

# Shock Size, Asymmetries, and State Dependent Pricing

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## Abstract

State dependent pricing models predict different real responses to shocks than time dependent pricing models. For sufficiently large shocks, the real effects of shocks are independent of their sign.

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# 1 Introduction

Hansen and Prescott (2002) and Sichel (1993) provide empirical evidence that positive shocks produce smaller positive output effects than negative shocks produce negative output effects. Elsewhere this has been explained using both capacity constraint models (Hansen and Prescott 2002; Danziger 2002; Danziger and Kreiner 2003) and sticky price models (Devereux and Siu 2003).<sup>3</sup> In this note, we show that the state dependent pricing mechanism of Dotsey, King and Wolman (1999) can explain this phenomenon, while time dependent pricing cannot. Further, for sufficiently large shocks, state dependent pricing implies identical real responses to positive and negative shocks.

## 2 The model

Consider a simple sticky price model with monopolistic competition in which the pricing decision of firms is state dependent.

### 2.1 Firms

As in Dotsey, King and Wolman (1999), firms enjoy constant returns to scale technology, and the real profit function of the firm in period  $t$  with price  $\hat{P}_{t-h}$  is

$$\pi_{t-h,t} = \left( \frac{\hat{P}_{t-h}}{P_t} \right)^{-\epsilon} \left( \frac{\hat{P}_{t-h} - MC_t}{P_t} \right) C_t \quad (1)$$

where  $\epsilon$  is the elasticity of substitution between goods.

Firms draw a lottery on the price adjustment cost  $\omega$  from a distribution  $G(\cdot)$  over a finite support  $[0, \bar{\omega}]$  after the realization of shocks each period, where the price adjustment cost is

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<sup>3</sup>Devereux and Siu (2003) assume that prices cannot remain fixed for more than two periods; in contrast, in our model, the maximum number of periods for which prices remain fixed is endogenous.

denominated in terms of output. Consider an individual firm that fixes its price for  $h$  periods.

Its real value function (after paying price adjustment costs) is given by

$$V_{t-h,t} = \pi_{t-h,t} + \beta E_t \left[ \alpha_{t-h,t+1} V_{t+1,t+1} + (1 - \alpha_{t-h,t+1}) V_{t-h,t+1} - \alpha_{t-h,t+1} \Omega_{t-h,t+1} \right] \quad (2)$$

where

$$\Omega_{t-h,t+1} = \int_0^{G^{-1}(\alpha_{t-h,t+1})} x g(x) dx \quad \text{for } h = 0, 1, \dots, J-1 \quad (3)$$

is the *ex ante* expected price adjustment cost in period  $t+1$  given that the price was last set in period  $t-h$ , where  $g(x)$  is the density function of the price adjustment cost.

Iterating forward on the value functions and taking first order conditions,

$$\hat{P}_t = \frac{\epsilon}{\epsilon - 1} \frac{E_t \left[ \sum_{h=0}^{J-1} \beta^h \eta_{t,t+h} C_{t+h} P_{t+h}^{\epsilon-1} MC_{t+h} \right]}{E_t \left[ \sum_{h=0}^{J-1} \beta^h \eta_{t,t+h} C_{t+h} P_{t+h}^{\epsilon-1} \right]} \quad (4)$$

is the price chosen by all adjusting firms where  $\eta_{t,t+h}$  is an individual firm's conditional probability that  $\hat{P}_t$  holds in period  $t+h$ .

## 2.2 Aggregate demand

The aggregate output is given by the quantity equation

$$Y_t = C_t + \bar{\Omega}_t, \quad (5)$$

where  $C_t = \frac{M_t}{P_t}$  and  $\bar{\Omega}_t$  is the aggregate menu cost.<sup>4</sup>

The nominal marginal cost facing each firm is the money supply,<sup>5</sup>

$$MC_t = M_t, \quad (6)$$

<sup>4</sup>Reported results are for  $\bar{\Omega} = \sum_h \theta_h \Omega_h$ . Qualitatively, the results also hold if menu costs have no output effects ( $\bar{\Omega} = 0$ ).

<sup>5</sup> $MC_t = M_t$  results from the utility function  $U(C, N) = \ln(C) - N$  together with linear production in labor.

which we assume follows the process

$$M_t = M_{t-1}(1 + \mu)(1 + \varepsilon_t), \quad (7)$$

where  $\mu$  is the money growth rate chosen by the central bank and  $\varepsilon_t$  is white noise.

