Traffic Grooming for WDM Rings with Dynamic Traffic

Chenming Zhao J.Q. Hu
Department of Manufacturing Engineering
Boston University
15 St. Mary's Street
Brookline, MA 02446

***Abstract*— We study the problem of traffic grooming in wavelength-division-multiplexing (WDM) rings with dynamic traffic. We consider both unidirectional and bi-directional rings. In our dynamic traffic model, traffic is represented by a set of different traffic requirements that the network needs to satisfy, but at different times. Our objective is to minimize the required number of electronic add/drop multiplexers (ADMs). We first formulate the problem as an integer linear programming (ILP) problem. However, in general, it is computational infeasible to solve the resulting ILP problem. Therefore, we propose a heuristic method in combination with the ILP formulation, in which we first use the ILP formulation to find a traffic grooming solution that minimizes the number of wavelengths, based on which we then construct a solution that uses as few ADMs as possible. Numerical results are provided to validate our method.**

Key Words: System design, Mathematical programming/optimization.

I. INTRODUCTION

Wavelength division multiplexing (WDM) is emerging as a dominant technology in optical networks and has been used to significantly increase network capacity. In a WDM network, each fiber link can carry high-rate traffic at many different wavelengths, thus multiple channels can be created within a single fiber. There are two basic architectures used in WDM networks: ring and mesh. The majority of optical networks in operation today have been built based on the ring architecture, though the mesh architecture has been increasingly considered by carriers as an alternative in their next generation networks. In this paper, we consider the traffic grooming problem for WDM rings with dynamic traffic. In a WDM ring, nodes are arranged in a ring configuration and each pair of adjacent nodes on the ring are then connected by a fiber (or multiple fibers). At each node, electronic add/drop multiplexers (ADMs) are used to multiplex lower rate traffic onto high-rate wavelength. For example, 64 OC-3 (155Mb/s) circuits can be multiplexed onto a single wavelength with capacity OC-192 (10 Gb/s). Since the cost of ADMs often makes up a significant portion of the total cost for a WDM ring, one of the important issues in the design of a WDM ring network is to decide how to multiplex lower rate traffic onto different wavelengths so that the number of ADMs required in the network is minimized. This problem is referred as the traffic grooming problem. Usually, an ADM for a wavelength is needed at a particular node only when the wavelength is dropped at the node (i.e., when a demand is multiplexed onto the wavelength at the node), and it is not needed when the wavelength is only passing through the node.

This work was supported in part by the National Science Foundation under Grant EEC-0088073

There are two types of ring network: unidirectional rings, such as UPSR (unidirectional path-switched ring), and bi-directional rings, such as BLSR (bi-directional line-switched ring). There are significant differences between unidirectional and bi-directional rings. For example, unidirectional rings usually offer dedicated protection (i.e., 1+1 protection) while bi-directional rings offer shared protection (i.e., 1:n protection). As such, bi-directional rings in general require less bandwidth for the same amount of traffic. An excellent introduction on unidirectional and bi-directional rings can be found in [13, Section 10.4.2]. In this paper, we consider both types of rings.

The traffic grooming problem for WDM rings has been studied by many researchers [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [14], [15], [16], [18], [19]. In the majority of these studies, it has been assumed that traffic is static, i.e., it is given and never changes. However, in practice, this is hardly the case and traffic is usually dynamic, e.g., it may change over time. The traffic grooming problem with dynamic traffic is considered only in a few papers (see [3], [7], [10], [12], [14]). There are three different models that have been used to characterize dynamic traffic:

Stochastic Model In this model, traffic requests (between a pair of nodes) arrive according to a stochastic point process and each request may last a random amount of time.

Deterministic Model In this model, traffic is represented by a set of different traffic requirements that the network needs to satisfy, but at different times. Each traffic requirement contains a set of demands. For example, the different traffic requirements can be a result of traffic fluctuation in different operation periods (morning, afternoon, and evening). Note that the static traffic case becomes a special case of this model in which there is only one traffic requirement for the network (and it never changes).

