The Real Effects of Inflation in Continuous versus Discrete Time Sticky Price Models

Wai-Yip Alex Ho*
Boston University

James Yetman†
The University of Hong Kong

15th March 2006

Abstract

We demonstrate the important implications of the assumption of discrete time in many sticky price models of the macroeconomy. For a given level of menu costs, discrete time models imply longer average contract length but smaller real effects of both trend inflation and monetary shocks than continuous time models. It is also feasible for a firm to enjoy full price flexibility in discrete time, while this would require paying infinite menu costs in continuous time, a distinction that is most important at high levels of trend inflation.

Keywords: Continuous time; Discrete time; Sticky price models

JEL Classification: E31

*Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215, USA; email hwyalex@bu.edu.

†Corresponding Author. School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong; Phone (852) 2857-8506; fax (852) 2548-1152; email jyetman@econ.hku.hk. I thank the Hong Kong Research Grants Council for financial support (HKU7240/04H).

1 Introduction

Many New Keynesian models incorporate time-dependent sticky prices. Most such models are based on discrete time (for example, Woodford (2003), Chapter 3), and effectively impose an arbitrary minimum length of time over which prices must be fixed. With many authors defining a period as corresponding to one quarter, this implies that complete price flexibility corresponds with prices being updated once a quarter. Removing this restriction results in a continuous time model, as in Calvo (1983). Here we compare the real effects of trend inflation and monetary shocks in discrete and continuous time versions of a simple model of the economy. We assume that nominal prices are sticky due to menu costs, but that firms optimally choose their average contract length, as argued in Ball, Mankiw and Romer (1988), Devereux and Yetman (2002), and Romer (1990). We find that for given menu costs continuous time implies shorter average contract lengths, and larger real effects of both trend inflation and monetary shocks, than discrete time unless inflation is very low. Further, while discrete time models result in complete price flexibility above some finite level of trend inflation, price flexibility only obtains asymmptotically in continuous time models. And if changing prices requires labor input, continuous time models imply large welfare costs of high rates of inflation, consistent with common perception, while discrete time models do not.

2 The model

We now develop a simple model to illustrate the important differences between discrete time and continuous time sticky price models. Suppose aggregate consumption is given by the quantity equation, expressed in logs as

$$c(t) = m(t) - p(t), \tag{1}$$

where $c(t)$, $m(t)$ and $p(t)$ are aggregate consumption, money supply and general price level respectively. Following Calvo (1983), prices are sticky, and firms reset their price with a constant probability that is independent of the length of time for which their price has already been fixed. As in Devereux and Yetman (2002), this probability is chosen optimally by the firm, subject to menu costs.

Suppose that the marginal cost of production is the money supply. Then, up to a constant, the desired price of profit maximizing firms is given by

$$p^*(t) = m(t). \tag{2}$$

Note that these assumptions are consistent with a utility function given by $U(C, N) = ln(C) - N$, with linear production technology ($C = Y = N$).

The loss function of a representative firm that is resetting its price in period t may be approximated by the squared deviation of the log price from the desired log price (see Walsh (1998)). In discrete time, this implies

$$L_D(t) = F + E_t\left[\sum_{j=0}^{\infty}(\beta_D\kappa)^j(\hat{p}_D(t) - p_D^*(t+j))^2 + \frac{1-\kappa}{\kappa}\sum_{j=1}^{\infty}(\beta_D\kappa)^j L_D(t+j)\right], \tag{3}$$

where F, $\beta_D \in [0,1]$, $\hat{p}_D(t)$ and $\kappa \in [0,1]$ are the menu cost, the discount rate, the price set by the firm in period t, and the probability that the firm will not change its price each period respectively. That is, each time a firm adjusts its price it must pay a fixed cost F. The probability that this price applies j periods later is given by κ^j. The cost to the firm of its price deviating from the desired price is approximated by the square of the difference between the two. And finally, there is a probability $(1-\kappa)\kappa^{j-1}$ that the firm will next adjust its price after j periods.

Following Calvo (1983), the loss function in continuous time is given by

$$L_C(t) = F + E_t\left[\int_0^{\infty} \exp(-(\beta_C + \delta)j)\Big((\hat{p}_C(t) - p_C^*(t+j))^2 dj + jL_C(t+j)\Big)dj\right], \tag{4}$$

where $\delta \exp(-\delta j)$ is the probability density function of the firm adjusting its price after j units of time, and $\beta_C \equiv \frac{1-\beta_D}{\beta_D}$ is the discount rate.