Aggregate prices are given by

$$P_t = \left[ \sum_{h=1}^J \left( \alpha_{t-h,t} \theta_{t-h,t} \hat{P}_t^{1-\epsilon} + (1 - \alpha_{t-h,t}) \theta_{t-h,t} \hat{P}_{t-h}^{1-\epsilon} \right) \right]^{\frac{1}{1-\epsilon}}, \quad (8)$$

where  $\theta_{t-h,t}$  is the portion of firms whose price was fixed in period  $t-h$  and  $\alpha_{t-h,t}$  is the fraction of firms who reset their price in period  $t$  after being fixed for  $h$  periods.

## 2.3 Solving the model

We consider parameter values similar to those used in other studies:  $\beta = 0.99$ ;  $\epsilon = 11$  (implying flexible-price mark-up of 10%); and  $\mu = 3\%$ . The price adjustment cost distribution is calibrated to a uniform distribution with a maximum possible menu cost of 3.75% of steady state output.

We first solve the non-stochastic steady state of the state dependent pricing model. Given our calibration, in steady state  $J = 3$ , implying that we require solution values of four variables: three values of  $\alpha$  (one for each cohort of firms), and the real value of  $\hat{P}$  (the price that adjusting firms set). For arbitrary initial  $\alpha$ 's we determine  $\hat{P}$ , and then update the  $\alpha$ 's repeatedly using  $\alpha_{t-h,t} = G^{-1}(V_{t,t} - V_{t-h,t})$ , a necessary condition for equilibrium, until they converge. We then update  $\hat{P}$ , and so on, until convergence, with a convergence criterion of  $10^{-8}$ . We then ensure that our solution satisfies the sufficient conditions for equilibrium using a two-dimensional grid-search over  $\{\alpha_1, \alpha_2\}$ .<sup>6</sup> We next include a shock and solve for the impulse response using the

<sup>6</sup>The sufficient condition for equilibrium is  $\{\alpha_{t,t+h}\}_{h=1}^J = \arg \max V_{t,t}(\{\alpha_{t-h,t}\}_{h=1}^J)$ ; see John and Wolman

same approach, adding an additional dimension to the numerical problem, since  $\alpha$  now depends on both the cohort of the firm, and the time period. Note that we are solving the full non-linear model numerically as in Burstein (2005), and so the results should be accurate up to our convergence criterion, except for rounding errors.

We also compute results with a model of time dependent pricing, where the probability of price adjustment at each horizon is taken to be the steady state probability.<sup>7</sup>

### 3 Results and Discussion

Figure 1 reports the response of the model in period 1, when the shock is realized, and illustrates that the real effects of nominal shocks on output are very different with state dependent pricing than with time dependent pricing. As in Devereux and Siu (2003), firms are more averse to goods being underpriced than overpriced, since the maximum loss from over-pricing a good is limited to making zero profits, while the maximum loss from under-pricing goods is potentially unbounded. This effect is further exacerbated by positive trend inflation, since a price that is too high today will depreciate with future inflation, while a price that is too low will only become more so over time. Firms are therefore quicker to respond to positive than negative marginal cost shocks. In our calibration, all firms increase prices in response to a 6% positive shock, while it requires a much larger (16%) negative shock for all firms to cut prices. Thus output barely rises in response to a positive nominal shock, while it may fall (by as much as 3%) in response to a negative shock, in agreement with the empirical evidence outlined earlier.

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(2004) for a discussion.

<sup>7</sup>This implies that  $\alpha_{t-h,t}$  is increasing in  $h$ . Similar results are obtained with Calvo (1983) pricing where  $\alpha_{t-h,t} = 0.25 \forall h$ .

In contrast, time dependent pricing implies that positive shocks result in larger real effects than negative shocks. Adjusting firms only increase their prices a little in response to a positive shock, but decrease their prices by more in response to a negative shock, due to competition from firms that cannot adjust their prices. While these “strategic complementarity” effects are also present with state dependent pricing, they are reduced since the number of adjusting firms is highly responsive to the size of the shock.

