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The Queueing Equivalence to a Manufacturing System with Failures

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***Abstract*—We consider optimal production rate control in a failure prone manufacturing system. It is well known that the hedging point policy is the optimum controller for such a system. We show that under the hedging point policy the system can be treated as an M/M/1 queue. Therefore, the existing results in queueing theory can be readily applied to obtaining the steady-state probability density function of the production surplus, based on which the optimal hedging point policy can be computed. To a large extent, our approach is based on sample path analysis. It not only provides an alternative way to solve the problem but also reveals some interesting insights. Furthermore, the approach can be potentially applied to problems which we may find hard to deal with using conventional means.**

I. INTRODUCTION

The problem of optimal production rate control in a failure prone manufacturing system has received considerable attention since the seminal work by Kimemia and Gershwin [1]. Based on a method proposed by Rishel [2], they showed that the optimal control has a special structure called *hedging* point policy. In such a policy, a nonnegative production surplus of part types is maintained during times of excess capacity availability to hedge against future capacity shortages brought about by machine failures. Later, Gershwin, Akella, and Choong [3] proposed an approximation procedure to solve the optimal control problem. The exact solution to the problem was first provided by Akella and Kumar [4] and Bielecki and Kumar [5] for a special case in which the system has a single machine and processes a single-part type. The result of [5] was later extended by Sharifnia [6] to the case of multiple machines and a single-part type. Algoet [7] considered the multiple machines and multiple-part type case. He derived the partial differential equation characterizing the steady-state probability density function under a given hedging point policy; however the resulting equation is in general intractable. As an alternative, Caramanis and Sharifnia [8] proposed a near-optimal controller design which is analytically tractable. Recently, Caramanis and Liberopoulos [9] successfully applied the technique of perturbation analysis to designing a nearby optimum controller.

In this note, we study the system in [5] which consists of one machine with two states (up and down) and one part type. Our main contribution is to show that the system under the hedging point policy is "equivalent" to a GI/G/1 queue, in which the service and interarrival times correspond to the down and up times of the system, respectively. Especially, if the down and up times are exponentially distributed as usually assumed in the literature, then the system is "equivalent" to an M/M/1 queue. To be more precise, we show that the surplus of the system at the instances when the machine is up and down is governed by a recursive equation which is the same as the so-called Lindley equation for a GI/G/1 queue. Our approach is based on sample path analysis. Upon establishing this relationship, we can easily obtain the steady-state distribution function of the production surplus, which can be used to find the value of the optimum hedging point. The previous approach to obtain the steady-state distribution function is to solve a differential equation of the density function ([5]). Our method is relatively simpler, and perhaps more important is that it can be applied to systems with more general up and down times to which the previous approaches are not applicable. In addition to revealing the interesting connection between the optimal control problem of failure prone manufacturing systems and single-server queueing systems, our results also support the possibility of using sample path analysis in future studies. Finally, we point out that Chen and Yao [13] used a similar method as ours to study a fluid system with random disruptions (also see Kella and Whitt [14]). Interestingly, their results are also similar to what we obtained in this note.

The rest of this note is organized as follows. In Section II, the system of interest is presented and the dynamic recursive equation describing the evolution of its sample path under the hedging policy is derived. Based on this recursive equation we show that the system is in fact "equivalent" to an M/M/1 queue. In Section III, we derive the steady-state probability

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Manuscript received March 15, 1991; revised September 6, 1991 and January 24, 1992. This work was supported by the National Science Foundation under Grant ECS-8806932 and by the Alexander Onassis Foundation under Grant GROUP L-89/90.

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IEEE Log Number 9204965.

0018-9286/93$03.00

distribution function of the production surplus using the standard queueing theory results. Section IV contains some concluding remarks.

