Separate Appendix (Not For Publication) to "The World Has More Than Two Countries: Implications of Multi-Country International Real Business Cycle Models"

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Contents

C Derivation

C.1 Models

Objective Function

$$E_0 \sum_{i=1}^{N} \sum_{t=0}^{\infty} \beta^t u(C_{i,t}, L_{i,t})$$

where

$$u(C_{i,t}, L_{i,t}) = \frac{(C_{i,t}^{\psi}(1-L_{i,t})^{1-\psi})^{1-\gamma}}{1-\gamma}.$$

subject to production technology and resource constraints.

(CD): Cobb=Douglas Production Function

$$Y_{i,t} = A_{i,t}\zeta(Z_{i,t})K_{i,t}^{\alpha}L_{i,t}^{1-\alpha}$$

TFP process is defined

$$A_{i,t} = A_0(1+g)^t\tilde{A}_{i,t} = \bar{A}_t\tilde{A}_{i,t}$$

where

$$\tilde{\boldsymbol{a}}_t = \boldsymbol{\Omega}\tilde{\boldsymbol{a}}_{t-1} + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\Omega}_{(i,i)} = \omega_{ii}$ and $\boldsymbol{\Omega}_{(i,h)} = \omega_{ih}$ where subscripts (i,h) stands for (i,h), non-diagonal, element of the matrix. ith row of $\tilde{\boldsymbol{a}}_t$ is $\log \tilde{A}_{i,t}$. ith row of $\boldsymbol{\varepsilon}_t$ is $\varepsilon_{i,t}$, $\boldsymbol{\varepsilon}_t \sim N$ jointly normal and i.i.d. across t, $E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$, $V(\boldsymbol{\varepsilon}_t)_{(i,i)} = \sigma_{ii}^2$ and $V(\boldsymbol{\varepsilon}_t)_{(i,h)} = \sigma_{ih}$.

Adding to this, combinations of following components generate variations of models:

Choices of Investment Friction

(NF): No Friction (Baseline)

$$K_{i,t+1} = (1-\delta(Z_{i,t}))K_{i,t} + I_{i,t}$$

(AF): Adjustment Friction

$$K_{i,t+1} = (1-\delta(Z_{i,t}))K_{i,t} + \phi\left(\frac{I_{i,t}}{K_{i,t}}\right)K_{i,t}$$

$$\phi\left(\frac{I}{K}\right) = \frac{I}{K}, \quad \phi'\left(\frac{I}{K}\right) = 1$$

(TB): Time-to-Build

$$\begin{aligned}
&K_{i,t+1} = (1-\delta(Z_{i,t}))K_{i,t} + V_{i,t,1} \\
&I_{i,t} = \frac{1}{J}\sum_{j=1}^{J} V_{i,t,j} \\
&V_{i,t+1,j} = V_{i,t,j+1} \quad \text{for } j = 1, ..., J-1.
\end{aligned}$$

Choices of Utilization

(CU): Constant Utilization

$$\delta(Z_{i,t}) = \delta$$

(VU): Variable Utilization

$$\delta(Z) = \delta, \quad Z = 1$$

Choice of Resource Constraint

(CM): Complete Market

$$\sum_i \pi_i [Y_{i,t} - C_{i,t} - I_{i,t}] = 0.$$

(TC): Complete Market with Trade Costs

$$\begin{aligned}
&\sum_i \pi_i [Y_{i,t} - C_{i,t} - I_{i,t} - \tau(Y_{i,t} - C_{i,t} - I_{i,t})] = 0. \\
&\tau(0) = 0, \tau'_{i,t} > 0,
\end{aligned}$$

(BM): Bond Market

$$\begin{aligned}
&Y_{i,t} + B_{i,t} = C_{i,t} + I_{i,t} + P_t B_{i,t+1} \\
&\sum_i \pi_i B_{i,t} = 0.
\end{aligned}$$

C.2 Equilibrium Conditions

Although the model is presented as a Social planner's problem, the resulting system coincides with competitive equilibria. The equilibrium conditions can be expressed as the appropriate combinations of following blocks of equations. All the model uses marginal utility (MU) condition and the production function (CD) condition. In addition, the model is characterized by one of the investment friction conditions ((NF), (AF) or (TB)), one of the utilization conditions ((CU) or (VU)), and one of the resource constraints ((CM), (TC) or (BM)).

Denote u_c and u_l first derivative of utility with respect to C and L. Λs are multipliers associate with various constraints.