Constrained Model In this model, traffic demands between nodes are not specified. Rather, only a set of constraints on the traffic requirement are provided, such that the total amount of traffic at each node does not exceed a certain limit and/or the total capacity requirement on each fiber link does not exceed a certain limit.

To the best of our knowledge, the traffic grooming problem with dynamic stochastic traffic has not been studied in literature, though the stochastic traffic model has been used in the study of other design problems for optical networks, such as the problem of wavelength conversion and blocking (e.g., see [1], [2], [17] and references therein). The constrained traffic model is used in [3], [7], [10], [14], where the focus is on obtaining lower and upper bounds on network costs (such as the number of ADMs required). The deterministic traffic model is first considered in [3], where a simple algorithm is proposed for the case of two traffic requirements. In [12], the traffic grooming problem with dynamic deterministic traffic is formulated as an integer linear programming problem for unidirectional rings.

In this paper, we adopt the deterministic traffic model. Our approach is based on a combination of the integer linear programming (ILP) formulation and heuristic algorithms. We note that the ILP formulation has been previously used in [6], [18], [12] for the static traffic grooming problem. In [6], the objective function considered is electronic routing, which is quite different from the number of ADMs considered in this paper, and also the goal there is to derive bounds based on the ILP formulation instead of finding optimal or near-optimal solutions. The problem setting in [18] is similar to ours, however, it is concluded there that the ILP formulation is not computationally feasible for rings with 8 nodes or more. Hence, it proposes instead to use methods based on simulated annealing and heuristics. In [12], a more efficient mixed ILP (MILP) formulation is proposed for unidirectional rings which results in significant reduction in computation time. The numerical results provided in [12] show that optimal or near-optimal solutions can usually be obtained in a few seconds or minutes for unidirectional rings with up to 16 nodes.

The ILP formulation in this paper for the dynamic traffic grooming problem is an extension of the work in [12]. As demonstrated in [12], the ILP formulation is very effective for solving the static traffic grooming problem for unidirectional rings, however, the computational complexity of the ILP formulation increases dramatically for rings with dynamic traffic, and it becomes much worse when rings are bi-directional (even in the case of static traffic). In fact, it is computationally infeasible to solve the dynamic traffic grooming problem based on the ILP formulation directly.

Therefore, we propose a method based on a combination of the ILP formulation and heuristics. We first use the ILP formulation to solve a slightly different traffic grooming problem in which the objective is to minimize the total number of wavelengths. This problem is much easier to solve than the traffic grooming problem whose objective function is the total number of ADMs. We note that it is well known that in general the traffic grooming solution with minimum number of ADMs does not necessarily use minimum number of wavelengths (e.g., see [5], [19]). On the other hand, for most rings we have studied so far, it is often the case that the minimum number of ADMs is achieved when the minimum number of wavelengths is used. In fact, in [5] this result is conjectured for the ring with all-to-all uniform traffic, and in [11] it is proven that the conjecture is true for $g = 4$. Once we have a traffic grooming solution with minimum number of wavelengths, we then use it to construct a solution with as few ADMs as possible based on a heuristic method. This two-phase approach of minimizing the number of ADMs was first used in [19] for the traffic grooming problem with static traffic. However, only heuristic algorithms were used in [19] to minimize the total number of wavelengths. It should be pointed out that the second phase of our method is very important since in many cases a traffic grooming solution with minimum number of wavelengths may use a very large number of ADMs. Our method can also be used in combination with the simulated annealing method.

The rest of the paper is organized as follows. In Section 2, we introduce the dynamic traffic grooming problem and its ILP formulation for both types of rings. In Section 3, we propose the heuristic method, in combination of the ILP formulation, to solve the dynamic traffic grooming problem. We also discuss how our method can be used in combination with the simulated annealing approach. Extensive numerical results are presented in Section 4 to validate our approach. Finally, a conclusion and some discussions are given in Section 5.

