Traffic Grooming, Routing, and Wavelength Assignment in Optical WDM Mesh Networks

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Abstract

In this paper, we consider the traffic grooming, routing, and wavelength assignment (GRWA) problem for optical mesh networks. In most previous studies on optical mesh networks, traffic demands are usually assumed to be wavelength demands, in which case no traffic grooming is needed. In practice, optical networks are typically required to carry a large number of lower rate (sub-wavelength) traffic demands. Hence, the issue of traffic grooming becomes very important since it can significantly impact the overall network cost. In our study, we consider traffic grooming in combination with traffic routing and wavelength assignment. Our objective is to minimize the total number of transponders required in the network. We first formulate the GRWA problem as an integer linear programming (ILP) problem. Unfortunately, for large networks it is computationally infeasible to solve the ILP problem. Therefore, we propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing problem and the wavelength assignment problem, which can then be solved much more efficiently. In general, the decomposition method only produces a near optimal solution for the GRWA problem. However, we also provide some sufficient conditions under which the decomposition method gives an optimal solution. Finally, some numerical results are provided to demonstrate the efficiency of our method.

Keywords: mesh optical networks, wavelength division multiplexing, traffic grooming and routing, wavelength assignment

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1 Introduction

Wavelength division multiplexing (WDM) is now being widely used for expanding capacity in optical networks. In a WDM network, each fiber link can carry high-rate traffic at many different wavelengths, thus multiple channels can be created within a single fiber. There are two basic architectures used in WDM networks: ring and mesh. The majority of optical networks in operation today have been built based on the ring architecture. However, carriers have increasingly considered the mesh architecture as an alternative for building their next generation networks. Various studies have shown that mesh networks have a compelling cost advantage over ring networks. Mesh networks are more resilient to various network failures and also more flexible in accommodating changes in traffic demands (e.g., see [7, 12, 25] and references therein). In order to capitalize on these advantages, effective design methodologies are required.

In the design of an optical mesh network, traffic grooming, routing, and wavelength assignment are some of the most important issues that need to be considered. The problem of traffic grooming and routing for mesh networks is to determine how to efficiently route traffic demands and at the same time to combine lower-rate (sub-wavelength) traffic demands onto a single wavelength. On the other hand, the problem of wavelength assignment is to determine how to assign specific wavelengths to lightpaths, usually under the wavelength continuity constraint. In previous studies on the routing and wavelength assignment (RWA) problem (e.g., see [17, Chapter 8] and references therein), the issue of grooming has largely been ignored, i.e., it has been assumed that each traffic demand takes up an entire wavelength. In practice, this is hardly the case, and networks are typically required to carry a large number of lower rate (sub-wavelength) traffic demands.

The traffic grooming problem has been considered by several researchers for ring networks (e.g. see, [4, 6, 8, 9, 10, 13, 14, 18, 19, 20, 22, 24]), and is only considered recently in [23] for mesh networks. The objective considered in [23] is either to maximize the network throughput or to minimize the connection-blocking probability, which are operational network-design problems. Alternatively, a strategic network-design problem is to minimize the total network cost.

Typically, the cost of a nation-wide optical network is dominated by optical transponders and optical amplifiers. If one assumes that the fiber routes are fixed, then the amplifier cost is constant, in which case one should concentrate on minimizing the number of transponders in the network. Grooming costs should also be considered. However, under realistic assumptions of either a low-cost interconnect between grooming equipment and transport equipment, or integrated (long-reach) transponders on the grooming equipment, then the relative cost of the grooming switch fabric is negligible, and minimizing transponders is still the correct objective. In addition, the advent of Ultra Long-Haul transmission permits optical pass-through at junction nodes, hence, requiring transponders only at the end of the lightpaths.

Though the number of transponders has been used as an objective function in many studies on ring networks, it has not been considered at all for mesh networks. The objective functions that have been considered for mesh networks so far include: the blocking probability, the total number of wavelengths required, and the total route distance.

In this paper, we consider the problem of traffic grooming, routing, and wavelength assignment (GRWA) with the objective of minimizing the number of transponders in the network. We first formulate the GRWA problem as an integer linear programming (ILP) problem. Unfortunately, the resulting ILP problem is usually very hard to solve computationally, in particular for large networks. To overcome this difficulty, we then propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing (GR) problem and the wavelength assignment (WA) problem. In the GR problem, we only consider how to groom and route traffic demands onto lightpaths (with the same objective of minimizing the number of transponders) and ignore the issue of how to assign specific wavelengths to lightpaths. Similar to the GRWA problem, we can formulate the GR problem as an ILP problem. The size of the GR ILP problem is much smaller than its corresponding GRWA ILP problem. Furthermore, we can significantly improve the computational efficiency for the GR ILP problem by relaxing some of its integer constraints, which usually leads near-optimal solutions for the GR problem. Once we solve the GR problem, we can then consider the WA problem, in which our goal is to derive a feasible wavelength assignment solution.