To close the model, the money supply process in the discrete time case is given by

$$m_D(t) = m_D(t-1) + \mu + u_D(t), \tag{5}$$

where $u_D(t) \sim N(0, \sigma^2)$ is an i.i.d. shock, while it's continuous time counterpart is

$$m_C(t) = \mu t + u_C(t), \tag{6}$$

where $u_C(t)$ is a standard Brownian motion with variance $t\sigma^2$.

Taking first order conditions of (3) and (4), the optimal nominal prices set by adjusting firms in discrete and continuous time are given by

$$\hat{p}_D(t) = (1 - \beta_D\kappa)E_t\left[\sum_{j=0}^{\infty}(\beta_D\kappa)^j p^*(t+j)\right] \tag{7}$$

and

$$\hat{p}_C(t) = (\beta_C + \delta)E_t\left[\int_0^{\infty} \exp(-(\beta_C + \delta)j)p^*(t+j)dj\right]. \tag{8}$$

To close the model, aggregate prices are given by

$$p_D(t) = (1-\kappa)\hat{p}_D(t) + \kappa p_D(t-1) \tag{9}$$

and

$$p_C(t) = \delta\left[\int_0^{\infty} \exp(-\delta j)\hat{p}_C(t-j)dj\right] \tag{10}$$

respectively.

Substituting (7) into (3) and (8) into (4) and taking unconditional expectations (that is, treating L_D and L_C as stationary), we have

$$L_D = (1-\beta_D\kappa)F + \mu^2\frac{\beta_D\kappa}{(1-\beta_D\kappa)^2} + \sigma^2\frac{\beta_D\kappa}{1-\beta_D\kappa} \tag{11}$$

and

$$L_C = \frac{\beta_C + \delta}{\beta_C} F + \mu^2 \frac{1}{\beta_C(\beta_C + \delta)^2} + \sigma^2 \frac{1}{\beta_C(\beta_C + \delta)}. \tag{12}$$

Firms are assumed to choose the average contract length (which is a function of κ and δ) optimally- that is, to minimize (11) and (12).[1]

3 Results and Discussion

We treat our model as quarterly, and calibrate as follows: $\beta_D = 0.985$, $\sigma = 0.01$, and menu costs are such that if annual inflation is 4%, firms change their prices once a year on average in the discrete time version of the model ($\kappa = 0.75$), implying $F = 0.0122$.

Figure 1 plots the consumption-inflation trade-off in our model. We can see here that as inflation increases from zero, consumption first rises, but then falls. With discrete time, as inflation rises beyond 43.9% (annualized), all firms adjust prices every period, and so consumption returns to the same level as at zero trend inflation. In contrast, with continuous time, full price flexibility only obtains asymptotically. Thus trend inflation continues to have real effects even at high levels of trend inflation.

Figure 2 provides intuition for this result, plotting the average contract length.[2] In discrete time, a firm that adjusts its price once every period has a price that is always exactly equal to its desired price, and effectively enjoys full price flexibility. Thus as inflation increases, firms decrease their average contract length to a minimum of one. In contrast, in continuous time, no matter how often a firm adjusts its price with non-zero inflation, it can only be equal to the firm's desired price for an instant. Since every price adjustment requires paying a menu cost, an increase in trend inflation will induce the firm to shorten its average contract length, but

[1] We determine the optimal κ and δ numerically.

[2] The average contract lengths are given by $\sum_{j=1}^{\infty} jk^{j-1} = 1/(1-\kappa)^2$ and $\int_0^{\infty} j \exp(-\delta j) dj = 1/\delta^2$ for discrete time and continuous time models respectively.

never to the point of full price flexibility.

Figures 3 plots the welfare-inflation trade-off. As inflation increases from zero, welfare initially declines as prices fall below efficient levels due to the discounting by firms of future profits. But as inflation rises still further, the average contract length shortens, reducing the degree to which average prices undercut efficient levels. But note the scale of the vertical axis: in percent terms, welfare varies little with trend inflation.

