respectively, and suppose that each unit of activity in these sectors, y_e, generates ε_e physical units of energy. Then the RPS can be expressed by the rationing constraint $\sum_{e \in \mathcal{E}^R} \varepsilon_e y_e \geq \rho \sum_{e=1}^{\mathcal{E}} \varepsilon_e y_e$. To comply with the standard, energy suppliers must collectively tax themselves to finance the production of ρ units of renewable energy for every unit of energy produced systemwide. The marginal financing charge per unit of aggregate energy supplied can be thought of as an endogenous tax, τ^{RPS}, whose proceeds are recycled to renewable energy producers. Every energy firms therefore pays an additional cost $\rho\tau^{\text{RPS}}$ per unit of energy produced, while renewable suppliers as a group receive the full τ^{RPS} per unit of energy they produce.[16]

An intuitive way of understanding this result is to think of the RPS as a tradable renewable energy credit scheme (see, e.g., Baron and Serret, 2002). A unit of energy supplied by a renewable producer generates one credit which may be sold, whereas a unit of energy produced—regardless of its origin—requires the purchase of ρ credits as a renewable financing charge. An important implication is that, in contrast to the rationing example, the RPS does not create pure rents—it merely redistributes revenue from conventional to renewable energy producers, with indirect impact on aggregate income which operates through the prices of energy commodities.[17] Accordingly, all the action occurs in the zero profit condition for industries:

$$p_j \leq \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} p_m^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} w_f^{1-\sigma_j} \right)^{1/(1-\sigma_j)} \qquad \perp y_j \quad j \in \mathcal{M}$$

$$p_j \leq \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} p_m^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} w_f^{1-\sigma_j} \right)^{1/(1-\sigma_j)} + \rho\tau^{\text{RPS}} \qquad \perp y_j \quad j \in \mathcal{E}^C$$

$$p_j + \tau^{\text{RPS}} \leq \left(\sum_{i=1}^{\mathcal{N}} \beta_{i,j}^{\sigma_j} p_m^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} w_f^{1-\sigma_j} \right)^{1/(1-\sigma_j)} + \rho\tau^{\text{RPS}} \qquad \perp y_j \quad j \in \mathcal{E}^R, \qquad (26a)$$

while the rationing constraint merely determines the value of the auxiliary financing charge:

$$\sum_{e \in \mathcal{E}^R} \varepsilon_e y_e \geq \rho \sum_{e=1}^{\mathcal{E}} \varepsilon_e y_e \qquad \perp \tau^{\text{RPS}}. \qquad (26g)$$

and the income definition (16f) and other equations remain unchanged.

The final policy I examine is a cap on aggregate emissions of CO_2. It is necessary to establish the relationship between the levels of production and demand activities and the quantity of emissions. The simplest way to proceed is to assume a fixed stoichiometric relationship between the quantity of emissions in the benchmark year and the value of aggregate demand for the fossil fuel commodities which generate them, expressed as a set of commodity-specific emission coefficients (ϕ_e). An tax on emissions (τ^{CO_2}) therefore creates a set of commodity taxes that are differentiated according to the carbon contents of energy goods, adding a markup to the price of each fossil fuel in the amount of $\tau^{\text{CO}_2}\phi_e$.

Let Q^{CO_2} denote a quantitative CO_2 target which sets an upper bound on the emissions from aggregate fossil fuel use. The shadow price on this constraint is the tax τ^{CO_2}, which can be thought of as the endogenous market-clearing price of emission allowances in an economy-wide cap-and-trade scheme. Interestingly, in the present setting the two main methods for allocating allowances—auctioning and grandfathering to firms—are modeled in the same way and generate identical welfare impacts. Grandfathering allowances is equivalent to defining a new factor of production that increases the profitability of firms but at the same time is also owned by the households, so that the returns to permits accrue as income to the representative agent. Likewise, auctioning allowances generates additional government revenue which is then immediately recycled to the representative agent in a lump sum.

As in (24), the price distortion simultaneously affects the zero profit and market clearance and income balance conditions:

$$p_j \leq \left(\sum_{e=1}^{\mathcal{E}} \beta_{e,j}^{\sigma_j} \left(p_e + \tau^{\mathrm{CO_2}} \phi_e\right)^{1-\sigma_j} + \sum_{m=1}^{\mathcal{M}} \beta_{m,j}^{\sigma_j} p_m^{1-\sigma_j} + \sum_{f=1}^{\mathcal{F}} \gamma_{f,j}^{\sigma_j} w_f^{1-\sigma_j} \right)^{1/(1-\sigma_j)} \quad \perp y_j \qquad (27a)$$

$$\theta \leq \left(\sum_{e=1}^{\mathcal{E}} \alpha_e^{\omega} \left(p_e + \tau^{\mathrm{CO_2}} \phi_e\right)^{1-\omega} + \sum_{m=1}^{\mathcal{M}} \alpha_m^{\omega} p_m^{1-\omega} \right)^{1/(1-\omega)} \quad \perp u \qquad (27b)$$

$$y_e \geq \sum_{j=1}^{\mathcal{N}} \beta_{e,j}^{\sigma_j} p_j^{\sigma_j} \left(p_e + \tau^{\mathrm{CO_2}} \phi_e\right)^{-\sigma_j} y_j + \alpha_e^{\omega} \theta^{\omega} \left(p_e + \tau^{\mathrm{CO_2}} \phi_e\right)^{-\omega} u + g_{e,O} \quad \perp p_e$$