We note that the WA problem has been studied by several researchers before (e.g., see [5, 1, 15, 16, 17, 21, 2] and references therein). However, the objective in all these studies has been to minimize the number of wavelengths required in a network, in some cases by using wavelength converters. In general, the use of additional wavelengths in a network only marginally increases the overall network cost as long as the total number of wavelengths used in the network does not exceed a given threshold (the wavelength capacity of a WDM system). This is mainly because the amplification cost is independent of the number of wavelengths. In recent years, the wavelength capacity for optical networks has increased dramatically. For example, with most advanced techniques, a single WDM system on a pair of fibers can carry up to 160 10G-wavelengths or 80 40G-wavelengths. Of course, once the wavelength capacity is exceeded, then a second parallel system (with another set of optical amplifiers) needs to be built, which would then substantially increase the network cost. Therefore, assuming a single WDM system on all fiber routes fixes the amplifier cost, then one should focus on minimizing the number of transponders in the network, which is already taken into consideration in the GR problem. In this setting, the objective in our WA problem is to find a feasible wavelength assignment solution under the wavelength capacity constraint.

It is clear that in general the decomposition method would not yield the optimal solution for the GRWA problem. However, we will provide sufficient conditions under which we show that the decomposition method does produce an optimal solution for the GRWA problem. Under these sufficient conditions, we also develop a simple algorithm that finds a wavelength assignment solution.

The rest of this paper is organized as follows. In Section 2, we present the GRWA problem and demonstrate how it can be formulated as an ILP problem. In section 3, we first present our decomposition method. We then provide an ILP formulation for the GR problem and develop an algorithm for solving the WA problem. We also discuss under what conditions the decomposition method produces an optimal solution for the GRWA problem. Some numerical results are provided in Section 4. Finally, a conclusion is given in Section 5.

2 The GRWA Problem

An optical network architecturally has two layers: a physical layer and an optical layer. The physical layer consists of fiber spans and nodes and the optical layer consists of lightpaths (optical links) and a subset of nodes contained in the physical layer. A lightpath in the optical layer is a path connecting a pair of nodes via a set of fiber spans in the physical layer. Throughout this paper, we assume that lightpaths and their routes in the physical layer are given. In practice, the selection of lightpaths is another important design issue that needs to be addressed, which is beyond the scope of this paper.

We use graph $G_f = (V_f, E)$ to represent the physical layer, where E is the set of edges representing fiber spans and V_f is the set of nodes representing locations which are connected via fiber spans. We use graph $G_o = (V_o, L)$ to represent the optical layer, where L is the set of edges representing lightpaths and $V_o \subset V_f$ is a subset of locations that are connected via lightpaths. Each edge in L corresponds to a path in G_f. For ease of exposition, we first assume that G_o is a directed graph (i.e., the lightpaths are unidirectional). The extension to the undirected graph case is quite straightforward and will be discussed later in this section (basically, we can simply replace every undirected edge with two directed edges).

The GRWA problem concerned in our study can be described as follows. Assuming that a set of traffic demands are given (some of them are of low rate, i.e., sub-wavelength), our goal is to find an optimal way to route and groom these demands in the optical layer, G_o, and also to assign a set of specific wavelengths to each lightpath so that the total number of transponders required is minimized. There are two key constraints we need to take into consideration in this problem: 1) the wavelength capacity constraint for each fiber span, and 2) the wavelength continuity constraint for every lightpath, i.e., the same wavelength(s) needs to be assigned to a lightpath over the fiber spans it traverses. In this problem setting, the number of transponders required for each lightpath is equal to twice the number of wavelengths assigned to it (one transponder for each end of each wavelength on a lightpath). Therefore, by grooming several low rate demands onto a single wavelength, we can potentially reduce the total number of wavelengths required by the lightpaths, thus the number of transponders.

