Traffic grooming in wavelength-division-multiplexing ring networks: a linear programming solution

Jian-Qiang Hu

Boston University, 15 St. Mary's Street, Brookline, Massachusetts 02446

hqiang@bu.edu

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We consider the problem of traffic grooming in wavelength-division-multiplexing (WDM) rings. Our objective is to minimize the required number of electronic add–drop multiplexers. We first formulate the problem as an integer linear programming (ILP) problem, and we then show that this ILP problem can be converted into an equivalent mixed ILP (MILP) problem in which a large number of integer variables in the original ILP can be relaxed to continuous variables. The resulting MILP problem is much easier to solve. For ring networks found in most applications (e.g., access and interoffice rings), which typically have less than 20 nodes, it can produce optimal or near-optimal solutions in a few seconds or minutes by use of commercially available linear programming software, such as CPLEX, on a PC. We also discuss how our ILP formulation can be extended to more-general traffic grooming problems, such as networks with dynamic traffic and how to take the number of wavelengths into consideration. Finally, numerical examples are presented.

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1. Introduction

Wavelength-division multiplexing (WDM) is now being widely used for expanding capacity in optical networks. In a WDM network, each fiber link can carry high-rate traffic at many different wavelengths; thus multiple channels can be created within a single fiber. In this paper we consider the traffic grooming problem for WDM rings. The ring architecture is commonly used in many of today's optical networks. In a WDM ring, nodes are arranged in a ring configuration, and each pair of adjacent nodes on the ring are then connected by a fiber (or multiple fibers). At each node, electronic add–drop multiplexers (ADMs) are used to multiplex lower-rate traffic onto individual wavelengths. For example, 64 OC-3 (155 Mbit/s) circuits can be multiplexed onto a single wavelength with capacity OC-192 (10 Gbit/s). Since the cost of ADMs often makes up a significant portion of the total cost for a WDM ring, one of the important issues in the design of a WDM ring network is to decide how to multiplex lower-rate traffic onto different wavelengths so that the number of ADMs required in the network is minimized. This problem is referred to as the traffic grooming problem. Usually, an ADM for a wavelength is needed at a particular node only when the wavelength is dropped at the node (i.e., when a demand is multiplexed onto the wavelength at the node), and it is not needed when the wavelength is only passing through the node.

There are two types of ring networks: UPSR (unidirectional path-switched ring) and BLSR (bidirectional line-switched ring). Furthermore, BLSRs include two-fiber (BLSR/2) and four-fiber (BLSR/4) rings. One significant difference between UPSR and BLSR is that UPSR offers dedicated protection (i.e., $1+1$ protection), whereas BLSR provides only shared protection (i.e., $1:n$ protection). As such, BLSR usually requires less bandwidth for the same amount of traffic. In general, UPSRs are often used in local access networks, whereas BLSRs are widely deployed in interoffice and long-haul networks. An excellent introduction on UPSR and BLSR can be found in Ref. 1, Sect. 10.4.2. In this paper, we consider only UPSRs, though the proposed method is believed to be applicable to BLSRs as well.

The problem of traffic grooming for WDM rings has been studied in several recent papers.[2–13] Most of these studies have focused on either the development of heuristic algorithms or special cases of WDM rings (e.g., rings with hub-type traffic or rings with all-to-all uniform traffic), and in some cases bounds on the minimum cost were developed (e.g., Refs. 4, 5, 13). In this paper, we consider the traffic grooming problem with general traffic input, and our goal is to obtain optimal or near-optimal solutions. The method we use is based on the integer linear programming (ILP) formulation. We note that the ILP formulation is also proposed in Refs. 5 and 12. However, the objective function considered in Ref. 5 is electronic routing, which is quite different from ours in this paper, and the goal in Ref. 5 is to derive bounds based on the ILP formulation instead of finding optimal or near-optimal solutions. The problem setting in Ref. 12 is similar to ours; however, it is concluded there that the ILP formulation is not computationally feasible for rings with eight or more nodes. Hence in Ref. 12 it is proposed instead to use methods based on simulated annealing and heuristics. In this paper we develop a much more efficient ILP formulation for the traffic grooming problem, which can be used to solve large rings. We first formulate the traffic grooming problem as an ILP problem. However, as indicated by our experimental results (see Section 5), this initial ILP formulation is effective only for medium-size networks. For example, by using commercially available linear programming software such as CPLEX, we can obtain optimal or near-optimal solutions in a few seconds or minutes for rings with less than ten nodes. This approach becomes computationally intractable for large ring networks. To overcome this difficulty, we modify the initial ILP formulation by relaxing the integer constraint on a large portion of its integer variables, which results in a mixed ILP (MILP) problem. The resulting MILP problem can be solved much faster, and our numerical results show that the improvement is quite significant in that it can produce better solutions in much less time. With the MILP formulation, we can easily deal with much larger rings.