Until now, our analysis has assumed that changing price implies a reduction in firm profits, but does not require labor input. Suppose instead that menu costs represent labor costs that firms must pay in order to adjust prices.[3] Figure 4 plots the welfare-inflation trade-off under this assumption, and shows that welfare is decreasing in trend inflation. As inflation increases, firms hire additional labor to change prices, implying increased disutility from work effort for consumers. With discrete time, this process continues only up until the point that firms change their prices every period; thereafter further increases in inflation have no welfare implications. But with continuous time firms will always change their prices more frequently as inflation increases, and so welfare declines monotonically with trend inflation. In this form, the continuous time model can illustrate the potential large real costs of high levels of inflation that are commonly believed to exist, while the discrete time model cannot.

As an additional comparison, we return to our original model and consider the effects of shocks. Figures 5 and 6 plot the impulse response of a one standard deviation shock to monetary policy in both discrete and continuous time for 1% trend inflation and 20% trend inflation respectively.[4] At low levels of trend inflation, there is little difference between the two sets of impulse responses. But as trend inflation increases, the immediate inflationary effects of the shock become relatively more pronounced with discrete time than with continuous time, and

[3] More precisely, $N = C + \Omega$ where $\Omega_D = (1-\kappa)F$ and $\Omega_C = \delta F$.

[4] Constructing the impulse response in discrete time follows intuitively from equations (5), (7), and (9). See the appendix for details of how the impulse response is constructed in continuous time.

the output effects of the shock less in terms of both impact and persistence. Thus relative to the continuous time model, the discrete time model exaggerates the nominal effects but understates the real effects of monetary shocks. Again, the difference between the two is increasing in trend inflation.

4 Conclusions

We have demonstrated the important implications of the assumption of discrete time, as opposed to continuous time, in many sticky price models of the macroeconomy. For a given level of menu costs, discrete time models imply longer average contract length but smaller real effects of both trend inflation and monetary shocks than continuous time models. It is also feasible for a firm to enjoy full price flexibility in discrete time, while this would require paying infinite menu costs in continuous time, a distinction that is most important at high levels of trend inflation.

Appendix

We now outline the construction of the impulse responses in the continuous time version of our model. Consider first the case without shocks. Then from equations (6), (8), and (10)

$$\hat{p}_C(t) = \mu[t + \frac{1}{\beta_C + \delta}], \tag{13}$$

$$p_C(t) = \mu[t - \frac{\beta_C}{\delta(\beta_C + \delta)}]. \tag{14}$$

In discrete time, a period is condensed into a single point in time, and so it makes sense to think of a shock occurring at a point in time. In continuous time, a shock that occurs at a single point in time would imply infinite variance at that point (see equation (6)). Thus we model the shock in continuous time as being revealed uniformly over one period. Suppose that there are no money shocks before time 0; there is then an unanticipated shock of one standard deviation that is realized uniformly between time 0 and time 1; and there are no further shocks after time 1. Then (from (14)) the price level at time 0 is given by

$$p_C(0) = \mu[-\frac{\beta_C}{\delta(\beta_C + \delta)}]. \tag{15}$$

Between time 0 and time 1, price setting firms have observed the part of the shock that has been realized already, and condition their price setting decision on this information. The price they set for $0 < t \leq 1$ is given by

$$\hat{p}_C(t) = \mu[t + \frac{1}{\beta_C + \delta}] + \sigma t. \tag{16}$$

For $t > 1$, they have observed the full shock, so set a price of

$$\hat{p}_C(t) = \mu[t + \frac{1}{\beta_C + \delta}] + \sigma. \tag{17}$$

It is now simply a matter of combining these prices set by firms to obtain the relevant price indices. Between time 0 and time 1,

$$\begin{aligned} p_C(t) &= \delta\left[\int_0^t e^{-\delta j}\hat{p}_C(t-j)dj\right] + \delta\left[\int_t^\infty e^{-\delta j}\hat{p}_C(t-j)dj\right] \\ &= \mu\left[t - \frac{\beta_C}{\delta(\beta_C+\delta)}\right] + \sigma\left[t - \frac{1}{\delta}(1-e^{-\delta t})\right], \end{aligned} \tag{18}$$

and after time 1, the price index is given by

$$\begin{aligned} p_C(t) &= \delta\left[\int_0^{t-1} e^{-\delta j}\hat{p}_C(t-j)dj\right] + \delta\left[\int_{t-1}^t e^{-\delta j}\hat{p}_C(t-j)dj\right] + \delta\left[\int_t^\infty e^{-\delta j}\hat{p}_C(t-j)dj\right] \\ &= \mu\left[t - \frac{\beta_C}{\delta(\beta_C+\delta)}\right] + \sigma\left[1 + \frac{1}{\delta}e^{-\delta t}(1-e^{\delta})\right]. \end{aligned} \tag{19}$$