The GRWA problem can be formulated as an integer linear programming (ILP) problem. First, we need to introduce some necessary notation:

W: the set of wavelengths available at each fiber;

D: the set of traffic demands;

s_d: the size of demand $d \in D$;

g: the capacity of a single wavelength;

A: $= [a_{v,l}]_{|V_o| \times |L|}$, the node-edge incidence matrix of graph G_o, where

$$a_{v,l} = \begin{cases} 1 & \text{if lightpath } l \text{ originates from node } v, \\ -1 & \text{if lightpath } l \text{ terminates at node } v, \\ 0 & \text{otherwise;} \end{cases}$$

B: $= [b_{e,l}]_{|E|\times|L|}$, the fiber-lightpath incidence matrix, where

$$b_{e,l} = \begin{cases} 1 & \text{if fiber span } e \text{ is on lightpath } l, \\ 0 & \text{otherwise;} \end{cases}$$

u_d: $= [u_{v,d}]_{v\in V_o}$, the source-destination column vector for $d \in D$, where

$$u_{v,d} = \begin{cases} 1 & \text{if } v \text{ is the starting node of } d, \\ -1 & \text{if } v \text{ is the end node of } d, \\ 0 & \text{otherwise;} \end{cases}$$

x_d: $= [x_{l,d}]_{l\in L}$, the column vector containing lightpath routing variables for $d \in D$, where

$$x_{l,d} = \begin{cases} 1 & \text{if demand } d \text{ traverses lightpath } l; \\ 0 & \text{otherwise;} \end{cases}$$

y_w: $= [y_{l,w}]_{l\in L}$, the column vector containing wavelength assignment variables for $w \in W$, where

$$y_{l,w} = \begin{cases} 1 & \text{if wavelength } w \text{ is assigned to lightpath } l; \\ 0 & \text{otherwise;} \end{cases}$$

$\mathbf{1}$: $= [1, 1, \ldots, 1]$, the unit column vector.

Then the GRWA problem can be formulated as the following ILP problem (which we shall refer as the GRWA ILP problem):

$$\begin{aligned} \min \quad & \sum_{w\in W, l\in L} y_{w,l} & & \\ \text{s.t.} \quad & Ax_d = u_d & d \in D & \quad (1) \\ & By_w \leq \mathbf{1} & w \in W & \quad (2) \\ & \sum_{d\in D} s_d x_{l,d} \leq g \sum_{w\in W} y_{l,w} & l \in L & \quad (3) \\ & x \text{ and } y \text{ are binary variables.} & & \end{aligned}$$

where the objective function $\sum_{w\in W,l\in L} y_{w,l}$ is the total number of wavelengths assigned to all lightpaths, which is equivalent to minimizing the total number of transponders needed. The three constraints are

- (1) is the flow balance equation, which guarantees that the lightpaths selected based on x_d constitute a path from the starting node of d to the end node of d.

- (2) implies a single wavelength along each fiber span can be assigned to no more than one lightpath.

- (3) is the capacity constraint for lightpath l, since $\sum_{d \in D} s_d x_{l,d}$ is the total amount of demands carried by lightpath l, and $g \sum_{w \in W} y_{l,w}$ is the total capacity of lightpath l.

We refer the type of the network considered above as the basic model. There are several variations of the basic model, which include

1. Networks with both protected and unprotected demands;

2. Networks in which lightpaths are undirected;

3. Networks with non-homogeneous fibers where different types of fiber may have different wavelength capacities;

4. Networks in which demand exceeds a single WDM system per fiber pair.

To illustrate how the ILP formulation for the basic model can be extended, we consider the first two cases in the remaining of this section.

2.1 Protected demands

In the basic model, we assume that each traffic demand only requires one path, i.e., it is unprotected. However, in many applications, some demands may need to be protected, i.e., they require two paths: one working path and one protection path, which do not share any common fiber span. Our ILP formulation can be easily modified to accommodate protected demands. However, we need some additional variables for protected demands.

D^p: $\subset D$, the subset of demands that need protection;

x^p_d: $= [x^p_{l,d}]_{l \in L}$, the column vector containing lightpath routing variables for the protection path of $d \in D^p$, where

$$x^p_{l,d} = \begin{cases} 1 & \text{if the protection path of demand } d \text{ traverses lightpath } l; \\ 0 & \text{otherwise;} \end{cases}$$

z_d: $= [z_{e,d}]_{e \in E}$, the column vector containing fiber routing variables for the working path of d, where

$$z_{e,d} = \begin{cases} 1 & \text{if (the working path of) } d \text{ traverses fiber } e; \\ 0 & \text{otherwise;} \end{cases}$$

z^p_d: $= [z^p_{e,d}]_{e \in E}$, the column vector containing fiber routing variables for the protection path of d, where

$$z^p_{e,d} = \begin{cases} 1 & \text{if the protection path of } d \text{ traverses fiber } e; \\ 0 & \text{otherwise.} \end{cases}$$