(MU): Marginal Utility

$$
\begin{aligned}
u_c(i,t) &= \psi C_{i,t}^{\psi(1-\gamma)-1} \left(1 - L_{i,t}\right)^{(1-\psi)(1-\gamma)} \\
u_l(i,t) &= -(1-\psi) C_{i,t}^{\psi(1-\gamma)} \left(1 - L_{i,t}\right)^{(1-\psi)(1-\gamma)-1} \\
\Lambda_{i,t}^{Y} &= \pi_i u_c(i,t)
\end{aligned}
$$

(CD): Cobb-Douglas Production Function

$$
\begin{aligned}
Y_{i,t} &= A_{i,t} Z_{i,t}^{\alpha} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \\
(1-\alpha)\frac{Y_{i,t}}{L_{i,t}} &= \frac{1-\psi}{\psi} \frac{C_{i,t}}{1-L_{i,t}}
\end{aligned}
$$

(NF): No Friction

$$
\begin{aligned}
K_{i,t+1} &= (1 - \delta(Z_{i,t}))K_{i,t} + I_{i,t} \\
u_c(i,t) &= E_t \beta u_c(i,t+1) \left[\alpha \frac{Y_{i,t+1}}{K_{i,t+1}} + 1 - \delta(Z_{i,t+1})\right] \\
\Lambda_{i,t}^{K} &= u_c(i,t) \\
\phi'(i,t) &= 1
\end{aligned}
$$

(AF): Adjustment Friction

$$
\begin{aligned}
&K_{i,t+1} = (1-\delta(Z_{i,t}))K_{i,t} + \phi(i,t)K_{i,t} \\
&\frac{u_c(i,t)}{\phi'(i,t)} = E_t\beta\left[\frac{u_c(i,t+1)}{\phi'(i,t+1)}\left(\phi'(i,t+1)\alpha\frac{Y_{i,t+1}}{K_{i,t+1}} + 1 - \delta(Z_{i,t+1}) + \phi(i,t+1) - \phi'(i,t+1)\frac{I_{i,t+1}}{K_{i,t+1}}\right)\right] \\
&\Lambda^K_{i,t} = u_c(i,t) \\
&\phi(i,t) = \phi(I_{i,t}/K_{i,t}) \\
&\phi'(i,t) = d\phi(i,t)/d(I_{i,t}/K_{i,t})
\end{aligned}
$$

(TB): Time-to-Build

$$
\begin{aligned}
&K_{i,t+1} = (1-\delta(Z_{i,t}))K_{i,t} + V_{i,t,1} \\
&I_{i,t} = \frac{1}{J}\sum_{j=1}^{J} V_{i,t,j} \\
&V_{i,t+1,j} = V_{i,t,j+1} \\
&\Lambda^K_{i,t,} = E_t\beta\left[\Lambda^Y_{i,t+1}\alpha Y_{i,t+1}K^{-1}_{i,t+1} + \Lambda^K_{i,t+1}(1-\delta(Z_{i,t+1}))\right] \\
&\frac{1}{J}\Lambda^Y_{i,t} = \Lambda^K_{i,t} - \frac{1}{G_4}\Lambda^V_{i,t-1,1} \\
&\frac{1}{J}\Lambda^Y_{i,t} = \Lambda^V_{i,t,1} - \frac{1}{G_4}\Lambda^V_{i,t-1,2} \\
&... \\
&\frac{1}{J}\Lambda^Y_{i,t} = \Lambda^V_{i,t,J-2} - \frac{1}{G_4}\Lambda^V_{i,t-1,J-1} \\
&\frac{1}{J}\Lambda^Y_{i,t} = \Lambda^V_{i,t,J-1} \\
&\phi'(i,t) = 1
\end{aligned}
$$

(CU): Constant Utilization

$$
\begin{aligned}
&Z_{i,t} = 1 \\
&\delta(Z_{i,t}) = \delta
\end{aligned}
$$

(VU): Variable Utilization

$$
\phi'(i,t)\Lambda^Y_{i,t}\alpha\frac{Y_{i,t}}{Z_{i,t}} = \Lambda^K_{i,t}\delta'(Z_{i,t})K_{i,t}
$$

(CM): Complete Market

$$\sum_i \pi_i \left(Y_{i,t} - I_{i,t} - C_{i,t}\right) = 0$$
$$u_c(i,t) = u_c(h,t)$$

(TC): Complete Market with Trade Costs

$$\sum_i \pi_i \left(Y_{i,t} - I_{i,t} - C_{i,t} - \tau(Y_{i,t} - I_{i,t} - C_{i,t})\right) = 0$$
$$\frac{u_c(i,t)}{1-\tau'_{i,t}} = \frac{u_c(h,t)}{1-\tau'_{h,t}}$$

(BM): Bond Market

$$\sum_i \pi_i B_{i,t} = 0$$
$$Y_{i,t} + B_{i,t} = C_{i,t} + I_{i,t} + P_t B_{i,t+1}$$
$$P_t = \beta E_t \frac{u_c(i,t+1)}{u_c(i,t)}$$

C.3 Transformed Equilibrium Conditions

Denote variables with tilde as detrended values. The table summarizes various factors to detrend variables.