II. PROBLEM FORMULATION

Consider a WDM ring with N nodes, labelled as $1, 2, \ldots, N$ clockwise. We assume that all available wavelengths have the same capacity and there may be multiple traffic circuits between a pair of nodes, but all traffic circuits have the same rate. The traffic granularity of the network is defined as the total number of low-rate traffic circuits that can be multiplexed onto a single wavelength. For example, if each circuit is OC-12 and the wavelength capacity is OC-48, then the traffic granularity is 4.

As we mentioned earlier, the key in designing a WDM ring is to determine which ADMs are needed at each node. This mainly depends on how low-rate traffic circuits are multiplexed onto high-rate wavelengths. An ADM for an individual wavelength is required at a node only when the wavelength needs to be added/dropped at the node, i.e., when a circuit originated from (or terminated at) the node is multiplexed onto the wavelength. If a wavelength only passes through a node, then no ADM for such a wavelength is needed at the node. Our objective is to find an optimal way to multiplex low-rate traffic circuits so as to minimize the total number of ADMs required in the network. However, it will become clear later that we can easily incorporate other considerations into our objective as well, such as the total number of wavelengths used in the ring.

Before presenting our ILP formulation, we first introduce the following notation:

N: the number of nodes in the ring;
L: the number of wavelengths available;
g: the traffic granularity;
R: the number of traffic requirements;
m_{ij}^r: the number of circuits from node i to j in the rth traffic requirement ($r = 1, 2, \ldots, R$).
(Without loss of generality, we assume that $m_{ij}^r = 0$ for $i \geq j$.)

We note that the static traffic grooming problem corresponds to $R = 1$.

A. Unidirectional Rings

In this subsection, we consider the dynamic traffic grooming problem for unidirectional rings. We first introduce the following decision variables:

x_{ijl}^r: the number of traffic circuits from node i to j in the rth traffic requirement that are multiplexed onto wavelength l ($0 \leq x_{ijl}^r \leq m_{ij}^r$);

$$y_{il} = \begin{cases} 1 & \text{if any circuit originated from or terminated at node } i \text{ is multiplexed onto wavelength } l; \\ 0 & \text{otherwise.} \end{cases}$$

We note that $y_{il} = 1$ if and only if there exists some $x_{ijl}^r > 0$ or $x_{jil}^r > 0$, which implies that wavelength l needs to be added/dropped at node i, i.e., an ADM for wavelength l is required at node i. Since our objective is to minimize $\sum_{i=1}^{N} \sum_{l=1}^{L} y_{il}$, the total number of ADMs required in the ring, the dynamic traffic grooming problem can be formulated as the following integer linear programming (ILP) problem:

($\mathrm{ILP}_1^{\mathsf{u}}$)

$$\min \quad \sum_{i=1}^{N} \sum_{l=1}^{L} y_{il}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ijl}^r \leq g \quad \forall\, l, r \qquad (1)$$

$$\sum_{l=1}^{L} x_{ijl}^r = m_{ij}^r \quad \forall\, i, j, r \qquad (2)$$

$$M y_{il} \geq \sum_{r=1}^{R} \sum_{j=1}^{N} (x_{ijl}^r + x_{jil}^r) \quad \forall i, l \qquad (3)$$

x_{ijl}^r non-negative integer variable
y_{il} binary variable,

where $M > 0$ is a very large positive constant. We denote the above ILP as ($\mathrm{ILP}_1^{\mathsf{u}}$). The three constraints in ($\mathrm{ILP}_1^{\mathsf{u}}$) are:

1. The total number of circuits multiplexed onto wavelength l should not exceed g for each traffic requirement.
2. Every circuit in each traffic requirement has to be assigned to one (and only one) wavelength.
3. Given that the objective is to minimize $\sum_{i=1}^{N} \sum_{l=1}^{L} y_{il}$ and y_{il} is binary variable, it is equivalent to the condition that $y_{il} = 1$ if and only if there exists some $x_{ijl}^r > 0$ or $x_{jil}^r > 0$ (which is the same as $\sum_{r=1}^{R} \sum_{j=1}^{N} (x_{ijl}^r + x_{jil}^r) > 0$).