Then the ILP formulation for the GRWA problem with both unprotected and protected demands is as follows:

$$\begin{aligned}
\min \quad & \sum_{w \in W, l \in L} y_{w,l} & & \\
\text{s.t.} \quad & Ax_d = u_d & d \in D & \\
& Ax^p_d = u_d & d \in D^p & \quad (4) \\
& Bx_d \le kz_d & d \in D^p & \\
& Bx^p_d \le kz^p_d & d \in D^p & \quad (5) \\
& z_d + z^p_d \le \mathbf{1} & d \in D^p & \quad (6) \\
& By_w \le \mathbf{1} & w \in W & \quad (7) \\
& \sum_{d \in D} s_d x_{l,d} + \sum_{d \in D^p} s_d x^p_{l,d} \le g \sum_{w \in W} y_{l,w} & l \in L & \quad (8) \\
& x,\ y, \text{ and } z \text{ are binary variables,} & &
\end{aligned}$$

where k is a very large positive constant, e.g., we can set $k = |W|$. Here are some explanations on the additional constraints we introduced:

- Each element in Bx_d (resp. Bx_d^p) indicates the number of lightpaths on the working (resp. protection) path of d that contain a particular fiber span. Note that a fiber span may be contained in more than one lightpath on a working or protection path, which implies that a demand may traverse a fiber span more than once. Since z_d is binary, k is needed in (5).

- (6) ensures that the working path and the protection path do not share any common fiber span.

2.2 Undirected Lightpaths

As pointed out earlier, in the basic model we assume that all lightpaths are unidirectional. We now consider the case in which all lightpaths are undirected, i.e., the optical layer graph G_o is undirected. We can easily convert this undirected model to the basic model. We first replace each undirected edge $l \in L$ with two opposite directed links l^1 and l^2, each representing a unidirectional lightpath and having the same end-nodes and the same fiber path as that of l. Let $x_d^i = [x_{l^i,d}]_{l \in L}$ $(i = 1, 2)$, where

$$x_{l^i,d} = \begin{cases} 1 & \text{if demand } d \text{ traverses lightpath } l^i; \\ 0 & \text{otherwise.} \end{cases}$$

Then the GRWA problem with undirected lightpaths can be formulated as the following ILP:

$$\begin{array}{lllr} \min & \displaystyle\sum_{w \in W, l \in L} y_{w,l} & & \\ \text{s.t.} & A(x_d^1 - x_d^2) = u_d & d \in D & (9) \\ & By_w \leq \mathbf{1} & w \in W & (10) \\ & \displaystyle\sum_{d \in D} s_d(x_d^1 + x_d^2) \leq g \sum_{w \in W} y_{l,w} & l \in L & (11) \\ & x \text{ and } y \text{ are binary variables.} & & \end{array}$$

3 A Decomposition Method

In the previous section, we formulated the GRWA problem as an ILP problem, however, it may not be computationally feasible to solve the ILP problem, particularly for large networks (e.g., see numerical results in Section 4). Therefore, it is necessary to find more efficient ways to solve the GRWA problem. In this section, we propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing (GR) problem and the wavelength assignment (WA) problem. In the GR problem, we only consider how to groom and route demands over lightpaths and ignore the issue of how to assign specific wavelengths to lightpaths. Based on the grooming and routing, we can then derive wavelength capacity requirements for all lightpaths. Similar to the GRWA problem, we formulate the GR problem as an ILP problem. The size of the GR ILP problem is much smaller than its corresponding GRWA ILP problem. Furthermore, we can significantly improve the computational efficiency for the GR ILP problem by relaxing some of its integer constraints, which usually leads to near-optimal solutions for the GR problem. Once we solve the GR problem, we can then consider the WA problem, in which our goal is to derive a feasible wavelength assignment solution that assigns specific wavelengths to lightpaths based on their capacity requirements derived in the GR problem.

It is obvious that in general the decomposition method would not yield the optimal solution for the GRWA problem. However, we will provide sufficient conditions under which we show that the decomposition method does produce an optimal solution for the GRWA problem. We also develop a simple algorithm that finds a wavelength assignment solution under these sufficient conditions.

For ease of exposition, in the rest of this section we focus exclusively on the basic model. However, similar to the GRWA problem, our results can be easily extended to more general models.