$$y_m \geq \sum_{j=1}^{\mathcal{N}} \beta_{m,j}^{\sigma_j} p_j^{\sigma_j} p_m^{-\sigma_j} y_j + \alpha_m^{\omega} \theta^{\omega} p_m^{-\omega} u + g_{m,O} \quad \perp p_m \qquad (27c)$$

$$\mu = \sum_{f=1}^{\mathcal{F}} w_f V_f - \left(\sum_{e=1}^{\mathcal{E}} (p_e + \tau^{\mathrm{CO_2}} \phi_e) g_{e,O} + \sum_{m=1}^{\mathcal{M}} p_m g_{m,O} \right) + \tau^{\mathrm{CO_2}} Q^{\mathrm{CO_2}} \quad \perp \mu \qquad (27f)$$

while the rationing constraint is denominated in terms of intermediate and final demands for fossil energy:

$$Q^{\mathrm{CO_2}} \geq \sum_{e=1}^{\mathcal{E}} \phi_e \left(\sum_{j=1}^{\mathcal{N}} \beta_{e,j}^{\sigma_j} p_j^{\sigma_j} \left(p_e + \tau^{\mathrm{CO_2}} \phi_e\right)^{-\sigma_j} y_j + \alpha_e^{\omega} \theta^{\omega} \left(p_e + \tau^{\mathrm{CO_2}} \phi_e\right)^{-\omega} u + g_{e,O} \right) \quad \perp \tau^{\mathrm{CO_2}}, \qquad (27g)$$

and the market clearance conditions for factors and utility are the same as in (24). Note that in the present closed-economy model the rationing constraint (27g) could have been expressed simply as

$Q^{\mathrm{CO}_2} \geq \sum_{e=1}^{\mathcal{E}} \phi_e y_e$. However, in an open-economy model where trade in energy goods creates a divergence between the production and consumption of fossil fuels, the real source of emissions is consumption, as specified in eq. (27g).

One final point deserves mention. With either a price or a quantity instrument, the direct effect of a policy on the welfare of the representative agent operates through two channels: the substitution effect in consumption induced by changes commodity prices, and the income effect of changes in factor remuneration induced by shifts in factor prices. The latter is indicated by the change in the magnitude of the first term on the right-hand side of the income definition equation, and can be thought of as a summary measure of the policy's primary economic burden in terms of its factor incidence. But it bears emphasizing that neither this quantity, nor GDP, nor even the "Harberger triangle" welfare approximation (which in the case of output taxes τ_j^Y that induce changes in production Δy_j is given by $\frac{1}{2}\sum_j \tau_j^Y \Delta y_j$—see Hines, 1999) is sufficient to capture the full range of general equilibrium impacts on consumers' utility. The theoretically correct summary welfare measure is the quantity of aggregate consumption indicated by the activity level u. The implication is that the choice of numeraire influences the measurement of policies' welfare effects (e.g., Hosoe, 2000): designating θ as the numeraire price equates utility with the expression for disposable income.

8 A Realistic Worked Example: The Impacts of Abating Fossil-Fuel CO_2 Emissions in the U.S.

This section undertakes a simple yet realistic application of the CES economy developed above. The goal is to shed light on the macroeconomic costs of reducing emissions of CO_2 from the combustion of fossil-fuels in the U.S.

8.1 Model Structure

The simulation is an extension of the CES economy developed in the previous sections. The structure of the economy is summarized in Figure 6(a). Firms are classified into eight broad sectoral groupings: coal mining, crude oil and gas mining, natural gas distribution, refined petroleum, electric power, energy-intensive manufacturing (an amalgam of the chemical, ferrous and non-ferrous metal, pulp and paper, and stone, clay and glass industries), purchased transportation, and a composite of the remaining manufacturing, service, and primary extractive industries in the economy. Households are modeled as a representative agent, who is endowed with fixed quantities of three primary factors: labor, capital, and primary energy resources. While the first two of these can be re-allocated among industries in response to inter-sectoral shifts in factor demand, energy resources play the role of sector-specific fixed factors, of which there is one type in coal mining, another in crude oil and gas and a third in electricity.

An important feature of the model is the presence of pre-existing distortions. Real-world GHG mitigation policies will generate interactions between the distortionary effects of quantitative limits or Pigovian fees on emissions and the pre-existing tax system, particularly taxes on labor, capital and fossil fuels. The simplest way of accounting for these impacts is to introduce pre-existing ad-valorem taxes on production and imports ($\overline{\tau}_j^Y$), which are assumed to be levied on the output of each industry. As before (cf. eq. (27f)), I assume that the revenue raised by both these taxes and the auctioning or grandfathering of emission allowances is recycled to the representative agent in a lump sum. However, a key result from the large literature on the impacts of environmental policies in the presence of prior tax distortions (see, e.g., Goulder, 2002) is that the alternative use of permit revenues to finance a revenue-neutral reduction in $\overline{\tau}_j^Y$ has the potential to significantly lower the welfare cost of the emission constraint.

Industries' outputs are produced by combining inputs of intermediate energy and non-energy goods with primary factors. A signal characteristic of climate change mitigation policies is that higher fossil-fuel prices induce an expansion of carbon-free sources of energy supply, the bulk of which occur in the electric power sector. Accordingly, I apply the single-level CES function of the previous sections to every sector except electric power (sector 5), where a bi-level nested CES function is used to model the substitution between fossil fuel electric generation (5(a)) and carbon-free primary electricity (5(b)—a composite of nuclear, hydro and renewables), each of which is represented by the CES functions used in other industries. To distinguish between these subsectors I assume that all conventional intermediate energy inputs to electric power are used by 5(a), while 5(b) is entirely responsible for the sector's demand for primary energy resources.