The rest of the paper is organized as follows. In Section 2 the traffic grooming problem and its ILP formulation are introduced. In Section 3 we see that the ILP problem can be modified to an equivalent MILP problem that can be solved much more efficiently. Some extensions of our ILP formulation are presented in Section 4. Numerical results are provided in Section 5, and finally a conclusion and some discussion are given in Section 6.

2. Problem Formulation

Consider a WDM ring with N nodes, labeled as $1, 2, \dots, N$ clockwise. We assume that all available wavelengths have the same capacity and that there may be multiple traffic circuits between a pair of end nodes but that all traffic circuits have the same rate. The traffic granularity of the network is defined as the total number of low-rate traffic circuits that can be multiplexed onto a single wavelength. For example, if each circuit is OC-12 and the wavelength capacity is OC-48, then the traffic granularity is 4.

In designing a WDM ring, the key is to determine which ADMs are needed at each node. This depends mainly on how lower-rate traffic circuits are multiplexed onto high-rate wavelengths. An ADM for an individual wavelength is needed at a node only when the wavelength needs to be dropped at the node, i.e., when a circuit with the node as one of its two end nodes is multiplexed onto the wavelength. If the wavelength only passes through the node, then no ADM for the wavelength is needed. Our objective is to find an optimal way to multiplex lower-rate traffic circuits so as to minimize the total number of ADMs required in the network. However, it becomes clear below that we can easily incorporate other considerations into our objective as well, such as the total number of wavelengths used in the network.

To present our ILP formulation, we need to introduce the following notation:

N the number of nodes in the ring;

L the number of wavelengths available;

g the traffic granularity;

m_{ij} the number of circuits between nodes i and j $(i, j = 1, 2, \ldots, N)$; since we consider only UPSRs, without loss of generality we can assume $m_{ij} = 0$ when $i \geq j$;

$$x_{ijsl} = \begin{cases} 1 & \text{if the } s\text{th circuit between nodes } i \text{ and } j \text{ is multiplexed onto wavelength } l \\ 0 & \text{otherwise} \end{cases};$$

$$\begin{aligned} y_{il} &= \max_{s,j}(x_{ijsl}, x_{jisl}) \\ &= \begin{cases} 1 & \text{if any circuit with node } i \text{ as one of its end nodes is multiplexed onto wavelength } l \\ 0 & \text{otherwise} \end{cases}. \end{aligned}$$

We note that if $y_{il} = 1$, then wavelength l needs to be dropped at node i, which implies that an ADM for wavelength l is required at node i. Since our objective is to minimize $\sum_{i=1}^{N}\sum_{l=1}^{L} y_{il}$, the total number of ADMs required in the ring, the traffic grooming problem can be formulated as the following ILP problem:

$$\begin{aligned}
(\text{ILP}_1) \qquad \min \quad & \sum_{i=1}^{N}\sum_{l=1}^{L} y_{il} \\
\text{s.t.} \quad & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} x_{ijsl} \leq g, && l = 1, 2, \ldots, L, && (1) \\
& \sum_{l=1}^{L} x_{ijsl} = 1, && \forall\, i, j, s, && (2) \\
& y_{il} \geq x_{ijsl}, \\
& y_{il} \geq x_{jisl}, && \forall\, i, j, s, l, && (3) \\
& x_{ijsl}, y_{il} \text{ are all binary variables.}
\end{aligned}$$

The above ILP is denoted as (ILP_1). The three constraints in (ILP_1) are

(1) The total number of circuits multiplexed onto wavelength l should not exceed g.