3.1 The GR Problem

Let $t = [t_l]_{l \in L}$, a column vector containing lightpath capacity decision variables, where $t_l = \sum_{w \in W} y_{l,w}$ is the number of wavelengths needed for lightpath $l \in L$. Then, the GR problem can be formulated as:

$$
\begin{aligned}
\min \quad & \sum_{l \in L} t_l & & \\
\text{s.t.} \quad & Ax_d = u_d & d \in D & \qquad (12) \\
& Bt \le |W|\mathbf{1} & & \qquad (13) \\
& \sum_{d \in D} s_d x_{l,d} \le g t_l & l \in L & \qquad (14) \\
& x \text{ binary variable and } t \text{ integer variable.} & &
\end{aligned}
$$

We refer the above ILP problem as the GR ILP problem. We now present the following result:

Theorem 1 *If x and y are feasible solutions for the GRWA ILP problem, then x and t are feasible solutions for the GR ILP problem, where $t = \sum_{w \in W} y_w$.*

Proof We first note that by summing over $w \in W$ in (2) it leads to (13). Secondly, (3) is the same as (14). Hence, the result follows. ∎

Based on Theorem 1, it is clear that

Theorem 2 *If x and t are the optimal solutions of the GR ILP problem, and there exists a binary y such that $\sum_{w \in W} y_w = t$ and $By_w \le \mathbf{1}$ for $w \in W$, then x and y are the optimal solutions of the GRWA ILP problem.*

Obviously, the GR ILP problem is much easier to solve than the GRWA ILP problem since it has less integer variables and less constraints (e.g., see numerical examples in Section 4). More importantly, we can now relax the integer constraint on t in the GR ILP problem and solve a relaxed mixed ILP problem and then round up the values of t to obtain a solution for the GR problem. This would dramatically improve the computational efficiency. On the other hand, the relaxation approach is much less effective for the GRWA ILP problem since all its decision variables are binary. Various additional improvements can be made for this relaxation method. In general, if most lightpaths have relatively high wavelength counts, then the relaxed GR ILP problem often produces very good solutions for the GR problem, as illustrated by our numerical examples in Section 4.

3.2 The WA Problem

The WA problem of our interest is to find a binary solution y such that

$$\sum_{w \in W} y_w = t \quad \text{and} \quad By_w \leq \mathbf{1} \text{ for } w \in W,$$

where t is a feasible (or optimal) solution of the GR problem. This problem can be viewed as an ILP problem (without an objective function), which is much easier to solve than the GRWA ILP and the (relaxed) GR ILP problems. For example, it can be solved for networks with a few hundred nodes and lightpaths in seconds or minutes by using commercially available LP software, e.g., CPLEX. Based on Theorem 2, we know that if x and t are optimal solutions of the GR problem and the WA problem has a feasible solution y, then x and y are optimal solutions of the GRWA problem. In case when we cannot find a feasible solution for the WA problem, we can either increase the number of wavelengths in W in the WA problem (note that we can always find a feasible solution for the WA problem if W has enough wavelengths), or we can use $W^* \subset W$ in the GR problem (specifically, replace $|W|$ with $|W^*|$ in (13)) but still use W in the WA problem. Obviously, the latter approach is preferred in which case the decomposition method provides a feasible solution for the GRWA problem. An alternative approach is to use wavelength conversion via lightpath regeneration, which is equivalent to modifying L by breaking select lightpaths into two or more lightpaths. In addition, there are other possible remedies available to alleviate the infeasibility of the WA problem.

Though the WA problem can be solved as an ILP problem, it is also possible to solve it directly based on some heuristic algorithms (e.g., see [5]). In what follows, we consider a special type of the GRWA problem, in which the lightpaths satisfy a certain condition. Under such a condition, we show that a feasible solution for the corresponding WA problem can always be found, and we also develop an algorithm for finding a feasible solution. Without loss of generality, we assume that the required capacity of every lightpath is one wavelength. For a lightpath whose required capacity is more than one wavelength, we can treat it as several identical parallel lightpaths, each of which has capacity of one wavelength. Let p_e ($e \in E$) be the number of lightpaths that traverse fiber span e, and $p = \max_{e \in E} p_e$, which is the minimum number of wavelengths required for the network.

We now convert the WA problem to a special coloring problem for a bipartite graph. First, we construct a bipartite graph $G = (U, V, C)$ based on the WA problem as follows:

1. Each node in U represents a lightpath;
2. Each node in V represents a fiber span;
3. Two nodes (one in U and one in V) are connected by an edge in C if and only if the corresponding fiber span is on the corresponding lightpath.