II. THE FAILURE PRONE SYSTEM AND ITS QUEUEING EQUIVALENCE

The system under consideration has a single machine and produces a single-part type. The system satisfies a constant demand rate d and backlog is allowed. The machine has two states: up and down. When the machine is up, it can produce at a maximum rate r. The machine state changes in continuous time according to a Markov chain, i.e., both the up and down times are exponentially distributed, say, with rates λ_u and λ_d, respectively. We use $\alpha \in \{0, 1\}$ to denote the state of the machine, where $0 =$ down and $1 =$ up. Denote the production surplus (positive or negative, negative surplus corresponds to a backlog) at time t by $X(t)$ and the state of the machine at time t by $\alpha(t)$. $\{X(t)\colon t \geq 0\}$ is a stochastic process characterized by the following differential equation

$$\frac{dX(t)}{dt} = u(t) - d \tag{1}$$

where $u(t)$ is the controlled production rate of the machine at time t. The allowable controls satisfy $0 \leq u(t) \leq \alpha(t)r$. Let $g(x)$ be the cost of maintaining a production surplus at x, where $g(x)$ is a convex function of x. Our goal is to minimize the long-run average expect cost

$$J = \lim_{T \to \infty} \frac{1}{T} E \int_0^T g(X(t))\, dt. \tag{2}$$

It can be proven that the optimal control of the above stochastic control problem takes the following form ([5]):

$$u(t) = \begin{cases} r & \text{if } x(t) < z \text{ and } \alpha(t) = 1; \\ d & \text{if } x(t) = z \text{ and } \alpha(t) = 1; \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where z is some constant. The control policy defined by (3) is the so-called hedging point policy and z is the hedging point. Unless otherwise stated, the rest of this note focuses on the hedging point policy. We show next that the system dynamics under the hedging point policy is very similar to those of an M/M/1 queue.

We now take a close look at a sample path of the system. For the sake of convenience, we assume $x(0) = z$ and $\alpha(0) = 0$. Denote by $t_{d,n}$ the length of the nth down time and by $t_{u,n}$ the length of the nth up time. Therefore, $t_{d,n}$'s and $t_{u,n}$'s are i.i.d. exponential random variables with rate λ_d and λ_u. Let $T_{d,n}$ be the beginning of the epoch of the nth time the machine is down and $T_{u,n}$ be the beginning of the epoch of the nth time the machine is up, i.e.,

$$T_{d,n} = \sum_{i=1}^{n-1} (t_{d,i} + t_{u,i}) \quad \text{and} \quad T_{u,n} = T_{d,n} + t_{d,n}. \tag{4}$$

For simplicity of notation, we denote

$$X_{d,n} \triangleq X(T_{d,n}) \quad \text{and} \quad X_{u,n} \triangleq X(T_{u,n}).$$

We then have the following recursive equations for $X_{d,n}$ and $X_{u,n}$

$$X_{u,n} = X_{d,n} - t_{d,n}d \tag{5}$$

$$X_{d,n+1} = \min(z, X_{u,n} + t_{u,n}(r - d)) \tag{6}$$

with $X_{d,1} = z$. Equation (5) represents the unique dynamics when the machine is down. The "min" in (6) represents the hedging point policy, namely when the machine is up the production surplus increases at rate $r - d$ until either it hits the level z where it remains until the machine breaks down or the machine breaks down before it hits the level z. Letting $Y_{d,n} = z - X_{d,n}$ and $Y_{u,n} = z - X_{u,n}$, it follows from (5) and (6) that

$$Y_{u,n} = Y_{d,n} + t_{d,n}d \tag{7}$$

$$Y_{d,n+1} = \max(0, Y_{u,n} - t_{u,n}(r - d)) \tag{8}$$

with $Y_{d,1} = 0$. The recursive equations (7) and (8) are identical to the Lindley recursive equation for a GI/G/1 queue with service times $\{t_{d,n}d\colon n = 1, 2, \cdots\}$ and interarrival times $\{t_{u,n}(r - d)\colon n = 1, 2, \cdots\}$. Since both $\{t_{d,n}d\colon n = 1, 2, \cdots\}$ and $\{t_{u,n}(r - d)\colon n = 1, 2, \cdots\}$ are i.i.d. exponential random variables, it is in fact an M/M/1 queue. $Y_{d,n}$ and $Y_{u,n}$ correspond to the waiting time and the system time of job n in the M/M/1 queue, respectively. After establishing this connection, we proceed to derive the steady-state distribution function of the production surplus. The results available from the M/M/1 queue analysis can be readily used once the relationship between the workload of the M/M/1 queue and the production surplus, X, has been established.