(2) Each circuit has to be assigned to one (and only one) wavelength.

(3) Given that the objective is to minimize $\sum_{i=1}^{N}\sum_{l=1}^{L} y_{il}$, it is equivalent to $y_{il} = \max_{s,j}(x_{ijsl}, x_{jisl})$.

In general, it is computationally infeasible to use (ILP_1) to solve the traffic grooming problem for large rings. For example, it takes hours for CPLEX to solve (ILP_1) for a 15-node ring with $m_{ij} = 1$ (all-to-all uniform traffic) and $g = 4$. Fortunately, WDM rings found in most applications (e.g., access rings and interoffice rings) are not very large, typically with less than a dozen nodes. In addition, we can modify (ILP_1) so that it can be solved more efficiently. First, we note that since y_{il} is a binary variable, constraint (3) is equivalent to

$$My_{il} \geq \sum_{j=1}^{N} \sum_{s=1}^{m_{ij}} (x_{ijsl} + x_{jisl}),$$

where $M > 0$ is a very large positive number. Therefore the following ILP problem is equivalent to (ILP$_1$):

$$
\begin{aligned}
(\mathrm{ILP}_1^{\mathsf{C}}) \qquad \min \quad & \sum_{i=1}^{N} \sum_{l=1}^{L} y_{il} & & \\
\text{s.t.} \quad & \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{s=1}^{m_{ij}} x_{ijsl} \leq g, & l = 1, 2, \ldots, L, & \qquad (4) \\
& \sum_{l=1}^{L} x_{ijsl} = 1, & \forall\, i, j, s, & \qquad (5) \\
& My_{il} \geq \sum_{j=1}^{N} \sum_{s=1}^{m_{ij}} (x_{ijsl} + x_{jisl}), & \forall\, i, l, & \qquad (6) \\
& x_{ijsl}, y_{il} \text{ are binary variables.} & &
\end{aligned}
$$

When (3) is replaced with (6), the number of constraints in (ILP$_1^{\mathsf{C}}$) is reduced dramatically; hence optimal or near-optimal solutions can be found much faster (in some cases hundreds times faster), as shown by our numerical results in Section 5. In addition to the number of constraints, we can further reduce the number of variables in (ILP$_1^{\mathsf{C}}$). Let us define

$$x_{ijl}^{\mathsf{S}} = \sum_{s=1}^{m_{ij}} x_{ijsl},$$

which is the number of traffic circuits between nodes i and j that are multiplexed onto wavelength l. Then we can rewrite (ILP$_1^{\mathsf{C}}$) as

$$
\begin{aligned}
(\mathrm{ILP}_1^{\mathsf{V}}) \qquad \min \quad & \sum_{i=1}^{N} \sum_{l=1}^{L} y_{il} & & \\
\text{s.t.} \quad & \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ijl}^{\mathsf{S}} \leq g, & l = 1, 2, \ldots, L, & \qquad (7) \\
& \sum_{l=1}^{L} x_{ijl}^{\mathsf{S}} = m_{ij}, & \forall\, i, j, & \qquad (8) \\
& My_{il} \geq \sum_{j=1}^{N} (x_{ijl}^{\mathsf{S}} + x_{jil}^{\mathsf{S}}), & \forall\, i, l, & \qquad (9) \\
& x_{ijl}^{\mathsf{S}} \text{ are nonnegative integer variables,} & & \\
& y_{il} \text{ are binary variables.} & &
\end{aligned}
$$

The reduction in the number of variables in (ILP$_1^{\mathsf{V}}$) is very significant when m_{ij}'s are large. Intuitively, it should be clear why it is easier to solve (ILP$_1^{\mathsf{V}}$) than (ILP$_1^{\mathsf{C}}$), since the number of possible ways to assign values to x's is smaller in (ILP$_1^{\mathsf{V}}$). In fact, it will become even more obvious why it is much more efficient to deal with (ILP$_1^{\mathsf{V}}$) in Section 3 when we examine how additional (and equally significant) improvement can be made on (ILP$_1^{\mathsf{C}}$) and (ILP$_1^{\mathsf{V}}$). The improvement is based on the idea of converting (ILP$_1^{\mathsf{C}}$) and (ILP$_1^{\mathsf{V}}$) to equivalent mixed ILPs (MILPs) in which the integer constraint on x_{ijsl} and x_{ijl}^{S} is relaxed.