However, even this simple structure significantly complicates the specification of the excess-demand correspondence. It is necessary to introduce new activity variables for the fossil and renewable subsectors $y_{5(\mathrm{a})}$ and $y_{5(\mathrm{b})}$, as well as complementary dual variables $p_{5(\mathrm{a})}$ and $p_{5(\mathrm{b})}$ to track the marginal costs of these activities. Then, incorporating the electricity subsectors into the set of activities as $j = \{1, \ldots, 5(\mathrm{a}), 5(\mathrm{b}), \ldots, 8\}$ while keeping electricity as a homogeneous commodity with $i = \{1, \ldots, 5, \ldots, 8\}$, the resulting model is:

$$p_j \leq \left(\sum_e \beta_{e,j}^{\sigma_j} \left((1 + \overline{\tau}_e^Y) p_e + \tau^{\mathrm{CO_2}} \phi_e \right)^{1-\sigma_j} + \sum_m \beta_{m,j}^{\sigma_j} \left((1 + \overline{\tau}_m^Y) p_m \right)^{1-\sigma_j} + \sum_{f=L,K} \gamma_{f,j}^{\sigma_j} w_f^{1-\sigma_j} + \gamma_{R,j}^{\sigma_j} w_{R,j}^{1-\sigma_j} \right)^{1/(1-\sigma_j)} \quad \perp y_j$$

$$p_5 \leq \left(\eta_{5(\mathrm{a}),5}^{\vartheta} \, p_{5(\mathrm{a})}^{1-\vartheta} + \eta_{5(\mathrm{b}),5}^{\vartheta} \, p_{5(\mathrm{b})}^{1-\vartheta} \right)^{1/(1-\vartheta)} \quad \perp y_5 \tag{28a}$$

$$\theta \leq \left(\sum_e \alpha_e^{\omega} \left((1 + \overline{\tau}_e^Y) p_e + \tau^{\mathrm{CO_2}} \phi_e \right)^{1-\omega} + \sum_m \alpha_m^{\omega} \left((1 + \overline{\tau}_m^Y) p_m \right)^{1-\omega} \right)^{1/(1-\omega)} \quad \perp u \tag{28b}$$

$$y_e \geq \sum_j \beta_{e,j}^{\sigma_j} p_j^{\sigma_j} \left((1 + \overline{\tau}_e^Y) p_e + \tau^{\mathrm{CO_2}} \phi_e \right)^{-\sigma_j} y_j$$

$$+ \alpha_e^\omega \theta^\omega \left((1+\overline{\tau}_e^Y)p_e + \tau^{\mathrm{CO_2}}\phi_e\right)^{-\omega} u + g_{e,O} \qquad \perp p_e$$

$$y_m \geq \sum_j \beta_{m,j}^{\sigma_j} p_j^{\sigma_j} \left((1+\overline{\tau}_m^Y)p_m\right)^{-\sigma_j} y_j$$

$$+ \alpha_m^\omega \theta^\omega \left((1+\overline{\tau}_m^Y)p_m\right)^{-\omega} u + g_{m,O} \qquad \perp p_m$$

$$y_i \geq \eta_{i,5}^{\vartheta} p_5^{\vartheta} p_i^{-\vartheta} y_5 \qquad \perp p_i,\ i = 5\text{(a)}, 5\text{(b)} \tag{28c}$$

$$V_f \geq \sum_j \gamma_{f,j}^{\sigma_j} p_j^{\sigma_j} w_f^{-\sigma_j} y_j \qquad \perp w_f,\ f = K, L$$

$$V_{R,j} \geq \gamma_{R,j}^{\sigma_j} p_j^{\sigma_j} w_{R,j}^{-\sigma_j} y_j \qquad \perp w_{R,j} \tag{28d}$$

$$u \geq \mu/\theta, \qquad \perp \theta = 1 \tag{28e}$$

$$\mu = \sum_{f=K,L} w_f V_f + \sum_j w_{R,j} V_{R,j}$$

$$- \sum_e \left((1+\overline{\tau}_e^Y)p_e + \tau^{\mathrm{CO_2}}\phi_e\right) g_{e,O}$$

$$- \sum_m \left((1+\overline{\tau}_m^Y)p_m\right) g_{m,O}$$

$$+ \tau^{\mathrm{CO_2}} Q^{\mathrm{CO_2}} + \sum_i \overline{\tau}_i^Y p_i y_i \qquad \perp \mu \tag{28f}$$

$$Q^{\mathrm{CO_2}} \geq \sum_e \phi_e \Bigg(\sum_j \beta_{e,j}^{\sigma_j} p_j^{\sigma_j} \left((1+\overline{\tau}_e^Y)p_e + \tau^{\mathrm{CO_2}}\phi_e\right)^{-\sigma_j} y_j$$

$$+ \alpha_e^\omega \theta^\omega \left((1+\overline{\tau}_e^Y)p_e + \tau^{\mathrm{CO_2}}\phi_e\right)^{-\omega} u + g_{e,O} \Bigg) \qquad \perp \tau^{\mathrm{CO_2}}, \tag{28g}$$

The final term on the right-hand side of (28f) represents the revenue from pre-existing taxes recycled to the representative agent. Interactions between $Q^{\mathrm{CO_2}}$ and $\overline{\tau}_j^Y$ occur through the effect of the former on industries' output prices and activity levels. We shall see that, apart from its direct impact on factor remuneration, the indirect effect of an emission limit is to provide additional income from recycled CO_2 permit-cum-tax revenues while at the same time attenuating the revenue from pre-existing taxes through its distortionary impact on commodity prices. With θ selected as the numeraire, the sum of these three effects determines the policy's aggregate welfare impact.

