Appendix to
Trade Costs and Business Cycles Transmission in a Multi-country, Multi-sector Model

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A.1 Additional statistics and figures

A.1.1 Reduced form gravity equations

Table A.1 contains the results obtained from reduced form gravity equations. These gravity equations are estimated by pooling all the sample.

A.1.2 IRBC statistics

Table A.2 shows international business statistics of additional cases. All the statistics are taken from the model G7 average.

- Complete market: the market assumption is complete market. The corresponding plot is the left-panel of Figure A.1.
- Financial autarky: the market assumption is financial autarky. The corresponding plot is the right-panel of Figure A.1.
- Same TC: Only cross-country variation is population. The corresponding plot is the left-panel of Figure A.2.
- Same Pop: Only cross-country variation is trade costs. The corresponding plot is the right-panel of Figure A.2.
- Symmetric countries: Perfectly symmetric case.

A.1.3 Additional robustness checks

- Figure A.3: Autocorrelation of productivity shock ρ_a
- Figure A.4: Autocorrelation of trade cost shock ρ_τ
- Figure A.5: CES parameter θ
- Figure A.6: CES parameter ρ
- Figure A.7: Capital stock adjustment friction parameter η_ϕ
- Figure A.8: Number of countries specified as the rest of the world. (Baseline is six.)

A.2 Data and mapping from data to model

A.2.1 Data for the business cycle calculations

The quarterly and monthly data are filtered by approximate band path filter (Baxter and King, 1999). For the quarterly data, the cutoff parameters are $(6, 32)$ and autoregressive order is 12. The annual data is filtered by HP filter, with $\lambda = 6.25$ (Hodrick and Prescott, 1997). The choice of the filters for annual and quarterly data does not alter basic implications. HP is employed for keeping longer observations in annual data. For the monthly data, the cutoff parameters are $(2, 96)$ and autoregressive order is 18 for extracting trend. For the monthly data, the cutoff parameters are $(18, 96)$ and autoregressive order is 18 for extracting business cycle component.

Quarterly national accounts and labor variables are taken from "Source OECD" quarterly national account and main economic indicators. If longer data is available through country's own source or older SNA system, main economic indicator data is extrapolated by growth rate of additional data. All the variables are quarterly and seasonally adjusted. Real values in the data set are used. Hours and employments are normalized by 2000 values (average of four quarters in 2000) separately, and then they are multiplied to obtain labor variable. Italian labor is calculated only using employment data since hours data is missing. Italian labor statistics is used for TFP calculation. Labor related statistics are based on 6 countries. Filtering is applied for the longest possible dataset, and then moments are calculated using limited sample period (1970, 1st quarter to 2006, 4th quarter). Germany and West Germany data are separately filtered. Then, statistics are obtained by pooling both Germany and West Germany. Excluding net exports, variables are in natural logs, and then applied filters. Net exports are filtered after divided by real GDP. Aggregate TFP is calculated by standard Solow accounting. After calculating autocorrelation by fitting AR1, TFP shock is calculated as residual.

Sector level productivity is calculated using Historic ISIC industrial production data base (ISIC2) and ISIC industrial production data base (ISIC3) obtained from "Source OECD". ISIC2 is used as basis and expanded by the data using growth rates of ISIC3. The sectors are manufacturing (3 in ISIC2, D in ISIC3), food (31 in ISIC2, 15/16 in ISIC3) and mining (2 in ISIC2, C in ISIC3). Seasonally adjusted production indices and labor indices (no capital stock information) are used. If data is provided only in raw series, X12 is applied. The productivity is calculated by production/labor$^{(2/3)}$. After calculating autocorrelations by fitting AR1, shock components are calculated as residuals.

Stopford (2009) compiles long time series of the freight cost. This freight index represents a price of the dry bulk and grain freight, based mainly on US gulf to Japan via Panama canal. The index is annual, and both nominal and inflation adjusted values are provided. The nominal index series is normalized by 1947 value, and then real value is obtained by using appropriate deflator. This paper employs real value index from 1947–2007.

Baltic exchange dry index (BDI) is a price index of dry bulk, compiled by Baltic Exchange. BDI is an average made up of various sizes and routes. The price is in (nominal) US dollar. The data is drawn from Datastream (Datastream code: BALTICF). The available index is daily (business dates) observations from May 1985. After taking a unweighted monthly average of the daily observations, the nominal price is adjusted by US monthly CPI (Bureau of Labor Statistics series: CUUR0000SA0).

A.2.2 Data for the estimations

A.2.2.1 Data set

National accounting is taken from Penn World Table mark 6.2 (Heston et al., 2008). Trade data is Feenstra et al. (2005). It spans whole countries and economic regions, 4-digit SITC (SITC rev. 2), from 1962 to 2000. I compress this data into 21 OECD countries, 1-digit SITC (10 categories of products), from 1962 to 2000. As a result, $163,800 = 21 \times 20 \times 10 \times 39$ observations are used. Distance variables are taken from Glick and Rose (2002) and Rose (2005). Since my focus is only OECD countries, used variables are log of the bilateral physical distance, and dummy variables. Dummies are of sharing border, of common language, of formal colonial relationship, if both in EU, if both in NAFTA, and if both in Australia-New Zealand FTA. Free trade agreement dummies are aggregated into a single dummy variable.

A.2.2.2 Data modifications

I made some modifications to data set.

National accounts of Belgium and Luxembourg

Since trade data treats Belgium and Luxembourg as a single country, national account data is modified. Weighted average is taken for the series for each year, where the population is used as a weight. It preserve aggregate real GDP as a sum of the two economies. Distance data treats two countries as a single entry before 1996 (basically using Belgium information). After 1997, there are two separated entries. Belgium information is used as Belgium-Luxembourg entries.

National accounts of Germany

PWT6.2 has no German entries before 1969, except for population and nominal exchange rate. Other entries are extrapolated by using West Germany growth rate series, based on 1970 observations. The growth rate is obtained from PWT 5.6. On trade data, entire Germany is used after reunification. Before unification, West Germany data is used. In a few years around reunification, there are double entries for West Germany and Germany. According to the data appendix (Feenstra et al., 2005), they are not double counted, so that they are summed up.

Imports of Canada in 1978

Canadian imports in 1978 in Feenstra et al. (2005) data set show obvious errors. Import values in 1978 are around half of 1977 and 1979 values. There is no obvious reason that there is such a huge temporal decline. The data is replaced by directly obtained data from UN comm trade data base.

Population

The model assumes constant population over time. The constant fraction of population to world population, π_i, is calculated as follows. First, I sum up the population of the sample countries for each year using PWT. Then, I calculate population fraction of each country in each year by dividing country's population by total of the sample countries. Finally, time series average of the fraction given period is taken.

A.2.2.3 Price conversion and national accounting

The model economy has no nominal friction, inflation, etc. Also, the model economy uses world common currency. Consumption bundle price level is different across countries, reflecting their productivity. The model "purchasing power parity" price measure, $P_{i,t}$, is different from actual data counterpart in PWT. The model is CES aggregator whereas PWT employs Geary-Khamis method. The mapping between them is highly nonlinear function. Yet, within OECD economies, the discrepancy among various construction methods of world consumption basket and price index is minimal (Deaton and Heston, 2008).

Given the difference, the basic strategy of conversion is following. First, I express (or use) statistics in current USD. Then, USD inflation is adjusted by USD deflator. For international consistency, inflation adjustment is by implicit US PPP deflator (obtained by fixed 2000 basket Lasperyres measure) in PWT. Finally, I adjust PPP. Namely,

$$\underbrace{\underbrace{\frac{\text{CULCU}}{\text{EXR}}}_{\text{CUUSD}}\frac{1}{\text{USDEF}}}_{\text{COUSD}} \Big/ \underbrace{\frac{\text{PPP}}{\text{EXR}}}_{\text{PPP in USD}} = \text{COPPP}$$

where, for the case of GDPPC (GDP per capita), CULCU is current local currency unit GDPPC, CUUSD is current GDPPC in USD, COUSD is constant price GDPPC in USD = $P_{i,t}Y_{i,t}$, COPPP is constant PPP adjusted GDPPC (in USD) = "RGDPL" = $Y_{i,t}$, EXR is

“XRAT”, nominal exchange rate (Local currency/USD), USDEF is USD deflater, calculated by “CGDP”/“RGDPL” (2000 USD = 1), PPP is “PPP”, expressed in terms of LCU by USD, and PPP/EXR is price level as if LCU is USD = $P_{i,t}$. The variabels with quotation marks are directly obtained from PWT. Given GDP statistics (RGDPL series), “RGDPL” × (“KC” +“KG”+ “KI”) = $Z_{i,t}$ where KC, KI, and KG are consumption, government spending, and investment shares in PWT.

The trade data is recorded in CIF, based on importing country’s report, in nominal US dollar, in current value. Let’s denote the value as CUUSD. US dollar deflator is constructed by USDEF=“CGDP”/“RGDPL” of US in PWT. Then

$$\frac{\text{CUUSD}}{\text{USDEF}} = ex_{j,i,m,t} = \pi_i p_{j,i,m,t} z_{j,i,m,t}$$

A.3 Estimation detail of the second step

The fixed effect estimation does not give $\bar{\Gamma}$, $\bar{\Lambda}$ and ν. Yet, there are N additional structural assumptions: intra-country trade costs are all unity. The basic idea of the following estimation strategy is to set these three parameters so as to minimize the deviations of N conditions, using non-linear least squares. The structural interpretation of the terms are

$$\exp\left(\Gamma_j\right) = p_j^{\frac{-\rho}{1-\rho}}, \tag{A.1}$$

$$\exp\left(\nu + x'_{j,i}\beta + \varepsilon_{j,i}\right) = \tau_{j,i}^{\frac{-\rho}{1-\rho}}, \tag{A.2}$$

$$\exp\left(\Lambda_i\right) = \left(\sum_{h=1}^{N} p_h^{\frac{-\rho}{1-\rho}} \tau_{h,i}^{\frac{-\rho}{1-\rho}}\right)^{\frac{\theta-\rho}{\rho(1-\theta)}}. \tag{A.3}$$

Hence, using the assumption that $\tau_{i,i} = 1$, it must hold

$$\exp\left(\frac{\rho(1-\theta)}{\theta-\rho}\Lambda_i\right) = \exp\left(\Gamma_i\right) + \sum_{h\neq i} \exp\left(\Gamma_h + \nu + x'_{h,i}\beta + \varepsilon_{h,i}\right). \tag{A.4}$$

If Γ_h is replaced by $\widehat{\ddot{\Gamma}}_h + \bar{\Gamma}$, Λ_i is replaced by $\widehat{\ddot{\Lambda}}_i + \bar{\Lambda}$, and $\nu + x'_{j,i}\beta + \varepsilon_{j,i}$ is replaced by $\nu + x'_{h,i}\hat{\beta} + \hat{\varepsilon}_{h,i} = y_{h,i} - \left(\widehat{\ddot{\Gamma}}_h + \bar{\Gamma} + \widehat{\ddot{\Lambda}}_i + \bar{\Lambda}\right)$,

$$\exp\left(\frac{\rho(1-\theta)}{\theta-\rho}\left(\hat{\Lambda}_i + \bar{\Lambda}\right)\right) = \exp\left(\widehat{\ddot{\Gamma}}_i + \bar{\Gamma}\right) + \sum_{h\neq i} \exp\left(y_{h,i} - \widehat{\ddot{\Lambda}}_i - \bar{\Lambda}\right), \tag{A.5}$$

or

$$\exp\left(\frac{\theta(1-\rho)}{\theta-\rho}\left(\widehat{\ddot{\Lambda}}_i + \bar{\Lambda}\right)\right) = \exp\left(\widehat{\ddot{\Gamma}}_i + \bar{\Gamma} + \widehat{\ddot{\Lambda}}_i + \bar{\Lambda}\right) + \sum_{h\neq i} \exp\left(y_{h,i}\right). \tag{A.6}$$

There are $i = 1, ..., N$ equations, but there are two unknowns $\bar{\Gamma}$ and $\bar{\Lambda}$. Define $v_i(\bar{\Gamma}, \bar{\Lambda})$ as:

$$v_i(\bar{\Gamma}, \bar{\Lambda}) = \sum_{h\neq i} \exp\left(y_{h,i}\right) + \exp\left(\widehat{\ddot{\Gamma}}_i + \bar{\Gamma} + \widehat{\ddot{\Lambda}}_i + \bar{\Lambda}\right) - \exp\left(\frac{\theta(1-\rho)}{\theta-\rho}\left(\widehat{\ddot{\Lambda}}_i + \bar{\Lambda}\right)\right). \tag{A.7}$$

Set a square distance objective function:

$$V(\bar{\Gamma}, \bar{\Lambda}) = \frac{1}{2}\sum_{i=1}^{N}\left(v_i(\bar{\Gamma}, \bar{\Lambda})\right)^2. \tag{A.8}$$

Then, two unknowns are calculated by a minimum distance estimation (non-linear least squares):

$$\{\widehat{\bar{\Gamma}}, \widehat{\bar{\Lambda}}\} = \underset{\{\bar{\Gamma}, \bar{\Lambda}\}}{\operatorname{argmin}} V(\bar{\Gamma}, \bar{\Lambda}). \tag{A.9}$$

I employ "fminunc" function of MATLAB optimization toolbox, setting $\bar{\Lambda} = \bar{\Gamma} = 0$ as initial values. Using obtained estimates, $\widehat{\Gamma}_j = \widehat{\bar{\Gamma}} + \widehat{\ddot{\Gamma}}_j$ and $\widehat{\Lambda}_i = \widehat{\bar{\Lambda}} + \widehat{\ddot{\Lambda}}_i$. Then,

$$\hat{\nu} = \frac{1}{(N-1)^2} \sum_j \sum_i y_{j,i} - \frac{1}{(N-1)^2} \sum_j \sum_i x'_{j,i} \hat{\beta} - \widehat{\bar{\Gamma}} - \widehat{\bar{\Lambda}}. \tag{A.10}$$

Since these are calculated for all (m, t), the above method gives $\widehat{\Gamma}_{j,m,t}$, $\widehat{\Lambda}_{i,m,t}$ and $\hat{\nu}_{m,t}$. Trade cost is then calculated by

$$\hat{\tau}_{j,i,m,t} = \exp\left(\frac{\rho - 1}{\rho}\left(y_{j,i,m,t} - \left(\widehat{\ddot{\Gamma}}_{j,m,t} + \widehat{\bar{\Gamma}}_{m,t} + \widehat{\ddot{\Lambda}}_{i,m,t} + \widehat{\bar{\Lambda}}_{m,t}\right)\right)\right). \tag{A.11}$$

The transformation gives a few observations (less than 1%) with trade cost being less than one. Trade costs of these observations are set to one.

