Fig. 3. Parameter estimation of $c_{21}$.

Fig. 4. Parameter estimation of $g_{12}$.

VIII. CONCLUSION

A simple method for recursive identification of MISO Wiener–Hammerstein model was performed. It is shown, by means of an attractive transformation, that parameters to be estimated are those of each subsystem of the initial and unique realization. Sufficient conditions are given to assure the convergence of the algorithm where the sequence $\{\lambda_n\}$ is introduced to enlarge the interval $[1 - \sqrt{\Delta_n}, 1 + \sqrt{\Delta_n}]$ and then to enhance the convergence of the proposed procedure, this was illustrated by means of a numerical example. Finally, some important remarks to deal with the initialization problem, in order to ensure condition (54), are given.

REFERENCES


Production Rate Control for Failure-Prone Production Systems With No Backlog Permitted

Jian-Qiang Hu

Abstract—Previously, the problem of optimal production rate control for failure-prone production systems has been studied exclusively under the assumption that backlog is permitted. It is well known that when backlog is permitted, the optimal control is usually the hedging point policy. In this note, we consider systems in which backlog is not allowed. We show that the hedging point policy is still optimal. For systems with backlog, it is usually quite straightforward to show that their optimal cost-to-go functions are convex—a key property that is needed for the hedging point policy to be optimal. With no backlog permitted, it becomes much more difficult to establish the convexity property, and the explicit formulas for the optimal hedging point and the optimal cost-to-go functions have to be obtained, based on which the convexity property can then be verified. The method we use in this note to derive these explicit formulas is mainly based on an interesting relationship between the inventory process of the system under the hedging point policy and some stochastic process which is well studied in queueing theory.

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I. INTRODUCTION

In this note, we consider the problem of optimal production rate control for failure-prone production systems. This problem has been studied extensively by many researchers in the literature (for a review and a comprehensive list of references on the subject, see a recent monograph by Gershwin [4]). A key assumption used in all the previous work is that backlog of demands is allowed. This assumption usually implies that the instantaneous cost function on inventory level (negative inventory being backlog) is convex (in most cases, it is either piecewise linear or quadratic function of inventory level), which can immediately lead to convexity of optimal cost-to-go functions based on a simple sample path argument. Using the convex property and the dynamic Bellman equation for the optimal cost-to-go functions, one can then easily show that the optimal control is a hedging point policy, in which a (nonnegative) production inventory level of part types is maintained during times of excess capacity availability to hedge against future capacity shortages brought about by machine failures. Intuitively, the hedging point is safety production inventory and is carried to protect the production process from the uncertainty in future capacity availability.

Although the backlog assumption is reasonable for many systems, it may not hold for a large class of production systems in which unsatisfied demands are lost. One such example is a production line in which machines are linked in tandem. In such a system, the demand rate for each machine (except the last one) is simply equal to the production rate of its downstream machine, and obviously, in this case, production inventory level between any two machines can never become negative (i.e., backlog is not permitted). This motivates us to consider the problem of optimal production rate control for systems in which backlog is not permitted. As we shall see, for a system with no backlog permitted, it becomes much more difficult to establish convexity for its optimal cost-to-go functions. This is simply because, in this case, the instantaneous cost function on inventory level is no longer convex, hence the simple sample path argument used before when backlog is permitted cannot be applied. Instead, the explicit formulas for the optimal cost-to-go functions have to be derived.

To derive the optimal cost-to-go functions, we first focus on hedging point policies and try to obtain the optimal hedging point policy and its associated cost-to-go functions. Clearly, if the optimal control policy is a hedging point policy, then the optimal cost-to-go functions are simply equal to the cost-to-go functions associated with the optimal hedging point policy. To obtain the optimal hedging point policy, we use a method which has recently been developed by Hu and Dong [5, 6] to study the systems with backlog permitted. The basic idea of the method is to establish a relationship between the inventory processes of failure-prone production systems and some well-studied stochastic processes in queueing systems. In our case, we show that the inventory process of the system under a hedging point policy is related to the workload process of a single-node queueing system with limited workload. Based on this relationship and existing results in queueing theory, we can obtain the probability distribution function of the inventory process, from which we can then find the optimal hedging point. Finally, with the optimal hedging policy and its cost-to-go functions, we can use the verification theorem to verify that the optimal hedging point policy is indeed optimal overall. We point out that it is also possible for one to obtain the probability distribution function of the inventory process for our system based on a different approach used in [1], [2], [8], in which a system of two differential equations needs to be solved under two boundary conditions. However, the method used in [1], [2], [8] can only be applied to Markov systems (i.e., both machine up and down times have to be exponentially distributed), while our method is applicable to non-Markov systems as well.