8.2 Data and Calibration

The SAM used to calibrate the model in (28) is constructed from the Bureau of Labor Statistics' 200-sector nominal make and use tables for the year 2004, using the industry technology assumption.[18] The components of value added are disaggregated using data on industries' shares of labor, capital, taxes and subsidies in GDP from the Bureau of Economic Analysis GDP by Industry accounts. The resulting benchmark flow table is aggregated to eight sectoral groupings outlined above, and scaled to approximate the U.S. economy in the year 2005 using the growth rate of real GDP. Adjustments were made to the intermediate transactions matrix to match Energy Information Administration (EIA) statistics on fossil fuel use, especially in the electric power sector, and to the factor supply matrix to disaggregate natural resource inputs from the returns to capital, following Sue Wing (2001). Finally, since 28.4% of the electricity generated in 2005 was supplied by carbon-free primary energy (nuclear, hydro and renewables), the electric power sector was split to disaggregate these sources of supply from fossil fuels.[19]

The final SAM is shown in Figure 6(b). While its structure is similar to Figure 2, it disaggregates the fossil fuel and carbon-free electricity subsectors, and includes an additional vector of benchmark payments of net taxes on production and imports in each industry ($\overline{\boldsymbol{\Psi}}$). These distortions affect the benchmark equilibrium, and therefore need to be taken into account in calibrating the model. As these flows are assumed to represent payments of taxes on industries' outputs, it is a simple matter to find the ad-valorem net tax rates implied by the SAM ($\overline{\tau}_j^Y = \overline{\psi}_j/\overline{y}_j$) and employ the result to compute the technical coefficients along the lines of eq. (17): $\alpha_{i,C} = \left(\overline{g}_{i,C}/\overline{G}_C\right)^{1/\omega}(1+\overline{\tau}_i^Y)$, and for the non-electric sectors, $\beta_{i,j} = \left(\overline{x}_{i,j}/\overline{y}_j\right)^{1/\sigma_j}(1+\overline{\tau}_j^Y)$ and $\gamma_{f,j} = \left(\overline{v}_{f,j}/\overline{y}_j\right)^{1/\sigma_j}(1+\overline{\tau}_j^Y)$.

To calibrate the electric power cost function we need to deal with the fact that the tax payments recorded in the SAM are not apportioned between the subsector activities. Indexing the subsectors by k = 5(a), 5(b), a simple solution is to define the gross-of tax level of activity of the aggregate

Figure 6: Model Sectoral Structure and Database

Energy (e)	**Non-energy** (m)	**Final Demands**
1. Coal mining	6. Energy-intensive industries	C Private consumption
2. Crude oil and gas	7. Transportation	O Other (Investment
3. Gas works and distribution	8. Other industries	+ Government + Net Exports)
4. Refined petroleum	**Primary factors** (f)	
5. Electric power	L Labor	
(a) Fossil fuel generation	K Capital	
(b) Carbon-free generation	R Primary energy resources	

(a) Sectoral structure

	1	2	3	4	5(a)	5(b)	6	7	8	C	O	Total
1	0.207	0.000	0.000	0.001	2.748	0.000	0.216	0.002	0.277	0.008	0.514	3.973
2	0.006	2.181	5.501	19.417	0.000	0.000	0.591	0.533	1.491	0.000	-15.072	14.648
3	0.000	0.018	0.105	0.294	3.343	0.000	0.408	0.059	1.956	4.287	0.023	10.494
4	0.093	0.153	0.104	4.369	0.513	0.000	2.375	4.858	15.241	8.871	-2.084	34.492
5	0.034	0.133	0.013	0.220	0.021	0.000	1.195	0.288	13.740	14.392	-0.035	30.002
6	0.063	0.312	0.023	0.692	0.179	0.071	16.679	0.348	48.898	21.713	-6.456	82.521
7	0.141	0.150	0.808	1.069	0.966	0.383	3.071	6.477	27.023	15.565	7.770	63.422
8	0.646	4.098	0.525	3.989	2.613	1.037	25.696	19.972	684.784	755.850	405.703	1904.913
L	1.009	1.078	0.823	0.968	2.908	1.154	15.897	20.179	667.040			711.058
K	0.972	3.168	2.160	3.320	7.248	2.875	15.484	9.081	365.512			409.819
R	0.648	2.592	0.000	0.000	0.000	0.533	0.000	0.000	0.000			3.773
$\overline{\psi}$	0.154	0.765	0.430	0.151	3.413		0.908	1.626	78.952			86.399
Total	3.973	14.648	10.494	34.492	30.002		82.521	63.422	1904.913	820.685	390.363	3355.512
q^{CO_2}	2,094		1,170	2,487								5,751
ϕ_e	0.053		0.011	0.007								

Monetary flows: 10^{10} 2004 dollars, CO_2 emissions ($\overline{q}^{CO_2}$): 10^6 tons, Emission coefficients (ϕ): tons CO_2 per dollar.
$\overline{\psi}$ Payments of state and federal taxes on production and imports net of subsidies.
GDP: \$12.1 Trillion, Gross output: \$33.6 Trillion.

(b) Benchmark social accounts for the year 2005 electric power sector as $\overline{y}_5 = \sum_k \overline{y}_k + \overline{\psi}_5$, where $\overline{y}_k = \sum_i \overline{x}_{i,k} + \sum_f \overline{v}_{f,k}$ denotes the net-of-tax levels of activity of the subsectors. The technical coefficients may then be computed on a gross-of-tax basis as $\eta_k = (\overline{y}_k/\overline{y}_5)^{1/\vartheta} (1 + \overline{\tau}_5^Y)$ at the upper level of the production hierarchy, and on a net-of-tax basis as $\beta_{i,k} = (\overline{x}_{i,k}/\overline{y}_k)^{1/\sigma_k}$ and $\gamma_{f,k} = (\overline{v}_{f,k}/\overline{y}_k)^{1/\sigma_k}$ at the lower level.