Detrending factors

detrend factor	variables
$\bar{A}_t^{\frac{1}{1-\alpha}}$	$Y_{i,t}, C_{i,t}, I_{i,t}, K_{i,t}, B_{i,t}, V_{i,t,j}$
$\bar{A}_t^{\frac{\psi(1-\gamma)}{1-\alpha}}$	$u_l(i,t), \Lambda_{i,t}^L, \Lambda_{i,t}^M$
$\bar{A}_t^{\frac{\psi(1-\gamma)-1}{1-\alpha}}$	$u_c(i,t), \Lambda_{i,t}^Y, \Lambda_{i,t}^K, \Lambda_{i,t,j}^V$
none	$L_{i,t}, Z_{i,t}$

For notational convenience, denote G's as growing factors, and define shortcut notation of adjustment friction functions

Growing factors

notation	value
G_1	$(1+g)$
G_2	$G_1^{\frac{1}{1-\alpha}}$
G_3	$\beta G_1^{\frac{\psi(1-\gamma)}{1-\alpha}}$
G_4	$\beta G_1^{\frac{\psi(1-\gamma)-1}{1-\alpha}}$

(MU)

$$\widetilde{u_c}(i,t) = \psi \widetilde{C}_{i,t}^{\psi(1-\gamma)-1}\left(1-\widetilde{L}_{i,t}\right)^{(1-\psi)(1-\gamma)}$$
$$\widetilde{u_l}(i,t) = -(1-\psi)\widetilde{C}_{i,t}^{\psi(1-\gamma)}\left(1-\widetilde{L}_{i,t}\right)^{(1-\psi)(1-\gamma)-1}$$
$$\widetilde{\Lambda}_{i,t}^Y = \pi_i \widetilde{u_c}(i,t)$$

(CD)

$$\widetilde{Y}_{i,t} = \widetilde{A}_{i,t}\widetilde{Z}_{i,t}^{\alpha}\widetilde{K}_{i,t}^{\alpha}\widetilde{L}_{i,t}^{1-\alpha}$$
$$(1-\alpha)\frac{\widetilde{Y}_{i,t}}{\widetilde{L}_{i,t}} = \frac{1-\psi}{\psi}\frac{\widetilde{C}_{i,t}}{1-\widetilde{L}_{i,t}}$$

(NF)

$$
\begin{aligned}
&G_2\widetilde{K}_{i,t+1} = (1-\delta(\widetilde{Z}_{i,t}))\widetilde{K}_{i,t} + \widetilde{I}_{i,t} \\
&\widetilde{u_c}(i,t) = G_4 E_t \left[\widetilde{u_c}(i,t+1)\left(\alpha\frac{\widetilde{Y}_{i,t+1}}{\widetilde{K}_{i,t+1}} + 1 - \delta(\widetilde{Z}_{i,t+1})\right)\right] \\
&\phi'(i,t) = 1
\end{aligned}
$$

(AF)

$$
\begin{aligned}
&G_2\widetilde{K}_{i,t+1} = (1-\delta(\widetilde{Z}_{i,t}))\widetilde{K}_{i,t} + \phi(i,t)\widetilde{K}_{i,t} \\
&\frac{\widetilde{u_c}(i,t)}{\phi'(i,t)} = G_4 E_t \left[\frac{\widetilde{u_c}(i,t+1)}{\phi'(i,t+1)}\left(\phi'(i,t+1)\alpha\frac{\widetilde{Y}_{i,t+1}}{\widetilde{K}_{i,t+1}} + 1 - \delta(\widetilde{Z}_{i,t+1}) + \phi(i,t+1) - \phi'(i,t+1)\frac{\widetilde{I}_{i,t+1}}{\widetilde{K}_{i,t+1}}\right)\right]
\end{aligned}
$$

(TB)