Thus it is possible to solve for the exact price level, and therefore the inflation rate and output (from (1)), at all points in time.

References

Ball, L.N., Mankiw, N.G., Romer, D., 1988. The New Keynesian economics and the output-inflation trade-off. Brookings Papers on Economic Activity 0, 1-65.

Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12, 383-398.

Devereux, M.B., Yetman, J., 2002. Menu costs and the long-run output-inflation trade-off. Economics Letters 76, 95-100.

Romer, D., 1990. Staggered price setting with endogenous frequency of adjustment. Economics Letters 32, 205-210.

Wash, C.E., 1998. Monetary Theory and Policy. MIT Press, Cambridge.

Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press, Princeton.

Figure 1. Consumption-inflation trade-off:
Consumption measured as percent deviation from zero trend inflation

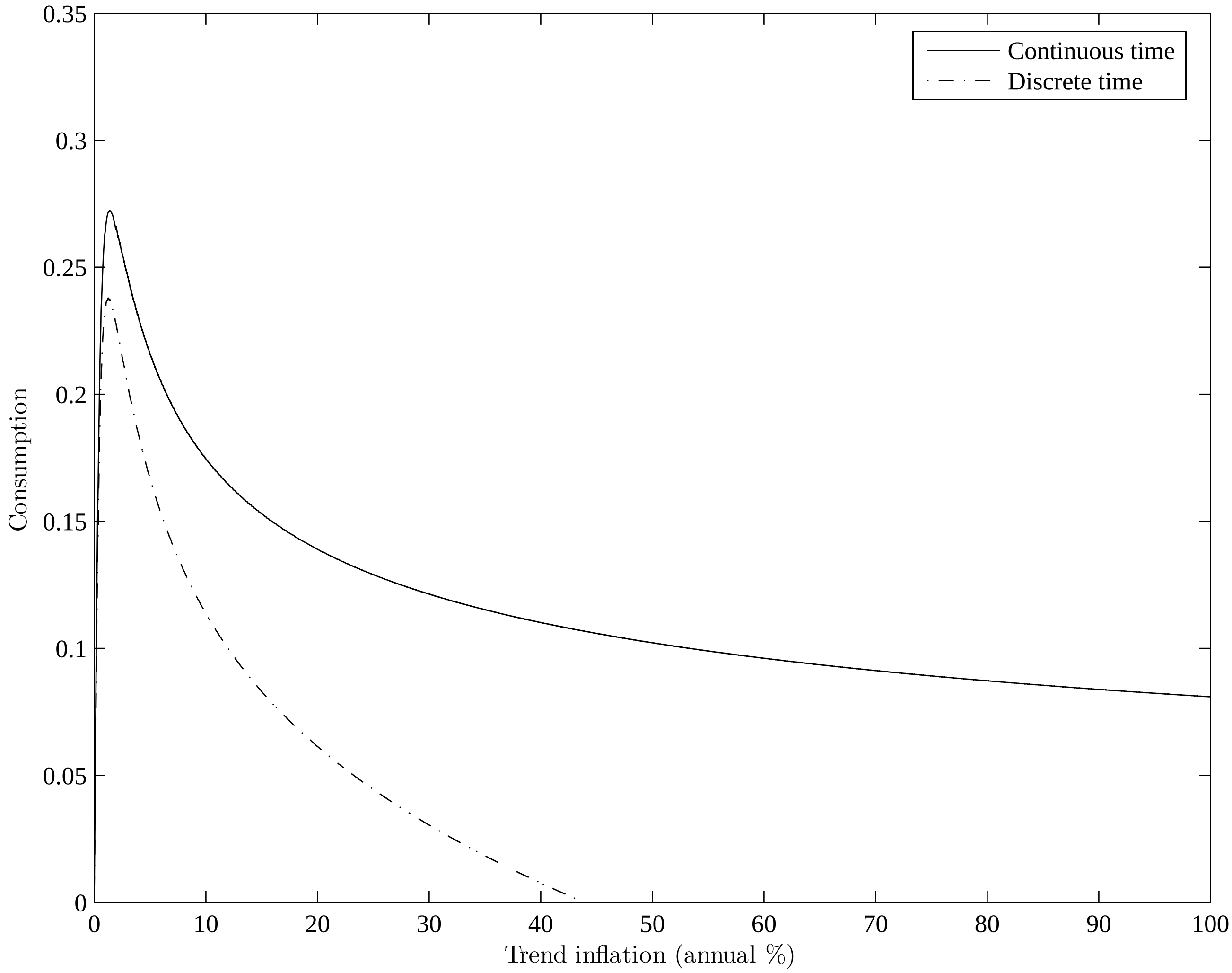

Figure 2. Average contract length

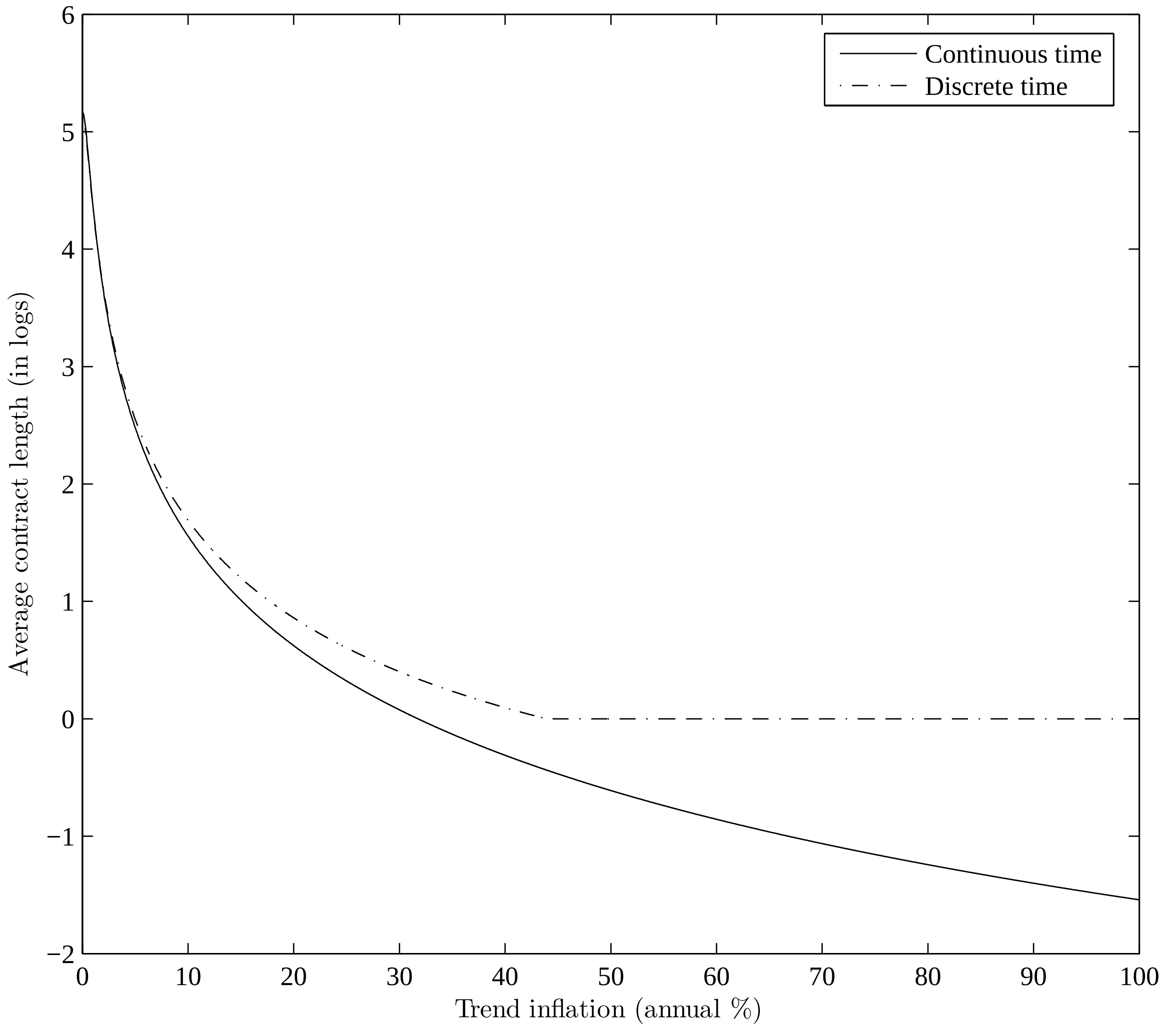

Figure 3. Welfare-inflation trade-off: Base Calibation
Welfare measured as percent deviation from zero trend inflation

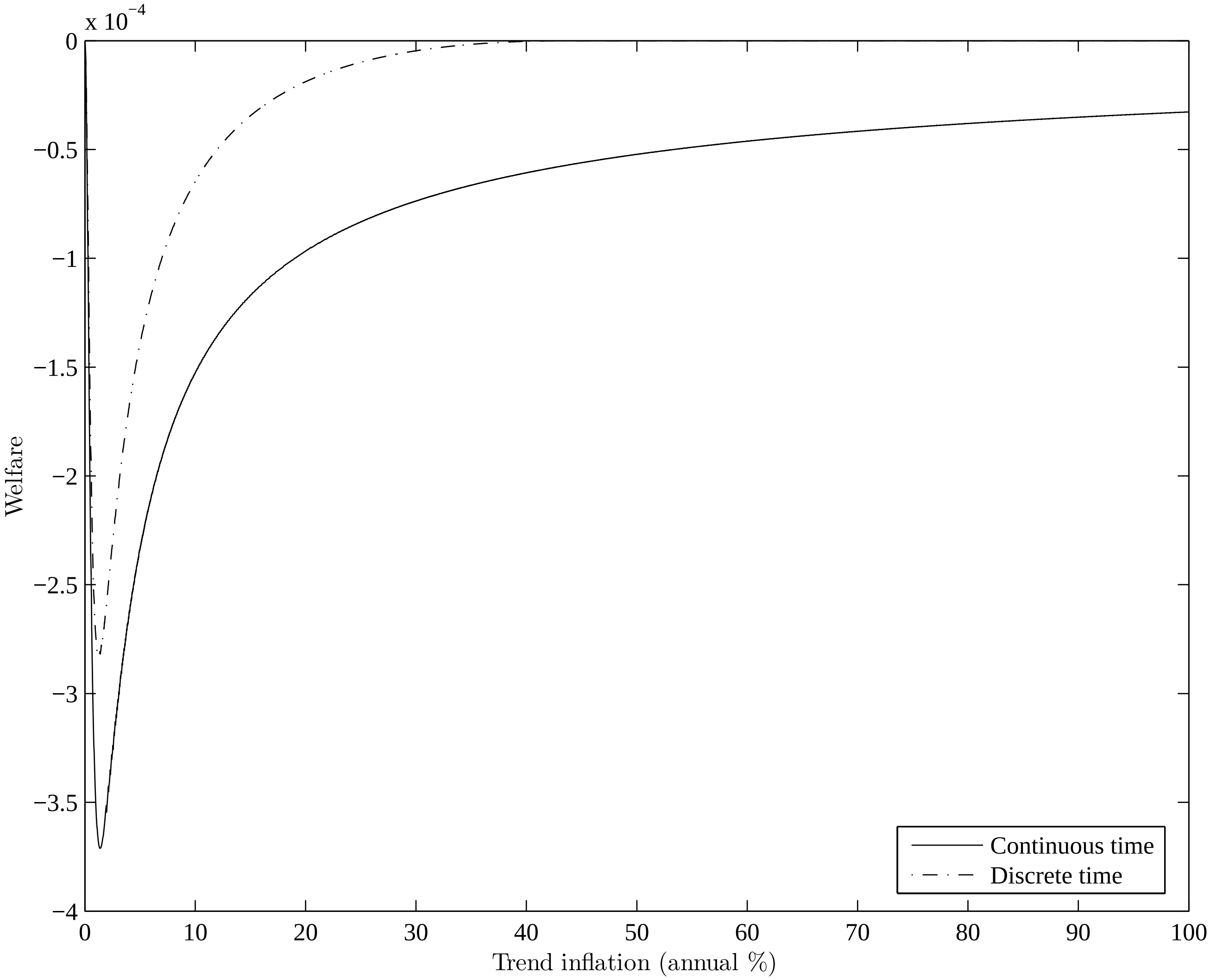

Figure 4. Welfare-inflation trade-off: Changing prices requires labor input
Welfare measured as percent deviation from zero trend inflation

