Inventory-Theoretic Money Demand and Relative Price Dynamics

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Abstract

We construct a two-goods inventory-theoretic money demand model, and find that the model implies, in a monetary contraction, the decline in the prices of low cash-intensity goods, durables, or luxuries outpaces that of high cash-intensity goods, non-durables, or necessities. Using U.S. data, we show that our model's predictions are consistent with the data.

1 Introduction

Increasing evidence has shown that disaggregated price responses to a shock to monetary policy are not uniform across economic sectors. Yet, the determinants of differences in the price response have not been fully accounted for in the literature (Bils, Klenow, and Kryvtsov [2003] and Boivin, Giannoni, and Mihov [2009]). To investigate the determinants, we develop a two-goods model that extends Alvarez, Atkeson, and Edmond (2009). We find that a monetary contraction leads to a change in the relative price of goods; the prices of low cash-intensity goods, durables, or luxuries decline by a greater extent than the prices of high cash-intensity goods, non-durables, or necessities. Using U.S. monthly disaggregated price series data, we show that our model's predictions are consistent with the data.

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Our model is a type of inventory model of money demand in which friction associated with portfolio rebalancing plays an important role in the economy.[1] In this model, households are only allowed infrequently to rebalance their money holdings between an account for the asset market (the "brokerage account") and one for the goods market (the "bank account"). As Alvarez, Atkeson, and Edmond (2009) show, following a monetary policy shock this model generates a sluggish price response, a change in the nominal interest rate, and a change in the aggregate real money balance, without introducing any frictions in adjusting the nominal price.

We incorporate multiple goods into the inventory model. The goods differ from each other in terms of cash-intensity, durability, or degree of luxury (hereafter, "luxuriousness"). Cash-intensity is a goods-specific fraction of household expenses that must be paid using cash. Durability is a goods-specific depreciation rate of the goods stock, and luxuriousness is characterized by a goods-specific elasticity parameter in a household's utility function, as defined in Bils and Klenow (1998). We then explore how these goods-specific characteristics affect the response of the goods prices to a monetary policy shock.

A monetary policy shock generates a change in the relative price of the disaggregated goods through two channels: changes in the nominal interest rate and the aggregate real money balance. In response to a contractionary monetary policy shock, the nominal interest rate rises, and the aggregate real money balance declines. A rise in the nominal interest rate raises the value of a dollar in the brokerage account. Households prefer purchasing goods with high cash-intensity ("cash goods") to those with low cash-intensity ("credit goods"), since the latter reduces dollars in the brokerage account by a greater amount. Reflecting this demand difference, the relative price of the low cash-intensity goods declines compared with the high cash-intensity goods. In addition, a drop in the aggregate real money balance induces households to reduce their demand with a low depreciation rate (or high luxuriousness) by a greater amount than those with a high depreciation rate (or low luxuriousness). Consequently, the relative price of durables or luxurious goods tends to decline compared with non-durables or necessity goods, respectively.

[1]Baumol (1952), Tobin (1956) and Grossman and Weiss (1983) develop the basic inventory models of money demand. Alvarez, Atkeson, and Edmond (2009) generalize Grossman and Weiss (1983) to quantitatively explain the observed velocity of money circulation in the United States. A related class of models with portfolio rebalancing frictions includes Alvarez, Lucas, and Weber (2001) and Occhino (2004, 2008). These studies examine models in which some households are permanently excluded from an asset market. Nakajima (2006) and King and Thomas (2007) include other extensions. See Edmond and Weill (2008) for a brief review of the literature.

Employing the U.S. disaggregated price series of the Personal Consumption Expenditures (PCE) data, we ask if our model correctly predicts how a disaggregated price responds to a monetary policy shock. To this end, first we set the degree of cash-intensity, durability, and luxuriousness for each PCE item,[2] using existing studies. We construct the goods-specific cash-intensity according to the credit card usage data compiled by Fisher (2003).[3] We borrow goods-specific durability and luxuriousness from Bils and Klenow (1998). Second, we estimate two sets of impulse responses of the disaggregated prices to a positive shock to the federal funds rate; one identification method is based on a factor-augmented vector autoregression, or FAVAR (Boivin, Giannoni, and Mihov [2009]) and the other employs the narrative records reported by Romer and Romer (2004). Finally, we study the statistical relationship between the constructed goods characteristics and the estimated impulse response. As the model predicts, the relative price of credit goods, durables, or luxuriousness declines compared with cash goods, non-durables, or necessity, respectively, after the contractionary monetary policy shock. These regression results are robust to both sets of impulse response identification methods.

Our paper follows the two strands of literature, and offers new insights on both of them. The first strand includes Alvarez, Atkeson, and Kehoe (2002), Occhino (2004, 2008), and Alvarez, Atkeson, and Edmond (2009) that analyze the implication of the friction associated with portfolio rebalancing for the economy. Contrary to the focus of these studies on the aggregate economy, our paper focuses on the model's implications for the disaggregated economy, and its empirical evaluation using the U.S. disaggregated data. The second strand includes works on the disaggregated price response to the monetary policy shock (e.g., Bils, Klenow, and Kryvtsov [2003], Bils and Klenow [2004], Carvalho [2006], Barsky, House, and Kimball [2007], and Boivin, Giannoni, and Mihov [2009]). While these studies explore the relationship between the frequency of price change and the price response based on sticky-price models, we shed light on the goods characteristic based on an inventory-theoretic money demand model. In our model, all prices are flexible and the difference in the frequency of price change does not play a role, but the difference in the goods characteristics has a significant impact on the price response to the monetary policy shock. Our model therefore provides a new way to grasp the determinants of price response to a monetary policy shock.

[2] We analyze both goods and services of PCE items.

[3] "Declaration of Franklin M. Fisher in Support of Plan of Allocation," *In re Visa Check/MasterMoney Antitrust Litigation*, No. 96-CV-5238, (E.D.N.Y. Aug. 14, 2003).

The rest of our paper is as follows. In Section 2, we describe the model. Section 3 provides the model's quantitative implications. We show that cash-intensity, durability, or luxuriousness affects the goods price responses to a monetary policy shock. In Section 4, we empirically evaluate the model using the U.S. PCE data. We explore the statistical relationship between the constructed goods' characteristics and the estimated price response to a monetary shock. Section 5 concludes.

2 The model

Our model is a two-goods extension of Alvarez, Atkeson, and Edmond (2009). The two goods differ in terms of cash-intensity, durability, or luxuriousness, and are inelastically supplied. An exogenously imposed nominal price friction does not exist in the economy, and the goods prices are flexibly determined to clear the goods markets.

2.1 Model setting

2.1.1 Environment

The setup of the economy follows Alvarez, Atkeson, and Edmond (2009), except that the economy consists of two goods. There are two separate markets, and each household has two financial accounts, one for each market. One market is the asset market where households trade interest-bearing assets using the "brokerage account." The other market is the goods market where households trade goods using the "bank account."

In each period, only a fraction of the households can rebalance assets between the two accounts. Specifically, the economy consists of $s = 0, ..., N-1$ equally divided types of the households, and only the type $s = 0$ households can transfer money (cash) from the brokerage account to the bank account. The type of the households shifts forward: the type 0 households in period t become type 1 households in the next period, type 1 households in period t become type 2 in the next period, . . . , and type $N-1$ households become type 0. The number s hence indicates the number of periods after the households' last rebalancing of the portfolio. Let s_0 denote the type of certain households in the initial period. The type of the households in period t is determined by the initial type s_0 and the number of periods from the initial period: $s = S(t, s_0)$. We call the type $s = 0$

households in a particular period "active households" during the period.[4]

The two goods are denoted by $j = 1, 2$. They are supplied inelastically and transacted in the goods markets. At the beginning of the period, every household receives an equal amount of endowments for goods j. Each household is then divided into a shopper-seller pair. The sellers sell their endowments to the shoppers of other households, while the shoppers purchase goods from the sellers. The transaction of the goods is conducted either in cash, credit, or both, depending on the cash-intensity of goods. The payments are deducted from the shoppers' bank accounts if they are paid in cash, and from the shoppers' brokerage accounts if they are paid in credit. The cash-intensity is measured by a parameter $\theta_j \in [0, 1]$ that indicates a goods-specific fraction of the payment that must be paid in cash. This cash-intensity parameter generalizes the dichotomy introduced by Lucas and Stokey (1987); $\theta_j = 0$ indicates that the goods j are credit goods, whereas $\theta_j = 1$ indicates the goods are cash goods. Durability of goods j is measured by the goods-specific depreciation rate, $\delta_j \in [0, 1]$. Luxuriousness is measured by the goods-specific elasticity parameter $\sigma_j > 0$ in the utility function, following the terminology used by Bils and Klenow (1998). To simplify the analysis, we hereafter assume that the first goods are cash goods ($\theta_1 = 1$), non-durables ($\delta_1 = 1$), and that they have a unity income elasticity ($\sigma_1 = 1$) in the utility function.