$$
\begin{aligned}
&G_2\widetilde{K}_{i,t+1} = (1-\delta(\widetilde{Z}_{i,t}))\widetilde{K}_{i,t} + \widetilde{V}_{i,t,1} \\
&\widetilde{I}_{i,t} = \frac{1}{J}\sum_{j=1}^{J}\widetilde{V}_{i,t,j} \\
&G_2\widetilde{V}_{i,t+1,j} = \widetilde{V}_{i,t,j+1} \\
&\widetilde{\Lambda}^K_{i,t,} = E_t G_4\left[\widetilde{\Lambda}^Y_{i,t+1}\alpha\widetilde{Y}_{i,t+1}\widetilde{K}^{-1}_{i,t+1} + \widetilde{\Lambda}^K_{i,t+1}(1-\delta(\widetilde{Z}_{i,t+1}))\right] \\
&\frac{1}{J}\widetilde{\Lambda}^Y_{i,t} = \widetilde{\Lambda}^K_{i,t} - \frac{1}{G_4}\widetilde{\Lambda}^V_{i,t-1,1} \\
&\frac{1}{J}\widetilde{\Lambda}^Y_{i,t} = \widetilde{\Lambda}^V_{i,t,1} - \frac{1}{G_4}\widetilde{\Lambda}^V_{i,t-1,2} \\
&\ldots \\
&\frac{1}{J}\widetilde{\Lambda}^Y_{i,t} = \widetilde{\Lambda}^V_{i,t,J-2} - \frac{1}{G_4}\widetilde{\Lambda}^V_{i,t-1,J-1} \\
&\frac{1}{J}\widetilde{\Lambda}^Y_{i,t} = \widetilde{\Lambda}^V_{i,t,J-1} \\
&\phi'(i,t) = 1
\end{aligned}
$$

(CU)

$$
\begin{aligned}
&\widetilde{Z}_{i,t} = 1 \\
&\delta(\widetilde{Z}_{i,t}) = \delta
\end{aligned}
$$

(VU)

$$\phi'(i,t)\widetilde{\Lambda}^{Y}_{i,t}\alpha\frac{\widetilde{Y}_{i,t}}{\widetilde{Z}_{i,t}} = \widetilde{\Lambda}^{K}_{i,t}\delta'(\widetilde{Z}_{i,t})\widetilde{K}_{i,t}$$

(CM)

$$\begin{aligned}
&\sum_i \pi_i \left[\widetilde{Y}_{i,t} - \widetilde{I}_{i,t} - \widetilde{C}_{i,t}\right] = 0 \\
&\widetilde{u_c}(i,t) = \widetilde{u_c}(h,t)
\end{aligned}$$

(TC)

$$\begin{aligned}
&\sum_i \pi_i \left[\widetilde{Y}_{i,t} - \widetilde{I}_{i,t} - \widetilde{C}_{i,t} - \tau(\widetilde{Y}_{i,t} - \widetilde{I}_{i,t} - \widetilde{C}_{i,t})\right] = 0 \\
&\frac{\widetilde{u_c}(i,t)}{1-\tau'_{i,t}} = \frac{\widetilde{u_c}(h,t)}{1-\tau'_{h,t}}
\end{aligned}$$

(BM)

$$\begin{aligned}
&\sum_i \pi_i \widetilde{B}_{i,t} = 0 \\
&\widetilde{Y}_{i,t} + \widetilde{B}_{i,t} = \widetilde{C}_{i,t} + \widetilde{I}_{i,t} + P_t G_2 \widetilde{B}_{i,t+1} \\
&P_t = \beta G_4 E_t \frac{\widetilde{u_c}(i,t+1)}{\widetilde{u_c}(i,t)}
\end{aligned}$$

C.4 Non-stochastic Steady State

At the non-stochastic steady state, market structure does not matter. Due to symmetric assumption, steady state values are the same across countries so that we drop country subscripts. Also, ad hoc capital adjustment friction and variable utilization do not affect by construction. Denote Ξ as the steady state value of $\widetilde{\Xi}_{i,t}$. The following system include cases with time-to-build (TB), adjustment friction (AF), and variable utilization (VU). If time-to-build is not used, set $J = 1$.

$$
\begin{aligned}
r &= \frac{1}{\alpha}\left(\frac{1}{G_4} - 1 + \delta\right) \\
L &= \left(1 + \frac{1-\psi}{\psi}\frac{1}{1-\alpha}\left(1 - \frac{1}{J}\frac{1-G_2^J}{1-G_2}(G_2 - 1 + \delta)r^{-1}\right)\right)^{-1} \\
K &= r^{\frac{1}{\alpha-1}} L \\
V_1 &= (G_2 - 1 + \delta)K \\
V_{j+1} &= G_2 V_j \\
I &= \frac{1}{J}\sum_{j=1}^{J} V_j \\
Y &= rK \\
C &= Y - I \\
Z &= 1 \\
\delta' &= \alpha r \\
u_c &= \psi C^{\psi(1-\gamma)-1}(1-L)^{(1-\psi)(1-\gamma)} \\
\Lambda^Y &= \pi u_c \\
\Lambda^V_{J-1} &= \frac{1}{J}\Lambda^Y \\
\Lambda^V_{J-2} &= (1 + \frac{1}{G_4})\frac{1}{J}\Lambda^Y \\
&\ldots \\
\Lambda^V_1 &= (1 + \ldots + \frac{1}{G_4^{J-2}})\frac{1}{J}\Lambda^Y \\
\Lambda^K &= (1 + \frac{1}{G_4^{J-1}})\frac{1}{J}\Lambda^Y \\
B &= 0 \\
P &= \beta G_4
\end{aligned}
$$