The steady state price is calculated, first by

$$p_{j,m,t} = \left(\exp\left(\hat{\Gamma}_{j,m,t}\right)\right)^{\frac{1-\rho}{-\rho}}, \tag{A.12}$$

then taking time series average of the obtained prices.

A.4 Derivations and model properties

A.4.1 Household

A.4.1.1 Household problem

Each representative agent living in i solves

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pr(s^t) u(C_{i,t}(s^t), L_{i,t}(s^t)) \tag{A.13}$$

where

$$u(C_{i,t}(s^t), L_{i,t}(s^t)) = \frac{\left(C_{i,t}(s^t)^{\psi}(1 - L_{i,t}(s^t))^{1-\psi}\right)^{1-\gamma}}{1-\gamma} \tag{A.14}$$

subject to

$$\sum_{m=1}^{M} l_{i,m,t}(s^t) = L_{i,t}(s^t) \tag{A.15}$$

$$k_{i,m,t+1}(s^t) = (1-\delta) k_{i,m,t}(s^{t-1}) + \phi\left(\frac{x_{i,m,t}(s^t)}{k_{i,m,t}(s^{t-1})}\right) k_{i,m,t}(s^{t-1}) \tag{A.16}$$

where ϕ is capital adjustment friction function:

$$\phi\left(\bar{x}/\bar{k}\right) = \bar{x}/\bar{k},\ \phi'\left(\bar{x}/\bar{k}\right) = 1,\ -\frac{\bar{x}_{i,m}\bar{\phi}''_{i,m}}{\bar{k}_{i,m}\bar{\phi}'_{i,m}} = \eta_{\phi},\ \text{constant.} \tag{A.17}$$

The budget constraint is one of the following. Under the complete market,

$$\begin{aligned} & P_{i,t}(s^t)C_{i,t}(s^t) + P_{i,t}(s^t)\sum_{m=1}^{M} x_{i,m,t}(s^t) + \sum_{s_{t+1}} Q_t(s^{t+1}|s^t) A_{i,t+1}(s^{t+1}, s^t) \\ = \ & A_{i,t}(s^t, s^{t-1}) + P_{i,t}(s^t)\left[W_{i,t}\sum_{m=1}^{M} l_{i,m,t} + \sum_{m=1}^{M} R_{i,m,t}(s^t) k_{i,m,t}(s^{t-1}) - G_{i,t}(s^t)\right]. \end{aligned} \tag{A.18}$$

where $P_{i,t}(s^t)$ is final good price, $W_{i,t}(s^t)$ is real wage, $R_{i,m,t}(s^t)$ is real rental rate,[1] $G_{i,t}(s^t)$ is lump-sum tax (being equal to government expenditure), $A_{i,t+1}(s^{t+1}, s^t)$ is state contingent claim contracted after realizing s^t by agent in i for rewarding a unit of currency if s^{t+1} occurs,

[1]Since labor is instantaneously adjustable, wage is equalized across sectors. The rental rate may not.

and $Q_t(s^{t+1}|s^t)$ is the price of the claim. Transversality conditions:

$$0 = E_0 \lim_{t\to\infty} \prod_{k=0}^{t} \sum_{s_{k+1}} Q_k(s^{k+1}|s^k) A_{i,k+1}(s^{k+1}, s^k). \tag{A.19}$$

If only available asset in the world is non-state contingent period ahead risk free claim, household budget constraint is

$$\begin{aligned} & P_{i,t}(s^t)\left[C_{i,t}(s^t) + \sum_{m=1}^{M} x_{i,m,t}(s^t)\right] + P_t^b(s^t)B_{i,t}(s^t) + \frac{\eta_t^b}{2} P_t^b(s^t)\left(B_{i,t}(s^t)\right)^2 - T_{i,t}^b(s^t) \\ = \; & B_{i,t-1}(s^{t-1}) + P_{i,t}(s^t)\left[W_{i,t}\sum_{m=1}^{M} l_{i,m,t} + \sum_{m=1}^{M} R_{i,m,t}(s^t)k_{i,m,t}(s^{t-1}) - G_{i,t}(s^t)\right]. \end{aligned} \tag{A.20}$$

where P_t^b is world price of non-state contingent bond, $B_{i,t}$ is amount of bond holdings, η_t^b is bond holdings adjustment tax parameter (which is time-dependent for having stationarity in transformed economy) and $T_{i,t}^b$ is lump-sum transfer financed by bond holding adjustment tax.[2] Also, corresponding transversality condition is imposed:

$$0 = E_0 \lim_{t\to\infty} \prod_{k=0}^{t} P_k^b(s^k) B_{i,t}(s^t). \tag{A.21}$$

Period-by-period trade balance assumption restricts possibility of asset trading that is balanced:

$$\begin{aligned} & P_{i,t}(s^t)\left[C_{i,t}(s^t) + \sum_{m=1}^{M} x_{i,m,t}(s^t)\right] \\ = \; & P_{i,t}(s^t)\left[W_{i,t}\sum_{m=1}^{M} l_{i,m,t} + \sum_{m=1}^{M} R_{i,m,t}(s^t)k_{i,m,t}(s^{t-1}) - G_{i,t}(s^t)\right]. \end{aligned} \tag{A.22}$$

The period-by-period trade balance is a straightforward extension of "financial autarky" assumption proposed by Heathcote and Perri (2002). If there are only two countries, a country-by-country goods side balance automatically implies a period-by-period financial side balance. In the multi-country framework, however, the economies are still possible to trade asset. The asset trading may not hold country-by-country financial side balance, but a financial account of the economy is balanced in the aggregate, ensuring period-by-period financial side balance.

[2]This type of tax and transfer is based on Ghironi and Melitz (2005).

A.4.1.2 Household maximization conditions

Given the household problem, first order conditions characterize the optimization. The intertemporal Euler equation with respect to capital investment,

$$\frac{u_{C,i,t}}{\phi'_{i,m,t}} = \beta \Pr(s^{t+1}|s^t) u_{C,i,t+1} \left(R_{i,m,t+1} + \frac{1-\delta+\phi_{i,m,t+1}}{\phi'_{i,m,t+1}} - \frac{x_{i,m,t+1}}{k_{i,m,t+1}} \right) \tag{A.23}$$

The labor-leisure choice

$$W_{i,t} = -\frac{u_{L,i,t}}{u_{C,i,t}} = \frac{1-\psi}{\psi} \frac{C_{i,t}}{1-L_{i,t}} \tag{A.24}$$

and intertemporal Euler equation with respect to cross country asset. If complete market

$$\frac{u_{C,i,t+1}/P_{i,t+1}}{u_{C,i,t}/P_{i,t}} = \frac{u_{C,j,t+1}/P_{j,t+1}}{u_{C,j,t}/P_{j,t}} \tag{A.25}$$

or

$$\frac{u_{C,i,t}}{\mu_i} = \frac{u_{C,j,t}}{\mu_j} \frac{P_{i,t}}{P_{j,t}} \tag{A.26}$$

where μ_i is the initial period (steady state) asset parameter.[3] If non-state contingent claim only,

$$P_t^b(s^t) \left(1 + \eta_t^b B_{i,t}(s^t)\right) = \beta E_t \left[\frac{u_{C,i,t+1}/P_{i,t+1}}{u_{C,i,t}/P_{i,t}} \right]. \tag{A.27}$$

There is no additional condition for the financial autarky case.

A.4.2 Final good producer

A.4.2.1 Final goods producer problem

Final goods producer in i solves

$$\max P_{i,t}(s^t) Z_{i,t}(s^t) - \sum_{m=1}^{M} \sum_{j=1}^{N} p_{j,i,m,t}(s^t) z_{j,i,m,t}(s^t) \tag{A.28}$$

[3] Namely, $\mu_i = u_{c,i,0}/P_{i,0}$. Or in other words, μ_i is the lagrange multiplier to 0-period budget constraint. Also, it is the inverse of Social planner weight.

subject to

$$Z_{i,t}(s^t) = \left[\sum_{m=1}^{M}\left[\sum_{j=1}^{N}\upsilon_{j,i}^{\rho} z_{j,i,m,t}(s^t)^{\rho}\right]^{\frac{\theta}{\rho}}\right]^{\frac{1}{\theta}} \tag{A.29}$$

where $p_{j,i,m,t}(s^t)$ is the price of an intermediate product $z_{j,i,m,t}(s^t)$ in country j. Let

$$q_{i,m,t}(s^t) = \left[\sum_{j=1}^{N}\upsilon_{j,i}^{\rho} z_{j,i,m,t}(s^t)^{\rho}\right]^{\frac{1}{\rho}} \tag{A.30}$$

and

$$p_{i,m,t}^{q} = \left[\sum_{j=1}^{N}\upsilon_{j,i}^{\frac{-\rho}{\rho-1}} p_{j,i,m,t}(s^t)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}. \tag{A.31}$$

A.4.2.2 Final goods producer maximization

The final good producer maximization condition is an inverse demand function

$$p_{j,i,m,t}(s^t) = P_{i,t}(s^t) Z_{i,t}(s^t)^{1-\theta} q_{i,m,t}(s^t)^{\theta-\rho}\upsilon_{j,i}^{\rho} z_{j,i,m,t}(s^t)^{\rho-1}. \tag{A.32}$$

Since perfect competition and constant returns to scale, it is also true that

$$P_{i,t}(s^t) Z_{i,t}(s^t) = \sum_{m=1}^{M} p_{i,m,t}^{q}(s^t) q_{i,m,t}(s^t) = \sum_{m=1}^{M}\sum_{j=1}^{N} p_{j,i,m,t}(s^t) z_{j,i,m,t}(s^t), \tag{A.33}$$

where

$$p_{i,m,t}^{q}(s^t) q_{i,m,t}(s^t) = \sum_{j=1}^{N} p_{j,i,m,t}(s^t) z_{j,i,m,t}(s^t) \tag{A.34}$$

Also,

$$P_{i,t}(s^t) = \left[\sum_{m=1}^{M} p_{i,m,t}^{q\ \frac{\theta}{\theta-1}}\right]^{\frac{\theta-1}{\theta}}, \tag{A.35}$$

and

$$p_{i,m,t}^{q}(s^t) = P_{i,t}(s^t) Z_{i,t}^{1-\theta} q_{i,m,t}(s^t)^{\theta-1}. \tag{A.36}$$

The implied (currency unit) value of trade flow is expressed:

$$\begin{aligned} ex_{j,i,m,t}(s^t) &\equiv \pi_i p_{j,i,m,t}(s^t) z_{j,i,m,t}(s^t) \\ &= \pi_i P_{i,t}(s^t)^{\frac{1}{1-\rho}} Z_{i,t}(s^t)^{\frac{1-\theta}{1-\rho}} q_{i,m,t}(s^t)^{\frac{\theta-\rho}{1-\rho}} \upsilon_{j,i}^{\frac{\rho}{1-\rho}} p_{j,i,m,t}(s^t)^{\frac{-\rho}{1-\rho}} \\ &= \pi_i P_{i,t}(s^t)^{\frac{1}{1-\theta}} Z_{i,t}(s^t) p^q_{i,m,t}(s^t)^{\frac{\rho-\theta}{(1-\rho)(1-\theta)}} \upsilon_{j,i}^{\frac{\rho}{1-\rho}} p_{j,i,m,t}(s^t)^{\frac{-\rho}{1-\rho}}. \end{aligned} \quad (A.37)$$

A.4.3 Intermediate goods producer, trade cost and distribution

In each country i, each intermediate producer producing (i, m) solves

$$\max p_{i,m,t}(s^t) z_{i,m,t}(s^t) - P_{i,t}(s^t) W_{i,t}(s^t) l_{i,m,t}(s^t) - P_{i,t}(s^t) R_{i,m,t}(s^t) k_{i,m,t}(s^t) \quad (A.38)$$

subject to

$$z_{i,m,t}(s^t) = a_{i,m,t}(s^t) k_{i,m,t}(s^t)^{\alpha} l_{i,m,t}(s^t)^{1-\alpha} \quad (A.39)$$