2.1.2 Households problem

Each household maximizes

$$\sum_{t=0}^{\infty} \sum_{h^t} \beta^t \Pr(h^t) u(c_{1,t}(s, h^t), k_{2,t}(s, h^t)), \quad s = S(t, s_0), \tag{1}$$

where h^t is the history to date t and $\Pr(h^t)$ is the probability that a particular history of the economy is realized. The utility depends on the flow of the first goods $c_{1,t}(s, h^t)$ and the stock of the second goods $k_{2,t}(s, h^t)$ since the first goods are non-durables. Following Bils and Klenow (1998), we assume that the temporal utility function has the following addilog form:

$$u(c_{1,t}(s, h^t), k_{2,t}(s, h^t)) = \alpha_1 \frac{c_{1,t}(s, h^t)^{1-\frac{1}{\sigma_1}} - 1}{1 - \frac{1}{\sigma_1}} + \alpha_2 \frac{k_{2,t}(s, h^t)^{1-\frac{1}{\sigma_2}} - 1}{1 - \frac{1}{\sigma_2}}, \tag{2}$$

[4] This exogenous timing of portfolio rebalancing opportunity assumption is relaxed by Chiu (2007), Khan and Thomas (2007), and Silva (2009). Under plausible calibration parameters, the endogenous timing models show a similar property to the exogenous timing models regarding the price sluggishness.

where $\alpha_j > 0$ stands for the utility weight of the goods j consumption. The goods-specific luxuriousness is captured by the Engel curve elasticity parameter σ_j. The higher σ_j is, the more luxurious the goods. The law of motion for the stock of the second goods held by type s households in period t is

$$k_{2,t}(s, h^t) = [1 - \delta_2]k_{2,t-1}(s - 1, h^{t-1}) + c_{2,t}(s, h^t), \tag{3}$$

where $\delta_2 \in [0, 1]$ denotes one-period depreciation rate of the second goods, and $c_{2,t}(s, h^t)$ is the amount of the second goods purchased by s in period t. All households receive an equal amount of endowments $y_{1,t}(h^t)$ and $y_{2,t}(h^t)$ in each period.

The households face constraints associated with two accounts and non-negativity constraints associated with the money and the goods. The constraints associated with the bank accounts are expressed by

$$\sum_{j=1}^{2} \theta_j P_{j,t}(h^t)c_{j,t}(s, h^t) + Z_t(s, h^t) = M_t(s, h^t), \tag{4}$$

$$M_t(s, h^t) = Z_{t-1}(s - 1, h^{t-1}) + \sum_{j=1}^{2} \gamma_j\theta_j P_{j,t-1}(h^{t-1})y_{j,t-1}(h^{t-1}) + \iota(s)P_{1,t}(h^t)x_t(h^t), \tag{5}$$

where $\gamma_j \in [0, 1]$ is the paycheck parameter, $P_{j,t}(h^t)$ is the price level of j goods, $Z_t(s, h^t)$ is the amount of money in the bank account carried over from t to $t+1$, $M_t(s, h^t)$ is the amount of money in the bank account by type s households in the beginning of period t, $x_t(h^t)$ is the amount of money transferred from the brokerage account in period t, and $\iota(s)$ is an index function taking one if $s = 0$ and zero otherwise. Equations (4) and (5) show how type s households receive and spend their money $M_t(s, h^t)$.

A household's money holding $M_t(s, h^t)$ comes from three different sources: (i) money that is carried over from the previous period $Z_{t-1}(s - 1, h^{t-1})$, (ii) received payments from selling the endowments in cash $\sum_{j=1}^{2} \gamma_j\theta_j P_{j,t-1}(h^{t-1})y_{j,t-1}(h^{t-1})$, and (iii) money withdrawn from the brokerage account, $P_{1,t}(h^t)x_t(h^t)$. The transfer is enumerated by the first goods price and is positive only for the active households.

Given $M_t(s, h^t)$, a household purchases the two goods using a portion of the money holding, and carries over the rest $Z_t(s, h^t)$ to the next period. Meanwhile, a seller of type s household sells the endowments of the two goods in the goods market.

The paycheck parameter γ_j determines fraction of household's income that is deposited in the bank account. Only a fraction $\gamma_j \theta_j$ of the earning is deposited in the bank account and the rest is deposited in the brokerage account. If the goods are cash goods ($\theta_j = 1$), then a fraction γ_j of the earnings is deposited in the bank account. If the goods are credit goods ($\theta_j = 0$), then all the earnings are deposited in the brokerage account.

The constraint associated with brokerage account is described as follows:

$$\begin{aligned} &\sum_{h_{t+1}} q_{t+1}^t(h^{t+1}) B_t(s, h^{t+1}) + A_t(s, h^t) + \iota(s) P_{1,t}(h^t) x_t(h^t) + \sum_{j=1}^{2} [1 - \theta_j] P_{j,t}(h^t) c_{j,t}(s, h^t) \\ &= B_{t-1}(s-1, h^t) + A_{t-1}(s-1, h^{t-1}) + \sum_{j=1}^{2} [1 - \gamma_j \theta_j] \, P_{j,t-1}(h^{t-1}) y_{j,t-1}(h^{t-1}) - P_{1,t}(h^t) \tau_t(h^t), \end{aligned} \tag{6}$$

where $q_{t+1}^t(h^{t+1})$ is the price of a one-period-ahead state-contingent government bond returning one dollar (conditional on a particular history h^{t+1} is realized), $B_t(s, h^{t+1})$ is the amount of bond held by type s households in t, $\tau_t(h^t)$ is lump-sum tax/transfer by the government, and $A_t(s, h^t)$ is the money holding in the brokerage account. This equation (6) shows accumulation process of the assets held by the type s households in period t. The assets of the households have three sources: (i) earnings from the bond ($B_{t-1}(s-1, h^t)$), (ii) earnings from the sales of endowments that are deposited in the brokerage account ($\sum_{j=1}^{2} [1 - \theta_j] P_{j,t}(h^t) c_{j,t}(s, h^t)$), and ($iii$) the lump-sum tax/transfer by the government, $\tau_t(h^t)$. Note also that a fraction of household's payment for purchasing goods, denoted by $[1 - \theta_j] P_{j,t}(h^t) c_{j,t}(s, h^t)$, is deducted directly from the brokerage account, while the rest of the payments are deducted from the bank account.

The households cannot borrow money from other households, and the purchases of goods need to be non-negative: $Z_t(s, h^t) \geq 0$, $M_t(s, h^t) \geq 0$, $A_t(s, h^t) \geq 0$ and $c_{j,t}(s, h^t) \geq 0$.

2.1.3 The government budget constraint and the equilibrium

The government faces the following budget constraint:

$$B_{t-1}(h^t) = M_t(h^t) - M_{t-1}(h^{t-1}) + P_{1,t}(h^t) \tau_t(h^t) + \sum_{h_{t+1}} q_{t+1}^t(h^{t+1}) B_t(h^{t+1}), \tag{7}$$

where $M_t(h^t)$ and $B_t(h^t)$ are the total amount of money and bonds in the economy, respectively. Let the growth rate of money be $\mu_t = M_t/M_{t-1}$. We assume μ_t follows an AR(1) process, where $\rho_M \in [0, 1)$:

$$\mu_t = \rho_M \mu_{t-1} + \varepsilon_t. \tag{8}$$

The monetary shock is specified by a disturbance in ε_t.[5]

The equilibrium is defined in a standard way:

Definition: A *competitive equilibrium* of this economy is prices $\left\{ \{P_{j,t}(h^t)\}_{j=1}^2, \{q_{t+1}^t(h^{t+1})\}_{h^{t+1}|h^t} \right\}$ and an allocation $\left\{ \left\{ \{c_{j,t}(s, h^t)\}_{j=1}^2 k_{2,t}(s, h^t), x_t(h^t), A_t(s, h^t), Z_t(s, h^t), M_t(s, h^t), \{B_t(s, h^{t+1})\}_{h^t+1|h^t}, \right\}_{s=0}^{N-1} \right\}$ for a given government policy $\{\tau_t(h^t), M_t(h^t), B_t(h^t)\}$, an endowment process $\left\{ \{y_{j,t}(s, h^t)\}_{j=1}^2 \right\}$ and initial conditions $\left\{ \{k_{j,-1}(s_0 - 1)\}_{j=1}^2, A_{-1}(s_0 - 1, .), Z_{-1}(s_0 - 1, .), B_{-1}(s_0 - 1, .) \right\}_{s_0=0}^{N-1}$ such that for all t and h^t :

(i) household maximizes utility taking the prices as given;

(ii) the government budget constraint holds;

(iii) markets clear:

$$\frac{1}{N} \sum_s c_{j,t}(s, h^t) = y_{j,t}(h^t) \text{ for } j = 1, 2, \tag{9}$$

$$\frac{1}{N} \sum_s B_t(s, h^{t+1}) = B_t(h^{t+1}) \text{ for all } h^{t+1} \text{ given } h^t, \tag{10}$$

$$\frac{1}{N} \sum_s [M_t(s, h^t) + A_t(s, h^t)] = M_t(h^t). \tag{11}$$

Here, we focus on an economy with a positive interest rate, so that $A_t(s, h^t) = Z_t(N-1, h^t) = 0$.[6]

[5] As shown in Alvarez, Atkeson, and Edmond (2009), the money growth rate rule delivers price sluggishness that is similar to a nominal interest rate rule, whereas the nominal interest rate rule introduces a computational complexity in the model. For the sake of simplicity, we choose the money growth rate rule and focus our analysis on the disaggregated price responses to shock to the monetary policy.

[6] Alvarez, Atkeson, and Edmond (2009) point out that when positive inflation prevails, type $s = N - 1$ households do not carry money into the subsequent period. This is because they can rebalance the assets in the next period and have no incentive to leave a positive amount of money in the bank accounts.

2.2 Key macroeconomic variables

The nominal interest rate and the aggregate real money balance are important for an understanding of the model mechanism. The nominal interest rate corresponds to a payment to a non-state contingent bond:

$$\frac{1}{1+i_t(h^t)} = \sum_{h^{t+1}|h^t} q_{t+1}^t(h^{t+1}). \tag{12}$$

As shown in Appendix B, this equation is expressed as

$$\frac{1}{1+i_t(h^t)} = \beta \sum_{h^{t+1}|h^t} \Pr(h^{t+1}|h^t) \frac{u_{1,t+1}\left(0,h^{t+1}\right)}{u_{1,t}\left(0,h^t\right)} \frac{P_{1,t}\left(h^t\right)}{P_{1,t+1}\left(h^{t+1}\right)}, \tag{13}$$

where $u_{j,t}(s,h^t)$ is the marginal utility of the type s household with respect to jth goods consumption. Since type $s=0$ households are the only households that can rebalance the money holding between the two accounts, the nominal interest rate is determined by the growth rate of marginal utility of consumption of the active households in periods t and $t+1$.