3. Equivalent Mixed Integer Linear Programming Formulation

As we pointed out above, ($\mathrm{ILP}_1^{\mathsf{C}}$) and ($\mathrm{ILP}_1^{\mathsf{V}}$) are much easier to solve than (ILP_1). For most small- and medium-size rings, they can produce optimal or near-optimal solutions in a few seconds or minutes. However, further improvement can still be made for larger rings. In this section we develop a more efficient method for solving ($\mathrm{ILP}_1^{\mathsf{C}}$) and ($\mathrm{ILP}_1^{\mathsf{V}}$). The crux of our method is based on removing the integer constraint on some decision variables. By reducing the number of integer variables, we can generally solve ($\mathrm{ILP}_1^{\mathsf{C}}$) and ($\mathrm{ILP}_1^{\mathsf{V}}$) more efficiently (for example, in the branch and bound algorithm for ILP, having fewer integer variables means that a smaller number of branches would be needed).

To present the idea, let us first consider (ILP_1). If we relax the integer constraint on x_{ijsl} with $x_{ijsl} \geq 0$, we then have the following mixed ILP, which we denote as (MILP_1):

$$
\begin{array}{llll}
(\mathrm{MILP}_1) & \min & \displaystyle\sum_{i=1}^{N}\sum_{l=1}^{L} y_{il} & \\
& \text{s.t.} & \displaystyle\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} x_{ijsl} \leq g, & l = 1,2,\ldots,L, \\
& & \displaystyle\sum_{l=1}^{L} x_{ijsl} = 1, & \forall\, i,j,s, \\
& & y_{il} \geq x_{ijsl}, & \\
& & y_{il} \geq x_{jisl}, & \forall\, i,j,s,l, \\
& & x_{ijsl} \geq 0, & \forall\, i,j,s,l, \\
& & y_{il} \text{ are binary variables.} &
\end{array}
$$

First note that because $\sum_{l=1}^{L} x_{ijsl} = 1$, $x_{ijsl} \geq 0$ also implies that $x_{ijsl} \leq 1$. In the following result we show that (MILP_1) has an integer optimal solution, which is obviously an optimal solution for (ILP_1) as well.

Theorem 1 *If g is an integer, then (MILP_1) has an integer optimal solution.*

Proof. Assume that (MILP_1) has an optimal solution that is not an integer solution, i.e., there exits at least one noninteger x_{ijsl}. Since $\sum_{l=1}^{L} x_{ijsl} = 1$, there must be another noninteger $x_{ijsl'}$ ($l' \neq l$). Also note that $\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} x_{ijsl} \leq g$, where g is an integer, we consider the following two cases:

- *Case 1.* All $x_{i'j's'l}$ $[(i',j',s') \neq (i,j,s)]$ are integers. In this case, if we decrease $x_{ijsl'}$ to zero while increasing x_{ijsl} to $x_{ijsl} + x_{ijsl'}$, then all the constraints in (MILP_1) are still satisfied; so the modified solution is still optimal (since all y_{il} remain the same), whereas the number of noninteger x_{ijsl} is decreased by at least one.
- *Case 2.* There exists another noninteger $x_{i'j's'l}$ $[(i',j',s') \neq (i,j,s)]$. In this case, there must exist another noninteger $x_{i'j's'l''}$, since $\sum_{l=1}^{L} x_{i'j's'l} = 1$. Repeating the two-case argument, we can easily see that there is a set of noninteger x_{ijsl}, denoted by I_n, such that (a) for every l with at most two exceptions, $\{x_{ijsl} :\ s = 1,2,\ldots,m_{ij}, i,j = 1,2,\ldots,N\}$ contains exactly either 0 or 2 elements in I_n, and (b) for each (i,j,s), $\{x_{ijsl} :\ l = 1,2,\ldots,L\}$ contains exactly either 0 or 2 elements in I_n. Let