The optimization implies

$$p_{i,m,t} \alpha \frac{z_{i,m,t}}{k_{i,m,t}} = P_{i,t}(s^t) R_{i,m,t}(s^t) \quad (A.40)$$

$$p_{i,m,t}(1-\alpha) \frac{z_{i,m,t}}{l_{i,m,t}} = P_{i,t}(s^t) W_{i,t}(s^t) \quad (A.41)$$

or two condition together implies

$$p_{i,m,t} \alpha^{\alpha} (1-\alpha)^{1-\alpha} a_{i,m,t} = P_{i,t}(s^t) R_{i,m,t}(s^t)^{\alpha} W_{i,t}(s^t)^{1-\alpha} \quad (A.42)$$

There is a pool of producers for each (i, m) goods. The goods are sold for any country subject to iceberg trade costs, $\tau_{j,i,m,t}$, where a sender needs to ship τ unit of goods to reach one unit of goods to receiver. For simplicity, no trade cost for internal distribution is assumed: $\tau_{i,i,m,t} = 1$. Hence, by arbitrage, price is equalized across destination after deducting trade costs:

$$\frac{p_{i,j,m,t}}{\tau_{i,j,m,t}} = \frac{p_{i,h,m,t}}{\tau_{i,h,m,t}} = p_{i,i,m,t} = p_{i,m,t} \quad (A.43)$$

Also, the total goods sales is subject to resource constraints

$$\sum_{j=1}^{N} \pi_j \tau_{i,j,m,t} z_{i,j,m,t}(s^t) = \pi_i z_{i,m,t}(s^t). \quad (A.44)$$

Perfect competition in exporting ensures zero profit in exporting and intermediate goods producer:

$$\sum_{j=1}^{N} \pi_j p_{i,j,m,t}(s^t) z_{i,j,m,t}(s^t) = \pi_i p_{i,m,t}(s^t) z_{i,m,t}(s^t). \tag{A.45}$$

A.4.4 Trade related variables

A.4.4.1 Total exports and total imports

Since the set of exporting goods from country i are $\{z_{i,j,m,t}\}_{j \neq i}$, country i's total exports sales value per capita is

$$EX_{i,t}(s^t) = \frac{1}{\pi_i P_{i,t}} \sum_m \sum_{j \neq i} \pi_j p_{i,j,m,t}(s^t) z_{i,j,m,t}(s^t). \tag{A.46}$$

Similarly, country i' total imports value is

$$IM_{i,t}(s^t) = \frac{1}{\pi_i P_{i,t}} \sum_m \sum_{j \neq i} \pi_i p_{j,i,m,t}(s^t) z_{j,i,m,t}(s^t). \tag{A.47}$$

$$NX_{i,t}(s^t) = EX_{i,t}(s^t) - IM_{i,t}(s^t). \tag{A.48}$$

A.4.4.2 Terms of trade

The set of exporting goods from country i are $\{z_{i,j,m,t}\}_{j \neq i}$. The price index of the exporting goods is the price of exports aggregated only by exporting goods and then evaluated by the aggregate price:

$$P^E_{i,t}(s^t) = \left[\sum_{m=1}^{M} \left(\sum_{j \neq i} \upsilon_{i,j}^{\frac{\rho}{\rho-1}} p_{i,j,m,t}^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}} / P_{i,t}(s^t). \tag{A.49}$$

Similarly, the price index of the importing goods is

$$P^I_{i,t}(s^t) = \left[\sum_{m=1}^{M} \left(\sum_{j \neq i} \upsilon_{j,i}^{\frac{\rho}{\rho-1}} p_{j,i,m,t}^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}} / P_{i,t}(s^t). \tag{A.50}$$

The terms of trade, defined as the relative price of imports to exports, is

$$ToT_{i,t}(s^t) = P^I_{i,t}(s^t) / P^E_{i,t}(s^t). \tag{A.51}$$

A.4.4.3 Real exchange rate

In this framework, nominal exchange rate is always unity by the assumption of common currency. Let $e_{i,j,t}(s^t)$ be the real exchange rate measuring value of j's consumption basket in i's consumption basket

$$e_{j,i,t}(s^t) = \frac{P_{j,t}(s^t)}{P_{i,t}(s^t)}. \tag{A.52}$$

By construction,

$$e_{j,i,t}(s^t) = 1/e_{i,j,t}(s^t) = \frac{e_{j,h,t}(s^t)}{e_{h,i,t}(s^t)}. \tag{A.53}$$

A.4.5 Market clearing and and the equilibrium

A.4.5.1 Market clearing

Followings are market clearing conditions of the economy:

$$\sum_{m=1}^{M} x_{i,m,t}(s^t) = X_{i,t}(s^t) \quad \forall (i,t,s^t) \tag{A.54}$$

$$\sum_{m=1}^{M} l_{i,m,t}(s^t) = L_{i,t}(s^t) \quad \forall (i,t,s^t) \tag{A.55}$$

$$\sum_{j=1}^{N} \pi_j \tau_{i,j,m,t} z_{i,j,m,t}(s^t) = \pi_i z_{i,m,t}(s^t) \quad \forall (i,m,t,s^t) \tag{A.56}$$

Country's resource constraint:

$$C_{i,t}(s^t) + X_{i,t}(s^t) + G_{i,t}(s^t) = Z_{i,t}(s^t) \quad \forall (i,t,s^t). \tag{A.57}$$

Market clearing of claims are, for all s^{t+1} given s^t,

$$0 = \sum_{i=1}^{N} \pi_i A_{i,t+1}(s^{t+1}, s^t). \tag{A.58}$$

Non-state contingent bond case:

$$0 = \sum_{i=1}^{N} \pi_i B_{i,t}(s^t). \tag{A.59}$$

and lump-sum transfer is financed by bond holding adjustment tax:

$$\frac{\eta_t^b}{2} P_t^b(s^t) \left(B_{i,t}(s^t)\right)^2 = T_{i,t}^b(s^t). \tag{A.60}$$

A.4.5.2 Equilibrium

Definition: The **competitive equilibrium** of the economy given exogenous shocks is a sequence of quantities and prices which are consistent with (1) household maximization problem, (2) final goods producer maximization problem, (3) intermediate goods producer maximization problem, (4) trade arbitrage conditions, and (5) market clearing conditions.

The competitive equilibrium of the economy with full state contingent claim is Pareto efficient, since markets are all competitive, there is no externality, and all the productions are constant returns to scale. The competitive equilibrium of the economy with non-state contingent claim is also shown to be equivalent to the allocation obtained by a constrained social planner's problem.

Claim: The competitive equilibrium with full state contingent claim is Pareto efficient.
Proof: The equilibrium conditions are coincide with conditions derived from social planner's problem. See below. (QED)

From now on, explicit history dependence of variable is dropped from the notation. Since there is one degree of freedom with respect to nominal price (across $P_{i,t}$, $p_{i,m,t}$ and $p_{j,i,m,t}$), $P_{N,t} = 1$ for all t is assumed as a normalization. The analysis focuses on an equilibrium in which steady state net exporting is zero. Namely, imposing

$$\bar{EX}_i = \bar{IM}_i \tag{A.61}$$

where variables with upper bar is of steady state values defined later. μ_i is adjusted so as to consistent with achieving the equilibrium.

A.4.6 Planner's problem

A.4.6.1 Planner's problem

Consider a Planner's problem with weight ω_i

$$\max \sum_i \omega_i \pi_i \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pr(s^t) u(C_{i,t}(s^t), L_{i,t}(s^t)) \tag{A.62}$$

where

$$u(C_{i,t}(s^t), L_{i,t}(s^t)) = \frac{\left[C_{i,t}(s^t)^{\psi}[1 - L_{i,t}(s^t)]^{1-\psi}\right]^{1-\gamma}}{1-\gamma} \tag{A.63}$$

s.t.

$$z_{i,m,t} = a_{i,m,t}(s^t) k_{i,m,t}(s^t)^{\alpha} l_{i,m,t}(s^t)^{1-\alpha} \tag{A.64}$$

$$k_{i,m,t+1}(s^t) = (1-\delta) k_{i,m,t}(s^{t-1}) + \phi\left(\frac{x_{i,m,t}(s^t)}{k_{i,m,t}(s^{t-1})}\right) k_{i,m,t}(s^{t-1}) \tag{A.65}$$

$$Z_{i,t}(s^t) = \left(\sum_{m=1}^{M}\left(\sum_{j=1}^{N} \upsilon_{j,i}^{\rho} z_{j,i,m,t}(s^t)^{\rho}\right)^{\frac{\theta}{\rho}}\right)^{\frac{1}{\theta}} \tag{A.66}$$

$$\pi_i z_{i,m,t}(s^t) = \sum_j \pi_j \tau_{i,j,m,t}(s^t) z_{i,j,m,t}(s^t) \tag{A.67}$$

$$X_{i,t}(s^t) = \sum_{m=1}^{M} x_{i,m,t}(s^t) \tag{A.68}$$

$$L_{i,t}(s^t) = \sum_{m=1}^{M} l_{i,m,t}(s^t) \tag{A.69}$$

Resource constraint:

$$0 = Z_{i,t}(s^t) - C_{i,t}(s^t) - X_{i,t}(s^t) - G_{i,t}(s^t). \tag{A.70}$$

A.4.6.2 FOCs of planner's problem

Dropping s^t from the notation for simplicity. Let the multiplier associated to production be $\lambda_{i,m,t}$ and the multiplier to resource constraint be $\Lambda_{i,t}$. FOCs are:

$$\begin{aligned}
&\Lambda_{i,t}(s^t) = \omega_i \beta^t \Pr(s^t) u_{C,i,t}(s^t) \\
&\frac{\lambda_{i,m,t}}{\Lambda_{i,t}}(1-\alpha)\frac{z_{i,m,t}}{l_{i,m,t}} = -\frac{u_{L,i,t}}{u_{C,i,t}} \\
&\frac{u_{C,i,t}}{\phi'_{i,m,t}} = \beta E_t \left[\lambda_{i,m,t+1}\alpha\frac{z_{i,m,t+1}}{k_{i,m,t+1}} + \frac{u_{C,i,t+1}}{\phi'_{i,m,t+1}}\left(1-\delta+\phi_{i,m,t+1}-\phi'_{i,m,t+1}\frac{x_{i,m,t+1}}{k_{i,m,t+1}}\right)\right] \\
&\pi_i \lambda_{j,m,t}\tau_{j,i,m,t} = \pi_j \Lambda_{i,t} Z_{i,t}^{1-\theta} q_{i,m,t}^{\theta-\rho} \upsilon_{j,i}^{\rho} z_{j,i,m,t}^{\rho-1} \\
&q_{i,m,t} = \left[\sum_j \upsilon_{j,i}^{\rho} z_{j,i,m,t}^{\rho}\right]^{\frac{1}{\rho}}.
\end{aligned}$$

If setting

$$\omega_i = 1/\mu_i \tag{A.71}$$

$$p_{i,m,t} = \lambda_{i,m,t}/\pi_i, \tag{A.72}$$

$$P_{i,t} = \Lambda_{i,t}/\pi_i, \tag{A.73}$$

$$W_{i,t} = \frac{\lambda_{i,m,t}}{\Lambda_{i,t}}(1-\alpha)\frac{z_{i,m,t}}{l_{i,m,t}} \tag{A.74}$$

$$R_{i,m,t} = \frac{\lambda_{i,m,t}}{\Lambda_{i,t}}\alpha\frac{z_{i,m,t}}{k_{i,m,t}} \tag{A.75}$$

Then, FOCs are consistent with equilibrium conditions under complete contingent claims.

A.4.7 Alternative specifications of budget constraint

The complete market specification is obtained by a full history time-zero budget constraint denominated by zero-period Arrow-Debreu price:

$$\begin{aligned} & \sum_{t=0}^{\infty}\sum_{s^t} P_{i,t}^0(s^t)\left[C_{i,t}(s^t) + \sum_{m=1}^{M} x_{i,m,t}(s^t)\right] \\ = & \sum_{t=0}^{\infty}\sum_{s^t} P_{i,t}^0(s^t)\left[W_{i,t}(s^t)\sum_{m=1}^{M} l_{i,m,t}(s^t) + \sum_{m=1}^{M} R_{i,m,t}(s^t)k_{i,m,t}(s^{t-1}) - G_{i,t}(s^t)\right]. \end{aligned}$$

Another interpretation of full contingent claim world is full access to other country's investment (but in this case, $\eta_\phi = 0$ needs to be assumed.):

$$\begin{aligned} & P_{i,t}(s^t)C_{i,t}(s^t) + \sum_j P_{j,t}(s^t)\sum_{m=1}^{M} x_{j,i,m,t}(s^t) \\ = & P_{i,t}(s^t)\left[W_{i,t}\sum_{m=1}^{M} l_{i,m,t} - G_{i,t}(s^t)\right] + \sum_j P_{j,t}(s^t)\sum_{m=1}^{M} R_{j,m,t}(s^t)k_{j,i,m,t}(s^{t-1}). \end{aligned}$$

where $x_{j,i,m,t}$ is investment of j's assembly by i, and capital stock is similarly indexed.