The aggregate real money balance evaluated by the first goods price is given by

$$\frac{M_t(h^t)}{P_{1,t}\left(h^t\right)} = \frac{\sum_s M_t(s,h^t)}{NP_{1,t}\left(h^t\right)}. \tag{14}$$

Suppose $\gamma_1 = \gamma_2 = 0$. Using equations (4) and (5), we have a simplified expression of the real money balance for each type s:

$$\frac{M_t(s,h^t)}{P_{1,t}\left(h^t\right)} = \frac{\sum_{k=0}^{N-1-s} \left(P_{1,t+k}\left(h^{t+k}\right) c_{1,t+k}(s+k,h^{t+k}) + \theta_2 P_{2,t+k}(h^{t+k}) c_{2,t+k}(s+k,h^{t+k})\right)}{P_{1,t}\left(h^t\right)}. \tag{15}$$

Notice that the household's real money balance equals to the sum of the consumption expenditures until the next rebalancing opportunity, evaluated by the current price. A decline in the real money balance, therefore, leads to a decrease in the consumption in real terms, and vice versa.

In addition, we give definitions of other aggregate variables. The aggregate consumer price index (CPI), $P_t(h^t)$ is a weighted average of the two disaggregate prices:

$$P_t(h^t) = P_{1,t}(h^t)^{\bar{\alpha}_1} P_{2,t}(h^t)^{\bar{\alpha}_2}, \tag{16}$$

where $\bar{\alpha}_j$ is the share of the aggregate expenditure of j goods at the non-stochastic steady state, which is defined in the next section. The inflation rate of the aggregate price index is given by

$$Inf_t(h^t) = \frac{P_t(h^t)}{P_{t-1}(h^{t-1})}. \tag{17}$$

2.3 Disaggregated prices in the model

In this model, the difference in households' demand yields the cross-sectional difference in the disaggregated price responses to a monetary shock. From the first-order conditions, the following user cost equation gives the type s households' demand for the disaggregated goods.[7]

$$\begin{aligned} p_{2,t}\left(h^t\right) = & \frac{u_{2,t}(s,h^t)}{\theta_2 u_{1,t}\left(s,h^t\right) + (1-\theta_2)u_{1,t}\left(0,h^t\right)} \\ & + \beta(1-\delta_2)\sum_{h^{t+1}|h^t} \Pr(h^{t+1}|h^t) p_{2,t+1}(h^{t+1}) \frac{\theta_2 u_{1,t+1}(s+1,h^{t+1}) + (1-\theta_2)u_{1,t+1}(0,h^{t+1})}{\theta_2 u_{1,t}\left(s,h^t\right) + (1-\theta_2)u_{1,t}\left(0,h^t\right)}, \end{aligned} \tag{18}$$

where

$$p_{2,t}\left(h^t\right) \equiv \frac{P_{2,t}\left(h^t\right)}{P_{1,t}\left(h^t\right)}. \tag{19}$$

This equation illustrates how households substitute the first goods from the second goods. Since the supplies of the two goods are exogenous and unchanged to a monetary policy shock, the disaggregated price response to the shock is determined by the households' demand. If price responses of two goods are identical, $p_{2,t}$ does not change. When goods' characteristics are not identical, the households demands one good more than the other. Consequently, the relative price $p_{2,t}$ changes to clear the goods market, making the short-run response of disaggregated prices to the shock different across goods. In what follows, we study how the response of the relative price $p_{2,t}$ depends on the goods characteristic parameters, θ_j, δ_j, and σ_j.

[7]See Appendix B for details. Here, we ignore the terms associated with non-negativity constraint of the $c_{2,t}$ for expositional simplicity.

3 Simulations

3.1 Simulation Procedure

To do the simulation, we first calculate the non-stochastic steady state in which the real variables stay constant and nominal variables grow at the rate of the money growth. To make the nominal variables stationary, we normalize them by the first goods price. The non-stochastic steady state is defined by the following.

Definition: The *non-stochastic steady state* of the economy is a competitive equilibrium with $A_t(s) = 0$, $Z_t(N-1) = 0$, such that for, all t, $\{\tau_t, M_t/P_{1,t},\ B_t/P_{1,t}\}$, $\{y_{j,t}\}_{j=1}^{2}$, $\{p_{2,t},\ q_{t+1}^{t}\}$, and $\{\{\{c_{j,t}(s)\}_{j=1}^{2}, k_{2,t}(s), Z_t(s)/P_{1,t}, M_t(s)/P_{1,t}, B_t(s)/P_{1,t}\}_{s=0}^{N-1}, x_t\}$ are constant, and $M_t/M_{t-1} = P_{1,t}/P_{1,t-1} = \bar{\mu}$.

We compute approximations to the solution of our model by log-linearizing the equations around the non-stochastic steady state.[8] We then simulate the model in response to a shock to the monetary policy rule. Our simulations consider a one-shot permanent unanticipated 1% decline in the money supply. The movement of the relative price is shown by the cumulative impulse response (CIR) of the relative price, which captures the accumulated impact of monetary shock to the relative price.[9]

The parameter values for the tastes and technology, presented in Table 1, are borrowed from Alvarez, Atkeson and Edmond (2009) whenever possible. The frequency of period is monthly, and the monthly discount factor β is $0.99^{(1/12)}$, the paycheck parameters γ_1 and γ_2 are both 0.6, and the growth rate of money at the steady state $\overline{\mu}$ is $1.01^{(1/12)}$. The utility weight parameters are set to $\alpha_1 = 2/3$ and $\alpha_2 = 1/3$, where α_1 is roughly calibrated to the expenditure share of the aggregate non-durables (non-durables plus services). As for the frequency of asset rebalancing, Alvarez, Atkeson and Edmond (2009) argue that $N = 15$ or $N = 38$ is consistent with the asset holding behaviors of the U.S. households. We set N equals to 15 in our benchmark simulation.

[8] See the separate appendix for the derivation and detail of the computational methodology.

[9] The CIR of ψ-periods after the shock is calculated by $CIR_{\psi} = \sum_{k=0}^{\psi} \hat{p}_{2,k}$ where $\hat{p}_{2,k}$ is the deviation of the relative price from the steady state at k-period after the shock. The empirical section also employs CIRs of the relative price to a monetary shock as the indicator of the price change.

3.2 Economy's response when two goods are identical

We first simulate the model economy under the assumptions that θ_2, δ_2, and, σ_2 are all unity. When three goods-specific parameters are all unity for the two goods (i.e., $\theta_2 = \delta_2 = \sigma_2 = 1$), the model response to a monetary policy shock becomes quite similar to that in Alvarez, Atkeson and Edmond (2009). In the next subsections, we study how goods characteristics affect the relative price response to the monetary policy shock by changing each of the parameter values from unity.

Figures 1 and 2 display the impulse response functions of macro variables to the contractionary monetary policy shock (the dashed lines in Figures 1 and 2). Since the government issues more bonds to withdraw money from the households in the asset market, the nominal interest rate rises to clear the market (the dotted line in Figure 1). In addition, the shock causes a temporal fall in the aggregate real money balance (the dotted line in Figure 2). Since money becomes scarce relative to goods, the price level declines.[10] Because households spend their money holdings only gradually, however, the price does not fall one for one with the money supply but falls sluggishly (the bold lines in Figures 1 and 2).[11] The relative price never changes after the shock (the lines with marked with stars in Figures 1 and 2) since the prices of the two goods respond in the same way.

3.3 Cash-intensity and price response

We now discuss the role of cash-intensity in the price responses to the monetary policy shock. To fucus on the cash-intensity, we consider a case where the second goods differ from the first goods only in terms of the cash-intensity parameter θ_2. Suppose that $\theta_2 = 0$ as Lucas and Stokey (1987) and Hodrick, Kocherlakota and Lucas (1991), then equation (18) is reduced to

$$u_{2,t}\left(s, h^t\right) = p_{2,t}\left(h^t\right) u_{1,t}\left(0, h^t\right). \tag{20}$$

[10]The price level slightly overshoots after N periods, as pointed out in other inventory models. For details, see Grossman and Weiss (1988) and Alvarez, Atkson and Edmond (2009).

[11]See Edmond and Weill (2008) and Alvarez, Atkeson, and Edmond (2009) for the mechanism as to how this inventory model with heterogeneous agents yields price sluggishness and other responses of the macro variables to a monetary shock.

Using the resource constraints (9), the response of the relative price to a monetary policy shock is given by

$$\hat{p}_{2,t}\left(h^{t}\right) = -\hat{u}_{1,t}\left(0, h^{t}\right), \tag{21}$$

where the variable with hat is the percentage deviation of the variable around the non-stochastic steady state.

Clearly, $\hat{p}_{2,t}\left(h^{t}\right)$ decreases with the active households' marginal utility $\hat{u}_{1,t}\left(0, h^{t}\right)$. To see this, recall equation (13), where the nominal interest rate is related to the marginal utility of the active households. According to this relationship, other things being equal, a higher current nominal interest rate implies a higher marginal utility of the active households in the current period, lowering the relative price of the second goods.

An intuitive way to interpret this is to pay attention to a change in the value of a dollar in the brokerage accounts compared with that in the bank accounts. When the money supply in the asset market is reduced, a dollar in the brokerage account becomes more precious than that in the bank account, as shown by a rise in the nominal interest rate. Since the purchase of the second goods involves a payment from the brokerage account, the households demand less for the second goods than for the first goods. Consequently, the relative price of the second goods declines if $\theta_2 < 1$.