It is shown in [12] that the integer constraint on x_{ijl}^r can be relaxed, i.e., the following mixed ILP (MILP) has at least one integer optimal solution:

($\mathrm{MILP}_1^{\mathsf{u}}$)

$$\min \quad \sum_{i=1}^{N} \sum_{l=1}^{L} y_{il}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ijl}^r \leq g \quad \forall\, l, r$$

$$\sum_{l=1}^{L} x_{ijl}^r = m_{ij}^r \quad \forall\, i, j, r$$

$$M y_{il} \geq \sum_{r=1}^{R} \sum_{j=1}^{N} (x_{ijl}^r + x_{jil}^r) \forall\, i, l$$

$$x_{ijl}^r \geq 0$$

y_{il} binary variable

It is much easier to solve (MILP$_1^{\text{u}}$) than (ILP$_1^{\text{u}}$). In most cases, the computation time required to solve (MILP$_1^{\text{u}}$) is only very small fraction of that required to solve (ILP$_1^{\text{u}}$) (see numerical experiments provided in [12]). Therefore, for unidirectional rings, we should simply use (MILP$_1^{\text{u}}$).

B. Bi-directional Rings

We now consider the dynamic traffic grooming problem for bi-directional rings. The decision variable x_{ijl}^r in the unidirectional ring case now needs to be replaced by two decision variables, x_{ijl1}^r and x_{ijl2}^r, one for clockwise direction and the other one for counter-clockwise direction, which are defined as follows:

x_{ijl1}^r: the number of traffic circuits from node i to j in the rth traffic requirement that are multiplexed onto wavelength l in clockwise direction (i.e., through nodes $i+1, \ldots, j-1$ for $i < j$);

x_{ijl2}^r: the number of traffic circuits from node i to j in the rth traffic requirement that are multiplexed onto wavelength l in counter-clockwise direction (i.e., through nodes $i-1, \ldots, 1, N, \ldots, j+1$ for $i < j$);

In the bi-directional ring case, wavelength l needs to be added/dropped at node i, i.e., $y_{il} = 1$, if and only if

$$\sum_{r=1}^{R}\sum_{j=1}^{N}\sum_{k=1}^{2}(x_{ijlk}^r + x_{jilk}^r) > 0,$$

which indicates that at least one circuit originated from or terminated at node i is multiplexed onto wavelength l in either clockwise or counter-clockwise direction. Therefore, the ILP problem for the dynamic traffic grooming problem for bi-directional rings is

(ILP$_1^{\text{b}}$)

$$\min \quad \sum_{i=1}^{N}\sum_{l=1}^{L} y_{il}$$

$$\text{s.t.} \quad \sum_{i<j, n\in[i,j)} x_{ijl1}^r + \sum_{i<j, n\notin[i,j)} x_{ijl2}^r \le g \qquad \forall\, n, l, r \quad (4)$$

$$\sum_{l=1}^{L}\sum_{k=1}^{2} x_{ijlk}^r = m_{ij}^r \qquad \forall\, i, j, r \quad (5)$$

$$M y_{il} \ge \sum_{r=1}^{R}\sum_{j=1}^{N}\sum_{k=1}^{2}(x_{ijlk}^r + x_{jilk}^r) \qquad \forall\, i, l \quad (6)$$

x_{ijlk}^r non-negative integer variable

y_{il} binary variable,

where $M > 0$ is a very large positive constant. We denote the above ILP as (ILP$_1^{\text{b}}$). It is clear that (4) in (ILP$_1^{\text{b}}$) is significantly different from (1) in (ILP$_1^{\text{u}}$). For unidirectional rings, the capacity load of each wavelength is the same for all fiber links. For bi-directional rings, the capacity load of each wavelength may be different for different fiber links. Therefore, (4) ensures that the total capacity assigned to wavelength l for fiber link n ($n = 1, \ldots, N$) does not exceed g.