Define

C_u: $= \{c \in C |\ u$ is an end-node of $c\}$, $u \in U$;

C_v: $= \{c \in C |\ v$ is an end-node of $c\}$, $v \in V$;

V_u: $= \{v \in V |\ (u, v) \in C\}$, $u \in U$;

U_v: $= \{u \in U |\ (u, v) \in C\}$, $v \in V$;

u_c: the end-node of $c \in C$ in U;

v_c: the end-node of $c \in C$ in V.

We now introduce the following coloring problem for the bipartite graph G.

Definition *(The Coloring Problem)* We want to color all nodes in U and all edges in C so that $\forall u \in U, v \in V$: 1) the edges in C_u all have the same color as u, and 2) no two edges in C_v have the same color.

If we let each color represent a wavelength, then it is not difficult to prove

Theorem 3 *The WA problem is equivalent to the coloring problem for G.*

It is worth noting that the WA problem can also be converted to a classical node coloring problem (e.g., see [17, Section 8.5]), which is different from the coloring problem we introduced above. We now present an algorithm for the coloring problem.

Algorithm 1 *(for the coloring problem)*

1. Select an initial node $u_0 \in U$ (arbitrarily), and color u_0 and C_{u_0} with one color.

2. Suppose $V_{u_0} = \{v_1, v_2, \ldots, v_k\}$. Set $U_0 = \{u_0\}$ and $C_0 = C_{u_0}$. For $i = 1$ to k, do

   (a) Color every edge $c \in C_{v_i} \setminus \cup_{0 \le j < i} C_j$ such that no two edges in C_{v_i} share the same color (note that $C_{v_i} \setminus \cup_{0 \le j < i} C_j$ is a subset of edges in C_{v_i} which are not colored yet).

   (b) For $c \in C_{v_i} \setminus \cup_{0 \le j < i} C_j$, color u_c and C_{u_c} with the same color as c (note $c \in C_{u_c}$).

   (c) Let

$$
\begin{aligned}
U_i &= \{u_c |\ c \in C_{v_i} \setminus \cup_{0 \le j < i} C_j\}, \\
C_i &= \cup_{u \in U_i} C_u \setminus C_{v_i}, \\
V_i &= \cup_{u \in U_i} V_u \setminus \{v_i\}.
\end{aligned}
$$

We note that U_i is the set of nodes that are colored in Step 2(b), C_i is the set of edges that are colored in Step 2(b), and V_i is the set of nodes in V that are connected to the edges in C_i.

3. For $i = 1, 2, \ldots, k$, apply the procedure in Step 2 to V_i (with V_{u_0} being replaced with V_i), and continue until all the elements in E and U are colored (note that since all edges connected to nodes in V_{u_0} have been colored in Step 2, we can simply replace V_i by $V_i \backslash V_{u_0}$).

The following properties associated with Algorithm 1 can be easily verified:

Proposition 1

1. *Every node in V_i is connected to at least one node in U_i via an edge in C_i.*
2. *For $1 \leq j \leq i$, $v_j \notin V_i$;*
3. *$U_i \cap U_j = \emptyset$ $(i \neq j)$;*
4. *If $u \in U_i$, then it is not connected to nodes $\{v_1, \ldots, v_{i-1}\}$;*
5. *If $V_i \cap V_j \neq \emptyset$ $(i \neq j)$, then there exists a cycle in G with one node in U_i and one node in U_j;*
6. *If a node is connected to one node in U_i and another node in U_j through two paths in the subgraph whose nodes are $(U \backslash \cup_{0 \leq h \leq k} U_h, V \backslash V_{u_0})$, then there exists a cycle in G with one node in V_i and one node in V_j.*

Proof First note that $U_i \subset U_{v_i}$ is the subset of nodes in U_{v_i} which are not colored yet, C_i is the subset of edges in $\cup_{u \in U_i} C_u$ which are not colored yet, and $V_i = \{v_c \in V | \, c \in C_i\}$.

1. By definition.

2. For $1 \leq j \leq i$, it is clear that all the edges in C_{v_j} have been colored at the end of Step 2(a) (for V_i). On the other hand, for any node $v \in V_i$, at least one edge in E_v is not colored at the end of Step 2(a) since $V_i = \{v_c \in V \mid c \in C_i\}$. Hence, $v_j \notin V_i$.

3. All nodes in U_j (and their associated edges) are colored at the end of Step 2(b) and they will not be considered again in later iterations.

4. It u and v_j were connected ($1 \leq j < i$), u would have been colored at the end of Step 2(b) for V_j, i.e., $u \in U_j$. But this contradicts to $U_i \cap U_j = \emptyset$.