The remainder of the note is organized as follows. In Section II, the problem of optimal production rate control for the failure-prone production system with no backlog permitted is formulated as an optimal control problem whose optimality condition is given by a dynamic Bellman equation. In Section III, we establish an important relationship between the inventory process of the system under a hedging point policy and the workload process of a single-node queue with bounded workload. Based on this relationship, we obtain the probability distribution function for the inventory process. We then proceed in Section IV to derive the explicit formulas for the optimal hedging point, the optimal value of the cost function, and the optimal cost-to-go functions. In Section V, we present the verification theorem, based on which we prove that the optimal hedging point policy obtained in Section IV is the optimal control policy. Finally, some discussions and future research directions are presented in Section VI.

II. PROBLEM FORMULATION

We consider a production system which has a single machine and produces a single part type. The system tries to meet a constant demand rate $d$, and backlog is not permitted, i.e., unsatisfied demands are lost. The machine has two states: up and down. When the machine is up, it can produce at any rate between zero and a maximum rate $r$. We assume that $r > d$. The machine state changes in continuous time according to a homogeneous Markov process: the state changes from down to up at a rate $q_d$, and from up to down at a rate $q_u$, i.e., both machine up and down times are exponentially distributed with rates $q_d$ and $q_u$, respectively. We use $i(x,t)$ to denote the state of the machine, where one corresponds to up and zero to down. Denote the production inventory at time $t$ by $x(t)$. Let $j(t)$ be the state of the machine at time $t$, and let $u(t)$ be the controlled production rate of the machine at time $t$. Then $x(t)$ is characterized by the following differential equation:

$$\frac{dx}{dt} = \begin{cases} u(t) - d, & \text{if } i(t) = 1, x(t) > 0 \\ 0, & \text{if } i(t) = 0 \text{ and } x(t) = 0 \end{cases}$$

(1)

where $u(t) \in [0, r]$ when $i(t) = 1$ and $u(t) \in [d, r]$ if $x(t) = 0$, and $u(t) = 0$ when $i(t) = 0$. The objective is to find a stationary, feedback control law, $u(x, i)$, so as to minimize the following long-run expected average cost:

$$J = \lim_{T \to \infty} \frac{1}{T} \mathbb{E} \left[ \int_0^T (c(x,t) + c_u(j(t) = 0, i(t) = 0)) \, dt \right]$$

(2)

where $c$ is the unit holding cost for inventory, $c_u$ is the unit shortage cost for unsatisfied demand, and $1(\cdot)$ is the indicator function. It should be pointed out that the instantaneous cost in the objective function, $g(x,i) = cx + 1(i = 0, j = 0)$, is not a convex function with respect to $x$.

Under suitable regularity conditions imposed on the control (e.g., [7]), the optimal feedback control of the system is stationary and characterized by the following Bellman equations:

$$\min_{i \geq 0} \left\{ \frac{dV_i(x)}{dx}(d - d) - q_u[V_i(x) - V_0(x)] + cx - J^* = 0, \right.$$

$$\text{for } x \geq 0$$

(3)

$$-\frac{dV_0(x)}{dx}d - q_u[V_0(x) - V_1(x)] + cx - J^* = 0,$$

$$\text{for } x > 0$$

(4)

where $V_i(x)$ ($i = 1, 0$) are (differential) cost-to-go functions and $J^*$ is the minimum long-run expected average cost associated (i.e.,
$J^* = \min J$). According to (3), if $V_i(x)$ is convex with minimum point $z^*$, then the optimal production rate satisfies

$$ u^*(x, i) = \begin{cases} 0, & \text{if } x > z^* \text{ and } i = 1 \\ d, & \text{if } x = z^* \text{ and } i = 1 \\ r, & \text{if } x < z^* \text{ and } i = 1 \\ \end{cases} \quad (5) $$

which is hedging point policy, and $z^*$ is the hedging point. If the instantaneous cost function were convex, then one could easily show that $V_i(x)$ is also convex based on a simple sample argument ([10]). However, as already mentioned above, the instantaneous cost function in our case is not convex, hence the explicit formula for $V_i(x)$ has to be obtained in order to establish its convexity.