We note that (4) makes it much more difficult to solve (ILP$_1^{\text{b}}$) than (ILP$_1^{\text{u}}$). Furthermore, the integer constraint on x_{ijlk}^r can no longer be relaxed as in (ILP$_1^{\text{u}}$). In other words, if we relax the integer constraint on x_{ijlk}^r in (ILP$_1^{\text{b}}$), then the resulting MILP problem may not have an integer optimal solution. Therefore, the dynamic traffic grooming problem is much more difficult to solve for bi-directional rings than for unidirectional rings

III. HEURISTIC METHOD

In the previous section, we presented the ILP and MILP formulations for the traffic grooming problem for both unidirectional and bi-directional rings. However, as we shall show later in our numerical results, in many cases, it is computationally infeasible to solve these ILP and MILP problems, which may take days and weeks, say, by using commercially available softwares, such as CPLEX. Therefore, in this section, we develop a heuristic method in combination with the ILP formulation to solve the traffic grooming problem. The basic idea of our method is as follows. Instead of minimizing the number of ADMs, we first try to find a solution that minimizes the number of wavelengths, based on which we then construct a solution with as few ADMs as possible. The traffic grooming problem with the number of wavelengths as the objective function is much easier to solve.

For ease of exposition, we focus on the bidirectional ring case throughout this section. The extension to the unidirectional ring case is straightforward.

A. *Minimizing the Number of Wavelengths*

To develop our heuristic method, we first consider the traffic grooming problem with the following objective function: a combination of the number of ADMs and the number of wavelengths. First, we introduce an additional decision variable:

$$z_l = \begin{cases} 1 & \text{if the } l\text{-th wavelength is used;} \\ 0 & \text{otherwise.} \end{cases}$$

Then the traffic grooming problem ($\text{ILP}_1^{\mathsf{b}}$) can be modified as follows:

($\text{ILP}_2^{\mathsf{b}}$)

$$\begin{aligned} \min \quad & c_1 \sum_{l=1}^{L} z_l + c_2 \sum_{i=1}^{N} \sum_{l=1}^{L} y_{il} \\ \text{s.t.} \quad & \sum_{i<j, n\in[i,j)} x_{ijl1}^r + \sum_{i<j, n\notin[i,j)} x_{ijl2}^r \\ & \qquad \le g z_l \qquad \forall\, n, l, r \\ & \sum_{l=1}^{L} \sum_{k=1}^{2} x_{ijlk}^r = m_{ij}^r \qquad \forall\, i, j, r \\ & M y_{il} \ge \sum_{r=1}^{R} \sum_{j=1}^{N} \sum_{k=1}^{2} (x_{ijlk}^r + x_{jilk}^r) \\ & \qquad \forall\, i, l \\ & y_{il}, x_{ijlk}^r, z_l \text{ are binary variables,} \end{aligned}$$

where c_1 is the weighted cost for the number of wavelengths and c_2 is the weighted cost for the number of ADMs. It is clear that ($\text{ILP}_1^{\mathsf{b}}$) is a special case of ($\text{ILP}_2^{\mathsf{b}}$) in which $c_1 = 0$. Furthermore, if $c_2 = 0$, then the objective function of ($\text{ILP}_2^{\mathsf{b}}$) is simply the number of wavelengths. ($\text{ILP}_2^{\mathsf{b}}$) with $c_2 = 0$ is much easier to solve than ($\text{ILP}_1^{\mathsf{b}}$). For example, for a ring with $N = 11$, $g = 4$, and all-to-all traffic, it takes more than 5 days to solve ($\text{ILP}_1^{\mathsf{b}}$) by using CPLEX 7.0 on a Pentium III 450-MHz PC while it only takes less a minute to solve ($\text{ILP}_2^{\mathsf{b}}$). (More numerical results will be provided in Section 4.)