5. Suppose $v \in V_i \cap V_j$. Based on (1), v is connected a node in U_i, say u_i, and another node in U_j, say u_j. Furthermore, u_i and u_j are connected to v_i and v_j, respectively, which are then connected to u_0. Therefore, we have a cycle with nodes $v, u_i, v_i, u_0, v_j, u_j$.

6. The same argument used in (5) can be applied here as well. ∎

In general, one needs to be careful about what colors to use in Step 2(a) of Algorithm 1, otherwise it is possible that it may not produce a feasible solution for the coloring problem. For example, consider the following example in which $V_{u_0} = \{v_1, v_2\}$, $U_1 = \{u_1\}$, $U_2 = \{u_2\}$, and $V_1 = V_2 = \{v\}$. If we use the same color to color u_1 and u_2, then we have to use the same color to color the corresponding two edges in C_v, which is not permissible. Therefore, we have to color u_1 and u_2 with different colors.

It is clear that the number of different colors needed in the coloring problem is at least p. In what follows, we provide a sufficient condition under which p different colors are enough to solve the coloring problem. First we need to introduce the following definition.

Definition G is a complete bipartite graph if every node in U is connected to every node in V. A cycle in G is a complete cycle if the subgraph produced by its nodes is a complete bipartite graph, otherwise, it is a non-complete cycle.

The bipartite graph G contains a non-complete cycle if and only if there exist a set of lightpaths $\{l_1, l_2, \ldots, l_k\} \subset L$ such that l_i and l_{i+1} ($i = 1, \ldots, k$ and $l_{k+1} = l_1$) share at least one common fiber span and there is at least one fiber span that is shared by some but not all lightpaths in $\{l_1, l_2, \ldots, l_k\}$. The network depicted in Figure 1 is an example whose corresponding bipartite graph G has a non-complete cycle. The network has four nodes (A, B, C, D), three fiber spans (A–B, B–C, C–D), and three lightpaths (A-B-C, C-B-D, D-B-A).

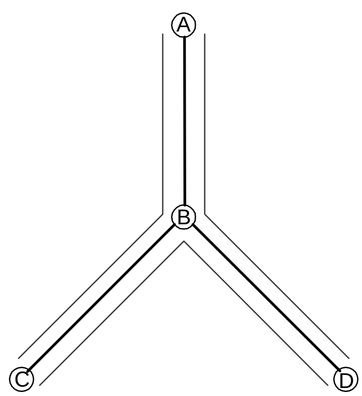

Figure 1: A 4-Node Network

Theorem 4 *If G does not contain any non-complete cycle, then Algorithm 1 can produce a feasible solution for the coloring problem which only needs p colors.*

Proof Since G does not contain any non-complete cycle, based on (4) and (5) in Proposition 1 we have $V_i \cap V_j = \emptyset$. Hence, when coloring edges in $C_{v_i} \setminus \cup_{0 \leq j < i} C_j$ ($i = 1, 2, \ldots, k$), we can use arbitrary colors, and it guarantees that the coloring method in Step 2(b) is permissible (i.e., no two edges in C_v have the same color, where $v \in \cup_{1 \leq i \leq k} V_i$). Using the same argument along with (6) in Proposition 1, it is not difficult to show that we can use arbitrary colors in Step 2(a) for V_i ($i = 1, 2, \ldots, k$)

and the coloring method in Step 2(b) is still permissible. Repeat this argument, we can show that in Algorithm 1 we can use arbitrary colors in Step 2(a) and obtain a feasible solution for the coloring problem. Since colors used in Step 2(a) can be arbitrary, the maximum number of different colors needed throughout Algorithm 1 should be no more than p. This completes our proof. ∎

Theorem 4 implies that if G does not contain any non-complete cycle, we can find a solution for the WA problem which only needs p wavelengths. In [5], the problem of whether the WA problem can be solved with p wavelengths was also studied. However, we believe that the result there (Theorem 2 in [5]) is incorrect, which states that if a network is acyclic then its WA problem can be solved with p wavelengths. The network in Figure 1 is a counterexample to this result. It is a tree (hence acyclic). Clearly we have $p = 2$, but need three wavelengths for its WA problem.