Figures 2 and 3 report the impulse responses of output and inflation for different shock sizes,  $\pm 1\%$  and  $\pm 20\%$ , for state dependent and time dependent pricing respectively. With state dependent pricing, the output responses to large shocks are identical, irrespective of the sign of the shock. This is because a sufficiently large shock induces all firms to adjust their prices immediately, and given that expected future states are the same for all firms, they choose the same real price regardless of the sign of the shock. In contrast, time dependent pricing implies an increasing difference in the impulse responses of positive and negative shocks as shock size increases.

## 4 References

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Figure 1. Same period responses to a shock.  
 — State Dependent Pricing; —.— Time Dependent Pricing

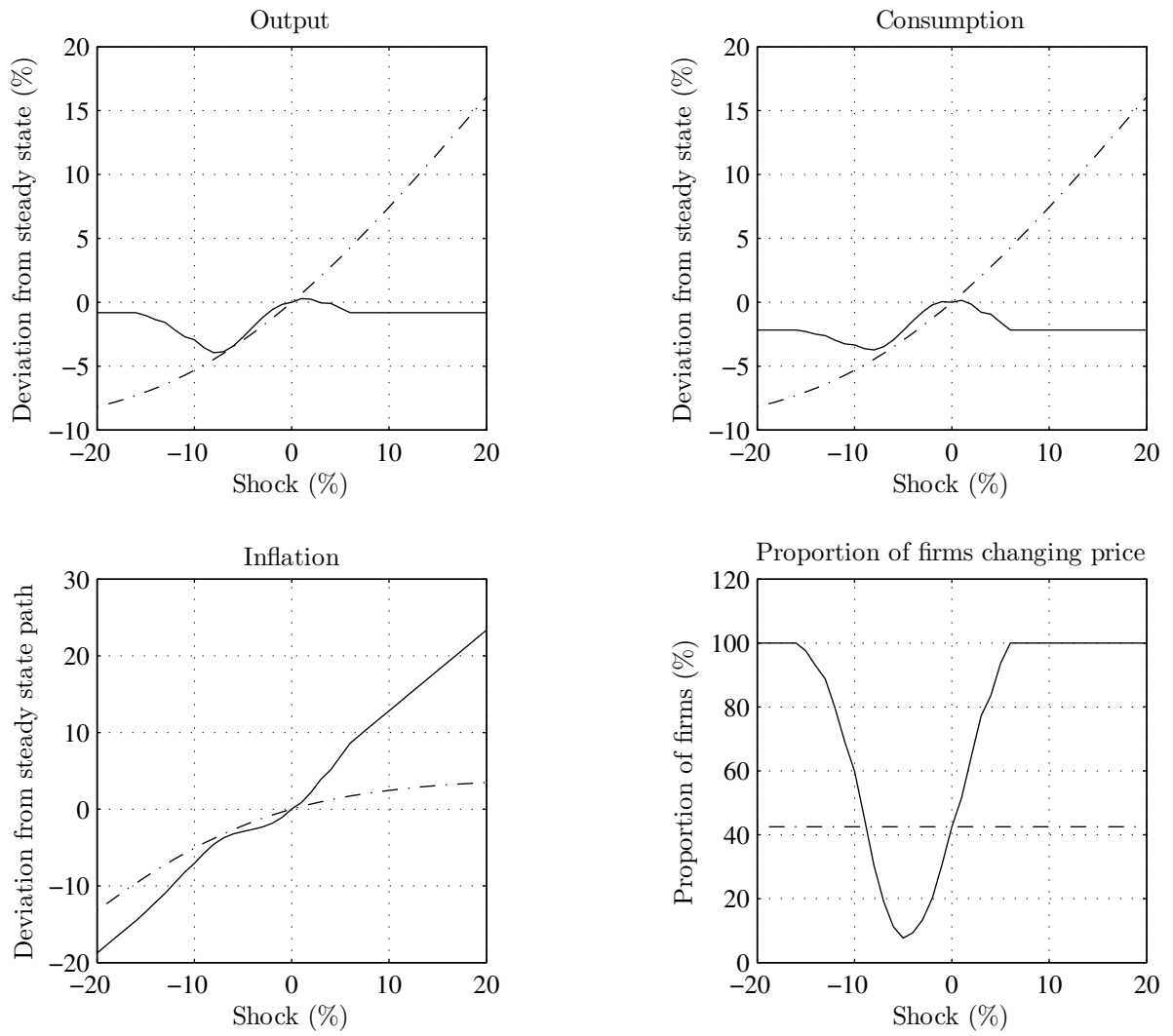


Figure 2. Impulse responses to a 1% and a 20% shocks with state dependent pricing  
 — Positive Shock; -.- Negative Shock