The final parameters necessary to calibrate the model are the substitution elasticities. In the absence of specific elasticity estimates I employ values in the range commonly found in the literature (cf. McKibbin and Wilcoxen, 1998). For simplicity I assume that substitution in the consumption of commodities is inelastic, and set $\omega = 0.5$. Substitution among inputs to production are also assumed to be uniformly inelastic, and to keep things simple I assume that the corresponding elasticities values are identical in all sectors: $\sigma_j = 0.8 \; \forall j$. The top-level elasticity in the electric power sector is different, however, because of the fossil fuel and carbon-free generation subsectors are near-perfect substitutes for one another in the production electricity. To reflect this fact, I set $\vartheta = 10$. If we plug these parameter values and the relevant flows from the SAM into the foregoing calibration equations, simulation of the resulting model replicates the initial distorted equilibrium in Figure 6(b).

8.3 Policy Analysis

The policy I consider is a limit on aggregate CO_2 emissions in the year 2012, which may be analyzed using a three-step procedure. The first step is to establish the link between emissions and the demands for the various fossil fuels solved for by the model in monetary terms. For this purpose I employ U.S. EPA (2007) data on the CO_2 emissions associated with the aggregate use of each fuel in 2005, indicated by $\overline{q}_e^{CO_2}$ in Figure 6(b). The emission coefficients are computed by dividing this quantity by the aggregate demand for each fossil fuel in the SAM ($\phi_e = \overline{q}_e^{CO_2}/\overline{y}_e$), which enables the calibration run of the model to replicate aggregate CO_2 emissions.

The second step is to project the future baseline emission level in 2010, by simulating the future expansion of the economy and the decline in its CO_2 intensity. Economic growth is modeled by scaling the benchmark endowments of primary factors upward at the average annual growth rate of GDP observed over the period 1999-2006 in the national income and product accounts (2.6%). This results in simulated GDP growth of approximately 15% from 2005 to 2012. To model the decline in aggregate emission intensity I scale the coefficients on energy in the model's cost and expenditure functions (α_e and $\beta_{e,j}$) downward at the average annual rate of decline in the CO_2-GDP ratio over the period 1999-2005 tabulated by EIA (-1.7%). This is essentially the same as introducing an index of autonomous energy efficiency improvement (AEEI) into the model. The AEEI is a popular device for simply representing the non-price induced secular decline in the aggregate energy- or emissions intensity commonly observed in developed economies.[20] Here, its effect is to reduce the simulated aggregate CO_2 intensity by just under 8% from 2005 to 2012.

Table 2(a) summarizes the characteristics of the no-policy "business-as-usual" (BAU) economy in 2012. Projected emissions from fossil fuels are 6183 million tons (MT) of CO_2, some 7% above 2005 levels, which represents a slightly faster growth of emissions than has been observed since 1999. The bulk of CO_2 emanates from the fossil electric subsector where the majority of the nation's coal is burned. The other significant contributions to aggregate emissions are made by the "rest-of-economy" sector, which is responsible for the bulk of petroleum demand, and household consumption, which uses substantial amounts of both petroleum and natural gas.

The third step is to solve the model with a quantity restriction on CO_2 emission as a counterfactual policy scenario. The emission target is loosely based on the proposed Climate Stewardship and Innovation Act of 2007 (S.280/H.R. 620), which seeks to limit annual emissions of a basket of six GHGs to 6130 MT over the period 2012-2019. As non-CO_2 GHGs are not accounted for within the model, an assumption needs to be made regarding the policy's impact on CO_2. For simplicity I assume that CO_2 emitted from fossil-fuel combustion is limited in the same proportion that fossil CO_2 contributes to aggregate GHG emissions in 2005: 79% of the 7,260 MT of total GHG emissions on a carbon-equivalent basis (U.S. EPA, 2007). The result is a CO_2 target of 4856