C.5 Log-linearized Equations

Let $\hat{\xi}_{i,t} = \log \tilde{\Xi}_{i,t} - \log \Xi$, deviation from the steady state, except for

$$\hat{b}_{i,t} = \frac{b_{i,t} - B}{Y}, \ \widehat{nx}_{i,t} = \frac{nx_{i,t} - nx}{Y}.$$

(MU)

$$\widehat{u_c}(i,t) = (\psi(1-\gamma) - 1)\hat{c}_{i,t} - (1-\psi)(1-\gamma)\frac{L}{1-L}\hat{l}_{i,t}$$

(CD)

$$\begin{aligned}
\hat{y}_{i,t} &= \hat{a}_{i,t} + \alpha \hat{z}_{i,t} + \alpha \hat{k}_{i,t} + (1-\alpha)\hat{l}_{i,t} \\
\hat{y}_{i,t} &= \hat{c}_{i,t} + \frac{1}{1-L}\hat{l}_{i,t}
\end{aligned}$$

(NF)

$$\begin{aligned}
&\hat{k}_{i,t+1} = \hat{k}_{i,t} - \frac{1}{G_2}\delta'(Z)Z\hat{z}_{i,t} + \frac{1}{G_2}\frac{I}{K}\left(\hat{\imath}_{i,t} - \hat{k}_{i,t}\right) \\
&\widehat{u_c}(i,t) = E\widehat{u_c}(i,t+1) + (1 - G_4(1-\delta))\left(E\hat{y}_{i,t+1} - E\hat{k}_{i,t+1}\right) - G_4\delta'(Z)ZE\hat{z}_{i,t+1} \\
&\hat{\lambda}^K_{i,t} = \widehat{u_c}(i,t) \\
&\widehat{\phi'}(i,t) = 0
\end{aligned}$$

(AF)

$$\begin{aligned}
&\hat{k}_{i,t+1} = \hat{k}_{i,t} - \frac{1}{G_2}\delta'(Z)Z\hat{z}_{i,t} + \frac{1}{G_2}\frac{I}{K}\left(\hat{\imath}_{i,t} - \hat{k}_{i,t}\right) \\
&\widehat{u_c}(i,t) - \widehat{\phi'}(i,t) = E\widehat{u_c}(i,t+1) + (1 - G_4(1-\delta))\left(E\hat{y}_{i,t+1} - E\hat{k}_{i,t+1}\right) \\
&\qquad\qquad - G_4\delta'(Z)ZE\hat{z}_{i,t+1} - G_4G_2E\widehat{\phi'}(i,t+1) \\
&\widehat{\phi'}(i,t) = -\frac{\phi''\frac{I}{K}}{\phi'}\left(\hat{k}_{i,t} - \hat{\imath}_{i,t}\right) \\
&\hat{\lambda}^K_{i,t} = \widehat{u_c}(i,t)
\end{aligned}$$

(TB)

$$
\begin{aligned}
&\hat{k}_{i,t+1} = \hat{k}_{i,t} - \frac{1}{G_2}\delta'(Z)Z\hat{z}_{i,t} + \frac{1}{G_2}\frac{V_1}{K}\left(\hat{v}_{i,t,1} - \hat{k}_{i,t}\right) \\
&I\hat{\imath}_{i,t} = \frac{1}{J}\sum_{j=1}^{J} V_j \hat{v}_{i,t,j} \\
&\hat{v}_{i,t+1,j} = \hat{v}_{i,t,j+1} \\
&\hat{\lambda}^K_{i,t} = (1 - G_4(1-\delta))\left(E\widehat{u_c}(i,t+1) + E\hat{y}_{i,t+1} - E\hat{k}_{i,t+1}\right) \\
&\qquad\qquad +G_4(1-\delta)E\hat{\lambda}^k_{i,t+1} - G_4\delta'(Z)ZE\hat{z}_{i,t+1} \\
&\Lambda^K\hat{\lambda}^K_{i,t} - \frac{1}{J}\Lambda^Y\widehat{u_c}(i,t) = \frac{1}{G_4}\Lambda^V_1\hat{\lambda}^V_{i,t-1,1} \\
&\Lambda^V_1\hat{\lambda}^V_{i,t,1} - \frac{1}{J}\Lambda^Y\widehat{u_c}(i,t) = \frac{1}{G_4}\Lambda^V_2\hat{\lambda}^V_{i,t-1,2} \\
&... \\
&\Lambda^V_{J-2}\hat{\lambda}^V_{i,t,J-2} - \frac{1}{J}\Lambda^Y\widehat{u_c}(i,t) = \frac{1}{G_4}\Lambda^V_{J-1}\hat{\lambda}^V_{i,t-1,J-1} \\
&\widehat{u_c}(i,t) = \hat{\lambda}^V_{i,t,J-1} \\
&\widehat{\phi'}(i,t) = 0
\end{aligned}
$$