III. THE STEADY-STATE PROBABILITY FUNCTION

As already pointed out, the optimal control is the hedging point policy described by (3). Therefore, in order to find the optimal control we only need to calculate the optimal value of the hedging point which minimizes J. If $\{X(t)\colon t \geq 0\}$ has steady-state probability distribution function $F_z(x)$ under the hedging point policy (the condition under which the process $\{X(t)\colon t \geq 0\}$ is stable and ergodic will be given in the later of this section), then the average expected cost J defined in (2) can be computed as the expected cost with respect to $F_z(x)$, i.e.,

$$J(z) = \int_{-\infty}^{\infty} g(x)\, dF_z(x). \tag{9}$$

Therefore, if we can obtain $F_z(x)$, the original optimal control problem becomes a simple optimization problem. In this section, we derive $F_z(x)$ using the relationship established in the last section.

Let $Y(t) = z - X(t)$ and define the following two random time transformations

$$\phi_d(t) = \int_0^t 1(\alpha(s) = 0)\, ds;$$

$$\phi_u(t) = \int_0^t 1(\alpha(s) = 1)\, ds$$

and their inverse functions

$$\tau_d(t) = \phi_d^{-1}(t) = \inf\{\tau\colon \phi_d(\tau) > t\};$$

$$\tau_u(t) = \phi_u^{-1}(t) = \inf\{\tau\colon \phi_u(\tau) > t\}.$$

Define also two processes $\{Y_d(t)\colon t \geq 0\}$ and $\{Y_u(t)\colon t \geq 0\}$ as

$$Y_d(t) = Y(\tau_d(t)) \quad \text{and} \quad Y_u(t) = Y(\tau_u(t)).$$

Intuitively, $\phi_d(t)$ [respectively $\phi_u(t)$] is the total down (respectively up) time of the machine during the time interval $[0, t]$. The process $\{Y_d(t)\colon t \geq 0\}$ (respectively $\{Y_u(t)\colon t \geq 0\}$) corresponds to the part of the process $\{Y(t)\colon t \geq 0\}$ when the machine is down (respectively up). This is illustrated in Fig. 1. Noticing that $Y_{d,n} = Y(T_{d,n})$ and $Y_{u,n} = Y(T_{u,n})$, we can easily see that $Y_u(t)$ is

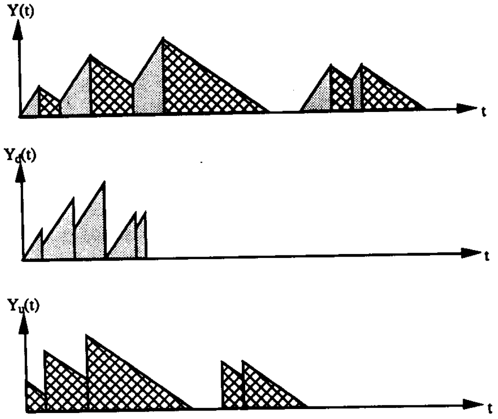

Fig. 1. The relationship between $Y(t)$, $Y_d(t)$, and $Y_u(t)$.

the workload process of the M/M/1 queue with service requirements $\{t_{d,n}d: n = 1, 2, \cdots\}$ and interarrival times $\{t_{u,n}(r-d): n = 1, 2, \cdots\}$ based on the recursive equations (7) and (8). (Strictly speaking, the M/M/1 queue has the interarrival times $\{t_{u,n}: n = 1, 2, \cdots\}$ and service rate $r - d$; however, the workload processes in the two queues have the same statistics in steady state.) Therefore, it immediately follows that $\{Y_u(t): t \geq 0\}$ has the steady-state probability distribution function

$$F_{Y_u}(x) = \begin{cases} 1 - \rho e^{-\mu(1-\rho)x} & x \geq 0; \\ 0 & x < 0, \end{cases} \tag{10}$$

and density function

$$f_{Y_u}(x) = \rho\mu(1-\rho)e^{-\mu(1-\rho)x} \tag{11}$$

where $\mu = (\lambda_d/d)$ and $\rho = (d\lambda_u/(r-d)\lambda_d)$ (e.g., see [10, pp. 196–206]). Furthermore, we know that the process $\{Y_u(t): t \geq 0\}$, as well as the process $\{Y(t): t \geq 0\}$, are stable and ergodic if and only if $\rho < 1$ (see [11, ch. 7]).