A.4.8 National account of the model economy

Combining household budget constraint, final goods producer accounting, intermediate goods producer accounting, and resource constraint in the model leads to the aggregate accounting identity. First, This economy excludes cross country investment and labor migration, meaning no net factor income, and GDP=GNP. Also, the current account equals the net exports. Let's denote the change in the net foreign asset ($FA_{i,t}$) per capita as:

$$FA_{i,t} = \begin{cases} A_{i,t}(s^t, s^{t-1}) - \sum_{s^{t+1}|s^t} Q_{i,t}(s^{t+1}|s^t)A_{i,t+1}(s^{t+1}, s^t) & \text{(Complete market)} \\ B_{i,t-1} - P_t^b B_{i,t} & \text{(Incomplete market)} \\ 0 & \text{(Period-by-period TB)} \end{cases} \tag{A.76}$$

Notice that by asset market clearing conditions,

$$\sum_i \pi_i FA_{i,t} = 0. \tag{A.77}$$

Let $Y_{i,t}$ be the real gross domestic products per capita of country i at t in this economy. The nominal gross domestic products is the value added (production accounts):

$$\pi_i P_{i,t} Y_{i,t} = \sum_{m=1}^{M} \pi_i p_{i,m,t} z_{i,m,t} \tag{A.78}$$

which is total factor payments (income accounts):

$$\pi_i P_{i,t} Y_{i,t} = \pi_i P_{i,t} W_{i,t} \sum_{m=1}^{M} l_{i,m,t} + \pi_i P_{i,t} \sum_{m=1}^{M} R_{i,m,t} k_{i,m,t}. \tag{A.79}$$

Using the definition of the income account, household budget constraint and investment market clearing conditions:

$$-\pi_i FA_{i,t} = \pi_i P_{i,t} Y_{i,t} - \pi_i P_{i,t} \left(C_{i,t} + X_{i,t} + G_{i,t}\right) \tag{A.80}$$

To show expenditure accounts, starting from production accounts:

$$\begin{aligned}
\pi_i P_{i,t} Y_{i,t} &= \sum_{m=1}^{M} \pi_i p_{i,m,t} z_{i,m,t} \\
&= \sum_{m=1}^{M} p_{i,m,t} \sum_{j=1}^{N} \pi_j \tau_{i,j,m,t} z_{i,j,m,t} \\
&= \sum_{m=1}^{M} \pi_i p_{i,i,m,t} z_{i,i,m,t} + \sum_{m=1}^{M} \sum_{j \neq i}^{N} \pi_j p_{i,m,t} \tau_{i,j,m,t} z_{i,j,m,t} \\
&= \pi_i P_{i,t} Z_{i,t} - \sum_{m=1}^{M} \sum_{j \neq i}^{N} \pi_i p_{j,i,m,t} z_{j,i,m,t} + \sum_{m=1}^{M} \sum_{j \neq i}^{N} \pi_j p_{i,m,t} \tau_{i,j,m,t} z_{i,j,m,t} \\
&= \underbrace{\pi_i P_{i,t} \left(C_{i,t} + X_{i,t} + G_{i,t}\right)}_{\text{Absorption}} - \underbrace{\sum_{j \neq i} \sum_{m=1}^{M} \pi_i p_{j,i,m,t} z_{j,i,m,t}}_{\text{Imports of inputs}} + \underbrace{\sum_{j \neq i} \sum_{m=1}^{M} \pi_j p_{i,j,m,t} z_{i,j,m,t}}_{\text{Exports of inputs}} \\
&= \pi_i P_{i,t} \left(C_{i,t} + X_{i,t} + G_{i,t}\right) - \pi_i P_{i,t} IM_{i,t} + \pi_i P_{i,t} EX_{i,t} && \text{(A.81)} \\
&= \pi_i P_{i,t} \left(C_{i,t} + X_{i,t} + G_{i,t}\right) + \pi_i P_{i,t} NX_{i,t}. && \text{(A.82)}
\end{aligned}$$

Comparing two equations, the balance of payments identity can be confirmed:

$$0 = \pi_i P_{i,t} NX_{i,t} + \pi_i FA_{i,t}. \tag{A.83}$$

Namely, trade deficit (if $NX_{i,t} < 0$) is financed by capital account surplus. In the steady state, trade balance (and hence financial account balance) is assumed:

$$0 = \pi_i \bar{P}_i \bar{NX}_i = \sum_{j \neq i} \sum_{m=1}^{M} \pi_j \bar{p}_{i,j,m} \bar{z}_{i,j,m} - \sum_{j \neq i} \sum_{m=1}^{M} \pi_i \bar{p}_{j,i,m} \bar{z}_{j,i,m} = \pi_i \bar{FA}_i. \tag{A.84}$$

Also,

$$\bar{Y}_i = \bar{C}_i + \bar{X}_i + \bar{G}_i = \bar{Z}_i. \tag{A.85}$$

The domestic absorption is financed by the domestic production.

Based on the aggregate GDP, implied aggregate variables are calculated. Aggregate capital stock per capita is

$$K_{i,t} = \sum_{m} k_{i,m,t}. \tag{A.86}$$

Then, implied aggregate TFP is

$$TFP_{i,t} = \frac{Y_{i,t}}{K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}}. \tag{A.87}$$

A.4.9 Detrended economy

A.4.9.1 Detrending

Let variables with tilde ˜ as detrended variables. It is defined, for a case of $Z_{i,t}$,

$$\tilde{Z}_{i,t} = Z_{i,t} / g_1^t \tag{A.88}$$

Other cases are summarized in the table:

Detrending factor	Value	Variables
g_a	g_a	$a_{i,m,t}$
g_τ	g_τ	$\tau_{i,j,m,t}$
g_τ^{-1}	g_τ^{-1}	$p_{i,m,t}$
g_1	$g_a^{\frac{1}{1-\alpha}} g_\tau^{\frac{-1}{1-\alpha}}$	$Z_{i,t}, C_{i,t}, X_{i,t}, G_{i,t}, x_{i,m,t}, k_{i,m,t}, z_{i,j,m,t}, B_{i,t}, W_{i,t}, ex_{j,i,m,t}, Y_{i,t}$
g_2	$g_a^{\frac{1}{1-\alpha}} g_\tau^{\frac{-\alpha}{1-\alpha}}$	$z_{i,m,t}$
g_3	$g_1^{\psi(1-\gamma)}$	$u_{L,i,t}$
g_4	$g_1^{\psi(1-\gamma)-1}$	$\Lambda_{i,t}, u_{C,i,t}$
1	–	$L_{i,t}, l_{i,m,t}, R_{i,m,t}, P_{i,t}, p_{i,j,m,t}, e_{j,i,t}$

Also, let $\eta_t^b = \eta^b g_1^{-t}$.

A.4.9.2 Detrended economy equilibrium conditions

$$\tilde{z}_{i,m,t} = \tilde{a}_{i,m,t}\tilde{k}_{i,m,t}^{\alpha}\tilde{l}_{i,m,t}^{1-\alpha} \tag{A.89}$$

$$\tilde{\phi}_{i,m,t} = \phi\left(\frac{\tilde{x}_{i,m,t}}{\tilde{k}_{i,m,t}}\right) \tag{A.90}$$

$$\tilde{\phi}'_{i,m,t} = \phi'\left(\frac{\tilde{x}_{i,m,t}}{\tilde{k}_{i,m,t}}\right) \tag{A.91}$$

$$g_1\tilde{k}_{i,m,t+1} = (1-\delta)\tilde{k}_{i,m,t} + \tilde{\phi}_{i,m,t}\tilde{k}_{i,m,t} \tag{A.92}$$

$$\tilde{Z}_{i,t} = \left(\sum_{m=1}^{M}\left(\sum_{j=1}^{N}\upsilon_{j,i}^{\rho}\tilde{z}_{j,i,m,t}^{\rho}\right)^{\frac{\theta}{\rho}}\right)^{\frac{1}{\theta}} \tag{A.93}$$

$$\pi_i\tilde{z}_{i,m,t} = \sum_j \pi_j\tilde{\tau}_{i,j,m,t}\tilde{z}_{i,j,m,t} \tag{A.94}$$

$$\tilde{p}_{i,m,t}(1-\alpha)\frac{\tilde{z}_{i,m,t}}{l_{i,m,t}} = \tilde{P}_{i,t}\tilde{W}_{i,t} = \tilde{P}_{i,t}\frac{-\tilde{u}_{L,i,t}}{\tilde{u}_{C,i,t}} \tag{A.95}$$

$$\frac{\tilde{u}_{C,i,t}}{\tilde{\phi}'_{i,m,t}} = \beta g_4 E_t\tilde{u}_{C,i,t+1}\left[\tilde{R}_{i,m,t+1} + \frac{1}{\tilde{\phi}'_{i,m,t+1}}\left(1-\delta+\tilde{\phi}_{i,m,t+1} - \tilde{\phi}'_{i,m,t+1}\frac{\tilde{x}_{i,m,t+1}}{\tilde{k}_{i,m,t+1}}\right)\right] \tag{A.96}$$

$$\tilde{q}_{i,m,t} = \left[\sum_j \upsilon_{j,i}^{\rho}\tilde{z}_{j,i,m,t}^{\rho}\right]^{\frac{1}{\rho}} \tag{A.97}$$

$$\tilde{p}_{j,i,m,t} = \tilde{P}_{i,t}\tilde{Z}_{i,t}^{1-\theta}\tilde{q}_{i,m,t}^{\theta-\rho}\upsilon_{j,i}^{\rho}\tilde{z}_{j,i,m,t}^{\rho-1} \tag{A.98}$$

$$\tilde{X}_{i,t} = \sum_{m=1}^{M} \tilde{x}_{i,m,t} \tag{A.99}$$

$$\tilde{L}_{i,t} = \sum_{m=1}^{M} \tilde{l}_{i,m,t} \tag{A.100}$$

$$0 = \tilde{Z}_{i,t} - \tilde{C}_{i,t} - \tilde{X}_{i,t} - \tilde{G}_{i,t}, \tag{A.101}$$

If complete market

$$\tilde{u}_{C,i,t}\tilde{P}_{i,t}/\mu_i = \tilde{u}_{C,j,t}\tilde{P}_{j,t}/\mu_j \tag{A.102}$$

If incomplete market,

$$0 = \sum_{i=1}^{N} \tilde{B}_{i,t} \tag{A.103}$$

$$\tilde{P^b}_t \left(1 + \eta^b \tilde{B}_{i,t}\right) = \beta g_4 E_t \left[\frac{\tilde{u}_{C,i,t+1}/\tilde{P}_{i,t+1}}{\tilde{u}_{C,i,t}/\tilde{P}_{i,t}} \right]. \tag{A.104}$$

$$\begin{aligned} & \tilde{P}_{i,t} \left[\tilde{C}_{i,t}(s^t) + \sum_{m=1}^{M} \tilde{x}_{i,m,t}(s^t) \right] + \tilde{P^b}_t(s^t)\tilde{B}_{i,t}(s^t) \\ = \quad & \tilde{B}_{i,t-1}(s^{t-1}) + \tilde{P}_{i,t}(s^t) \left[\tilde{W}_{i,t} \sum_{m=1}^{M} \tilde{l}_{i,m,t} + \sum_{m=1}^{M} \tilde{R}_{i,m,t}(s^t)\tilde{k}_{i,m,t}(s^{t-1}) - \tilde{G}_{i,t}(s^t) \right]. \end{aligned} \tag{A.105}$$

If financial autarky,

$$\tilde{Z}_{i,t} = \tilde{W}_{i,t} \sum_{m=1}^{M} \tilde{l}_{i,m,t} + \sum_{m=1}^{M} \tilde{R}_{i,m,t}(s^t)\tilde{k}_{i,m,t}(s^{t-1}). \tag{A.106}$$

A.4.10 Properties of the non-stochastic steady state

Claim: The non-stochastic steady state of complete market economy with steady state zero net exporting, the non-stochastic steady state of limited asset market economy, and the non-stochastic steady state of the period-by-period trade balance economy are equivalent.

Proof: Notice that the only differences are in the asset holdings Euler equation and household budget constraints. The asset holdings Euler equation in the complete market is used only for determining μ_i, which is assumed to be consistent with zero net exporting in the steady state. The risk-free bond holding Euler equation, $\bar{P}^b(1+\eta^b\bar{B}_i) = \beta g_4$, implies $\bar{B}_i = \bar{B}$ for all i. But the bond market clearing condition requires the summation to be zero. Hence, $\bar{B}_i = 0$ for all i. This is consistent with zero net exporting in the steady state, assumed as in the complete market case. Period-by-period trade balance always requires zero net exporting by construction, and hence in the steady state. (QED)

The bond holding Euler equation implies $\bar{P}^b = \beta g_4$.

A.4.10.1 Normalization of the price

Setting $\bar{P}_N = 1$ as normalization.