Table 2: The Economic Impacts of a CO_2 Emission Target

	1	2	3	4	5	5(a)	5(b)	6	7	8	Hhold.	Total
	(a) The no-policy "business-as-usual" scenario for 2012											
Prices[a]	0.96	0.95	0.97	0.95	0.96	0.95	0.98	1.00	0.99	1.01	–	–
Activity levels[b]	4.18	15.14	11.25	37.89	33.03	23.61	5.67	100.18	76.54	2293.14	992.61	1386.00[e]
Consumption[b]	0.01	0.00	4.68	9.78	15.78	–	–	26.35	18.93	913.36	–	–
Energy demand[b]												
Coal	0.20	0.00	0.00	0.00	2.88	2.88	–	0.24	0.00	0.31	0.01	4.18
Petroleum	0.09	0.14	0.10	4.32	0.54	0.54	–	2.67	5.42	17.04	9.78	37.89
Natural gas	0.00	0.02	0.10	0.29	3.48	3.48	–	0.45	0.07	2.15	4.68	11.25
Electricity	0.03	0.12	0.01	0.22	0.02	0.02	–	1.33	0.32	15.23	15.78	33.03
Tax revenue[b]	0.16	0.75	0.45	0.16	3.62	–	–	1.10	1.95	95.57	–	103.75
Pre-existing	0.16	0.75	0.45	0.16	3.62	–	–	1.10	1.95	95.57	–	103.75
CO_2 permits	–	–	–	–	–	–	–	–	–	–	–	–
Emissions[c]	109.39	12.13	18.47	343.88	1945.44	1945.44	–	369.68	399.47	1630.64	1231.74	6183.82
	(b) Policy impacts: changes from business-as-usual values (%, unless indicated otherwise)											
Prices	87.66	-5.78	16.85	11.11	9.19	10.84	4.26	0.25	0.34	-0.35	–	–
Activity levels	-40.77	-15.92	-13.83	-9.65	-6.06	-19.18	49.16	-0.75	-0.91	-0.10	-0.16	-0.14[e]
Consumption	-27.12	0.00	-7.64	-5.29	-4.46	–	–	-0.29	-0.33	0.01	–	–
Energy demand												
Coal	-66.51	-51.55	-49.24	-46.31	-46.96	-46.96	–	-39.89	-39.95	-39.80	-27.12	-40.77
Oil	-49.06	-26.31	-22.80	-18.35	-19.34	-19.34	–	-8.59	-8.67	-8.44	-5.29	-9.65
Gas	-51.07	-29.22	-25.85	-21.58	-22.52	-22.52	–	-12.20	-12.28	-12.06	-7.64	-13.83
Electricity	-48.35	-25.28	-21.71	-17.20	-18.20	-18.20	–	-7.30	-7.39	-7.15	-4.46	-6.06
Tax revenue	-3.15	-18.73	-11.13	297.52	57.43	57.43	–	46.56	32.06	2.16	2.01	7.50
Pre-existing	-45.50	-20.78	-16.54	-11.55	2.57	–	–	-0.50	-0.57	-0.46	–	-0.66
CO_2 permits[d]	42.34	2.05	5.42	309.07	54.86	54.86	–	47.06	32.63	2.62	2.01[f]	8.16
Emissions	-65.49	-27.06	-24.65	-18.70	-41.54	-41.54	–	-19.84	-8.82	-12.08	-6.37	-21.60
Abatement[c]	-71.64	-3.28	-4.55	-64.30	-808.13	-808.13	–	-73.34	-35.24	-197.00	-78.44	-1335.92

[a] Index: year 2005 = 1.00, [b] 10^{10} 2004 dollars, [c] Million tons CO_2, [d] Calculated as a percentage of revenues raised by pre-existing taxes, [e] Aggregate activity level proxied for by GDP, [f] Value of allowance purchases by the representative agent in 10^{10} 2004 dollars (there is no revenue from pre-existing taxes from which a percentage may be calculated).

MT, approximately 16% below the 2005 emission level and 22% below the BAU scenario.

The impacts of the policy are shown in Table 2(b). Constraining emissions induces significant increases in fossil fuel prices, particularly the price of coal, which almost doubles. The price of electricity rises as well, but only by 9%, reflecting both power generators' ability to substitute non-energy intermediate goods and labor and capital for fossil fuels, and the ability of carbon-free electric generators to expand supply. Thus, on a percentage basis the electricity price increase is smaller than the rise in the marginal cost of fossil generation, but is twice as large as the rise in the marginal cost of carbon-free power. Prices of non-energy commodities exhibit a negligible response, while the price of crude oil and gas mining falls with the demand for that sector's output.[21]

Overall, the output of the energy sectors is sharply curtailed. At one extreme, coal production declines by 41%, with double-digit declines in the demand for this fuel in every sector. At the other extreme, the generation of electric power declines by a mere six percent, with reductions in demand of a similar magnitude in non-energy sectors and three to eight times that in the energy sectors. However, this aggregate picture belies the fact that the fossil fuel subsector declines by 19%, while the carbon-free subsector experiences a massive expansion in its output, which increases nearly 50%. As before, the impact on production in non-energy sectors is very slight, with slight declines in output of less than one percent.

The most vigorous CO_2 abatement occurs in coal mining and fossil-fuel electricity generation, while the largest quantities of emissions are reduced by the fossil power and rest-of-economy sectors, with household consumption, transportation, coal mining and petroleum accounting for most of the remaining cuts. For the most part, these reductions have a negative impact on the revenue raised by pre-existing taxes, especially in energy industries. The signal exception is electric power, whose output is buoyed and prices are moderated by the expansion of carbon-free generation, but the overall effect of the emission target on revenue from benchmark taxes is less than 1%. This outcome is in contrast to the recycled revenues from CO_2 allowances, which account for an 8%

increase in tax revenue over the BAU scenario, and mostly emanate from the electric power, rest-of-economy and final consumption sectors.

Finally, looking at the impact on the economy as a whole, the shadow price on the emission target is modest: \$17.50 per ton, GDP falls by 0.14% relative to its BAU level, while the decline in aggregate consumption is slightly larger: 0.16%, all of which suggest that the macroeconomic costs of the emission target are modest. However, both the price of CO_2 and the attendant welfare losses are much smaller than those computed by Paltsev et al. (2007) using the MIT-EPPA model, a large-scale multi-regional simulation that resolves emissions of non-CO_2 GHGs and their abatement possibilities in addition to representing the frictions associated with the "putty-clay" character of capital adjustments (Paltsev et al., 2005). Exploring these and other sources of divergence helps to shed light on the limitations of the modeling approach pursued thus far, as well as initiate discussion of methods for addressing them which fall under the rubric of advanced topics that space constraints prevent me from dealing with here.