Define $D_x(t)$ to be the downcrossing counting process of $Y(t)$ at level x during the interval $[0, t]$ and $U_x(t)$ the upcrossing counting process. Notice the total time that the process $Y(t)$ takes value between the interval $[x, x + \delta x)$ during $[0, t]$ is equal to

$$\frac{U_x(t)\delta x}{d} + \frac{D_x(t)\delta x}{r-d}.$$

Therefore, the steady-state probability density function of $\{Y(t): t \geq 0\}$ is given by

$$f_Y(x) = \lim_{t\to\infty} \frac{1}{t}\left(\frac{U_x(t)}{d} + \frac{D_x(t)}{r-d}\right) \quad \text{for } x > 0. \tag{12}$$

(The above formula can be derived rigorously by using the standard analysis of level crossings, e.g., see [12, pp. 363–373].) Since

$$\lim_{t\to\infty} \frac{U_x(t)}{t} = \lim_{t\to\infty} \frac{D_x(t)}{t}$$

we have from (12) that

$$\begin{aligned} f_Y(x) &= \frac{r}{d} \lim_{t\to\infty} \frac{\phi_u(t)}{t} \frac{D_x(t)}{(r-d)\phi_u(t)} \\ &= \frac{r}{d} \frac{\lambda_d}{\lambda_d + \lambda_u} \lim_{\phi_u(t)\to\infty} \frac{D_x(t)}{(r-d)\phi_u(t)}. \end{aligned} \tag{13}$$

$D_x(t)$ is also the downcrossing process of $Y_u(\phi_u(t))$ since $Y_u(\phi_u(t)) = Y(t)$, when $\alpha(t) = 1$. This implies that the limit in the most right-hand side of (13) is equal to $f_{Y_u}(x)$, which is given by (11). Therefore, we have

$$f_Y(x) = \frac{r}{d} \frac{\lambda_d}{\lambda_d + \lambda_u} \rho\mu(1-\rho)e^{-\mu(1-\rho)x} \quad \text{for } x > 0 \tag{14}$$

which yields

$$F_Y(x) = \begin{cases} 1 - \dfrac{r}{d} \dfrac{\lambda_d}{\lambda_d + \lambda_u} \rho e^{-\mu(1-\rho)x} & x \geq 0; \\ 0 & x < 0. \end{cases} \tag{15}$$

Since $X(t) = z - Y(t)$, we finally obtain

$$F_z(x) = \begin{cases} 1 - \dfrac{r}{d} \dfrac{\lambda_d}{\lambda_d + \lambda_u} \rho e^{-\mu(1-\rho)(z-x)} & x \leq z; \\ 0 & x > z, \end{cases} \tag{16}$$

which is consistent with the result in [5].

Having obtained the steady-state probability distribution of $X(t)$, we can now easily calculate the optimum value of the hedge point z. For example, if $g(x) = x^2$, then

$$\begin{aligned} J(z) &= \int_{-\infty}^{\infty} g(x)\, dF_z(x) \\ &= z^2 - \frac{2r\lambda_d\rho}{\mu(1-\rho)d(\lambda_d + \lambda_u)} z \\ &\quad + \frac{2r\lambda_d\rho}{\mu^2(1-\rho)^2 d(\lambda_d + \lambda_u)}. \end{aligned}$$

Therefore, solving $dJ(z)/dz = 0$, we obtain the optimum hedge point

$$z^* = \frac{r\lambda_d\rho}{\mu(1-\rho)d(\lambda_d + \lambda_u)}.$$

IV. CONCLUDING REMARKS

We have shown that the failure prone manufacturing system under the hedging point policy is "equivalent" to an M/M/1 queue. We can obtain the steady-state probability distribution of the production surplus by utilizing the standard queueing theory results. The equivalence is established using sample path analysis. It is quite clear that our analysis can also be applied to systems with more general down and up time distributions. We should especially point out that if the system has general instead of exponential down time distribution, which has a reasonable representation of reality, it is then equivalent to an M/G/1 queue whose steady-state workload distribution has a known simple solution ([10]). It should be noted in this case that the steady-state distribution can not be obtained using past approaches ([5]–[7]). Of course, it has not been proven that the hedging point policy is still optimal for systems with general down times, though we strongly believe it is true. This is an ongoing research topic.