Figure 3 shows the CIR of the relative price of second goods to the contractionary monetary policy shock for various values of θ_2. If $\theta_2 = 1$, there is no heterogeneity across the two goods and the relative price does not change after the shock (the black line with circles). For $\theta_2 < 0$, where the second goods purchase requires a payment from the brokerage account, the price of the second goods declines relative to that of the first goods. The magnitude of the reduction of the relative price is larger if θ_2 is smaller, indicating that the price of goods with smaller cash-intensity declines by a greater amount.

3.4 Durability and price response

Next, we examine the role of durability in the relative price response. Assuming that the second goods differ from the first goods only in terms of durability, then equation (18) is reduced to

$$\frac{c_{1,t}\left(s, h^{t}\right)}{k_{2,t}\left(s, h^{t}\right)} = p_{2,t}\left(h^{t}\right) - \beta\left[1 - \delta_2\right] \sum_{h^{t+1}|h^{t}} \Pr(h^{t+1}|h^{t}) p_{2,t+1}\left(h^{t+1}\right) \frac{c_{1,t}\left(s, h^{t}\right)}{c_{1,t+1}\left(s+1, h^{t+1}\right)}. \tag{22}$$

As discussed in Bils and Klenow (1998), the consumption theory implies that to an income shock the expenditure flow of durables $c_{2,t}(s, h^t)$ is more cyclical than that of non-durables $c_{1,t}(s, h^t)$. This is because a households receive utility from the stock of durables, which is larger than the expenditure flow of durables.

A contractionary monetary shock reduces the real money balance, driving down the real consumption expenditure as indicated by equation (15). Since households try to reduce the stock of durables as much as the non-durable consumption, the demand for the durable expenditure shrinks by a greater amount than the non-durable consumption, leading to a decline in the relative price of durables.

Figure 4 displays the CIRs of the relative price of second goods to the contractionary monetary policy shock for various values of δ_2. If $\delta_2 = 1$, there is no heterogeneity across the two goods and the relative price does not change after the shock (the black line with circles). For $\delta < 1$, where the households receive utility from the stock of the second goods, the relative price declines in the monetary contraction. As the goods becomes more durable (lower δ_2), the above mechanism works more significantly, causing a larger decline in the relative price.

3.5 Luxuriousness and price response

Lastly, we consider whether the distinction between luxuries and necessities plays a role in the relative price responses. Similar to Bils and Klenow (1998), luxuriousness is controlled by the parameter σ_j in the utility function (2), where a larger σ_j implies the goods j are more luxurious. Considering the case where the second goods differ from the first goods only in terms of luxuriousness, (18) is expressed as

$$\hat{p}_{2,t}(h^t) = \hat{c}_{1,t}\left(s, h^t\right) - \frac{1}{\sigma_2}\hat{c}_{2,t}\left(s, h^t\right). \tag{23}$$

Suppose that $\sigma_2 > 1$ so that the second goods have greater luxuriousness relative to the first goods. If the relative price $\hat{p}_{2,t}(h^t)$ does not change after the shock, the households increase the expenditure for the second goods more than the first goods when they become richer.[12] The relative price adjusts to clear the goods market, reflecting the demand difference for the two goods, because our economy is an endowment economy.

[12] Similar to Bils and Klenow (1998), under our utility function, the household increases the consumption expenditure for both of the goods when their income increases.

After a contractionary monetary shock, the real money balance declines, causing a decline in the households' expenditure. The households reduce the expenditure for the two goods, and reduce the demand for the luxuries more than for the necessities.

Figure 5 displays the CIRs of the relative price of the second goods to the contractionary monetary policy shock for various values of σ_2. When $\sigma_2 = 1$, there is no heterogeneity across the two goods and the relative price does not change after the shock (the black line with circles). The relative price of second goods declines if $\sigma_2 > \sigma_1 = 1$. This is because the luxuriousness of the second goods is greater than the first goods. In contrast, the second goods price rises if the luxuriousness of the second good is smaller than the first goods ($\sigma_2 < \sigma_1 = 1$).

3.6 Sensitivity analysis

The baseline simulation results show that the values of the goods-specific parameters θ_2 and δ_2 are positively related to the CIRs of the relative price $p_{2,t}$, and σ_2 is negatively related to the CIR. To see the robustness for these implications, we study the simulation results under different settings for the three parameters: (1) the autoregressive parameter of the money growth rate ρ_M; (2) the frequency with which households rebalance their assets between their brokerage and bank accounts N; and (3) the paycheck parameter γ_j.

We calculate the CIRs of the relative price $p_{2,t}$ to the contractionary monetary policy shock for various combinations of these three parameters, and investigate whether the implications are changed. The results based on the parameters $\rho_M = \{0, 0.5, 0.9\}$, $N = \{2, 15, 38\}$ and $\gamma_j = \{0, 0.6\}$ are reported in the separate appendix.[13] To summarize, the qualitative relationship between the goods' characteristics and the relative price response to the monetary policy shock is unaffected by the use of alternative values for ρ_M, N, and γ_j.

It is notable, however, that the size of the parameters ρ_M, N, and γ_j have an impact on the size of the CIRs that is quantitatively nonnegligible. Other things being equal, the absolute values of CIRs become larger as the persistency parameter ρ_M increases. A higher value of ρ_M implies a larger reduction of money supply in the subsequent periods, which increases the degree of severity of the contractionary shock's effect. When N increases, the CIRs also take larger absolute values.

[13]We examine $N = 2$ because the model of Alvarez, Atkeson, and Edmond (2009) with $N = 2$ is the model of Grossman and Weiss (1983). Alvarez, Atkeson, and Edmond (2009) suggest either a set of two parameters $N = 15$ and $\gamma = 0$ or another set $N = 38$ and $\gamma = 0.6$ is consistent with the U.S. data.

A higher value of N implies a greater degree of friction in rebalancing the portfolio—as indicated in Alvarez, Atkeson, and Edmond (2009)—thus increasing the effect of a monetary policy shock on the economy. Similarly, a smaller value of γ_j implies the greater degree of friction, as less cash is returned to the bank accounts. As γ_j decreases, hence, the CIRs take a larger absolute value.

4 Empirical Analysis

In this section, we evaluate the model's implications using the U.S. data. To do this, we conduct a cross-sectional regression analysis, focusing on the relationship between the goods characteristics on the one hand, and the response of disaggregated prices to a monetary policy shock on the other. We first construct empirical values of parameters θ_j, δ_j, and σ_j based on Fisher (2003) and Bils and Klenow (1998). We then estimate empirical CIRs of relative prices to a contractionary monetary policy shock. To make empirical outcomes comparable to the model, we define the relative prices as the ratio of disaggregated PCE prices to an aggregate price index of non-durable goods and services. Using the estimated CIRs and the calculated values of the goods characteristic parameters, we run the following cross-sectional regression.

$$CIR_{\psi,j} = \alpha_\psi + \beta_\psi \times \text{characteristic}_j + \varepsilon_{\psi,j}, \tag{24}$$

where subscript ψ denotes the number of periods after the shocks over which the cumulative sum is calculated. "characteristic$_j$" is either θ_j, δ_j, or σ_j, α_ψ and β_ψ are coefficients to be estimated, and $\varepsilon_{\psi,j}$ is an error term. In evaluating the model's prediction we use the sign of the estimated coefficient $\hat{\beta}_\psi$.

4.1 Data

4.1.1 Goods characteristics

We construct the goods-specific cash-intensity (θ_j) according to the credit card usage data compiled by Fisher (2003). He calculates Visa and MasterCard credit card shares for 95 categories of merchants from 1992 to 2003. We construct a measure of goods-specific cash-intensity for PCE items, by mapping from the merchant category to the PCE item category.[14] We obtain cash-intensity measures for 84 PCE items, because of the incomparability of categories. Use of the credit card shares provides quantitative measures of the goods-specific cash-intensity. In fact, most of the items are paid for using both cash and credit, and large cross-sectional variations in cash-intensity appear in the summary statistics (Table 2).[15]

For the values of durability and luxuriousness, we follow Bils and Klenow (1998). They present the "expected life of service time" of 43 goods based on "Fixed Reproducible Tangible Wealth, 1925-89" released by the U.S. Bureau of Economic Analysis (BEA) and an interoffice memorandum of a major U.S. property casualty insurance company. We map from their goods' category to the PCE category, and then calculate the monthly depreciation rate of PCE items from the reported expected lives. In addition, we set full depreciation $\delta_j = 1$ for all of the PCE items that belong to foods, fuels and services. Consequently, we obtain the monthly measure of durability δ_j for 140 items of the PCE data. The median of δ_j is unity, because many of the PCE items are services that we assume are perfectly perishable (Table 2). Bils and Klenow (1998) also report the "Engel curve" of 43 goods based on the cross-sectional data for household spending reported in the Consumer Expenditure Survey released by the Bureau of Labor Statistics. Their "Engel curve" corresponds to the elasticity parameter σ_j of goods j in our model. We obtain the values of luxuriousness for 41 PCE items.

4.1.2 Impulse response function of disaggregated prices

We estimate two sets of the CIR functions to the contractionary monetary policy shock using two different identification schemes for robustness.[16] The first estimation employs a FAVAR model proposed in Boivin, Giannoni, and Mihov (2009). We use their dataset and closely follow their estimation methodology. The data period extends from January 1976 to June 2005.

The second estimation exploits the monetary policy shock series provided by Romer and Romer (2004), based on the narrative record of the Federal Open Market Committee. Here, we first

[14] The details of the calculation including a mapping methodology from a merchant category to a PCE item category are explained in Appendix A.

[15] The literature (e.g., Ogaki [1988], Kakkar and Ogaki [2002]) use a dummy variable approach, which is consistent with the cash-credit dichotomy assumption introduced by Lucas and Stokey (1987).