A.4.10.2 Rental rate and capital labor ratio

By assumption, $\phi(\bar{x}/\bar{k}) = \bar{x}/\bar{k}$ and $\phi'(\bar{x}/\bar{k}) = 1$. From Euler equation,

$$\bar{R}_{i,m} = \bar{R} = \frac{1}{\beta g_4} - 1 + \delta. \tag{A.107}$$

Capital-labor ratio (denoting κ_i, which does not depend on m) is expressed by

$$\bar{\kappa}_i = \frac{\bar{k}_{i,m}}{\bar{l}_{i,m}} = \frac{\alpha}{1-\alpha}\frac{\bar{W}_i}{\bar{R}}. \tag{A.108}$$

A.4.10.3 Investment

Capital accumulation implies:

$$(g_1 - 1 + \delta)\bar{k}_{i,m} = \bar{x}_{i,m}, \tag{A.109}$$

but combining with capital-labor ratio,

$$\bar{x}_{i,m} = (g_1 - 1 + \delta)\bar{\kappa}_i\bar{l}_{i,m} \tag{A.110}$$

and summing up over m,

$$\bar{X}_i = (g_1 - 1 + \delta)\bar{\kappa}_i\bar{L}_i, \tag{A.111}$$

or

$$\bar{X}_i = (g_1 - 1 + \delta)\frac{\alpha}{1-\alpha}\frac{\bar{W}_i}{\bar{R}}\bar{L}_i, \tag{A.112}$$

A.4.10.4 Absorption

From household budget constraint and resource constraint,

$$\begin{aligned}\bar{Z}_i &= \bar{W}_i\bar{L}_i + \bar{R}\sum_m \bar{k}_{i,m} \\ &= \bar{W}_i\bar{L}_i + \bar{R}\sum_m \frac{\alpha}{1-\alpha}\frac{\bar{W}_i}{\bar{R}}\bar{l}_{i,m} \\ &= \frac{1}{1-\alpha}\bar{W}_i\bar{L}_i \qquad (A.113)\end{aligned}$$

With resource constraint and investment condition,

$$\frac{1-(g_1-1+\delta)\frac{\alpha}{\bar{R}}}{1-\alpha}\bar{W}_i\bar{L}_i = \bar{C}_i + \bar{G}_i. \qquad (A.114)$$

A.4.10.5 Labor

Combining two equations

$$\bar{W}_i = \frac{1-\psi}{\psi}\frac{\bar{C}_i}{1-\bar{L}_i}, \qquad (A.115)$$

$$\frac{1-(g_1-1+\delta)\frac{\alpha}{\bar{R}}}{1-\alpha}\bar{W}_i\bar{L}_i = \bar{C}_i + \bar{G}_i. \qquad (A.116)$$

to obtain

$$\bar{L}_i = L(\bar{W}_i) = \frac{\dfrac{\psi}{1-\psi} + \dfrac{\bar{G}_i}{\bar{W}_i}}{\dfrac{1-(g_1-1+\delta)\alpha/\bar{R}}{1-\alpha} + \dfrac{\psi}{1-\psi}}. \qquad (A.117)$$

A.4.10.6 Wage and price level 1

From

$$\bar{P}_i = \left(\sum_{m=1}^{M}\left(\sum_{j=1}^{N} \upsilon_{j,i}^{\frac{-\rho}{\rho-1}}\bar{p}_{j,i,m}^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}, \qquad (A.118)$$

$$\begin{aligned}\bar{p}_{j,i,m} &= \bar{p}_{j,m}\bar{\tau}_{j,i,m} \\ &= \bar{p}_{j,m}\bar{\tau}_{j,i,m},\end{aligned}$$

and

$$\bar{p}_{j,m}\alpha^{\alpha}(1-\alpha)^{1-\alpha}\bar{a}_{j,m} = \bar{P}_j\bar{R}^{\alpha}\bar{W}_j^{1-\alpha}, \tag{A.119}$$

Let

$$\Psi \equiv \bar{R}^{\alpha}\alpha^{-\alpha}(1-\alpha)^{-[1-\alpha]}. \tag{A.120}$$

To obtain

$$\bar{P}_i = \left(\sum_{m=1}^{M}\left(\sum_{j=1}^{N}\upsilon_{j,i}^{\frac{-\rho}{\rho-1}}\left(\Psi\bar{a}_{j,m}^{-1}\bar{P}_j\bar{W}_j^{1-\alpha}\bar{\tau}_{j,i,m}\right)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}\frac{\theta}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}, \tag{A.121}$$

or

$$\bar{P}_i = \Psi\left[\sum_{m}\left(\sum_{j}\left(\bar{P}_j\bar{W}_j^{1-\alpha}\frac{\bar{\tau}_{j,i,m}}{\upsilon_{j,i}\bar{a}_{j,m}}\right)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\theta-1}\frac{\theta}{\rho}}\right]^{\frac{\theta-1}{\theta}}. \tag{A.122}$$

A.4.10.7 Wage and price level 2

From $\bar{NX}_i = 0$,

$$\pi_i\bar{P}_i\bar{Z}_i = \sum_{j=1}^{N}\sum_{m=1}^{M}\pi_i\bar{p}_{j,i,m}\bar{z}_{j,i,m} = \sum_{j=1}^{N}\sum_{m=1}^{M}\pi_j\bar{p}_{i,j,m}\bar{z}_{i,j,m} \tag{A.123}$$

Since

$$\begin{aligned}
\bar{p}_{i,j,m}\bar{z}_{i,j,m} &= \bar{P}_j^{\frac{1}{1-\rho}}\bar{Z}_j^{\frac{1-\theta}{1-\rho}}\bar{q}_{j,m}^{\frac{\theta-\rho}{1-\rho}}\upsilon_{i,j}^{\frac{\rho}{1-\rho}}\bar{p}_{i,j,m}^{\frac{-\rho}{1-\rho}} && \text{(A.124)}\\
&= \bar{P}_j^{\frac{1}{1-\theta}}\bar{Z}_j\left[\sum_{h}\upsilon_{h,j}^{\frac{\rho}{1-\rho}}\bar{p}_{h,j,m}^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho}}\upsilon_{i,j}^{\frac{\rho}{1-\rho}}\bar{p}_{i,j,m}^{\frac{\rho}{\rho-1}}\\
&= \bar{P}_j^{\frac{1}{1-\theta}}\bar{Z}_j\left[\sum_{h}\upsilon_{h,j}^{\frac{\rho}{1-\rho}}\left(\Psi\bar{a}_{h,m}^{-1}\bar{P}_h\bar{W}_h^{1-\alpha}\bar{\tau}_{h,j,m}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho}}\upsilon_{i,j}^{\frac{\rho}{1-\rho}}\left(\Psi\bar{a}_{i,m}^{-1}\bar{P}_i\bar{W}_i^{1-\alpha}\bar{\tau}_{i,j,m}\right)^{\frac{\rho}{\rho-1}},
\end{aligned}$$

Also,

$$\begin{aligned}
\bar{Z}_h &= \frac{1}{1-\alpha}\bar{W}_h\bar{L}_h && \text{(A.125)}\\
&= \frac{1}{1-\alpha}\bar{W}_h\bar{L}(\bar{W}_h) && \text{(A.126)}
\end{aligned}$$

where $\bar{L}(\bar{W}_h)$ is a function derived above. Thus

$$\pi_i \bar{P}_i \bar{W}_i \bar{L}(\bar{W}_i) \tag{A.127}$$

$$= \Psi^{\frac{\theta}{\theta-1}} \sum_{j=1}^{N} \sum_{m=1}^{M} \pi_j \bar{P}_j^{\frac{1}{1-\theta}} \bar{W}_j \bar{L}(\bar{W}_j) \left[\sum_h \left(\bar{P}_h \bar{W}_h^{1-\alpha} \frac{\bar{\tau}_{h,j,m}}{\upsilon_{h,j} \bar{a}_{h,m}} \right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho}} \left(\bar{P}_i \bar{W}_i^{1-\alpha} \frac{\bar{\tau}_{i,j,m}}{\upsilon_{i,j} \bar{a}_{i,m}} \right)^{\frac{\rho}{\rho-1}}$$

A.4.10.8 Excess demand function

For simplifying the notation, define following:

$$Q_i = \bar{P}_i \bar{W}_i^{1-\alpha}, \tag{A.128}$$

$$\boldsymbol{Q} = \{Q_1, ..., Q_N\}. \tag{A.129}$$

$$\Upsilon_{i,j,m} = \left(\frac{\bar{\tau}_{i,j,m}}{\upsilon_{i,j} \bar{a}_{i,m}} \right)^{\frac{\rho}{\rho-1}} \tag{A.130}$$

$$\Omega_{i,m}(\boldsymbol{Q}) = \left[\sum_h \left(\bar{P}_h \bar{W}_h^{1-\alpha} \frac{\bar{\tau}_{h,i,m}}{\upsilon_{h,i} \bar{a}_{h,m}} \right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} = \left[\sum_h Q_h^{\frac{\rho}{\rho-1}} \Upsilon_{h,i,m} \right]^{\frac{\rho-1}{\rho}} \tag{A.131}$$

$$\begin{aligned} \Phi_i(\boldsymbol{Q}) = \bar{P}_i/\Psi &= \left[\sum_m \left(\sum_j \left(Q_j \frac{\bar{\tau}_{j,i,m}}{\upsilon_{j,i} \bar{a}_{j,m}} \right)^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\theta-1}\frac{\theta}{\rho}} \right]^{\frac{\theta-1}{\theta}} \\ &= \left[\sum_m \Omega_{i,m}(\boldsymbol{Q})^{\frac{\theta}{\theta-1}} \right]^{\frac{\theta-1}{\theta}}. \end{aligned} \tag{A.132}$$

$$\bar{L}_i = L(\bar{W}_i) = L\left(Q_i^{\frac{1}{1-\alpha}} (\Phi_i(\boldsymbol{Q}))^{\frac{-1}{1-\alpha}} \right) = L_i(\boldsymbol{Q}) = \frac{\dfrac{\psi}{1-\psi} + \bar{G}_i \Psi^{\frac{1}{1-\alpha}} (\Phi_i(\boldsymbol{Q}))^{\frac{1}{1-\alpha}} Q_i^{\frac{-1}{1-\alpha}}}{\dfrac{1-(g_1-1+\delta)\alpha/\bar{R}}{1-\alpha} + \dfrac{\psi}{1-\psi}}. \tag{A.133}$$

Then, (A.127) can be expressed as

$$\begin{aligned} & \pi_i \bar{L}_i(\boldsymbol{Q}) Q_i^{\frac{1}{1-\alpha}} (\Phi_i(\boldsymbol{Q}))^{\frac{-\alpha}{1-\alpha}} \\ = & \sum_{j=1}^{N}\sum_{m=1}^{M} \pi_j (\Phi_j(\boldsymbol{Q}))^{\frac{\theta-\alpha}{(1-\theta)(1-\alpha)}} Q_j^{\frac{1}{1-\alpha}} \bar{L}_j(\boldsymbol{Q}) \left(\Omega_{j,m}(\boldsymbol{Q})\right)^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho-1}} Q_i^{\frac{\rho}{\rho-1}} \Upsilon_{i,j,m} \end{aligned} \tag{A.134}$$

There are N equations for $i = 1, ..., N$. Regarding Q as the price of labor (since $Q^{\frac{1}{1-\alpha}}$ is basket adjusted real wage by definition), the above equation can be regarded as the reduced form labor market clearing equation. Solving this N system to obtain Q_i. Then, (A.127) gives $\bar{P}_i$.

This equation can be interpreted as supply of the labor in the left-hand-side whereas demand of the labor in the right-hand-side. Dividing both sides by Q_i and define $J_i((Q))$ as the difference of left-hand-side and right-hand-side:

$$\begin{aligned} & J_i(\boldsymbol{Q}) \\ = & Q_i^{\frac{1}{\rho-1}} \sum_{j=1}^{N}\sum_{m=1}^{M} \pi_j (\Phi_j(\boldsymbol{Q}))^{\frac{\theta-\alpha}{(1-\theta)(1-\alpha)}} Q_j^{\frac{1}{1-\alpha}} \bar{L}_j(\boldsymbol{Q}) \left(\Omega_{j,m}(\boldsymbol{Q})\right)^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho-1}} \Upsilon_{i,j,m} \\ & -\pi_i \bar{L}_i(\boldsymbol{Q}) Q_i^{\frac{\alpha}{1-\alpha}} (\Phi_i(\boldsymbol{Q}))^{\frac{-\alpha}{1-\alpha}} \end{aligned} \tag{A.135}$$

This function, $J(\boldsymbol{Q})$, is homogeneous of degree zero and $\sum_{i=1}^{N} Q_i J_i(\boldsymbol{Q}) = 0$, hence it can be interpreted as an excess demand function.

A.4.10.9 Homogeneity

$\Omega_{i,m}(\boldsymbol{Q})$ is homogenous of degree one. $\Phi_i(\boldsymbol{Q})$ is homogenous of degree one. $L_i(\boldsymbol{Q})$ is homogenous of degree zero. By straightforward calculation, $J_i(\boldsymbol{Q})$ is homogenous of degree zero.