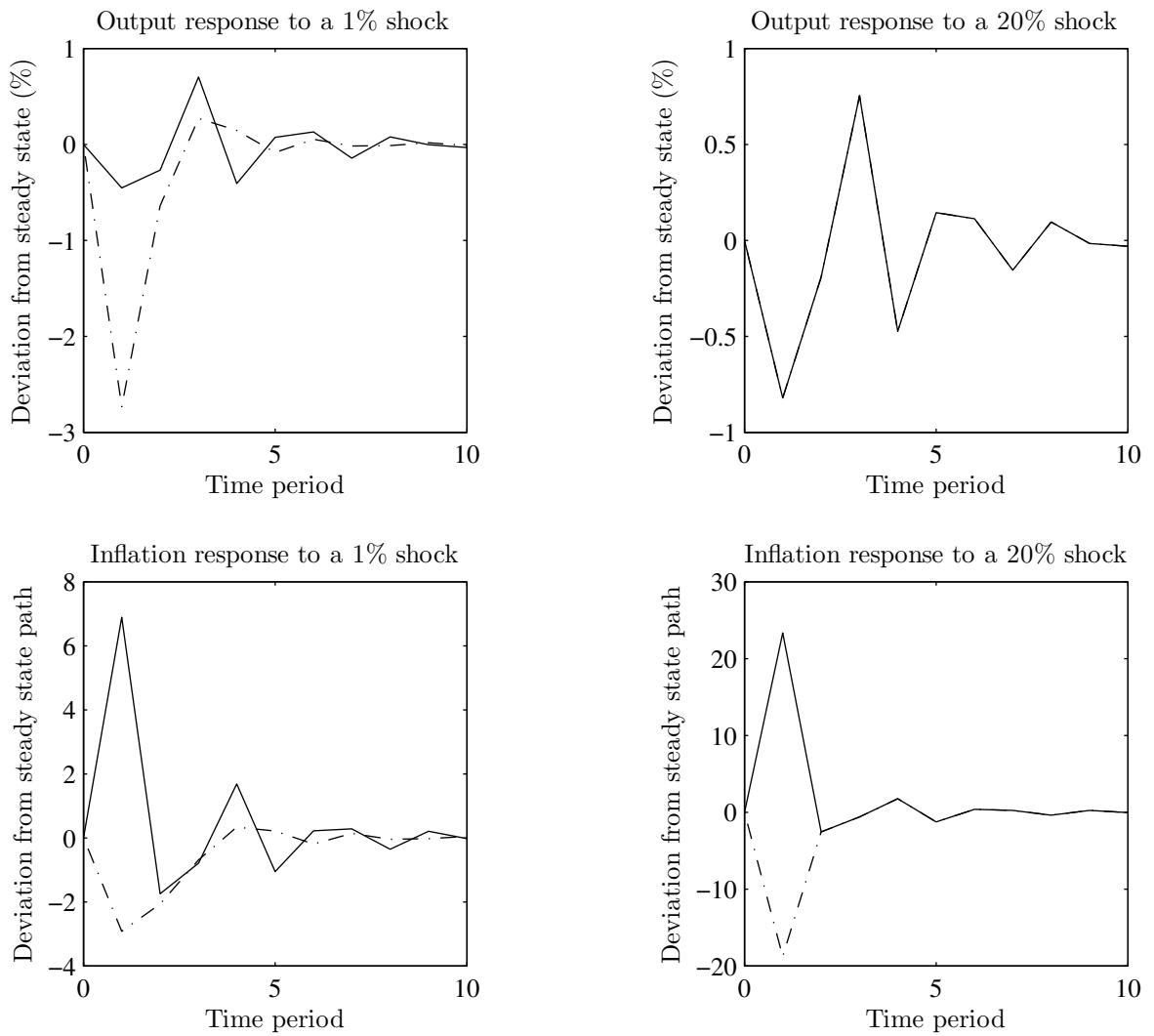


Figure 3. Impulse responses to a 1% and a 20% shocks with time dependent pricing  
 — Positive Shock; -.- Negative Shock