8.4 Caveats, and Potential Remedies

Returning to the black-box critique, it is useful to note that underlying the results in Table 2 are several driving forces whose precise effects on the macroeconomic costs of the policy shock have not been explicitly quantified. One one hand, decomposition analysis is a structured method for undertaking this kind of investigation which is gaining popularity because of its ability to accommodate simulations with large numbers of sectors, regions and exogenous parameters (see, e.g., Harrison et al., 2000; Paltsev, 2001; Böhringer and Rutherford, 2002). On the other hand, constructing highly stylized maquette models with a simplified structure and few sectors can also go a long way toward making the constituent economic interactions transparent. But this also implies that the present results should be taken with a grain of salt: simplified models such as the CES economy inevitably gloss over important real-world features of the economy that have potentially important implications for the effects of the policy under consideration. I go on to discuss come of these caveats below.

The first limitation is that consumption is the only price-responsive category of final uses. The constant "other final demand" vector implies that the economy's net export position and level of investment are both invariant to the emission limit, which is highly unrealistic. Addressing this shortcoming requires the modeler to disaggregate both gross trade flows and investment and model them as endogenous variables, with imports and exports specified as functions of the joint effects of changes in aggregate income and the level of gross-of-carbon-tax domestic prices in relation to world prices, and investment responding to the forward-looking behavior of households and the adjustment of saving and investment behavior to the policy shock. The model can then be re-cast in the format of a small open economy (e.g., Harrison et al., 1997), with imports and exports linked by a balance-of-payments constraint, and commodity inputs to production and final uses represented as Armington (1969) composites of imported and domestically-produced varieties.

Specifying and calibrating a fully forward-looking CGE model in the complementarity format of equilibrium is too complex an undertaking discuss here, and in any case Lau et al. (2002) provide an excellent introduction to the fundamentals. Recursive dynamic CGE models, which solve for a sequence of static equilibria chained together by intertemporal equations that update the economy's primary factor endowments and adjust the values of key time-varying parameters, have proven far more popular due to their comparative simplicity. The core of these models' dynamic process is an investment equation that uses the values of current-period variables to approximate the theoretically correct intertemporal demand for new capital formation. The realism of the present model could be markedly improved by enabling aggregate investment to adjust endogenously through the incorporation of a similar investment demand scheme.

A second limitation is the model's neglect of the important influences of capital malleability (the ability to adjust the factor proportions of production processes which employ extant capital) and intersectoral capital mobility on the short-run costs of emission constraints (Jacoby and Sue

Wing, 1999). At issue is the treatment of capital as homogeneous and capable of being frictionlessly reallocated as relative prices change, which casues to production exhibit complete reversibility. But in reality, changes in production activity of the magnitude seen in Table 2 would likely necessitate the scrappage and retrofit of energy-using capital on a massive scale, incurring substantial costs of adjustment. Such frictions may be captured by designating a portion of each sector's capital input as extant capital which is responsible for the production of output using a fixed input proportions technology. The likely consequence will be a substantial reduction in the mobility of—and returns to—capital, especially in declining sectors, with concomitantly larger abatement costs and reductions in welfare.

A third limitation is that, like capital, labor is modeled as being in inelastic supply. This, combined with the full employment assumption typical of many CGE models, implies that the reduction in the labor demanded by declining fossil fuel and energy-using sectors cannot result in unemployment. Instead, the wage falls, allowing the labor market to clear and surplus labor to move to other sectors, where it is re-absorbed. But in reality labor is likely to be far less mobile, implying that these types of price and quantity adjustments will occur more slowly, with the appearance of frictional unemployment in the interim. This phenomenon is easily simulated by introducing a labor supply curve into the model, through which the fall in the wage reduces the representative agent's endowment of labor.[22] Depending on the value of the labor supply elasticity the distorted equilibrium may exhibit significant unemployment, but general equilibrium interactions make it difficult to predict whether the welfare loss from an emission limit will be larger or smaller than in the inelastic labor supply case.

Lastly, perhaps the biggest deficiency of the current model is the CES assumption itself. Real-world policy analysis models routinely represent consumers' and producers' substitution possibilities using nested CES functions whose substitution elasticities that vary simultaneously among levels of the nesting structure and across sectors. The present model therefore underestimates the degree of inter-sectoral heterogeneity in substitution possibilities, implying that the results in Table 2 are subject to a range of biases in different directions. When faced with these sorts of issues, analysts typically undertake a sensitivity analysis to compare the results of simulations with different combinations of values for the various parameters in their models. However, the application of structured uncertainty analysis techniques that employ empirically-derived probability distributions over input parameters (e.g., Webster and Cho, 2006) has the potential to dramatically enhance our understanding of the scope and consequences of uncertainties in CGE models' structure and assumptions, and thereby generate robust insights into policies' economic impacts.

9 Summary

This paper has provided a lucid, rigorous and practically-oriented introduction to the fundamentals of computable general equilibrium modeling. The objective has been to de-mystify CGE models and their use in analyzing energy and climate policies by developing a simple, transparent and comprehensive framework within which to conceptualize their structural underpinnings, numerical parameterization, mechanisms of solution and techniques of application. Beginning with the circular flow of the economy, the logic and rules of social accounting matrices were developed, and it was demonstrated how imposing the axioms of producer and consumer maximization on this conceptual edifice facilitation the construction of a synthetic economy that could then be calibrated on these data. There followed a description of the techniques of numerical calibration and solution techniques, and a discussion of their implications for the uniqueness and stability of the simulated equilibria. The focus then shifted to techniques of application, introducing the kinds of structural modifications that allow CGE models to analyze the economy-wide impacts of various price and quantity distortions that arise in energy and environmental policy, which culminated in a practical demonstration using realistic numerical example.