III. HEDGING POINT POLICY

In this section, we focus exclusively on the hedging point policy defined by (5) with hedging point $z$. We want to derive the steady-state probability distribution function for the inventory process under the hedging point policy. Our method consists of two basic steps. First, we establish a relationship between the inventory process and the workload process of a single-node queueing system with bounded workload. Second, we use this relationship to derive the probability distribution function for the inventory process based on existing results in queueing theory and a level crossing technique. This method was first used by Hu and Dong [5], [6] to study systems with backlog. It is worth pointing out that a different approach proposed in [1], [2], [8] can also be used to obtain the probability distribution function in our case. The basic idea of their method is to solve a set of differential equations. However, we note that the method of [1], [2], [8] can only be applied to Markov systems, while ours is applicable to non-Markov systems as well (i.e., the machine up and down times do not have to be exponentially distributed).

To start, we take a close look at a sample path of the system under the hedging point policy. For simplicity, let us assume $x(0) = z$ and $i(0) = 0$. Denote by $d_u$ the length of the $nth$ down time and by $t_{u,n}$ the length of the $nth$ up time. Therefore, $d_u$ and $t_{u,n}$ are i.i.d. exponential random variables with rates $\gamma_0$ and $\gamma_1$. Let $d_{u,n}$ be the beginning of the epoch of the $nth$ time the machine is down and $t_{u,n}$ be the beginning of the epoch of the $nth$ time the machine is up, i.e.,

$$ T_{d_{u,n}} = \sum_{i=1}^{n-1} (d_u + t_{u,i}) \quad \text{and} \quad T_{t_{u,n}} = T_{d_{u,n}} + t_{d,u,n}. \quad (6) $$

To simplify notation, we denote

$$ x_{d,u}, \overset{\Delta}{=} x(T_{d_{u,n}}) \quad \text{and} \quad x_{t,u}, \overset{\Delta}{=} x(T_{t_{u,n}}). $$

We then have the following recursive equations for $x_{d,u}$ and $x_{t,u}$;

$$ x_{d,u} = \max \left( x_{d,u} - t_{d,u} d, 0 \right) \quad (7) $$

$$ x_{t,u+1} = \min \left( x_{t,u} + t_{u,n} \rho \right) \quad (8) $$

with $x_{d,1} = z$. Equation (7) represents the unique dynamics when the machine is down, where "max" means that the inventory level cannot fall below zero (i.e., backlog is not permitted). The "min" in (8) represents the hedging point policy, namely, when the machine is up, the inventory level increases at rate $r - d$ until either it hits the level $z$ where it remains until the machine breaks down, or the machine breaks down before it hits the level $z$. The trajectory of $x(t)$ is illustrated in Fig. 1.

Define the following two random time transformations:

$$ \tau_u(t) = \int_0^t {1(i(s) = 0)} ds $$

and their inverse functions

$$ \tau_u(t) = \rho^{-1}(t) = \inf \{ r : \tau_u(r) > t \} $$

$$ \tau_u(t) = \rho^{-1}(t) = \inf \{ r : \tau_u(r) > t \}. $$

Define also two processes $\{ x_d(t) ; t \geq 0 \}$ and $\{ x_u(t) ; t \geq 0 \}$ as

$$ x_d(t) = x(\tau_d(t)) \quad \text{and} \quad x_u(t) = x(\tau_u(t)). $$

Intuitively, $\rho_u(t)$ (respectively, $\rho_u(t)$) is the total down (respectively, up) time of the machine during the time interval $[0, t]$. The process $\{ x_d(t) ; t \geq 0 \}$ (respectively, $\{ x_u(t) ; t \geq 0 \}$) corresponds to the part of the process $\{ x(t) ; t \geq 0 \}$ when the machine is down (respectively, up).

The process $\{ x_d(t) ; t \geq 0 \}$ is illustrated in Fig. 2.