[16] For details of the dataset and the estimations, see Appendix A.

estimate the following equation:

$$\Delta p_{j,t} = c_0 + \sum_{l=1}^{24} b_l \Delta p_{j,t-l} + \sum_{m=1}^{48} c_m S_{t-m} + e_{j,t}, \tag{25}$$

where $\Delta p_{j,t}$ is the log-difference of the relative price of goods j, S_t is the measure of monetary policy shock provided in Romer and Romer (2004), c_0, b_l, and, c_m are the coefficients to be estimated, and $e_{j,t}$ is an error term. We then calculate changes in the price to a positive shock to S_t following the methodology employed by Romer and Romer (2004).

A CIR of the relative price is negative if the price of the item responds to a greater degree than the aggregate non-durable price index, since the shock is contractionary monetary shock. Table 2 shows the summary statistics of obtained $CIR_{\psi,j}$, where ψ are six- and 12-month. The mean and median of $CIR_{\psi,j}$ of various disaggregated relative prices are negative in the six-month cumulation periods, indicating the mean and median disaggregated prices responds more than the aggregate non-durable price index after a monetary contraction. The constructed $CIR_{\psi,j}$ shows some variations across items.

We expect that the coefficient β_ψ of θ_j is positive, because our model predicts that the prices of higher cash-intensity goods respond to a lesser degree than those of lower cash-intensity goods. Similarly, the coefficient of δ_j is expected to be positive, because durables have smaller δ_j. On the contrary, the model predicts that the coefficient of σ_j is negative, because luxuries have larger σ_j.

4.2 Empirical results

For each of the three goods' characteristics θ_j, δ_j, and σ_j, and for each of the two estimation methods of the CIRs, we run 12 univariate regressions for the different accumulation length $\psi = 1, ...12$. Figure 6 displays the regression coefficients of (24) using the CIRs of PCE items as dependent variables, and our values of cash-intensity, durability, or luxuriousness as independent variables. The horizontal axis of each panel indicates ψ, and we plot the estimated coefficients of the goods' characteristics (β_ψ) with one- and two-standard-error bands. The standard errors are calculated by a bootstrap method to avoid a potential bias in the analytical standard errors due to generated dependent variables. The bold lines in the middle display the estimated coefficient for each ψ, the dashed lines show one-standard-error band, and the outer dash-dotted lines show two-standard-error band.

The signs of estimated coefficients are consistent with the model's prediction, regardless of the goods characteristics and the methodology of estimating CIRs. The coefficients of θ_j are positive, those of δ_j are positive, and those of σ_j are negative. Namely, after a monetary contraction, the relative price of low cash-intensity goods (credit goods) tends to decline, that of durables tends to decrease, and that of luxuries tends to drop. Moreover, as shown in the standard error bands, the regressions significantly support the model's implications.

5 Conclusion

In this paper, we study implications of an inventory-theoretic money demand model for the disaggregated price responses of goods to a monetary policy shock. We develop a two-goods version of the model of Alvarez, Atkeson, and Edmond (2009). The two goods differ in terms of cash-intensity, durability, or luxuriousness. We then explore how these goods' characteristics affect the disaggregated price response of the goods to the shock. We find that, in a monetary contraction, the decline in the prices of low cash-intensity goods, durables, or luxuries outpaces that of high cash-intensity goods, non-durables, or necessities.

Using the U.S. PCE data, we evaluate the validity of the model's implications. We construct the goods-specific cash-intensity based on the credit usage data (Fisher [2003]), and the goods-specific durability and luxuriousness based on Bils and Klenow (1998). We then study the empirical relationships between these goods characteristics and the disaggregated price response to the monetary policy shock. We find that the model's predictions are consistent with the estimation results.

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A Description of the data

A.1 Disaggregated price data and the CIR estimations

The disaggregated price data used in the current paper are seasonally adjusted monthly price series of the PCE indices (base year 2000 = 100), released by the BEA. To obtain the relative prices of these items, we construct the aggregate non-durable price index from the non-durable goods series and the service series by a Tornqvist approximation (see Whelan [2000]), and divide each of the disaggregated price indices by the constructed index.

For the estimation using FAVAR, the sample period is from January 1976 to June 2005. The estimation using the narrative record uses sample from January 1976 to December 1996 because of the limitation of the narrative record data.[17]

The category for food in our measure of cash-intensity, durability, and luxuriousness is broader than the PCE item category. To map between the two categories, we aggregate 18 PCE items of "food sold in stores" to "food" using the Tornqvist approximation.[18]

A.2 Credit card usage data

Our measure of cash-intensity is based on the share of credit card usage over total sales calculated by Fisher (2003). He compiled Visa and MasterCard shares of total sales for approximately 95 categories of merchants from 1992 to 2003, based on the Visa Payment System Panel Study, Visa Transaction Database, and the Nilson Report.[19] We use Visa and MasterCard credit shares of total sales from his tables.[20] The share of each card for each category of merchants is calculated

[17]Romer and Romer (2004) use a data series that is not seasonally adjusted in their estimation, and control the seasonal effects by including dummy variables. We use the seasonally adjusted series of the dependent variables to maintain the consistency with the analysis based on the FAVAR.

[18]Items are cereals; bakery products; beef & veal; pork; other meats; poultry; fish & seafood; eggs; fresh milk & cream; processed dairy products; fresh fruits; fresh vegetables; processed fruits and vegetables; juices & nonalcoholic drinks; coffee, tea & beverage materials; fats & oils; sugar & sweets; and other foods.

[19]His data is prepared for the allocation plan of the settlement funds among a class of plaintiffs in *In re Visa Check/MasterMoney Antitrust Litigation, No. 96-CV-5238, 297 F. Supp. 2d 503, 2003 U.S. Dist., LEXIS 22898 (E.D.N.Y. Dec. 19, 2003).* The class action, which consisted of approximately five million merchants, including Wal-Mart and other major retailers, restaurants, and service agencies, alleged that Visa and MasterCard illegally tied their debit products to their credit cards in violation of antitrust laws. The settlement plan, which was approved in 2003, required the card companies to pay the class more than US$3 billion (see, e.g., Constantine, Shinder, and Coughlin [2005], Green [2005], and Fisher [2008] for details). The approved plan called for allocating money from the settlement funds to the class members proportional to the merchants' debit and credit purchase volume. To achieve a fair allocation, Fisher (2003) calculates the shares of transactions through credit and debit cards for each category of merchants around the periods, together with other values used for estimating total damages of the class.

[20]Exhibit FAD-10C and FAD-10D in Fisher (2003).

by the arithmetic average over time. To maximize the number of observations, we include all the periods in his table (excluding years when the data are not available). Cash-intensity of a category of merchants is calculated by one minus the sum of the shares of Visa and MasterCard.[21] We then calculate an item's cash-intensity by mapping from the merchant category to the PCE item category. The correspondence is, for example, a merchant category "drug store" being mapped to "prescription drugs" and "medical supplies" in the PCE data, and "gasoline station" in the merchant list to "gasoline & other motor fuel" in the PCE data. The complete information about the mapping is available upon request.

We drop automobiles from the sample of the cash-intensity regressions. While Fisher (2003) includes the merchant category "auto dealership," the calculated sum of the credit card shares is less than 5%. In contrast, the literature agrees that automobiles are paid by private borrowings, not by cash (e.g., G.20, "Finance Companies" in the Federal Reserve Release).[22]

A.3 Durability and luxuriousness

Bils and Klenow (1998) consider the reciprocal to the expected life of service time as annual depreciation rate (δ_{annual}). The corresponding monthly value, δ, is given by $\delta = 1-(1-\delta_{\text{annual}})^{1/12}$. Our estimation result is little changed by the usage of the annual depreciation rate instead of the monthly depreciation rate. For luxuriousness, they report two sets of Engel curve parameters. We use the first set, but the results are little changed by the use of the second set.

B Derivation of the key equations of the model

B.1 Optimality conditions

First, the specified form of the utility function and non-durability of $c_{1,t}(s,h^t)$ imply that non-negativity of $c_{1,t}(s,h^t)$ is always satisfied. The non-negativity of $c_{j,t}(s,h^t)$ and $Z_t(s,h^t)$ ensure the non-negativity of $M_t(s,h^t)$ by equation (4). The optimality conditions for a type s household are

[21] We cross-check this cash-intensity of a category of merchants by comparing with the data used by Ching and Hayashi (2010). See the separate appendix for details.

[22] http://www.federalreserve.gov/releases/g20/current/g20.htm

following first order conditions (for all t and h^t):

$$\forall s \ c_{1,t}(s,h^t) : \beta^t \Pr(h^t) u_{1,t}(s,h^t) = P_{1,t}(h^t)\eta_t(s,h^t) \tag{B1}$$

$$\forall s \ c_{2,t}(s,h^t) : \theta_2 P_{2,t}(h^t)\eta_t(s,h^t) + [1-\theta_2]P_{2,t}(h^t)\lambda_t(s,h^t) = \lambda_t^K(s,h^t) + \lambda_{2,t}(s,h^t) \tag{B2}$$

$$\forall s \ k_{2,t}(s,h^t) : \beta^t \Pr(h^t) u_{2,t}(s,h^t) + [1-\delta_2]\lambda_{t+1}^K(s+1,h^{t+1}) = \lambda_t^K(s,h^t) \tag{B3}$$

$$\forall s \ Z_t(s,h^t) : \eta_t(s,h^t) = \eta_{t+1}(s+1,h^{t+1}) + \lambda_t^Z(s,h^t) \tag{B4}$$

$$\forall s \ A_t(s,h^t) : \lambda_t(s,h^t) = \lambda_{t+1}(s+1,h^{t+1}) + \lambda_t^A(s,h^t), \tag{B5}$$

$$x_t(h^t) : \eta_t(0,h^t) = \lambda_t(0,h^t) \tag{B6}$$

$$\forall (s,h^{t+1}) \ B_t(s,h^{t+1}) : q_{t+1}^t(h^{t+1})\lambda_t(s,h^t) = \lambda_{t+1}(s+1,h^{t+1}) \tag{B7}$$

and the Transversality condition ($\lim_{T\to\infty} \lambda_T B_T = 0$). Here, $\eta_t(s,h^t)$, $\lambda_t(s,h^t)$ and $\lambda_t^K(s,h^t)$ are the Lagrange multipliers on the type s household's bank account constraint (4), the brokerage account constraint (6), and the law of motion for the stock of second goods (3), respectively. The Lagrange multipliers $\lambda_t^Z(s,h^t)$, $\lambda_t^A(s,h^t)$, and $\lambda_{2,t}(s,h^t)$ are associated with non-negativity constraints for money carried over in the bank account $Z_t(s,h^t)$, the money holdings in the brokerage account $A_t(s,h^t)$, and the expenditure of the second goods $c_{2,t}(s,h^t)$, respectively. We denote marginal utility of the type s household with respect to the goods j consumption by $u_{j,t}(s,h^t)$.