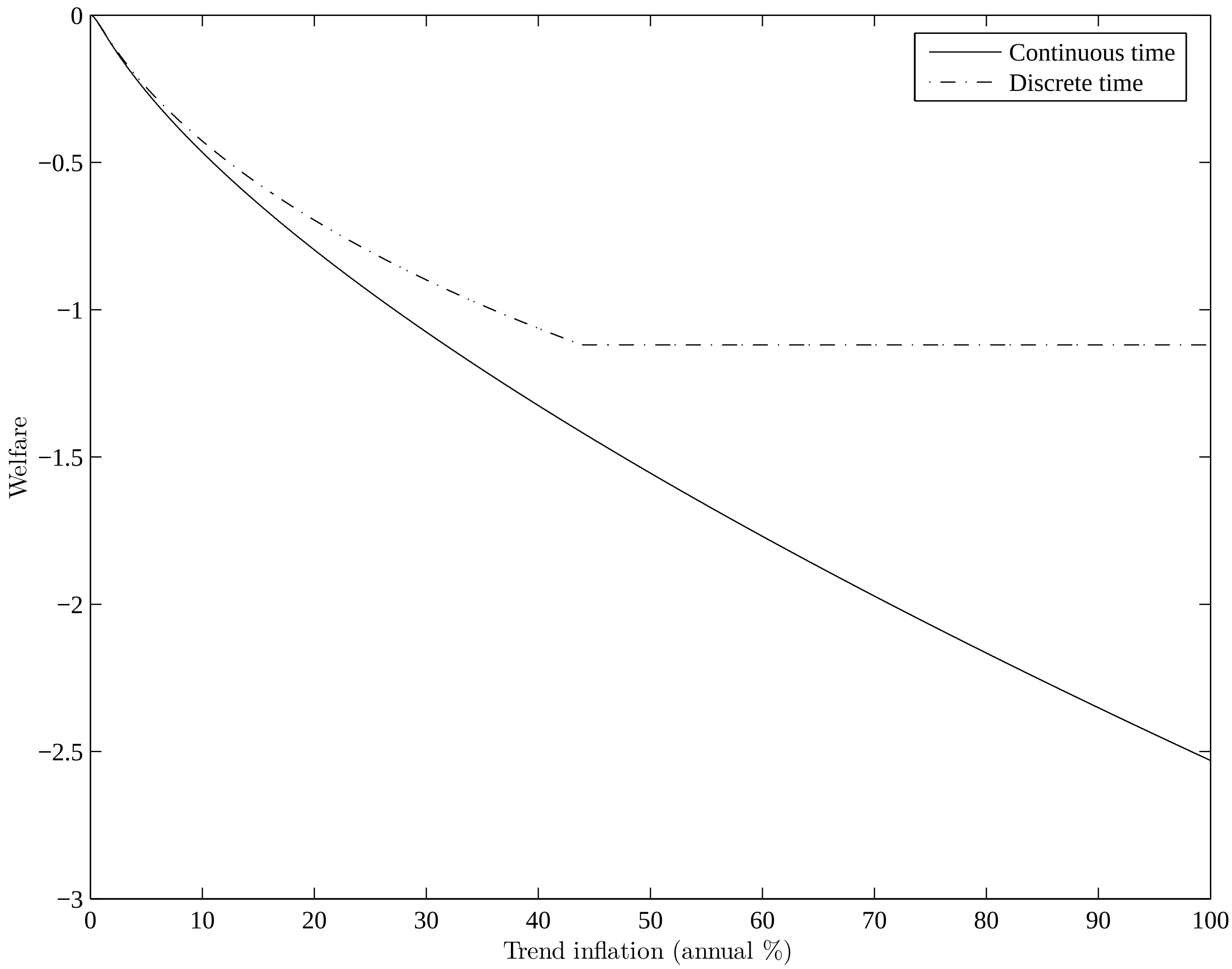

Figure 5. Impulse response to Monetary Shock: $\mu = 1\%$
All variables measured as deviation from trend