It is not too difficult to verify that $x_d(t)$ is in fact the same as the workload process of an M/M/1 queue with backlogged workload, in which the interarrival times are $\{ t_{d,u} ; n = 1, 2, \ldots \}$, the service requirements are $\{ t_{u,n} ; r - d ; n = 1, 2, \ldots \}$, the service rate is $d$, and the upper bound on the workload process is $z$. Hence, the steady-state probability distribution function of the process $\{ x_d(t) ; t \geq 0 \}$ is given by

$$ F_{x_d}(x) = \begin{cases} 1, & x \geq z \\ \frac{1 - e^{-\mu(x - z)}}{1 - e^{-\mu z}} & 0 \leq x < z \\ 0 & x < 0 \end{cases} \quad (9) $$

where

$$ \mu = \frac{\gamma_1}{\rho - d} \quad \text{and} \quad \rho = \frac{\gamma_0 (r - d)}{q_0 d}. $$

(See Takacs [9]). Next we use $F_{x_d}(x)$ to calculate $F_{x_u}(x)$, the steady-state probability distribution function of the process $\{ x_u(t) ; t \geq 0 \}$. We use $f_{x_d}(x)$ and $F_{x_d}(x)$ to denote the density functions of $F_{x_d}(x)$ and $F_{x_u}(x)$, respectively.

Define $D_r(t)$ to be the downcrossing counting process of $x(t)$ at level $r$ during the interval $[0, t]$, and $C_r(t)$ the upcrossing counting process. Define $t_0 = 0$, and

$$ t_n = \inf \{ t : x(t) = z, \ i(t) = 0, t > t_{n-1} \} \quad \text{for } n \geq 1. $$

Fig. 1. The trajectory of $x(t)$.

Fig. 2. The trajectory of $x_d(t)$. 
By definition, it is clear that \( \{x(t), t \geq 0 \} \) is a regenerative process with regenerative points \( \{t_n, n \geq 0 \} \). Based on the regenerative theorem, we have

\[
F_X(x) = \frac{E\left[ \int_0^{t_1} 1(x(s) < x) \, ds \right]}{E[t_1]}. 
\]

Note that

\[
\int_0^{t_1} 1(x(s) < x + \delta x) \, ds - \int_0^{t_1} 1(x(s) < x) \, ds 
\]

is the total time that the process \( x(t) \) takes value between the interval \([r, x + \delta x]\) during \([0, t_1]\), which is also equal to

\[
\frac{U_{\delta x}(t_1) \delta x + D_{\delta x}(t_1) \delta x}{d} 
\]

for \( 0 < x < z \).

We further note that \( U_{\delta x}(t_1) = D_{\delta x}(t_1) \) for \( 0 < x < z \). Therefore, we have

\[
\frac{d}{dx} \int_0^{t_1} 1(x(s) < x) \, ds = \frac{r}{(r - d) d} E[D_{\delta x}(t_1)]. 
\]

This gives us

\[
f_X(x) = \frac{dF_X(x)}{dx} = \frac{r}{(r - d) d} \frac{E[D_{\delta x}(t_1)]}{E[t_1]}. 
\]

It is worth noting that \( z^* \) is the unique solution of \( b(z) = 0 \) (because \( db(z)/dz > 0 \)). However, \( z^* = 0 \) if the unique solution of \( b(z) = 0 \) is negative (which is the case if \( b(0) = c - c_0q_1 > 0 \)). Since we have assumed that the optimal control is the hedging point policy, we have \( J^* = J^* \) and furthermore, the Bellman equations (3) and (4) reduce to

\[
\frac{dV(x)}{dx} = \begin{cases} 
A_1 (V(x) - b_1[J^* - cx]), & \text{for } 0 < x \leq z^* \\
A_2 (V(x) - b_2[J^* - cx]), & \text{for } x \geq z^* 
\end{cases} 
\]

where

\[
A_1 = \left( \frac{2\rho}{b_1} - \frac{1}{b_1} \right), \quad A_2 = \left( \frac{2\rho}{b_2} - \frac{1}{b_2} \right) 
\]

\[
V(x) = \begin{cases} 
V_l(x), & \text{for } 0 < x \leq z^* \\
V_h(x), & \text{for } x \geq z^* 
\end{cases} 
\]

with boundary conditions

\[
c_0 - J^* + q_0 (V_l(0) - V_h(0)) = 0 \quad (16) 
\]

\[
c_0^* - J^* - q_1 (V_l(z^*) - V_h(z^*)) = 0. \quad (17) 
\]