A.4.10.10 Walras' law

The multiplying by Q_i and the switching the summation order gives:

$$
\begin{aligned}
&\sum_{i=1}^{N} Q_i J_i(\boldsymbol{Q}) \\
=\;& \sum_{i=1}^{N} Q_i Q_i^{\frac{1}{\rho-1}} \sum_{j=1}^{N}\sum_{m=1}^{M} \pi_j (\Phi_j(\boldsymbol{Q}))^{\frac{\theta-\alpha}{(1-\theta)(1-\alpha)}} Q_j^{\frac{1}{1-\alpha}} \bar{L}_j(\boldsymbol{Q}) \left(\Omega_{j,m}(\boldsymbol{Q})\right)^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho-1}} \Upsilon_{i,j,m} \\
&-\sum_{i=1}^{N} Q_i \pi_i \bar{L}_i(\boldsymbol{Q}) Q_i^{\frac{\alpha}{1-\alpha}} (\Phi_i(\boldsymbol{Q}))^{\frac{-\alpha}{1-\alpha}} \\
=\;& \sum_{j=1}^{N} \pi_j (\Phi_j(\boldsymbol{Q}))^{\frac{\theta-\alpha}{(1-\theta)(1-\alpha)}} Q_j^{\frac{1}{1-\alpha}} \bar{L}_j(\boldsymbol{Q}) \sum_{m=1}^{M} \left(\Omega_{j,m}(\boldsymbol{Q})\right)^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho-1}} \underbrace{\sum_{i=1}^{N} Q_i^{\frac{\rho}{\rho-1}} \Upsilon_{i,j,m}}_{(\Omega_{j,m}(\boldsymbol{Q}))^{\rho/(\rho-1)}} \\
&-\sum_{i=1}^{N} \pi_i \bar{L}_i(\boldsymbol{Q}) Q_i^{\frac{1}{1-\alpha}} (\Phi_i(\boldsymbol{Q}))^{\frac{-\alpha}{1-\alpha}} \\
=\;& \sum_{j=1}^{N} \pi_j (\Phi_j(\boldsymbol{Q}))^{\frac{-\alpha}{1-\alpha}} Q_j^{\frac{1}{1-\alpha}} \bar{L}_j(\boldsymbol{Q}) - \sum_{i=1}^{N} \pi_i \bar{L}_i(\boldsymbol{Q}) Q_i^{\frac{1}{1-\alpha}} (\Phi_i(\boldsymbol{Q}))^{\frac{-\alpha}{1-\alpha}} = 0
\end{aligned}
\tag{A.136}
$$

A.4.11 Calculating steady state

A.4.11.1 Feeding productivities

After feeding all the parameters in the model, the above excess demand function is used for calculating Q_i. Then, $\bar{P}_i$ is calculated using (A.122). Then, $\bar{W}_i$ is calculated. Using $\bar{W}_i$ and $\bar{Q}_i$, $\bar{L}_i$ is calculated. Then, the rest shares the next case.

A.4.11.2 Feeding price estimates

The empirical estimation gives $\bar{\tau}_{j,i,m}$ and $\bar{p}_{j,m}$, but not $\bar{a}_{j,m}$. Hence, when using empirical values, calculation procedure is following.

- Step 1: Using $\bar{\tau}_{j,i,m}$ and $\bar{p}_{j,m}$ to obtain $\bar{P}_i$ by (A.118).
- Step 2: Solving the system of modified version of (A.127), but in this case, $\bar{Z}_i$ is calculated:

$$
\pi_i \bar{P}_i \bar{Z}_i = \sum_{j=1}^{N} \pi_j \bar{P}_j^{\frac{1}{1-\theta}} \bar{Z}_j \sum_{m=1}^{M} \left[\sum_{h} \left(\bar{\tau}_{h,j,m} \bar{p}_{h,m}\right)^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho}} \left(\bar{\tau}_{i,j,m} \bar{p}_{i,m}\right)^{\frac{\rho}{\rho-1}} \tag{A.137}
$$

Employing a notation that

$$\tilde{\Upsilon}_{i,j} = \sum_{m=1}^{M} \left[\sum_{h} (\bar{\tau}_{h,j,m} \bar{p}_{h,m})^{\frac{\rho}{\rho-1}} \right]^{\frac{\rho-\theta}{\theta-1}\frac{1}{\rho}} (\bar{\tau}_{i,j,m} \bar{p}_{i,m})^{\frac{\rho}{\rho-1}} \tag{A.138}$$

and vector $\bar{\boldsymbol{Z}} = [\bar{Z}_1 \ ... \ \bar{Z}_N]'$

$$\mathbf{0} = \left(\begin{bmatrix} \pi_1 \bar{P}_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \pi_N \bar{P}_N \end{bmatrix} - \begin{bmatrix} \tilde{\Upsilon}_{1,1} \pi_1 \bar{P}_1^{\frac{1}{1-\theta}} & \cdots & \tilde{\Upsilon}_{1,N} \pi_N \bar{P}_N^{\frac{1}{1-\theta}} \\ \vdots & \ddots & \vdots \\ \tilde{\Upsilon}_{N,1} \pi_1 \bar{P}_1^{\frac{1}{1-\theta}} & \cdots & \tilde{\Upsilon}_{N,N} \pi_N \bar{P}_N^{\frac{1}{1-\theta}} \end{bmatrix} \right) \bar{\boldsymbol{Z}} \tag{A.139}$$

or

$$\mathbf{0} = \boldsymbol{A} \bar{\boldsymbol{Z}} \tag{A.140}$$

Note that the system is linearly dependent due to "Walras law" in the excess demand property of the equation as discussed in the last subsection. $\bar{Z}_1$ is normalized to be unity. The actual calculation is implemented by solving linear system:

$$\mathbf{0} = \begin{bmatrix} a_{1,1} \\ \vdots \\ a_{1,N} \end{bmatrix} + \begin{bmatrix} a_{1,2} & \cdots & a_{1,N} \\ \vdots & \ddots & \vdots \\ a_{N,2} & \cdots & a_{N,N} \end{bmatrix} \begin{bmatrix} \bar{Z}_2 \\ \vdots \\ \bar{Z}_N \end{bmatrix} \tag{A.141}$$

where $a_{i,j}$ is (i,j) element of the matrix $\boldsymbol{A}$.

- Step 3: $\bar{W}_i$ and $\bar{L}_i$ is obtained.
- Step 4: $\bar{a}_{j,m}$ is calculated by (A.119).

A.4.11.3 Consumption and absorption

Given $\bar{W}_i$ and $\bar{L}_i$,

$$\bar{C}_i = \frac{\psi}{1-\psi} \bar{W}_i (1 - \bar{L}_i), \tag{A.142}$$

$$\bar{Z}_i = (1-\alpha) \bar{W}_i \bar{L}_i \tag{A.143}$$

A.4.11.4 Capital-labor ratio, marginal utility, absorption

$$\bar{\kappa}_i = \frac{\alpha}{1-\alpha} \frac{\bar{W}_i}{\bar{R}}. \tag{A.144}$$

$$\bar{X}_i = (g_1 - 1 + \delta)\bar{\kappa}_i \bar{L}_i, \tag{A.145}$$

$$\bar{u}_{C,i} = \psi \bar{C}_i^{\psi(1-\gamma)-1}[1 - \bar{L}_i]^{(1-\psi)(1-\gamma)}, \tag{A.146}$$

$$\bar{u}_{L,i} = -\bar{u}_{C,i}\bar{W}_i, \tag{A.147}$$

Check

$$\bar{Z}_i = \bar{C}_i + \bar{X}_i + \bar{G}_i. \tag{A.148}$$

A.4.11.5 Budget constraint multiplier

After normalizing to $\sum_i \mu_i = 1$,

$$\bar{u}_{C,i}\bar{P}_i\mu_i^{-1} = \bar{u}_{C,j}\bar{P}_j\mu_j^{-1} \tag{A.149}$$

gives $\mu_1, ..., \mu_N$.

A.4.11.6 Goods production, exporting, and relative price

The price of goods m in j is

$$\bar{p}_{j,m} = \frac{1}{1-\alpha}\bar{a}_{j,m}^{-1}\bar{\kappa}_j^{-\alpha}\bar{W}_j\bar{P}_j. \tag{A.150}$$

Then, the price at country i is

$$\bar{p}_{j,i,m} = \bar{p}_{j,m}\bar{\tau}_{j,i,m}. \tag{A.151}$$

Using price aggregator,

$$\bar{Z}_i^{1-\theta}\bar{q}_{i,m}^{\theta-1} = \frac{\bar{p}_{i,m}^q}{\bar{P}_i} = \left[\upsilon_{j,i}^{\frac{-\rho}{\rho-1}}\left(\frac{\bar{p}_{j,i,m}}{\bar{P}_i}\right)^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\rho}}, \tag{A.152}$$

implying

$$\bar{q}_{i,m} = \bar{P}_i^{\frac{1}{1-\theta}}\bar{Z}_i\left[\sum_j \upsilon_{j,i}^{\frac{-\rho}{\rho-1}}\bar{p}_{j,i,m}^{\frac{\rho}{\rho-1}}\right]^{\frac{\rho-1}{\theta-1}\frac{1}{\rho}} \tag{A.153}$$

and then

$$\bar{z}_{j,i,m} = \left[\bar{P}_i \bar{Z}_i^{1-\theta} \bar{q}_{i,m}^{\theta-\rho} \upsilon_{j,i}^{\rho} \bar{p}_{j,i,m}^{-1}\right]^{\frac{1}{1-\rho}}. \tag{A.154}$$

A.4.11.7 Output and investment

$$\bar{z}_{i,m} = \sum_j \pi_j \bar{\tau}_{i,j,m} \bar{z}_{i,j,m} / \pi_i \tag{A.155}$$

$$\bar{l}_{i,m} = (1-\alpha) \frac{\bar{z}_{i,m}}{\bar{W}_i} \frac{\bar{p}_{i,m}}{\bar{P}_i} \tag{A.156}$$

$$\bar{k}_{i,m} = \bar{\kappa}_i \bar{l}_{i,m} \tag{A.157}$$

$$\bar{x}_{i,m} = (g_1 - 1 + \delta) \bar{k}_{i,m} \tag{A.158}$$

$$\bar{Y}_i = \bar{Z}_i \tag{A.159}$$

$$\bar{K}_i = \bar{k}_{i,m} \tag{A.160}$$

$$T\bar{F}P_i = \frac{\bar{Y}_i}{\bar{K}_i^{\alpha} \bar{L}_i^{1-\alpha}} \tag{A.161}$$

A.4.11.8 Trade values

$$e\bar{x}_{j,i,m} = \pi_i \bar{p}_{j,i,m} \bar{z}_{j,i,m} \tag{A.162}$$

$$I\bar{M}_i = \frac{1}{\pi_i} \sum_{j \neq i} \sum_m e\bar{x}_{j,i,m} / \bar{P}_i \tag{A.163}$$

$$E\bar{X}_i = \frac{1}{\pi_i} \sum_{j \neq i} \sum_m e\bar{x}_{i,j,m} / \bar{P}_i \tag{A.164}$$

$$\bar{P_i^I} = \left(\sum_{m=1}^{M} \left(\sum_{j \neq i} \upsilon_{j,i}^{\frac{-\rho}{\rho-1}} \bar{p}_{j,i,m}^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}\frac{\theta}{\theta-1}} \right)^{\frac{\theta-1}{\theta}} / \bar{P}_i, \tag{A.165}$$

$$\bar{P_i^E} = \left(\sum_{m=1}^{M} \left(\sum_{j \neq i} \upsilon_{i,j}^{\frac{-\rho}{\rho-1}} \bar{p}_{i,j,m}^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}\frac{\theta}{\theta-1}} \right)^{\frac{\theta-1}{\theta}} / \bar{P}_i \tag{A.166}$$

A.4.12 Log-linearization

The economy's dynamics is analyzed by first order log-linearized economy.

- # number of equations
- ♡ combining to reduce number of variables
- ♢ redundant conditions (not used)

All the variables excluding $\hat{B}_{i,t}$ is log-deviations from the stead state, e.g.,

$$\hat{C}_{i,t} = \log(C_{i,t}/\bar{C}_i) \tag{A.167}$$

Bond holding is linear approximation (not log-linear approximation):

$$\hat{B}_{i,t} = B_{i,t} - \bar{B}_i = B_{i,t}. \tag{A.168}$$

A.4.12.1 HH problem

$$(\#N) : 0 = -\hat{u}_{C,i,t} + (\psi(1-\gamma) - 1)\hat{C}_{i,t} - (1-\psi)(1-\gamma)\frac{\bar{L}_i}{1-\bar{L}_i}\hat{L}_{i,t} \tag{A.169}$$

$$(\#N) : 0 = -\hat{W}_{i,t} + \hat{C}_{i,t} + \frac{\bar{L}_i}{1-\bar{L}_i}\hat{L}_{i,t} \tag{A.170}$$

$$\begin{aligned}(\#NM) : 0 \quad &= \quad -\hat{u}_{C,i,t} - \eta_\phi \left(\hat{x}_{i,m,t} - \hat{k}_{i,m,t}\right) \\ &\quad +E_t\hat{u}_{C,i,t+1} + \beta g_4 \bar{R} E_t \hat{R}_{i,m,t+1} + \beta g_4 \eta_\phi g_1 E_t \left(\hat{x}_{i,m,t+1} - \hat{k}_{i,m,t+1}\right)\end{aligned} \tag{A.171}$$

$$(\#NM) : 0 = -g_1\hat{k}_{i,m,t+1} + (1-\delta)\hat{k}_{i,m,t} + (g_1 - 1 + \delta)\hat{x}_{i,m,t} \tag{A.172}$$