(CU)

$$\hat{z}_{i,t} = 0$$

(VU)

$$\widehat{u_c}(i,t) + \widehat{\phi'}(i,t) + \hat{y}_{i,t} = \left(1 + \frac{\delta'' Z}{\delta'}\right)\hat{z}_{i,t} + \hat{k}_{i,t} + \hat{\lambda}^K(i,t)$$

(CM)

$$
\begin{aligned}
&\sum_i \pi_i\left(Y\hat{y}_{i,t} - I\hat{\imath}_{i,t} - C\hat{c}_{i,t}\right) = 0 \\
&\widehat{u_c}(i,t) = \widehat{u_c}(h,t)
\end{aligned}
$$

(TC)

$$
\begin{aligned}
&\sum_i \pi_i\left(Y\hat{y}_{i,t} - I\hat{\imath}_{i,t} - C\hat{c}_{i,t}\right) = 0 \\
&\widehat{u_c}(i,t) + \tau_i''(0)\left(Y\hat{y}_{i,t} - I\hat{\imath}_{i,t} - C\hat{c}_{i,t}\right) = \widehat{u_c}(h,t) + \tau_h''(0)\left(Y\hat{y}_{h,t} - I\hat{\imath}_{h,t} - C\hat{c}_{h,t}\right)
\end{aligned}
$$

(BM)

$$
\begin{aligned}
&\sum_i \pi_i \hat{b}_{i,t} = 0 \\
&\widehat{u_c}(i,t) - E\widehat{u_c}(i,t+1) = \widehat{u_c}(h,t) - E\widehat{u_c}(h,t+1) \\
&\hat{y}_{i,t} + \hat{b}_{i,t} = \frac{C}{Y}\hat{c}_{i,t} + \frac{I}{Y}\hat{\imath}_{i,t} + G_3\hat{b}_{i,t+1}
\end{aligned}
$$

TFP process

$$\widehat{\boldsymbol{a}}_t = \boldsymbol{\Omega}\widehat{\boldsymbol{a}}_{t-1} + \widehat{\boldsymbol{\varepsilon}}_t$$

D Robustness Tables

Following tables are the tables similar to Table 3 in the main text, for the models shown in Figures 2–5.

Table D1: Business Cycle Moments of Backus et al. (1992) Model

N	2	3	5	10	25	A	B	C	D	E	
Size	0.5	0.33	0.2	0.1	0.04	0.1	0.9	0.1	0.8	0.1	Data
Standard deviation relative to standard deviation of output											
Consumption	1.01	0.75	0.60	0.51	0.47	0.66	1.84	0.58	1.47	0.50	0.91
Investment	5.68	5.46	5.39	5.37	5.37	5.18	6.60	5.23	6.03	5.37	2.60
Labor	0.87	0.75	0.69	0.65	0.64	0.85	1.25	0.79	0.97	0.64	1.04
Net exports	1.38	1.38	1.38	1.38	1.38	1.65	0.46	1.55	0.72	1.37	0.61
Correlation to output											
Consumption	0.20	0.25	0.30	0.35	0.38	−0.06	0.28	0.00	0.39	0.37	0.77
Investment	0.06	0.10	0.13	0.16	0.17	−0.09	0.34	0.03	0.30	0.17	0.75
Labor	0.63	0.75	0.82	0.86	0.89	0.84	0.26	0.87	0.30	0.87	0.74
Net exports	0.56	0.52	0.50	0.48	0.47	0.69	0.11	0.62	0.17	0.47	−0.36
Autocorrelation											
Output	0.90	0.91	0.91	0.92	0.92	0.91	0.86	0.92	0.88	0.92	0.93
Consumption	0.93	0.93	0.93	0.93	0.92	0.93	0.93	0.93	0.93	0.93	0.94
Investment	0.89	0.88	0.87	0.87	0.87	0.88	0.91	0.87	0.90	0.87	0.93
Labor	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.93
Net exports	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.91
Cross-country correlation											
Output	−0.76	−0.40	−0.19	−0.07	−0.01	−0.49	−0.49	0.20	−0.47	−0.08	0.37
Consumption	0.93	0.90	0.88	0.85	0.83	0.94	0.94	0.91	0.92	0.84	0.28
Investment	−0.38	−0.22	−0.10	−0.02	0.03	−0.20	−0.20	0.24	−0.27	−0.03	0.23
Labor	0.27	0.20	0.19	0.19	0.19	0.57	0.57	0.59	0.44	0.16	0.23
Net exports	−1.00	−0.50	−0.24	−0.11	−0.04	−1.00	−1.00	0.32	−0.81	−0.12	−0.02