Solving (15), we have, for \( 0 \leq x \leq z^* \),

\[
V_l(x) = \left[ I - \frac{A_1}{k_1} + \frac{A_1\rho}{k_1} (e^{k_1 x} - 1) \right] V_l(0) + \frac{A_1}{k_1} \left( J^* x - \frac{1}{2} c x^2 \right) + \frac{A_1 b_1}{k_1^2} \left[ J^* (1 - e^{k_1 x}) - c x + \frac{c}{k_1} (e^{k_1 x} - 1) \right] 
\]

where \( k_1 = \rho (1 - \rho) \), and for \( z^* \geq z^* \),

\[
V_h(x) = \left[ I - \frac{A_2}{k_2} + \frac{A_2\rho}{k_2} (e^{k_2 x - r}) \right] V_h(z^*) 
\]

\[
- \left( J^* (x - z^*) - \frac{1}{2} c x^2 (z^*)^2 \right) b_2 \left( \frac{1}{2} \right) 
\]

Finally, we calculate the long-run expected average cost \( J^* \) associated with the hedging point policy. Based on (2), it is quite clear that

\[
J^* = \int_0^r x f_X(x) \, dx + c_0 F_X(0) 
\]

\[
= \frac{1}{(q_0 + q_1)(1 - \rho e^{-\rho(1 - \rho)x})} \left[ c_0q_1(1 - \rho) + cr \rho \left( 1 - e^{-\rho(1 - \rho)x} \right) - (q_0 + q_1) e^{-\rho(1 - \rho)x} \right] 
\]

IV. OPTIMAL COST-TO-GO FUNCTIONS

To obtain the optimal cost-to-go functions, let us assume throughout this section that the optimal control is the hedging point policy. This will be verified later in the next section. Let \( z^* \) be the optimal hedging point, then (12) tells us that it must satisfy the following equation:

\[
b(z) = \frac{c_0q_1(1 - \rho)}{1 - \rho} + \frac{c}{(1 - \rho)^2} [d - (r + d) \rho + r (r - d + \rho) e^{-\rho(r - d)}] - c_0q_1 = 0 
\]

This is simply due to \( d J_0/dz = 0 \) and furthermore,

\[
J^* = c_0^* + \frac{cd}{q_1 + q_1} 
\]

It is worth noting that \( z^* \) is the unique solution of \( b(z) = 0 \) (because \( db(z)/dz > 0 \)). However, \( z^* = 0 \) if the unique solution of \( b(z) = 0 \) is negative (which is the case if \( b(0) = c - c_0q_1 > 0 \)). Since we have assumed that the optimal control is the hedging point policy, we have \( J^* = J^* \) and furthermore, the Bellman equations (3) and (4) reduce to

\[
\frac{dV(x)}{dx} = \begin{cases} 
A_1 (V(x) - b_1[J^* - cx]), & \text{for } 0 < x \leq z^* \\
A_2 (V(x) - b_2[J^* - cx]), & \text{for } x \geq z^* 
\end{cases} 
\]

where

\[
A_1 = \left( \frac{2\rho}{b_1} - \frac{1}{b_1} \right), \quad A_2 = \left( \frac{2\rho}{b_2} - \frac{1}{b_2} \right) 
\]

\[
V(x) = \begin{cases} 
V_l(x), & \text{for } 0 < x \leq z^* \\
V_h(x), & \text{for } x \geq z^* 
\end{cases} 
\]

with boundary conditions

\[
c_0 - J^* + q_0 (V_l(0) - V_h(0)) = 0 \quad (16) 
\]

\[
c_0^* - J^* - q_1 (V_l(z^*) - V_h(z^*)) = 0. \quad (17) 
\]