B.2 Derivation of the key equations

The derivation of the interest rate expression closely follows Alvarez et al. (2009). The Lagrange multiplier associated with asset $\lambda_t(s,h^t)$ is recursively calculated as:

$$\begin{aligned}
\lambda_t(S(t,s_0),h^t) &= q_t^{t-1}(h^t)\lambda_{t-1}(s-1,h^{t-1}) \\
&= q_t^{t-1}(h^t)q_{t-1}^{t-2}(h^{t-1})\lambda_{t-2}(s-2,h^{t-2}) \\
&= ... \\
&= \lambda_0(s_0,h^0)\prod_{k=0}^{t-1} q_{k+1}^k(h^{k+1}).
\end{aligned} \tag{B8}$$

For households whose type are $S(t, s_0) = 0$, by equations (B1) and (B6):

$$\lambda_0(s_0, h^0) \prod_{k=0}^{t-1} q_{k+1}^k(h^{k+1}) = \lambda_t(0, h^t) = \beta^t \Pr(h^t) u_{1,t}(0, h^t) \frac{1}{P_{1,t}(h^t)}. \tag{B9}$$

By combining a similar equation for $s_0 + 1$, we have

$$\begin{aligned} q_{t+1}^t(h^{t+1}) &= \lambda_{t+1}(s+1, h^{t+1})/\lambda_t(s, h^t) \\ &= \frac{\lambda_0(s_0, h^0)}{\lambda_0(s_0+1, h^0)} \beta \Pr(h^{t+1}|h^t t) \frac{u_{1,t+1}(0, h^{t+1})}{u_{1,t}(0, h^t)} \frac{P_{1,t}(h^t)}{P_{1,t+1}(h^{t+1})} \end{aligned} \tag{B10}$$

Following Alvarez et al. (2009), we examine equilibria in which the initial distribution of assets are such that $\lambda_0(s_0, h^0) = \lambda_0(s_0 + 1, h^0)$. This assumption implies

$$\lambda_t(s, h^t) = \lambda_t(0, h^t). \tag{B11}$$

Hence we have

$$q_{t+1}^t(h^{t+1}) = \beta \Pr(h^{t+1}|h^t) \frac{u_{1,t+1}(0, h^{t+1})}{u_{1,t}(0, h^t)} \frac{P_{1,t}(h^t)}{P_{1,t+1}(h^{t+1})} \tag{B12}$$

Substituting the obtained asset price into equation (12) leads to equation (13):

$$\frac{1}{1+i_t} = \sum_{h^{t+1}|h^t} \beta^t \Pr(h^{t+1}|h^t) \frac{u_{1,t+1}(0, h^{t+1})}{u_{1,t}(0, h^t)} \frac{P_{1,t}(h^t)}{P_{1,t+1}(h^{t+1})}.$$

By combining equations (B1) and (B4), an intertemporal Euler equation is obtained as

$$\frac{u_{1,t}(s, h^t)}{P_{1,t}(h^t)} = \beta \sum_{h^{t+1}|h^t} \Pr(h^{t+1}|h^t) \frac{u_{1,t+1}(s+1, h^{t+1})}{P_{1,t+1}(h^{t+1})} + \lambda_t^Z(s, h^t). \tag{B13}$$

When $Z_t(s, h^t) > 0$, the last term drops.

Finally, by combining equations (B1), (B2), (B3) and (B11), the relative price is expressed in the marginal utility of consumptions, which is equation (18):

$$
\begin{aligned}
p_{2,t}\left(h^t\right) &= \frac{u_{2,t}(s,h^t)+\lambda_{2,t}(s,h^t)}{\theta_2 u_{1,t}\left(s,h^t\right)+(1-\theta_2)u_{1,t}\left(0,h^t\right)} \\
&+ \beta(1-\delta_2)\sum_{h^{t+1}|h^t}\Pr(h^{t+1}|h^t)\Bigg\{\frac{p_{2,t+1}(h^{t+1})(\theta_2 u_{1,t+1}(s+1,h^{t+1})+(1-\theta_2)u_{1,t+1}(0,h^{t+1}))}{\theta_2 u_{1,t}\left(s,h^t\right)+(1-\theta_2)u_{1,t}\left(0,h^t\right)} \\
&\qquad - \frac{\lambda_{2,t+1}(s+1,h^{t+1})}{\theta_2 u_{1,t}\left(s,h^t\right)+(1-\theta_2)u_{1,t}\left(0,h^t\right)}\Bigg\}. \qquad \text{(B14)}
\end{aligned}
$$

Table 1: Parameter values used in the simulation

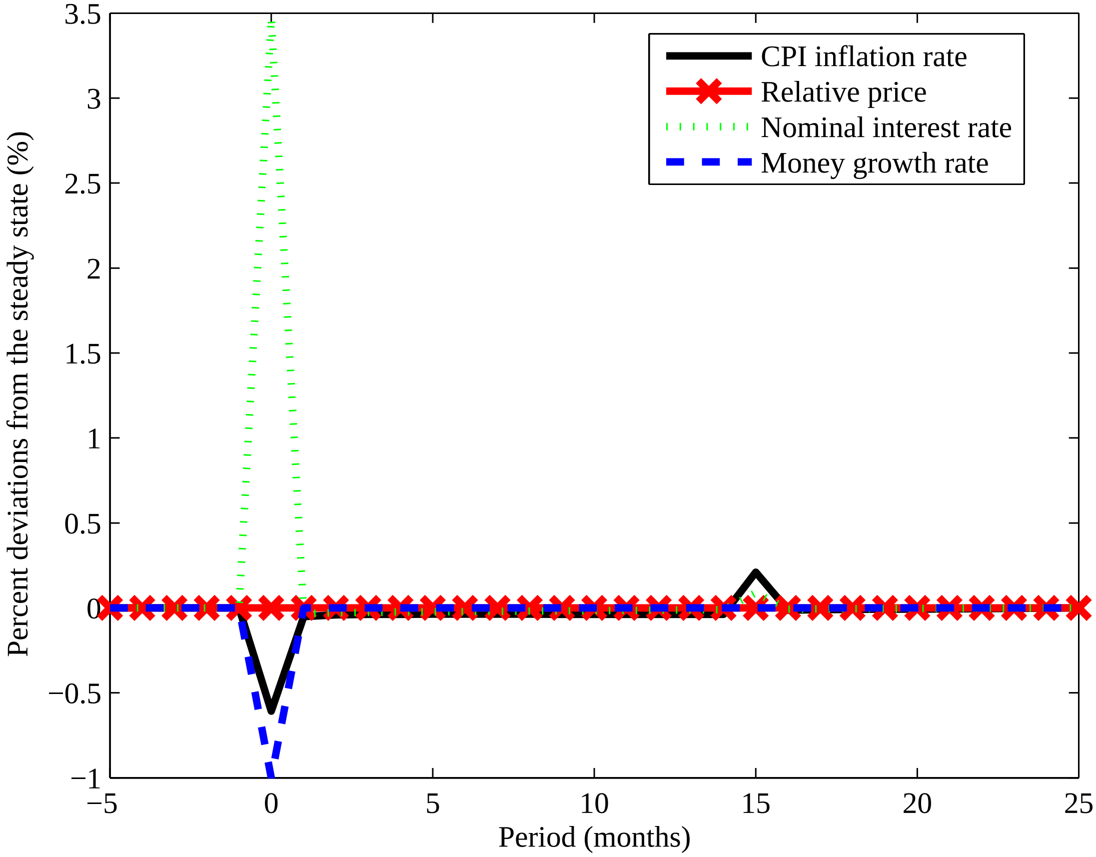
Parameter	Values	Description
N	15	Frequency of asset rebalance
β	$0.99^{(1/12)}$	Discount factor
$\bar{\mu}$	$1.01^{(1/12)}$	Steady-state money growth rate
ρ_M	0	Autocorrelation of money growth shock
γ_1	0.6	Ratio of first goods cash sales sent to the bank account
γ_2	0.6	Ratio of second goods cash sales sent to the bank account
α_1	2/3	Utility share of first goods
α_2	1/3	Utility share of second goods
θ_1	1.0	Cash-intensity of first goods
θ_2	Varies	Cash-intensity of second goods
δ_1	1.0	Depreciation rate of first goods
δ_2	Varies	Depreciation rate of second goods
σ_1	1.0	Utility elasticity parameter of first goods
σ_2	Varies	Utility elasticity parameter of second goods

We simulate the economy with different values of θ_2, δ_2 and σ_2 in Figures 1–5.