$$\varepsilon = \min_{(i,j,s,l)\in I_n} \{x_{ijsl}, 1 - x_{ijsl}\}.$$

Then we can decrease half the elements in I_n by ε while increasing the other half by ε such that the new solution is still optimal to (MILP$_1$) while the number of noninteger x_{ijsl} is decreased by at least one.

To summarize, if (MILP$_1$) has a noninteger optimal solution, we can then construct another optimal solution with one less noninteger variable. If we continue this procedure, then we can eventually obtain an integer optimal. This completes the proof.

The proof of Theorem 1 in fact provides us with a procedure for constructing an integer optimal solution from any optimal solution for (MILP$_1$). It is quite obvious that Theorem 1 holds for (ILP$_1^{\mathsf{C}}$) and (ILP$_1^{\mathsf{V}}$) as well; i.e., the following two MILPs also have integer optimal solutions:

$$
\begin{aligned}
(\text{MILP}_1^{\mathsf{C}}) \qquad \min \quad & \sum_{i=1}^{N}\sum_{l=1}^{L} y_{il} \\
\text{s.t.} \quad & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} x_{ijsl} \le g, && l = 1,2,\ldots,L, \\
& \sum_{l=1}^{L} x_{ijsl} = 1, && \forall\ i,j,s, \\
& My_{il} \ge \sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} (x_{ijsl} + x_{jisl}), && \forall\ i,l, \\
& x_{ijsl} \ge 0, && \forall\ i,j,s,l, \\
& y_{il} \text{ are binary variables,}
\end{aligned}
$$

$$
\begin{aligned}
(\text{MILP}_1^{\mathsf{V}}) \qquad \min \quad & \sum_{i=1}^{N}\sum_{l=1}^{L} y_{il} \\
\text{s.t.} \quad & \sum_{i=1}^{N}\sum_{j=1}^{N} x_{ijl}^{\mathsf{S}} \le g, && l = 1,2,\ldots,L, \\
& \sum_{l=1}^{L} x_{ijl}^{\mathsf{S}} = m_{ij}, && \forall\ i,j, \\
& My_{il} \ge \sum_{j=1}^{N} (x_{ijl}^{\mathsf{S}} + x_{jil}^{\mathsf{S}}), && \forall\ i,l, \\
& x_{ijl}^{\mathsf{S}} \ge 0, && \forall\ i,j,l, \\
& y_{il} \text{ are binary variables.}
\end{aligned}
$$

4. Extensions

In this section we want to show how our ILP formulation can be used to model more-general traffic grooming problems. Here we consider only three cases: (a) nonuniform traffic, (b) minimizing the number of wavelengths, and (c) dynamic traffic. We also discuss in each case whether the ILP problem can be relaxed to an equivalent MILP problem.

4.A. Nonuniform-Size Traffic

So far, we have assumed that all the traffic circuits have the same size. However, our ILP formulation can be easily modified to deal with the nonuniform traffic case. Let d_{ijs} be the size of the sth traffic circuit between note i and node j ($s = 1,\ldots,m_{ij}$, $i, j = 1,2,\ldots,N$) and g be the wavelength capacity. Then it is not difficult to see that the traffic grooming problem with nonuniform traffic can be formulated as

$$
\begin{aligned}
(\text{ILP}_2) \qquad \min \quad & \sum_{i=1}^{N}\sum_{l=1}^{L} y_{il} \\
\text{s.t.} \quad & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} d_{ijs}x_{ijsl} \le g, && l = 1,2,\ldots,L, \\
& \sum_{l=1}^{L} x_{ijsl} = 1, && \forall\, i,j,s, \\
& My_{il} \ge \sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} (x_{ijsl} + x_{jisl}), && \forall\, i,j,s,l, \\
& x_{ijsl}, y_{il} \text{ are all binary variables.}
\end{aligned}
$$