A.4.12.2 Final goods producer

$$(\#N): 0 = -\bar{Z}_i^{\theta}\hat{Z}_{i,t} + \sum_m \bar{q}_{i,m}^{\theta}\hat{q}_{i,m,t} \tag{A.173}$$

$$(\#NM): 0 = -\bar{q}_{i,m}^{\rho}\hat{q}_{i,m,t} + \sum_j \upsilon_{j,i}^{\rho}\bar{z}_{j,i,m}^{\rho}\hat{z}_{j,i,m,t} \tag{A.174}$$

$$(\#N\diamondsuit): 0 = -\bar{P}_i\hat{P}_{i,t} - \bar{Z}_i\hat{Z}_{i,t} + \sum_m \sum_j \bar{p}_{j,i,m}\bar{z}_{j,i,m}\left(\hat{p}_{j,i,m,t} + \hat{z}_{j,i,m,t}\right) \tag{A.175}$$

$$(\#N\diamondsuit): 0 = -\bar{P}_i^{\frac{\theta}{\theta-1}}\hat{P}_{i,t} + \sum_m \left(\sum_h \upsilon_{h,i}^{\frac{-\rho}{\rho-1}}\bar{p}_{h,i,m}^{\frac{\rho}{\rho-1}}\right)^{\frac{\theta-\rho}{\rho(1-\theta)}} \sum_j \upsilon_{j,i}^{\frac{-\rho}{\rho-1}}\bar{p}_{j,i,m}^{\frac{\rho}{\rho-1}}\hat{p}_{j,i,m,t} \tag{A.176}$$

$$(\#N^2M\heartsuit): 0 = -\hat{p}_{j,i,m,t} + \hat{P}_{i,t} + (1-\theta)\hat{Z}_{i,t} + (\theta-\rho)\hat{q}_{i,m,t} + (\rho-1)\hat{z}_{j,i,m,t} \tag{A.177}$$

A.4.12.3 Intermediate goods producer and trade

$$(\#NM): 0 = -\hat{z}_{i,m,t} + \hat{a}_{i,m,t} + \alpha\hat{k}_{i,m,t} + (1-\alpha)\hat{l}_{i,m,t} \tag{A.178}$$

$$(\#NM): 0 = -\hat{W}_{i,t} - \hat{P}_{i,t} + \hat{p}_{i,m,t} + \hat{z}_{i,m,t} - \hat{l}_{i,m,t} \tag{A.179}$$

$$(\#NM): 0 = -\hat{R}_{i,m,t} - \hat{P}_{i,t} + \hat{p}_{i,m,t} + \hat{z}_{i,m,t} - \hat{k}_{i,m,t} \tag{A.180}$$

$$(\#NM): 0 = -\pi_i\bar{z}_{i,m}\hat{z}_{i,m,t} + \sum_j \pi_j\bar{\tau}_{i,j,m}\bar{z}_{i,j,m}\left(\hat{z}_{i,j,m,t} + \hat{\tau}_{i,j,m,t}\right) \tag{A.181}$$

$$(\#N^2M\heartsuit): 0 = -\hat{p}_{i,j,m,t} + \hat{p}_{i,m,t} + \hat{\tau}_{i,j,m,t} \tag{A.182}$$

$$(\#NM\diamondsuit): 0 = -\pi_i\bar{p}_{i,m}\bar{z}_{i,m}\left(\hat{p}_{i,m,t} + \hat{z}_{i,m,t}\right) + \sum_j \pi_j\bar{p}_{i,j,m}\bar{z}_{i,j,m}\left(\hat{p}_{i,j,m,t} + \hat{z}_{i,j,m,t}\right) \tag{A.183}$$

A.4.12.4 Resource constraints

$$(\#N) : 0 = -\bar{L}_i \hat{L}_{i,t} + \sum_m \bar{l}_{i,m} \hat{l}_{i,m,t} \tag{A.184}$$

$$(\#N) : 0 = -\bar{X}_i \hat{X}_{i,t} + \sum_m \bar{x}_{i,m} \hat{x}_{i,m,t} \tag{A.185}$$

$$(\#N) : 0 = \bar{Z}_i \hat{Z}_{i,t} - \bar{C}_i \hat{C}_{i,t} - \bar{X}_i \hat{X}_{i,t} - \bar{G}_i \hat{G}_{i,t} \tag{A.186}$$

A.4.12.5 Price normalization

$$(\#1) : 0 = \hat{P}_{N,t} \tag{A.187}$$

A.4.12.6 Complete market case

$$(\#N-1) : 0 = -\hat{u}_{C,i,t} + \hat{P}_{i,t} + \hat{u}_{C,j,t} - \hat{P}_{j,t} \tag{A.188}$$

A.4.12.7 Incomplete market case

$$\begin{aligned}(\#N-1) : 0 = \; & - \left(E_t \hat{u}_{C,i,t+1} - \hat{u}_{C,i,t} - E_t \hat{P}_{i,t+1} + \hat{P}_{i,t} - \eta^b \bar{P}^b \hat{B}_{i,t} \right) \\ & + \left(E_t \hat{u}_{C,j,t+1} - \hat{u}_{C,j,t} - E_t \hat{P}_{j,t+1} + \hat{P}_{j,t} - \eta^b \bar{P}^b \hat{B}_{j,t} \right)\end{aligned} \tag{A.189}$$

$$\begin{aligned}(\#N) : 0 = \; & - \bar{P}_i \bar{Z}_i \hat{Z}_{i,t} - \bar{P}^b \hat{B}_{i,t} + \hat{B}_{i,t-1} \\ & + \bar{P}_i \bar{W}_i \bar{L}_i \left(\hat{W}_{i,t} + \hat{L}_{i,t} \right) + \bar{P}_i \bar{R} \sum_m \bar{k}_{i,m} \left(\hat{R}_{i,m,t} + \hat{k}_{i,m,t} \right)\end{aligned} \tag{A.190}$$

$$(\# : 1\diamondsuit) : 0 = \sum_i \pi_i \hat{B}_{i,t} \tag{A.191}$$

A.4.12.8 Financial autarky case

(One of N equation is not used.)

$$(\#N-1(+1\diamondsuit)) : 0 = -\bar{P}_i \bar{Z}_i \hat{Z}_{i,t} + \bar{P}_i \bar{W}_i \bar{L}_i \left(\hat{W}_{i,t} + \hat{L}_{i,t} \right) + \bar{P}_i \bar{R} \sum_m \bar{k}_{i,m} \left(\hat{R}_{i,m,t} + \hat{k}_{i,m,t} \right) \tag{A.192}$$

A.4.12.9 Shock process

$$(\#N) : \hat{G}_{i,t} \sim N(0, \sigma_G^2) \tag{A.193}$$

$$(\#NM) : \hat{a}_{i,m,t} = \rho_a \hat{a}_{i,m,t-1} + \sigma_\varepsilon \varepsilon_{i,m,t} \tag{A.194}$$

$$(\#NM) : \varepsilon_{i,m,t} \sim N(0, 1) \tag{A.195}$$

$$(\#N^2M) : \hat{\tau}_{i,j,m,t} = \rho_\tau \hat{\tau}_{i,j,m,t-1} + \sigma_\epsilon \epsilon_{i,j,m,t} \tag{A.196}$$

$$(\#N^2M) : \epsilon_{i,j,m,t} \sim N(0, 1) \tag{A.197}$$

A.4.12.10 # of state variables, equations, and total variables

- State variables ($\#2NM + N^2M$): $\hat{k}_{i,m,t}$, $\hat{a}_{i,m,t}$, $\hat{\tau}_{i,j,m,t}$
- # of equations: $9N + 8NM + 2N^2M - (2N + NM + N^2M)$
- # of variables: $7N + 7NM + 2N^2M - (N^2M)$
- $(\#N \times 7) : \hat{u}_{C,i,t}, \hat{C}_{i,t}, \hat{L}_{i,t}, \hat{X}_{i,t}, \hat{Z}_{i,t}, \hat{W}_{i,t}, \hat{P}_{i,t}$
- $(\#NM \times 7) : \hat{l}_{i,m,t}, \hat{x}_{i,m,t}, \hat{k}_{i,m,t}, \hat{q}_{i,m,t}, \hat{z}_{i,m,t}, \hat{p}_{i,m,t}, \hat{R}_{i,m,t},$
- $(\#N^2M \times (2-1)) : \hat{z}_{i,j,m,t}, \hat{p}_{i,j,m,t}$ (by $\heartsuit$, $\hat{p}_{i,j,m,t}$ is dropped.)
- Exogenous shocks: $\hat{a}_{i,m,t}, \hat{\tau}_{i,j,m,t}, \hat{\varepsilon}_{i,m,t}, \epsilon_{i,j,m,t}, \hat{G}_{i,t}$
- Incomplete market case: N additional state variables, $\hat{B}_{i,t}$, N additional equations.

Remarks

- One of the $\#N$ equations is redundant.
- $\hat{p}_{j,i,m,t}$ can be dropped from the system.

A.4.12.11 Additional variables of interests

$$\bar{Y}_i \hat{Y}_{i,t} = \bar{W}_i \bar{L}_i \left(\hat{W}_{i,t} + \hat{L}_{i,t} \right) + \bar{R} \sum_m \bar{k}_{i,m} \left(\hat{R}_{i,m,t} + \hat{k}_{i,m,t} \right) \tag{A.198}$$

$$\widehat{ex}_{i,j,m,t} = \hat{p}_{i,j,m,t} + \hat{z}_{i,j,m,t} \tag{A.199}$$

$$\widehat{EX}_{i,t} = \frac{1}{\pi_i \bar{P}_i \bar{EX}_i} \sum_{j \neq i} \sum_m \bar{ex}_{i,j,m} \widehat{ex}_{i,j,m,t} - \hat{P}_{i,t} \tag{A.200}$$

$$\widehat{IM}_{i,t} = \frac{1}{\pi_i \bar{P}_i \bar{IM}_i} \sum_{j \neq i} \sum_m \bar{ex}_{j,i,m} \widehat{ex}_{j,i,m,t} - \hat{P}_{i,t} \tag{A.201}$$

$$\widehat{NX}_{i,t} = \frac{\bar{EX}_i + \bar{EX}_i \widehat{EX}_{i,t} - \bar{IM}_i - \bar{IM}_i \widehat{IM}_{i,t}}{\bar{Y}_i + \bar{Y}_i \hat{Y}_{i,t}} \tag{A.202}$$

$$\hat{P}^I_{i,t} = \frac{\theta - 1}{\theta} \left(\bar{P}^I_i \bar{P}_i \right)^{\frac{\theta}{1-\theta}} \sum_m \left(\sum_{h \neq i} \upsilon_{h,i}^{\frac{-\rho}{\rho-1}} \bar{p}_{h,i,m}^{\frac{\rho}{\rho-1}} \right)^{\frac{\theta-\rho}{\rho(1-\theta)}} \sum_{j \neq i} \upsilon_{j,i}^{\frac{-\rho}{\rho-1}} \bar{p}_{j,i,m}^{\frac{\rho}{\rho-1}} \hat{p}_{j,i,m,t} - \hat{P}_{i,t} \tag{A.203}$$

$$\hat{P}^E_{i,t} = \frac{\theta - 1}{\theta} \left(\bar{P}^E_i \bar{P}_i \right)^{\frac{\theta}{1-\theta}} \sum_m \left(\sum_{h \neq i} \upsilon_{i,h}^{\frac{-\rho}{\rho-1}} \bar{p}_{i,h,m}^{\frac{\rho}{\rho-1}} \right)^{\frac{\theta-\rho}{\rho(1-\theta)}} \sum_{j \neq i} \upsilon_{i,j}^{\frac{-\rho}{\rho-1}} \bar{p}_{i,j,m}^{\frac{\rho}{\rho-1}} \hat{p}_{i,j,m,t} - \hat{P}_{i,t} \tag{A.204}$$

$$\widehat{ToT}_{i,t} = \hat{P}^I_{i,t} - \hat{P}^E_{i,t} \tag{A.205}$$

$$\hat{K}_{i,t} = \frac{1}{\bar{K}_i} \sum_m \bar{k}_{i,m} \hat{k}_{i,m,t} \tag{A.206}$$

$$\widehat{TFP}_{i,t} = \hat{Y}_{i,t} - \alpha \hat{K}_{i,t} - (1 - \alpha) \hat{L}_{i,t} \tag{A.207}$$

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Table A.1: Reduced form gravity equations

	(1)	(2)	(3)	(4)	(5)
	$\ln(ex_{j,i,m,t})$	$\ln(ex_{j,i,m,t})$	$\ln(ex_{j,i,m,t})$	$\ln\left(\frac{ex_{j,i,m,t}}{\pi_i Z_{i,t} P_{i,t}^{1/(1-\theta)}}\right)$	$\ln\left(\frac{ex_{j,i,m,t}}{\pi_i Z_{i,t} P_{i,t}^{1/(1-\theta)}}\right)$
$\ln Z_{j,t}$	1.95***	2.22***			
	(0.02)	(0.06)			
$\ln P_{j,t}$	1.13***	1.99***			
	(0.02)	(0.06)			
$\ln \pi_j$	0.93***	1.24***			
	(0.00)	(0.01)			
$\ln Z_{i,t}$	1.31***	1.91***	1.74***		
	(0.02)	(0.06)	(0.05)		
$\ln P_{i,t}$	0.69***	1.07***	1.25***		
	(0.02)	(0.06)	(0.05)		
$\ln \pi_i$	0.80***	1.09***	1.10***		
	(0.00)	(0.01)	(0.01)		
$\text{lnDist}_{j,i}$	7.99***	−2.43***	9.41***	4.62***	2.01*
	(0.33)	(0.90)	(0.98)	(0.98)	(1.18)
$(\text{lnDist}_{j,i})^2$	−1.27***	0.24*	−1.39***	−0.72***	−0.07
	(0.05)	(0.13)	(0.14)	(0.14)	(0.16)
$(\text{lnDist}_{j,i})^3$	0.06***	−0.01**	0.06***	0.03***	−0.02**
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
$\text{Border}_{j,i}$	0.09***	−0.61***	−0.33***	−0.27***	0.20***
	(0.01)	(0.03)	(0.03)	(0.03)	(0.04)
$\text{Lang}_{j,i}$	0.32***	0.76***	0.87***	0.91***	0.38***
	(0.01)	(0.03)	(0.03)	(0.03)	(0.03)
$\text{Colony}_{j,i}$	0.89***	1.05***	0.96***	0.96***	1.05***
	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)
$\text{RTA}_{j,i,t}$	0.34***	0.35***	0.47***	0.47***	0.03
	(0.01)	(0.03)	(0.03)	(0.03)	(0.03)
(j,m,t) FE	no	no	yes	yes	yes
(i,m,t) FE	no	no	no	no	yes
(m,t) FE	yes	yes	–	–	–
Restriction†	no	no	no	yes	yes
Missing	drop	50K	50K	50K	50K
Observations	159865	163800	163800	163800	163800
R^2	0.748	0.470	0.576	0.534	0.110

Robust standard errors in parentheses. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.01$.