Table 2: Explanations of empirical variables and descriptive statistics

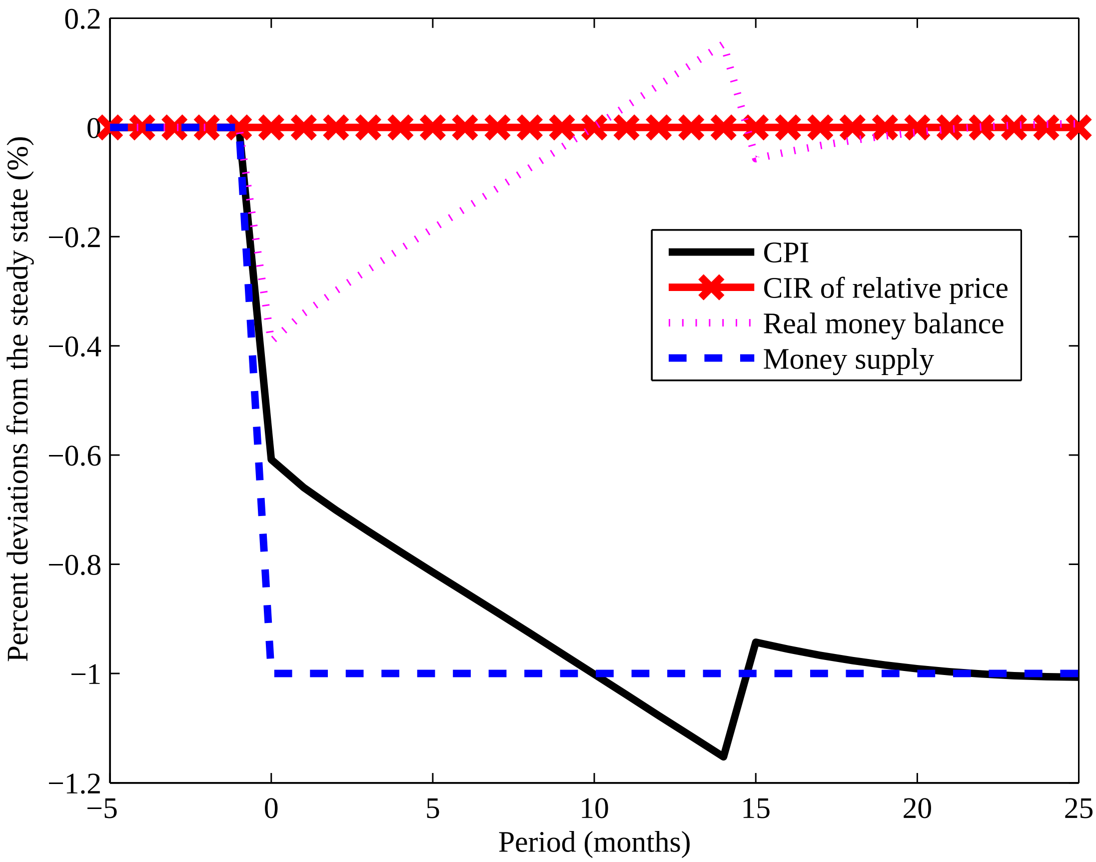
Variable	Obs.	Mean	S.D.	Median	Min	Max	Description
FAVAR6	173	−0.023	0.196	−0.0041	−0.981	0.601	CIR of relative price after a negative monetary shock, six-month cumulative, estimated by FAVAR
FAVAR12	173	0.025	0.582	0.077	−2.45	1.91	CIR of relative price after a negative monetary shock, 12-month cumulative, estimated by FAVAR
Narr.6	173	−0.0004	0.028	−0.0024	−0.136	0.164	CIR of relative price after a negative monetary shock, six-month cumulative, estimated by narrative records shocks
Narr.12	173	0.01	0.10	0.0078	−0.75	0.50	CIR of relative price after a negative monetary shock, 12-month cumulative, estimated by narrative records shocks
θ	84	0.81	0.13	0.82	0.51	0.995	Cash-intensity, calculated based on the credit card shares based on Fisher (2003)
δ	140	0.79	0.41	1	0.0049	1	Depreciation rate (Bils and Klenow [1998]), monthly values
σ	41	1.34	0.39	1.42	0.52	2.21	Curvature of the Engel curve (Bils and Klenow [1998])

Figure 1: Impulse response functions to a monetary shock (1) (Two goods are identical)



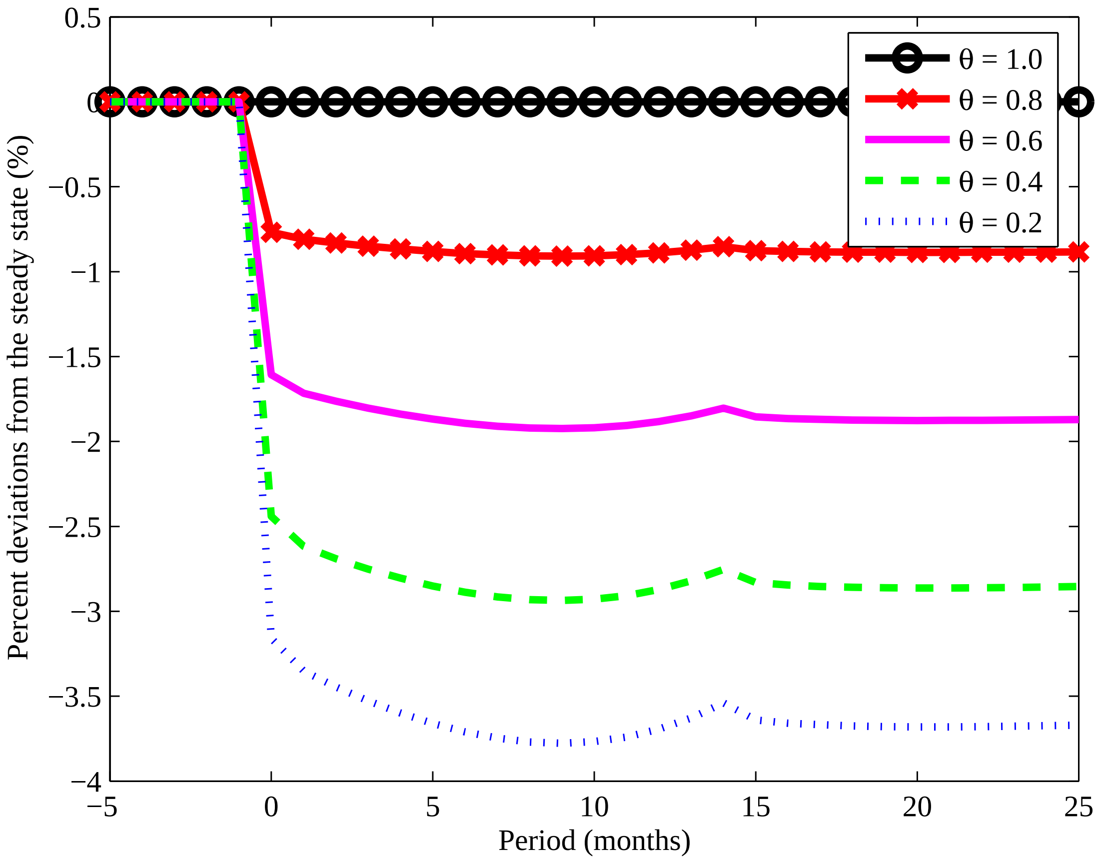
The lines are impulse response functions to 1 %, one-time negative money growth rate shock. The parameters of the second goods are: $\theta_2 = \delta_2 = \sigma_2 = 1$. The values of other parameters are shown in Table 1.

Figure 2: Impulse response functions to a monetary shock (2) (Two goods are identical)



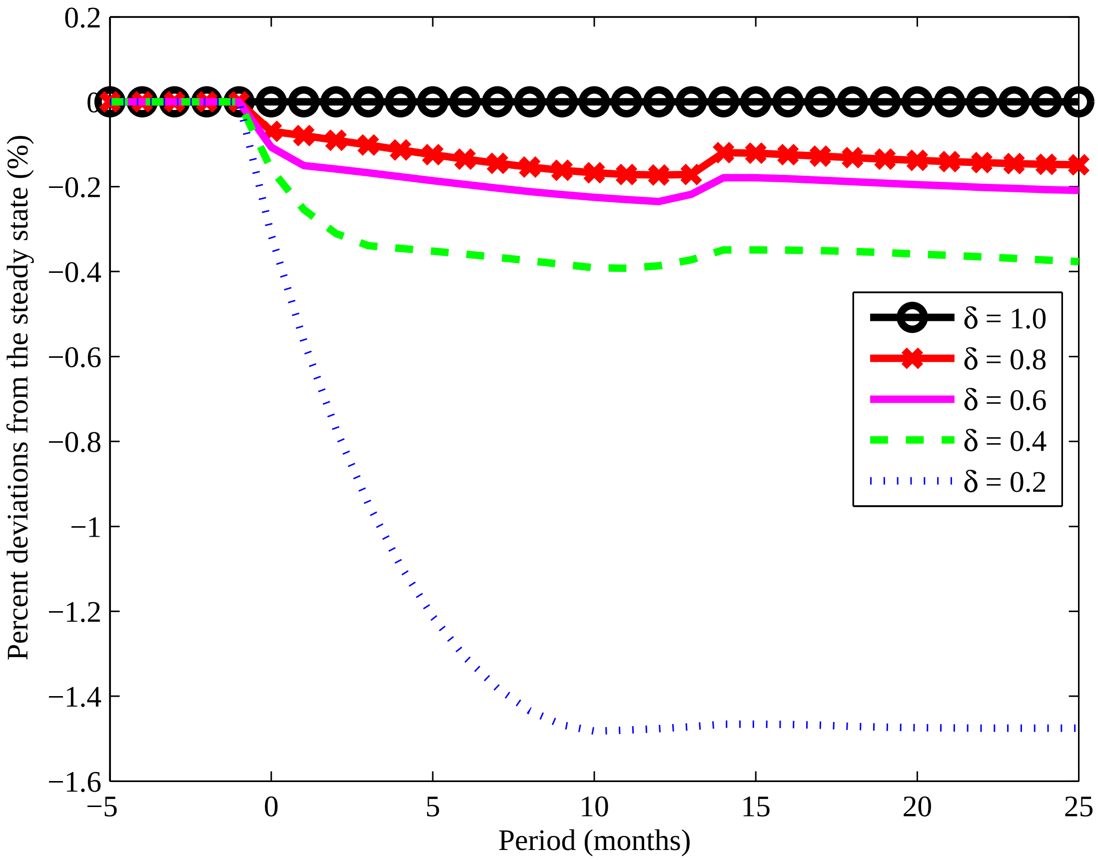
The lines are impulse response functions to 1%, one-time negative money growth rate shock. The parameters of the second goods are: $\theta_2 = \delta_2 = \sigma_2 = 1$. The values of other parameters are shown in Table 1.