We should note that the result of Theorem 1 no longer holds for (ILP_2). Even though the corresponding MILP problem for (ILP_2) may not have an integer optimal solution in general, it can still be used to provide a lower bound for (ILP_2), which can be quite valuable in many applications. Also, we have the following result:

Corollary 1 *If g is an integer, then the corresponding MILP problem for (ILP_2) has an optimal solution in which each $d_{ijs}x_{ijsl}$ is an integer.*

The proof of Corollary 1 is essentially the same as that of Theorem 1; hence we omit it here.

4.B. Minimizing the Number of Wavelengths

A different objective function other than the cost of ADMs considered in the earlier research for the traffic grooming problem is the total number of wavelengths required. It has been shown that in general minimizing the number of ADMs does not necessarily minimize the cost of wavelengths; e.g., see.[4,13] We should also point out that if the wavelength capacity is OC-192 or OC-48 and the circuit sizes are OC-3, OC-12, and OC-48, then the problem of minimizing the number of wavelengths is trivial. In fact, this is often the case in most applications. So a more realistic problem is to consider a combination of the number of ADMs and the number of wavelengths (again, we note that in general it is not possible to minimize both at the same time). Let z_l ($l = 1,2,...,L$) be a binary variable such that

$$z_l = \begin{cases} 1 & \text{if wavelength } l \text{ is needed} \\ 0 & \text{otherwise} \end{cases} .$$

Then the ILP formulation for the traffic grooming problem with the objective function of minimizing the combination of the number of ADMs and the number of wavelengths is given as follows:

$$
\begin{aligned}
(\mathrm{ILP}_3) \qquad \min \quad & \sum_{i=1}^{N}\sum_{l=1}^{L} c_y y_{il} + \sum_{l=1}^{L} c_z z_l \\
\text{s.t.} \quad & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} x_{ijsl} \le g z_l, && l = 1,2,\ldots,L, \\
& \sum_{l=1}^{L} x_{ijsl} = 1, && \forall\, i,j,s, \\
& M y_{il} \ge \sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} (x_{ijsl} + x_{jisl}), && \forall\, i,j,s,l, \\
& x_{ijsl}, y_{il}, z_l \text{ are binary variables,}
\end{aligned}
$$

where $c_y \ge 0$ and $c_z \ge 0$ are weighted costs for ADMs and wavelengths, respectively. (ILP_3) can be easily modified to accommodate the nonuniform traffic case.

An alternative for minimizing the combination of wavelengths and ADMs is to minimize the number of wavelengths with an upper limit on the number of ADMs. This problem can be formulated as

$$
\begin{aligned}
(\mathrm{ILP}_4) \qquad \min \quad & \sum_{l=1}^{L} z_l \\
\text{s.t.} \quad & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} x_{ijsl} \le g z_l, && l = 1,2,\ldots,L, \\
& \sum_{l=1}^{L} x_{ijsl} = 1, && \forall\, i,j,s, \\
& \sum_{i=1}^{N}\sum_{l=1}^{L} y_{il} \le A, \\
& M y_{il} \ge \sum_{j=1}^{N}\sum_{s=1}^{m_{ij}} (x_{ijsl} + x_{jisl}), && \forall\, i,j,s,l, \\
& x_{ijsl}, y_{il}, z_l \text{ are binary variables,}
\end{aligned}
$$

where A is the upper limit on the number of ADMs.

It is clear that in both cases Theorem 1 still holds if the traffic is uniform. Before closing this section, we want to note that even though for the traffic grooming problem minimizing the number of ADMs does not necessarily result in the minimum number of wavelengths required, for most rings that we have studied so far it is often the case that the minimum number of ADMs is achieved when the minimum number of wavelengths is used. In fact, in Ref. 4 this result is conjectured for the ring with all-to-all uniform traffic, and in Ref. 14 it is proven that the conjecture is true for $g = 4$.