N and size patterns: The first five columns represent models of symmetric size. Sizes are $1/N$. A, B: $N = 2, \{0.1, 0.9\}$. Intra-country moments are for the first (A) and second (B) countries. C, D: $N = 3, \{0.1, 0.1, 0.8\}$. Intra-country moments are for the small (C) and large (D) countries. Cross-country correlations are of the two small countries (C), and the large and one of the small countries (D). E: $N = 19, \{\{0.1\} \times 9, \{0.01\} \times 10\}$. Cross-country correlations are of first two countries. The data (median of G7 countries) is taken from Table 1 in the main text.

Table D2: Business Cycle Moments of Baxter and Crucini (1995) Model

N	2	3	5	10	25	A	B	C	D	E	
Size	0.5	0.33	0.2	0.1	0.04	0.1	0.9	0.1	0.8	0.1	Data
Standard deviation relative to standard deviation of output											
Consumption	1.00	1.07	1.14	1.19	1.22	1.18	0.83	1.19	0.87	1.19	0.91
Investment	2.40	2.65	2.86	3.02	3.11	3.12	1.77	3.11	1.91	3.01	2.60
Labor	0.22	0.25	0.27	0.28	0.29	0.44	0.12	0.41	0.11	0.28	1.04
Net exports	0.56	0.67	0.75	0.81	0.85	1.05	0.11	1.01	0.19	0.81	0.61
Correlation to output											
Consumption	0.95	0.94	0.94	0.94	0.95	0.84	1.00	0.86	0.99	0.94	0.77
Investment	0.89	0.88	0.88	0.89	0.89	0.73	0.99	0.76	0.98	0.89	0.75
Labor	0.16	−0.04	−0.18	−0.29	−0.35	0.01	0.94	−0.04	0.83	−0.30	0.74
Net exports	−0.43	−0.51	−0.57	−0.62	−0.65	−0.30	−0.54	−0.35	−0.61	−0.62	−0.36
Autocorrelation											
Output	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.93
Consumption	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.94
Investment	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.93
Labor	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.93
Net exports	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.91
Cross-country correlation											
Output	0.46	0.43	0.41	0.38	0.37	0.48	0.48	0.41	0.46	0.38	0.37
Consumption	−0.08	0.01	0.07	0.11	0.14	−0.05	−0.05	0.15	−0.03	0.11	0.28
Investment	−0.32	−0.14	−0.03	0.04	0.08	−0.24	−0.24	0.13	−0.21	0.04	0.23
Labor	−0.37	−0.17	−0.05	0.02	0.06	0.55	0.55	0.56	0.36	0.00	0.23
Net exports	−1.00	−0.50	−0.24	−0.11	−0.04	−1.00	−1.00	0.32	−0.81	−0.12	−0.02

N and size patterns: The first five columns represent models of symmetric size. Sizes are $1/N$. A, B: $N = 2, \{0.1, 0.9\}$. Intra-country moments are for the first (A) and second (B) countries. C, D: $N = 3, \{0.1, 0.1, 0.8\}$. Intra-country moments are for the small (C) and large (D) countries. Cross-country correlations are of the two small countries (C), and the large and one of the small countries (D). E: $N = 19, \{\{0.1\} \times 9, \{0.01\} \times 10\}$. Cross-country correlations are of first two countries. The data (median of G7 countries) is taken from Table 1 in the main text.

Table D3: Business Cycle Moments of Baxter and Farr (2005) Model

N	2	3	5	10	25	A	B	C	D	E	
Size	0.5	0.33	0.2	0.1	0.04	0.1	0.9	0.1	0.8	0.1	Data
Standard deviation relative to standard deviation of output											
Consumption	0.88	0.91	0.93	0.96	0.97	0.94	0.81	0.95	0.83	0.96	0.91
Investment	1.91	2.02	2.10	2.17	2.21	2.16	1.69	2.16	1.74	2.17	2.60
Labor	0.13	0.12	0.11	0.11	0.10	0.18	0.13	0.17	0.12	0.10	1.04
Net exports	0.23	0.27	0.30	0.33	0.34	0.43	0.04	0.41	0.08	0.32	0.61
Correlation to output											
Consumption	0.99	0.99	0.99	0.99	0.99	0.96	1.00	0.97	1.00	0.99	0.77
Investment	0.98	0.97	0.97	0.97	0.97	0.92	1.00	0.94	1.00	0.97	0.75
Labor	0.73	0.60	0.48	0.36	0.29	0.35	0.99	0.34	0.98	0.36	0.74
Net exports	−0.50	−0.59	−0.66	−0.71	−0.74	−0.41	−0.57	−0.46	−0.65	−0.72	−0.36
Autocorrelation											
Output	0.90	0.91	0.91	0.91	0.91	0.91	0.90	0.91	0.90	0.91	0.93
Consumption	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.94
Investment	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.93
Labor	0.91	0.91	0.91	0.91	0.91	0.90	0.90	0.91	0.90	0.91	0.93
Net exports	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.91
Cross-country correlation											
Output	0.45	0.42	0.39	0.37	0.36	0.45	0.46	0.39	0.44	0.37	0.37
Consumption	0.19	0.20	0.21	0.22	0.23	0.19	0.19	0.22	0.19	0.22	0.28
Investment	0.06	0.11	0.14	0.17	0.18	0.07	0.07	0.18	0.08	0.17	0.23
Labor	0.86	0.83	0.80	0.77	0.74	0.93	0.93	0.91	0.92	0.76	0.23
Net exports	−1.00	−0.50	−0.24	−0.11	−0.04	−1.00	−1.00	0.32	−0.81	−0.12	−0.02