Figure 3: Cumulative impulse response functions of the relative price to a monetary shock (Effect of cash-intensity)



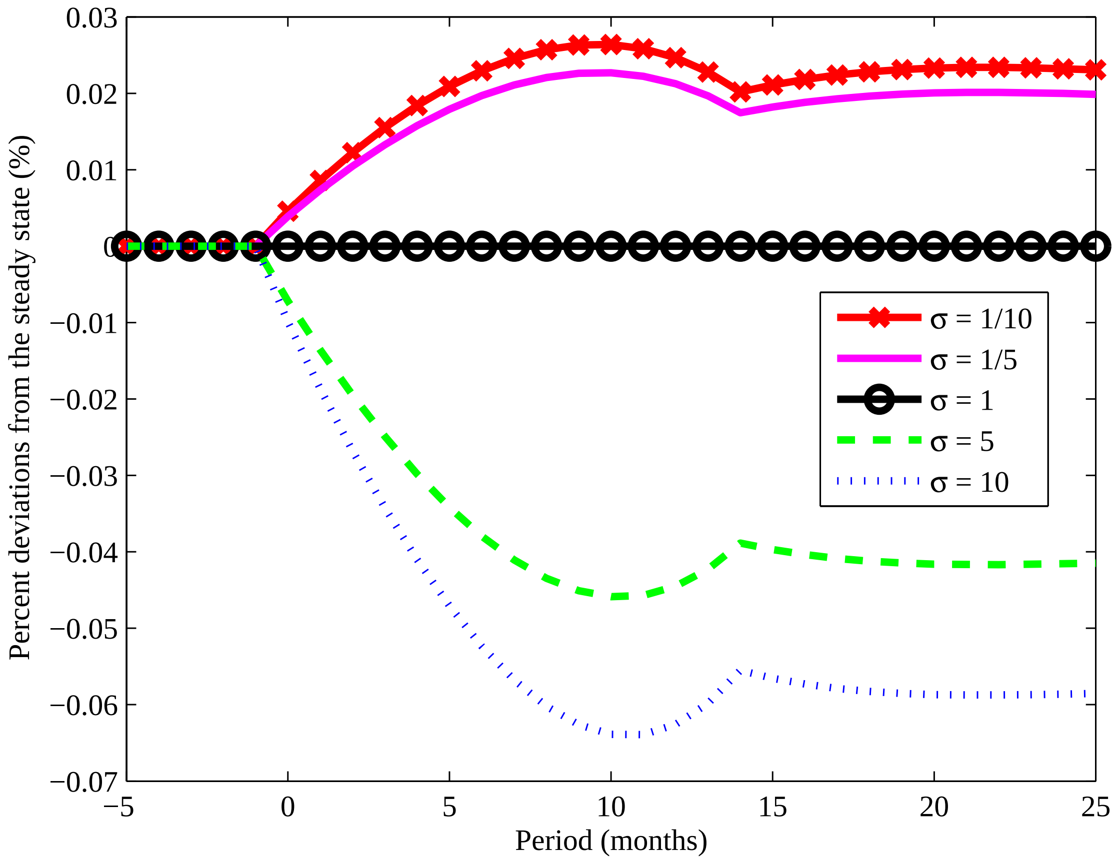
The lines are the cumulative impulse response (CIR) of the relative price (the price of the second goods to the price of the first goods) to 1%, one-time negative money growth rate shock. Each line shows CIR calculated under the indicated value of θ_2 (cash-intensity of the second goods). $\delta_2 = \sigma_2 = 1$. The values of other parameters are shown in Table 1.

Figure 4: Cumulative impulse response functions of the relative price to a monetary shock (Effect of durability)



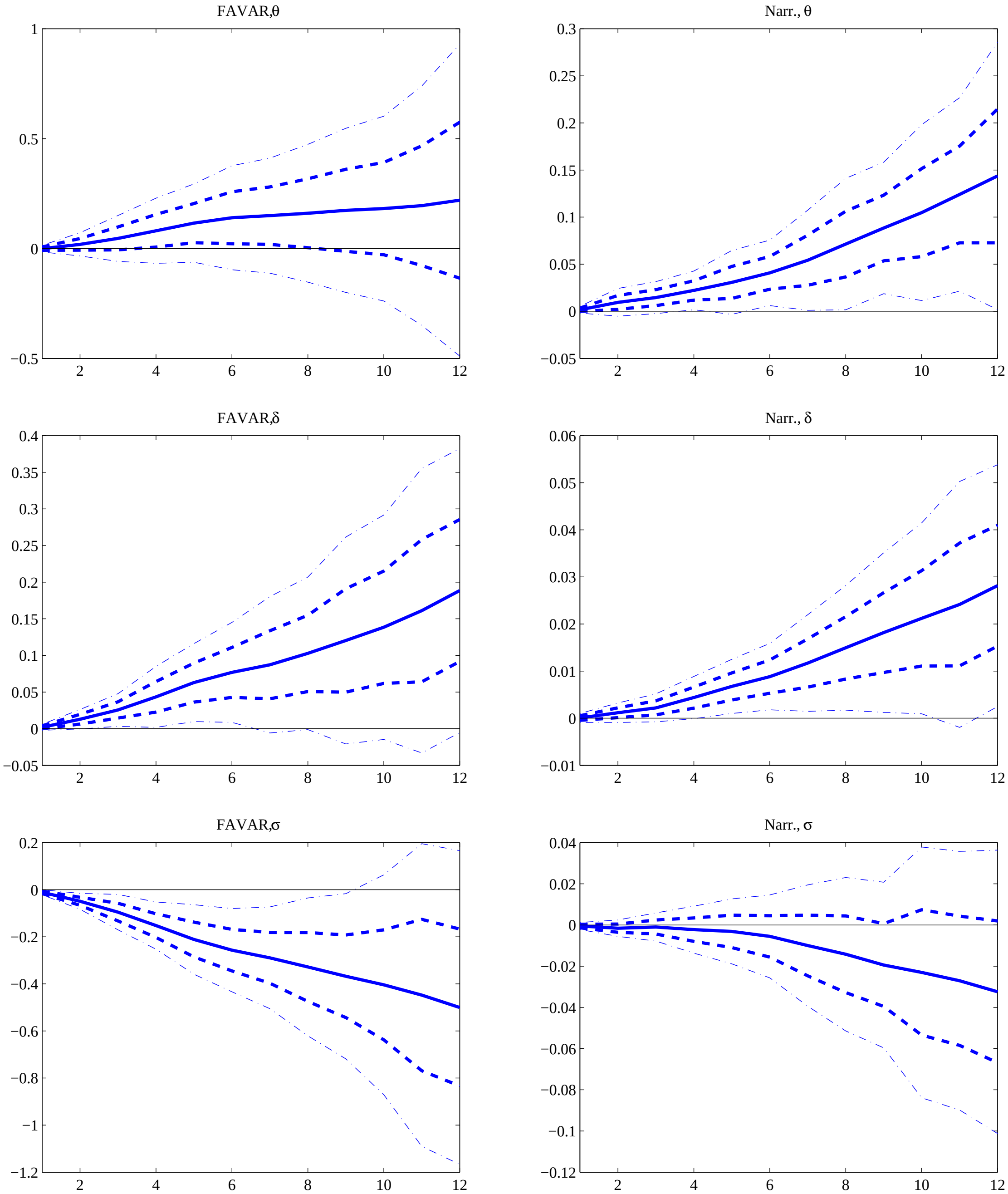
The lines are the cumulative impulse response (CIR) of the relative price (the price of the second goods to the price of the first goods) to 1%, one-time negative money growth rate shock. Each line shows CIR calculated under the indicated value of δ_2 (the depreciation rate of the second goods). $\theta_2 = \sigma_2 = 1$. The values of other parameters are shown in Table 1.

Figure 5: Cumulative impulse response functions of the relative price to a monetary shock (Effect of luxuriousness)



The lines are the cumulative impulse response (CIR) of the relative price (the price of the second goods to the price of the first goods) to 1%, one-time negative money growth rate shock. Each line shows CIR calculated under the indicated value of σ_2 (luxuriousness of the second goods). $\theta_2 = \delta_2 = 1$. The values of other parameters are shown in Table 1.

Figure 6: Regression coefficients of univariate regressions



These panels display regression coefficients and their standard error ranges. The center lines show estimated coefficients. The inner dashed lines show the range of one standard errors, and the outer dashed lines show the range of two standard errors. The dependent variables of the regressions are CIRs of relative prices (the price of the aggregate non-durables plus services to the price of the interested items) after 1%, one-time negative monetary shock. The CIRs are estimated by FAVAR (the left panels) and a narrative approach (the right panels). The independent variables are cash-intensity (the upper panels), depreciation rate (the middle panels), and the curvature of the Engel curve (the bottom panels). The horizontal axis of each panel shows cumulation periods (months) of CIRs.