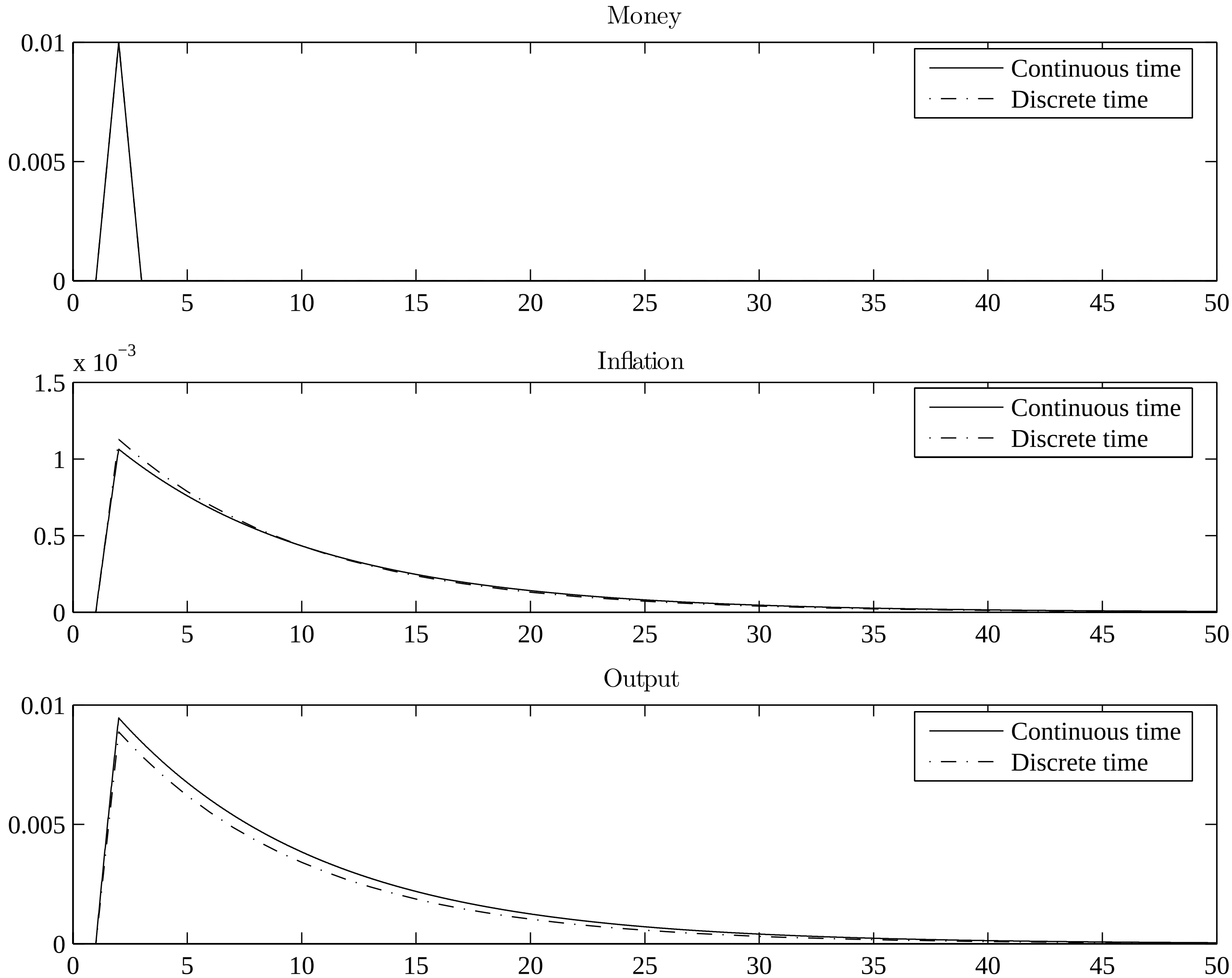

Figure 6. Impulse response to Monetary Shock: $\mu = 20\%$
All variables measured as deviation from trend