N and size patterns: The first five columns represent models of symmetric size. Sizes are $1/N$. A, B: $N = 2, \{0.1, 0.9\}$. Intra-country moments are for the first (A) and second (B) countries. C, D: $N = 3, \{0.1, 0.1, 0.8\}$. Intra-country moments are for the small (C) and large (D) countries. Cross-country correlations are of the two small countries (C), and the large and one of the small countries (D). E: $N = 19, \{\{0.1\} \times 9, \{0.01\} \times 10\}$. Cross-country correlations are of first two countries. The data (median of G7 countries) is taken from Table 1 in the main text.

Table D4: Business Cycle Moments of Time-to-Build and Variable Utilization Model

N	2	3	5	10	25	A	B	C	D	E	
Size	0.5	0.33	0.2	0.1	0.04	0.1	0.9	0.1	0.8	0.1	Data
Standard deviation relative to standard deviation of output											
Consumption	0.20	0.19	0.18	0.17	0.17	0.19	0.20	0.18	0.19	0.17	0.91
Investment	4.34	4.47	4.59	4.68	4.74	4.45	4.15	4.47	4.22	4.69	2.60
Labor	0.69	0.67	0.66	0.65	0.64	0.69	0.70	0.68	0.69	0.65	1.04
Net exports	0.37	0.45	0.50	0.54	0.56	0.64	0.07	0.63	0.13	0.54	0.61
Correlation to output											
Consumption	0.08	0.19	0.29	0.38	0.43	0.04	−0.01	0.11	0.06	0.39	0.77
Investment	0.93	0.90	0.88	0.87	0.87	0.82	0.98	0.82	0.98	0.88	0.75
Labor	0.98	0.98	0.98	0.99	0.99	0.98	0.98	0.98	0.98	0.99	0.74
Net exports	−0.03	−0.06	−0.09	−0.12	−0.13	0.15	−0.20	0.11	−0.27	−0.13	−0.36
Autocorrelation											
Output	0.92	0.92	0.91	0.91	0.91	0.92	0.93	0.92	0.92	0.91	0.93
Consumption	0.95	0.95	0.94	0.94	0.94	0.95	0.95	0.95	0.95	0.94	0.94
Investment	0.93	0.93	0.92	0.92	0.92	0.93	0.94	0.93	0.94	0.92	0.93
Labor	0.94	0.93	0.93	0.92	0.92	0.94	0.94	0.94	0.94	0.92	0.93
Net exports	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.91
Cross-country correlation											
Output	0.47	0.40	0.36	0.34	0.33	0.50	0.50	0.46	0.45	0.34	0.37
Consumption	0.75	0.69	0.65	0.61	0.58	0.76	0.76	0.71	0.72	0.60	0.28
Investment	0.33	0.27	0.25	0.25	0.24	0.37	0.37	0.34	0.30	0.24	0.23
Labor	0.60	0.52	0.47	0.44	0.41	0.64	0.63	0.58	0.58	0.43	0.23
Net exports	−1.00	−0.50	−0.24	−0.10	−0.04	−1.00	−1.00	0.32	−0.81	−0.11	−0.02

N and size patterns: The first five columns represent models of symmetric size. Sizes are $1/N$. A, B: $N = 2, \{0.1, 0.9\}$. Intra-country moments are for the first (A) and second (B) countries. C, D: $N = 3, \{0.1, 0.1, 0.8\}$. Intra-country moments are for the small (C) and large (D) countries. Cross-country correlations are of the two small countries (C), and the large and one of the small countries (D). E: $N = 19, \{\{0.1\} \times 9, \{0.01\} \times 10\}$. Cross-country correlations are of first two countries. The data (median of G7 countries) is taken from Table 1 in the main text.