Despite the broad swath of territory covered by this survey, space constraints have precluded discussion of many of the methodological tricks of the trade that are standard in CGE analyses of energy and climate policy. In particular, this chapter's closed-economy focus has paid scant attention to important issues of trade closure rules, model calibration in the presence of pre-existing import tariffs and/or export levies, or the specification and calibration of multi-region models which combine SAMs for individual economies with data on interregional trade flows. My hope is that the base of practical and theoretical knowledge developed here lays the groundwork for the study of these and other advanced topics in applied general equilibrium analysis.

Notes

[1]e.g., Panagariya and Duttagupta (2001): "Unearthing the features of CGE models that drive [their results] is often a time-consuming exercise. This is because their sheer size, facilitated by recent advances in computer technology, makes it difficult to pinpoint the precise source of a particular result. They often remain a black box. Indeed, frequently, authors are themselves unable to explain their results intuitively and, when pressed, resort to uninformative answers..."

[2]Of the numerous articles that use CGE simulations, the vast majority document only those attributes of their models that are relevant to the application at hand, or merely write down the model's equations with a minimum of explanation. On the other hand, expository books and manuals are often exhaustively detailed (e.g., Shoven and Whalley, 1992; Ginsburgh and Keyzer, 1997; Lofgren et al., 2002), and those in articles focused on applied numerical optimization (e.g., Rutherford 1995; Ferris and Pang 1997) often involve a high level of mathematical abstraction, neither of which make it easy for the uninitiated to quickly grasp the basics. Finally, although pedagogic articles (e.g., Devarajan et al 1997; Rutherford 1999; Rutherford and Paltsev 1999; Paltsev 2004) often provide a lucid introduction to the fundamentals, they tend to emphasize either models' structural descriptions or the details of the mathematical software packages used to build them, and have given short shrift to CGE models' theoretical basis or procedures for calibration.

[3]Together, these conditions imply that with unfettered competition firms will continue to enter

the economy's markets for goods until profits are competed away to zero.

[4]See also the simple example by Paltsev (2004).

[5]This simply a mathematical statement of Walras' Law (see e.g., Varian (1992, p. 343).)

[6]See Kehoe (1998a) for description of an alternative procedure when price data are available.

[7]Cf. Arndt et al. (2002), who develop a maximum-entropy data assimilation technique for calibrating substitution elasticities based on auxiliary information on prices and subjective bounds on parameter values.

[8]Econometric CGE models have been developed by Jorgenson (1984), McKibbin and Wilcoxen (1998), McKitrick (1998) and Fisher-Vanden and Ho (2007).

[9]See, e.g., Wilcoxen (1988) p. 127, especially footnote 2.

[10]See Dirkse and Ferris (1995); Ferris et al. (2000); Ferris and Kanzow (2002) for details of the algorithms and discussions of their convergence properties. The seminal theoretical work on this topic is Garcia and Zangwill (1981).

[11]Kehoe (1985) indicates that early findings along these lines by Foster and Sonnenschein (1970) and Hatta (1977) require at least one commodity to be an inferior good, a phenomenon which practically never arises in applied work on energy policy.

[12]I am grateful to Tim Kehoe for this insight.

[13]Kehoe and Whalley (1985) find no evidence of multiplicity in the Fullerton et al. (1981) and Kehoe and Serra-Puche (1983) tax models, while reports of multiple equilibria are restricted to models with increasing returns (Mercenier, 1995; Denny et al., 1997).

[14]Note that, for positive tax rates the tax revenue terms in these expressions are will be non-negative, satisfying the condition for uniqueness discussed in section 6.2.

[15]Section 5's discussion of the invariance of models' benchmark replication to the values of their substitution parameters figures prominently here. Imagine two static models, each with different substitution elasticities, calibrated so as to reproduce the same benchmark SAM in the absence of policy-induced distortions. An absolute emission limit can be imposed on the first model, and the

value of GDP in the resulting distorted equilibrium used to compute an ex-ante equivalent intensity target. Imposing this target on the first model will yield the same distorted equilibrium, but constraining the second model with this target will have different impacts as a result the alternative parameterization's effect on GDP.

[16]Observe that the revenue raised from all producers is $\sum_{e=1}^{\mathcal{E}} \rho\tau^{\text{RPS}}\varepsilon_e y_e$, while that received by renewable producers is $\sum_{e\in\mathcal{E}^R} \tau^{\text{RPS}}\varepsilon_e y_e$. Equating these expressions and canceling τ^{RPS} on both sides of the resulting expression yields the rationing constraint in the text. The implication is that the marginal financing charge exhibits complementary slackness with respect to the RPS constraint: the latter is either binding and $\tau^{\text{RPS}} > 0$, or it is non-binding and $\tau^{\text{RPS}} = 0$.

[17]I am grateful to Tom Rutherford for pointing this out to me.

[18]For details see, e.g., Reinert and Roland-Holst (1992). Gabriel Medeiros of the Bureau of Economic Analysis provided sterling assistance with the procedure.

[19]The column disaggregation of the sector was performed very simply: 28.4% of labor, capital and non-energy intermediate inputs, as well as all of the primary energy resource inputs, were allocated to carbon-free electricity generation, while the remaining inputs of intermediate goods and primary factors were allocated to fossil-fuel electricity generation. Sue Wing (2008) develops a more sophisticated method of disaggregating individual technologies from an aggregate economic sector.

[20]Sue Wing and Eckaus (2007) provide an in-depth discussion of the AEEI's conceptual shortcomings, empirical basis and use in CGE models.

[21]Note that the price increases shown in the table correspond to the *gross*-of-CO_2 markup prices of fossil fuels. *Net*-of-markup fossil fuel prices decline sharply as a consequence of shrinking demand.

[22]Balistreri (2002) develops a sophisticated external economies approach in which a slump in aggregate labor demand induces greater unemployment.

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