† If yes, the model implied restrictions are imposed by transforming the dependent variable with $\theta = 1/3$.

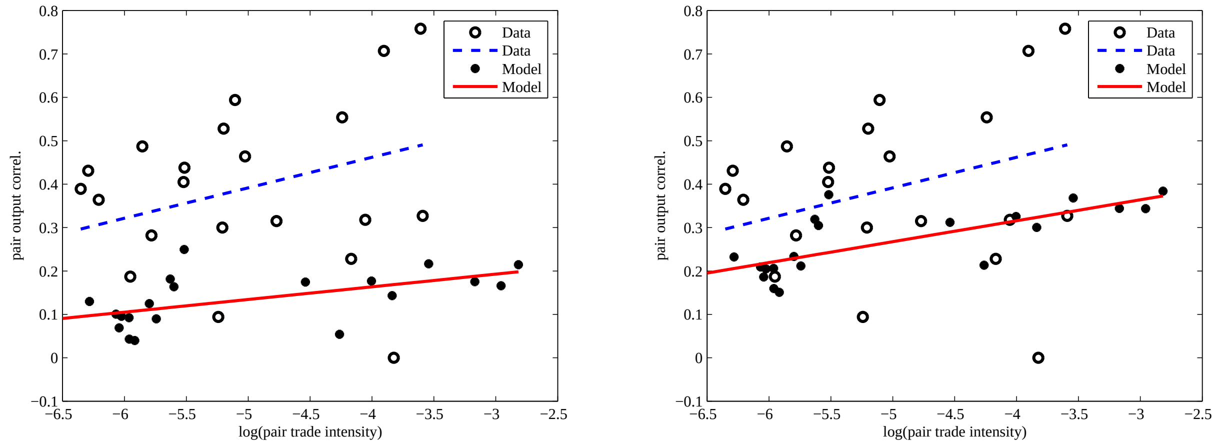
Missings are replaced by nominal $50,000.

The replacement value by 1K, 10K and 100K give the same results.

Table A.2: International business cycle moments

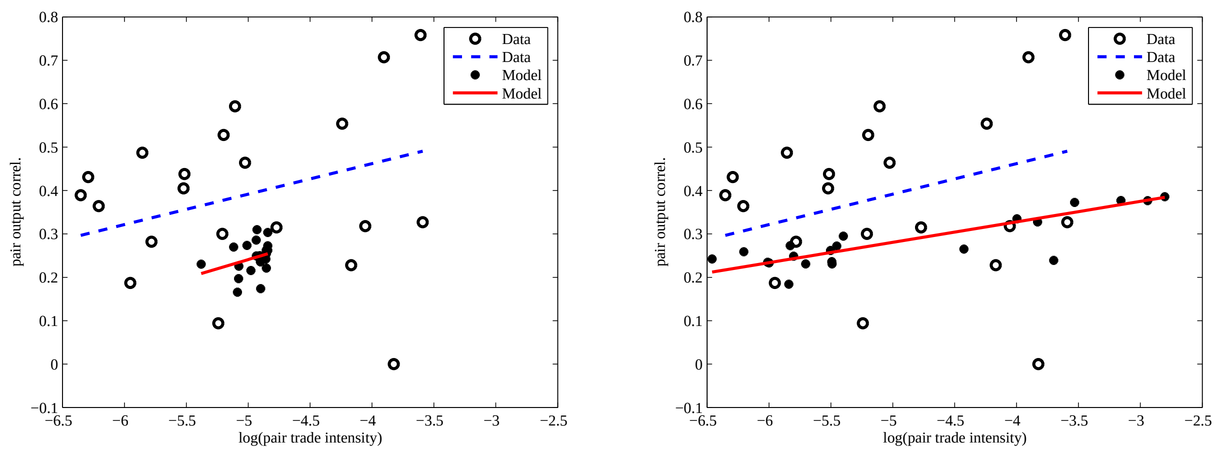
	G7	Baseline		Complete	Financial	Same	Same	Symmetric
	Mean	US	G7	market	autarky	TC	pop	countries
Cross country correl.								
Output (Y)	0.41	0.20	0.28	0.13	0.27	0.24	0.28	0.24
Consumption (C)	0.21	0.16	0.22	0.58	0.25	0.19	0.22	0.19
Investment (X)	0.23	0.14	0.22	0.07	0.30	0.19	0.22	0.19
Labor (L)	0.23	0.25	0.33	−0.08	0.37	0.39	0.34	0.40
Net exports (NX)	−0.02	−0.17	−0.11	−0.10	−0.10	−0.09	−0.10	−0.09
Aggregate TFP shock	0.10	0.14	0.23	0.08	0.31	0.19	0.23	0.19
Correl. to output								
Exports (EX)	0.42	0.28	0.32	0.65	0.32	0.28	0.32	0.28
Imports (IM)	0.72	0.29	0.32	−0.19	0.32	0.28	0.32	0.28
Net exports (NX)	−0.32	−0.05	−0.01	0.90	–	−0.01	−0.01	−0.02
Ave. bilat. trade int. (%)	0.97	0.46	1.35	1.35	1.35	0.71	1.45	0.80
Agg. Trade-GDP ratio	0.19	0.04	0.14	0.14	0.14	0.09	0.13	0.10
Trade cost	–	1.76	1.74	1.74	1.74	1.72	1.74	1.72
Slope (× 100)	7.02	–	5.18	2.92	4.82	8.49	4.71	–
SE (×100)	5.93	–	0.58	0.62	0.58	0.50	0.44	–

Figure A.1: G7 bilateral trade and output correlations



Left panel: Complete market case. Right panel: Financial autarky case.

Figure A.2: G7 bilateral trade and output correlations



Left panel: Symmetric trade cost case (population is different). Right panel: Symmetric population case (trade costs are different).

Figure A.3: Varying autocorrelation of productivity shock

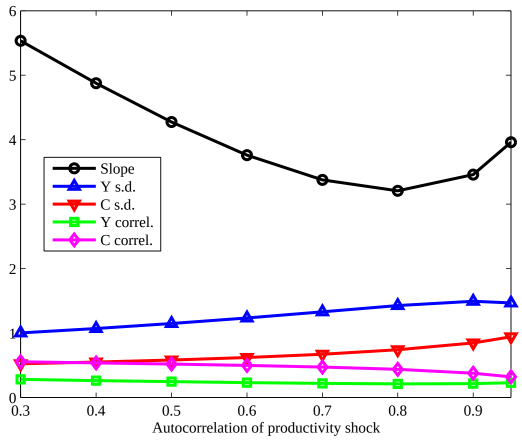
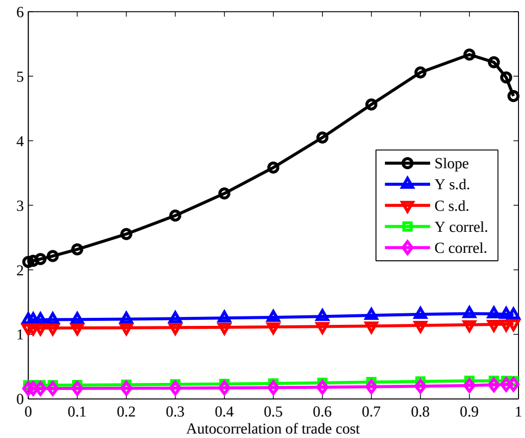


Figure A.4: Varying autocorrelation of trade cost shock



Left panel: baseline parameterization, with varying autocorrelation of productivity shock (ρ_a). Right panel: baseline parameterization, with varying autocorrelation of trade cost shock (ρ_τ). "Slope" is the slope coefficient ($\times$ 100) of trade-comovement regression. "Y s.d." and "C s.d." are standard deviations of model GDP and consumption (mean of model G7 countries), respectively. "Y correl." and "C correl." are cross-country correlations of model GDP and consumption, among G7 countries, respectively.

Figure A.5: Varying CES parameter θ

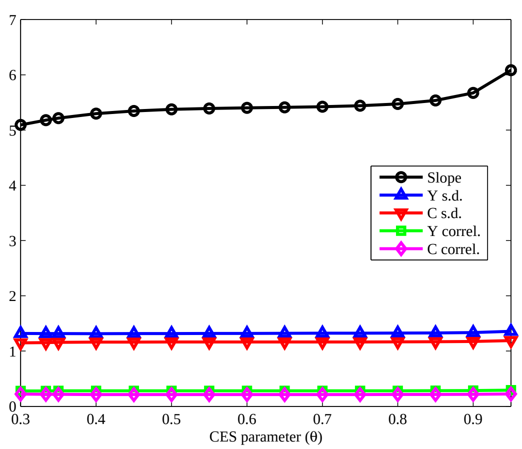
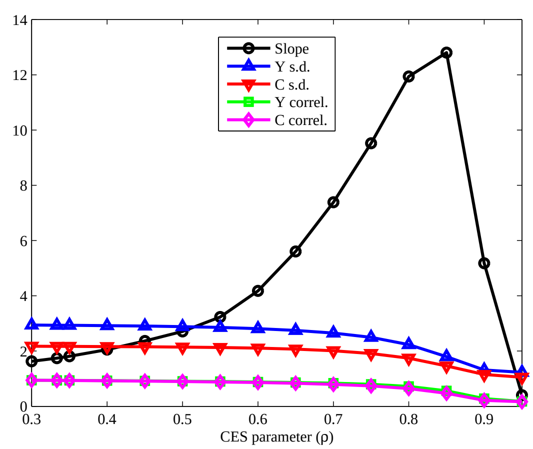


Figure A.6: Varying CES parameter ρ



Left panel: baseline parameterization, with varying elasticity of substitution across different products (θ). Right panel: baseline parameterization, with varying elasticity of substitution across different origins (ρ). "Slope" is the slope coefficient ($\times$ 100) of trade-comovement regression. "Y s.d." and "C s.d." are standard deviations of model GDP and consumption (mean of model G7 countries), respectively. "Y correl." and "C correl." are cross-country correlations of model GDP and consumption, among G7 countries, respectively.

Figure A.7: Varying capital stock adjustment friction η_ϕ

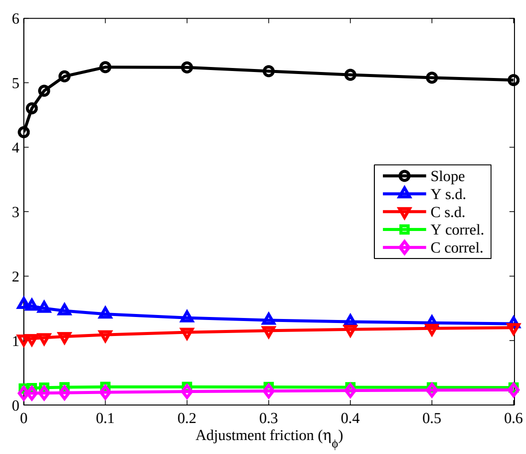
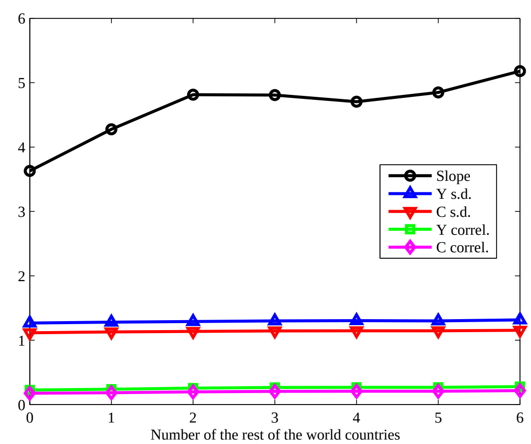


Figure A.8: Varying number of "rest of the world" countries



Left panel: baseline parameterization, with varying elasticity of capital stock adjustment friction (η_ϕ). Right panel: baseline parameterization, with varying number of "rest of the world" countries. "Slope" is the slope coefficient ($\times$ 100) of trade-comovement regression. "Y s.d." and "C s.d." are standard deviations of model GDP and consumption (mean of model G7 countries), respectively. "Y correl." and "C correl." are cross-country correlations of model GDP and consumption, among G7 countries, respectively.