4.C. Dynamic Traffic

We now consider the traffic grooming problem with dynamic traffic. The problem of designing optical networks with dynamic traffic has been studied in several papers (e.g., see Refs. 2, 8, 15–18). There are two types of dynamic traffic considered: stochastic traffic and deterministic traffic. In the case of stochastic traffic, each traffic request (between two nodes) arrives according to a stochastic point process, and the duration of the traffic request may be a random variable. In the case of deterministic traffic, there is a set of different traffic requirements that the network needs to satisfy, but at different times. For example, the different traffic requirements can be a result of traffic fluctuation in different operation periods (morning, afternoon, and evening). Here we consider the deterministic case. Assume that there is a total of T traffic requirements. We use the superscript t ($t = 1, 2, \ldots, T$) to indicate quantities associated with traffic requirement t. For example, m^t_{ij} is the number of circuits between nodes i and j in traffic requirement t, and x^t_{ijsl} is the binary variable indicating whether the sth circuit between nodes i and j in traffic requirement t is assigned to wavelength l. Then the dynamic traffic grooming problem is

$$
\begin{aligned}
(\mathrm{ILP}_5) \quad \min \quad & \sum_{i=1}^{N}\sum_{l=1}^{L} y_{il} & \\
\text{s.t.} \quad & \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{s=1}^{m^t_{ij}} x^t_{ijsl} \le g, & l = 1, 2, \ldots, L, t = 1, 2, \ldots, T, \\
& \sum_{l=1}^{L} x^t_{ijsl} = 1, & \forall\, i, j, s, t, \\
& M y_{il} \ge \sum_{t=1}^{T}\sum_{j=1}^{N}\sum_{s=1}^{m^t_{ij}} (x^t_{ijsl} + x^t_{jisl}), & \forall\, i, l, \\
& x^t_{ijsl}, y_{il} \text{ are binary variables} &
\end{aligned}
$$

Again, Theorem 1 holds for (ILP_5).

5. Numerical Results

In this section we examine two sets of numerical examples. All ILPs and MILPs were solved by use of CPLEX 7.0 on a Pentium III 800-MHz PC. In the first example we compare $\mathrm{MILP}_1^{\mathsf{C}}$, MILP_1, and ILP_1 and also their solutions with optimal solutions. In the second example, we compare $\mathrm{MILP}_1^{\mathsf{C}}$ and $\mathrm{MILP}_1^{\mathsf{V}}$.

Example 1. Here we consider a ring network with all-to-all uniform traffic (i.e., $m_{ij} = 1$ for $i < j$) and $g = 4$. For this network it can be shown (Ref. 14) that the minimum number of ADMs required is $N(N-1)/2$ (recall that N is the number of nodes). We choose this network for testing since its optimal solution can be obtained analytically. The results obtained from the three different formulations $\mathrm{MILP}_1^{\mathsf{C}}$, MILP_1, and ILP_1 along with the optimal solutions are given in Table 1. In all the cases except $N = 15$, we forced the CPLEX program to terminate after 1000 s if it was still running, and therefore the best solution obtained at termination may not be optimal. For $N = 15$, the termination time was set to 3000 s. For each formulation and every N, the best solution found when the CPLEX program was terminated is provided in Table 1, along with the time (in seconds) it took the CPLEX program to obtain the solution. For example, for $\mathrm{MILP}_1^{\mathsf{C}}$ and $N = 10$, the best solution is 45, which was obtained in 160 s (and it remained as the best solution when the CPLEX program was terminated in 1000 s). We mark the time with an asterisk if it is the time when the CPLEX program stopped by itself, in which case the corresponding solution must be optimal. For example, for MILP_1 and $N = 6$ the CPLEX program terminated in 33 s with the optimal solution 15. From Table 1, the following observations can be made:

- Even for $N = 7$ the CPLEX program was still running after 1000 s. Hence it is computationally infeasible to run the CPLEX program until it stops by itself (in which

Table 1. Numerical Results for Example 1 (all-to-all traffic and $g = 4$)