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Hautus-Type Conditions for Controllability of Implicit Linear Discrete-Time Systems Revisited

F. L. Lewis and K. M. Przyłuski

Abstract—**New simple proofs of validity of recently established Hautus-type conditions for controllability of (not necessarily regular) implicit linear discrete-time systems are given. The new proofs do not require any previous knowledge of implicit systems theory.**

I. INTRODUCTION

In a recent paper [3] Hautus-type conditions for controllability of implicit linear discrete-time systems have been established. The purpose of the present note is chiefly pedagogical: we give here very direct proofs of validity of these conditions. The content of the note is as follows. In the next section we specify the class of systems we consider. In Section III we define four (in general, not equivalent) concepts of controllability we will study. Then, in Section V, after auxiliary results of Section IV, the corresponding Hautus-type conditions are provided. The last section of the note (Section VI) suggests some possible applications of these conditions.

For an implicit system given by a triple (E, F, G) the Hautus-type conditions for controllability are as follows: $\operatorname{rank}[sE - F \vdots G] = \operatorname{rank}[E \vdots F \vdots G]$, for all s; $\operatorname{rank}[sE - F \vdots G] = \operatorname{rank}[E \vdots F \vdots G] = \operatorname{rank}[E \vdots G]$, for all s; the matrix $[sE - F \vdots G]$ has full-row rank for all s; the last condition holds and in addition the matrix $[E \vdots G]$ has full-row rank. Let us note that such conditions are also encountered in the theory of implicit systems when studying problems of realization and equivalence (see [8], [15]).

Since it is widely accepted that Hautus tests for controllability of standard systems are useful and powerful tools for their theoretical analysis (see [9], [11]) the same is to be expected in the case of implicit systems. This motivates the need for simple proofs of the validity of Hautus-type tests for controllability of implicit systems. As we previously mentioned, Hautus-type conditions for controllability of implicit systems have been established in [3]. Whereas the proofs reported in [3] are not difficult, their good understanding requires quite serious knowledge of various results and concepts from the existing theory of implicit systems. This includes familiarity with geometrical notions of these theory as given in [2] and several results of [5]. In opposition to [3], the present paper is a self contained and very elementary exposition of the subject. In particular, here we give new direct proofs of all of the auxiliary results we need to avoid by way of long chains of logical dependence. This should make the content of our note easily accessible to the general control-theoretic audience.

The note is intended to be a short one and therefore we do not discuss here related results and methods of the theory of implicit systems. However, in Sections III and V, we provide brief remarks establishing a connection between the concepts we are studying and those known from other works. The interested reader will find more information on implicit systems in [1], [7], [12]–[14].

Lastly, it is fair to mention that in the present note we are making free use of many general ideas of [2], [3], [5].

II. SYSTEMS DESCRIPTION

Let E, F, and G be fixed matrices with real entries. We assume that all these matrices have the same number of rows, say q, and that E and F have the same number of columns, denoted by n. (The number of columns of the matrix G is unimportant for us.) By (E, F, G) we shall denote the following *implicit system*:

$$Ex_{k+1} = Fx_k + Gu_k. \tag{1}$$

In the next section, we recall four (in general, not equivalent) definitions of controllability for the class of systems we want to consider. Two of these definitions are standard in the sense that they directly refer to the solutions of (1). It happens however that for implicit systems those standard concepts of controllability are too weak to be dual to naturally defined notions of observability (see [3], [4]). It is not difficult to observe that this is caused by the fact that the familiar concepts of controllability do not take explicitly into account the space in which (1) residues. One way to cope with this difficulty is to introduce an extra term into (1). This leads to the following *implicit system with disturbances*:

$$Ex_{k+1} = Fx_k + Gu_k + z_k. \tag{2}$$

Manuscript received May 31, 1991; revised April 3, 1992. This work was supported in part by the National Science Foundation under Grant ECS-8805932.

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IEEE Log Number 9204967.