Since t in the WA problem is a feasible solution for the GR problem, i.e., $Bt \leq W\mathbf{1}$. Hence, we have $p \leq |W|$. This, together with Theorem 4, leads to the following results:

Theorem 5 *If graph G does not contain any non-complete cycle, Algorithm 1 produces a feasible solution for the WA problem, and the decomposition method gives an optimal solution for the GRWA problem.*

In case that G contains non-complete cycles, let c^* be the minimum number of nodes in U (i.e., the minimum number of lightpaths) that need to be removed from G so that the remaining portion of G does not contain any non-complete cycles. Then we have

Theorem 6 *There exists a feasible solution for the WA problem which required at most $c^* + p$ wavelengths. Therefore, if $c^* + p \leq |W|$, then we can find a feasible solution for the WA problem and the decomposition method still gives an optimal solution for the GRWA problem.*

Before closing this section, we should point out that the result in Theorem 6 may be further refined, which can lead to better upper bounds on the number of wavelengths required for the WA problem.

4 Numerical Results

In this section, we present four sets of numerical examples. All ILPs and mixed ILPs were solved by using CPLEX 7.0 on a Dell Precision 420 PC with two 1GHz processors. We compare the numerical results obtained based on the three methods proposed in the previous two sections: the GRWA ILP formulation, the decomposition method combined with the GR ILP formulation, and the decomposition method combined with the relaxed GR ILP formulation. The run time for the decomposition method includes the run times for both the (relaxed) GR ILP problem and the WA problem. The run time for the WA problem in all four examples is very fast (it is less than a second in the first three cases and less than 3 seconds in the last case). Our numerical results clearly indicate that the decomposition method combined with the relaxed GR ILP formulation produces quite good results with reasonably fast run times.

Example 1. This is relatively small network with 12 nodes, 17 fiber spans, 24 lightpaths, and 104 traffic demands. For this example, we were able to obtain the optimal solution based on the GRWA ILP formulation. The results are presented in Table 1.

	Run Time	Solution
GRWA ILP	400 seconds	128
Decomposition with GR ILP	80 seconds	128
Decomposition with Relaxed GR ILP	2 seconds	136

Table 1: Numerical results for Example 1.

Example 2. The network we consider in this example has 30 nodes, 38 fiber spans, 47 lightpaths, and 242 demands. The results are presented in Table 2. For the GRWA

ILP problem, we stopped the CPLEX program after 75 hours and obtained a feasible solution with objective value 249.

	Run Time	Solution
GRWA ILP	>75 hours	249
Decomposition with GR ILP	37 hours	189
Decomposition with Relaxed GR ILP	12 seconds	202

Table 2: Numerical results for Example 2.

Example 3. The network in this example has 49 nodes, 75 fiber spans, 155 lightpaths, and 238 demands. It is a medium size network. For this example, the decomposition method based on the relaxed GR ILP problem produced a solution with value 345 in about 13 minutes, while the CPLEX program did not even return a feasible solution for the GRWA ILP and GR ILP problems after 40 hours (at which point we stopped the program). We were also able to obtain a lower bound (based on the GR ILP problem) 328 for the objective function. Hence, the solution provided by the relaxed GR ILP based decomposition method is within 5% of the lower bound. We also point out that the WA problem was solved in 0.37 seconds for this example.

	Run Time	Solution
GRWA ILP	>40 hours	No Solution
Decomposition with GR ILP	>40 hours	No Solution
Decomposition with Relaxed GR ILP	13 minutes	345

Table 3: Numerical results for Example 3.

Example 4. The network in this example has 144 nodes, 162 fiber spans, 299 lightpaths, and 600 demands. It is a relatively large network (a typical size for a nation-wide network). For this example, the method based on the relaxed GR ILP problem produced a solution in about 38 minutes, and the CPLEX program did not even return a feasible solution for the GRWA ILP and GR ILP problems after 100 hours (at which point we stopped the program). The WA problem in this case was solved in 2.67 seconds for this example.

	Run Time	Solution
GRWA ILP	>100 hours	No Solution
Decomposition with GR ILP	>100 hours	No Solution
Decomposition with Relaxed GR ILP	38 minutes	431

Table 4: Numerical results for Example 4.

5 Conclusion

We studied the GRWA problem for optical mesh networks and proposed a decomposition method based on both ILP formulation and its relaxed version. In the decomposition method, we divided the GRWA problem into two smaller problems: the GR problem and the WA problem, both of which are much easier to solve compared to the original GRWA problem. We also provided some necessary conditions under which we proved that the decomposition method in fact produces an optimal solution for the GRWA problem. In general, our numerical results showed that the decomposition method produces quite good approximate solutions with relatively short run times and it can be used to solve the GRWA problem for large optical mesh networks (with a few hundred nodes and fiber spans).

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