		$MILP_1^C$		$MILP_1$		ILP_1	
N	Optimum	Solution	Time	Solution	Time	Solution	Time
5	10	10	1*	10	1*	10	1*
6	15	15	15*	15	33*	15	20*
7	21	21	1	21	90	21	215
8	28	28	1	29	2	30	400
9	36	36	16	38	2	38	25
10	45	45	160	47	4	68	81
11	55	56	830	60	210	61	200
12	66	67	12	74	10	77	450
13	78	80	100	86	22	89	920
14	91	93	800	104	110	N/A	1000
15	105	108	1950	118	70	N/A	3000

case we know for sure that the solution obtained is optimal). Therefore in practice we can run the program only for a fixed amount of time and obtain a solution that may not be optimal. However, it is also clear from Table 1 that in most cases the solutions obtained from $MILP_1^C$ are quite good after the program is run for a few minutes, and they are all within 5% of the optimal solutions.

- Among the three formulations, $MILP_1^C$ is clearly the best. For large N, it can provide a better solution within much shorter time. For example, for $N = 13$, $MILP_1^C$ gives a solution with a value 80 in 100 s, whereas $MILP_1$ (resp. ILP_1) returns a solution with a value of 86 (resp. 89) when the program was terminated in 1000 s. For $N = 14$, ILP_1 does not produce a feasible solution even after 1000 s.

Example 2. We now consider a ring network from Refs. 10 and 4 with $g = 16$. The traffic in this ring network is distance dependent, and the amount of traffic between two nodes is inversely related to the hop distance separating them. Specifically, we have $m_{ij} = \lceil (N+1)/2 \rceil - \text{distance}(i, j)$ for $i < j$. For example, for $N = 6$ we have

$$(m_{ij}) = \begin{pmatrix} 0 & 3 & 2 & 1 & 2 & 3 \\ 0 & 0 & 3 & 2 & 1 & 2 \\ 0 & 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

In this example, we compare solutions obtained from $MILP_1^V$ and $MILP_1^C$. The solutions and their run times (in seconds), presented in Table 2, clearly indicate that $MILP_1^V$ is much more efficient than $MILP_1^C$ as N increases. As an example, for $N = 16$, $MILP_1^C$ took 907 s to find a solution with a value of 152, whereas $MILP_1^V$ only took 287 s to find a much better solution with a value of 130.

Before closing this section, we should point out that we did not make any attempt to optimize our linear programming implementation in CPLEX, which in some cases could dramatically reduce the run time. In addition, there are various parameters in CPLEX one can adjust to speed up the program. Our emphasis here is to compare the efficiency of different ILP and MILP formulations we developed for the traffic grooming problem and to demonstrate the computational feasibility of our method.

Table 2. Numerical Results for Example 2 (distance dependent traffic and $g = 16$)

N	$MILP_1^V$ Solution	$MILP_1^V$ Time	$MILP_1^C$ Solution	$MILP_1^C$ Time
11	46	3	47	65
12	61	10	62	60
13	71	41	75	330
14	97	263	106	271
15	103	121	112	387
16	130	287	152	907

6. Conclusion

In this paper we have studied the traffic grooming problem for WDM rings. An ILP formulation was proposed for the problem and it was also demonstrated how the ILP problem can be converted into an equivalent MILP problem in which the number of constraints and the number of integer decision variables can be reduced significantly. Not only does it take less time to solve the resulting MILP problem; better solutions can be obtained. Numerical results were also provided that demonstrate that for WDM rings found in most applications (often with no more than 16 nodes) our method can produce optimal or near-optimal solutions in a few seconds or minutes (not hours or days).

There are several interesting future research directions indicated by the research in this paper. First, we can significantly improve the presented linear programming method by exploring more efficient ways to solve our MILP problems; e.g., there are various ways to improve the linear programming implementation in CPLEX to reduce the run time. Second, the linear programming method can be used in combination with heuristic and other methods; e.g., heuristic methods may be used to enhance the branch and bound search procedure in solving our MILP problems. Third, we considered just the UPSR problem in this paper; however, it would be interesting to apply the method to the BLSR problem as well. Finally, additional numerical studies are needed to compare the presented method with other methods that have been proposed for WDM rings.

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