Solving (15), we have, for \( 0 \leq x \leq z^* \),

\[
V_l(x) = \left[ I - \frac{A_1}{k_1} + \frac{A_1\rho}{k_1} (e^{k_1 x} - 1) \right] V_l(0) + \frac{A_1}{k_1} \left( J^* x - \frac{1}{2} c x^2 \right) + \frac{A_1 b_1}{k_1^2} \left[ J^* (1 - e^{k_1 x}) - c x + \frac{c}{k_1} (e^{k_1 x} - 1) \right] 
\]

where \( k_1 = \rho (1 - \rho) \), and for \( z^* \geq z^* \),

\[
V_h(x) = \left[ I - \frac{A_2}{k_2} + \frac{A_2\rho}{k_2} (e^{k_2 x - r}) \right] V_h(z^*) 
\]

\[
- \left( J^* (x - z^*) - \frac{1}{2} c x^2 (z^*)^2 \right) b_2 \left( \frac{1}{2} \right) 
\]
and
\[
\frac{dV_i(x)}{dx} = \frac{q_i(q_i + q_1)}{k_i d(r - d)^2} J^* - \frac{c}{k_i} \frac{q_i}{d(r - d)} (1 - e^{-k_i x}) - \frac{q_i}{k_i} \frac{q_i}{d(r - d)} (1 - e^{-k_i x}),
\]
(21)

Substituting \( r_0 \) and \( J^* \) from (13) and (14) into (21), we obtain
\[
\frac{dV_i(x)}{dx} = \frac{c(q_i + q_1)}{q_i d(r - d)} (1 - e^{-k_i x}) - \frac{c(q_i + q_1)}{k_i d(r - d)} (z^* - x), \quad \text{for } 0 \leq x \leq z^*.
\]
(22)

Similarly, by combining (19), (17), and (14), we obtain for \( x \geq z^* \)
\[
V_i(x) = V_i(z^*) + J^* - \frac{c}{k_i d} \left[ e^{k_i (x - z^*)} - 1 \right] + \frac{1}{2} \frac{c}{k_i d} (x^2 - (z^*)^2) - J^* (x - z^*)
\]
(23)

and
\[
\frac{dV_i(x)}{dx} = -\frac{c}{q_i + q_1} \left( e^{k_i (x - z^*)} - 1 \right) + \frac{c}{d} (x - z^*).
\]
(24)

V. THE VERIFICATION THEOREM

In this section we verify that the optimal hedging point policy we obtained in the previous section is the optimal control. We first need the following verification theorem.

Verification Theorem: For the system under consideration, suppose the control \( u_i(x, i) \) is defined by (5) where \( z^* \) is given by (13). If there exist continuously differentiable functions \( V_i(x) \) on \( x \in (0, \infty) \) and a constant \( J^* \) such that the Bellman equations (3) and (4) are satisfied and \( V_i(x) \leq \alpha x^2 + b \) (where \( \alpha \) and \( b \) are two constants), then \( u^*(x, i) \) is the optimal control which minimizes the long-run expected average cost defined by (2).

The proof of the verification theorem is essentially the same as the one given in [2] which uses the Dynkin formula. Hence, we will not repeat it here. For the optimal cost-to-go function \( V_i(x) \), which we obtained in the previous section, we can easily verify [based on (22) and (24)] that it is a convex function with minimum \( z^* \), and also it is continuously differentiable. Hence, based on the above verification theorem, the optimal hedging point policy is indeed optimal.

VI. CONCLUSION

We proved that even when backlog is not permitted, the optimal control for the failure-prone production system remains to be a hedging point policy. We demonstrated that the traditional sample path argument is no longer valid in proving that the optimal cost-to-go functions are convex functions; instead, the explicit formulas have to be obtained. To obtain the optimal hedging point policy and the optimal cost-to-go functions, we first established a relationship between the inventory process of the system and a hedging point policy and the workload process of a single-node queueing system with limited workload, and then used it to obtain the probability distribution function of the inventory process based on some existing results in queueing theory and the level crossing technique.

An immediate application of our results in this note is to failure-prone production systems with many machines connected in network. One way to study the optimal control problem for these complex systems is to decompose them into many simple subsystems, e.g., systems with only one machine. The interaction among these simple subsystems can be formulated through the total demand rate to each subsystem. Clearly, for some subsystems, backlog of demands will not be permitted, especially if they are suppliers to other subsystems. One example is a production line in which machines are linked together in tandem. In such a system, the demand rate for each machine (except the last one) is simply equal to the production rate of its downstream machine, hence backlog will not be permitted. Our results can give optimal instantaneous production rates for those subsystems in which backlog is not permitted. Hence, they can be used to develop optimal or near-optimal designs on production rate control for complex systems when combined with other methods.

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