We note that the traffic grooming problem with the number of wavelengths as its objective function is trivial for unidirectional rings, in which case the minimum number of wavelengths required is simply equal to

$$\max_{1\le r\le R} \left\lceil \frac{\sum_{i,j} m_{ij}^r}{g} \right\rceil,$$

where $\lceil w \rceil$ is the smallest integer which is no less than w.

B. *Minimizing the Number of ADMs: A Heuristic Method*

We now present a heuristic method for the traffic grooming problem with the objective of minimizing the number of ADMs. The first step of our heuristic method is to solve ($\text{ILP}_2^{\mathsf{b}}$) with $g = 1$. In general, we set $c_2 = 0$ in ($\text{ILP}_2^{\mathsf{b}}$). However, in many cases, we find that results can be improved if we choose $c_1 >> c_2 > 0$. First, $c_1 >> c_2$ means that the number of wavelengths is the dominant factor in the objective function, which makes it relatively easier to solve ($\text{ILP}_2^{\mathsf{b}}$). Secondly, by choosing $c_2 > 0$, we take into account the number of the ADMs in the objective function, which can lead to a better initial solution for our heuristic method. We call each wavelength in the solution to this problem as a *subwavelength circle*. (We note that for unidirectional rings, each subwavelength circle only contains one demand.) For each subwavelength circle, there are a number of nodes associated with it where demands are added/dropped. We call these nodes *associated nodes* (with the subwavelength circle). It is clear that if we groom a subwavelength onto a full wavelength, then ADMs of the full wavelength are needed at the associated nodes.

Once all subwavelength circles are obtained, our next goal is to combine (groom) these subwavelength circles into full wavelengths, i.e., g subwavelength circles for each full wavelength, such that the total number of ADMs needed is minimized as much as possible. We develop a heuristic method to do this. To help fix ideas, we first present our heuristic method for the static traffic case (i.e., $R = 1$) in the following algorithm.

```
Algorithm 1 (for $R = 1$): combine subwavelength circles into wavelengths
  • Input: A set of subwavelength circles $S_w$, obtained based on (ILP$_2^{\text{b}}$) with $g = 1$.
  • Step 1. Select one subwavelength circle $s \in S_w$ with the largest number of associated nodes. Assign $s$ to a wavelength $l$ that has not been used, and delete $s$ from $S_w$. Set $c_l = g - 1$.
  • Step 2. For each $s \in S_w$:
      (2a) Compute $n_{s,1}$, the number of additional ADMs needed for wavelength $l$ if $s$ is combined into wavelength $l$.
      (2b) Set $n_s = n_{s,2} - n_{s,1}$, where $n_{s,2}$ is the number of associated nodes for $s$.
  • Step 3. Select $s \in S_w$ with the largest $n_s$, and in case of a tie, select the one with the smallest $n_{s,1}$. Assign subwavelength circle $s$ to wavelength $l$, and delete $s$ from $S_w$ and set $c_l = c_l - 1$.
  • Step 4. If $c_l = 0$ go to Step 1, otherwise go to Step 2 (until $S_w = \emptyset$).
```

We now extend our heuristic method to $R > 1$. For two different traffic requirements r_1 and r_2, we say that they have $\max(m_{ij}^{r_1}, m_{ij}^{r_2})$ common demands from node i to j. If $m_{ij}^{r_1} > m_{ij}^{r_2}$, then traffic requirement r_1 has $m_{ij}^{r_1} - m_{ij}^{r_2}$ additional demands (with respective to r_2) from node i to j, otherwise traffic requirement r_2 has $m_{ij}^{r_2} - m_{ij}^{r_1}$ additional demands (with respective to r_1) from i to j. Let $D_{r_1 \cap r_2}$ be the set of all common demands between r_1 and r_2, and $D_{r_1 \setminus r_2}$ be the set of all additional demands of traffic requirement r_1 with respect to r_2. We now present the following algorithm for $R > 1$.

```
Algorithm 2 (for $R > 1$)
  • Step 1. Solve (ILP$_2^{\text{b}}$) for the first traffic requirement ($r = 1$) with $g = 1$, based on which a set of subwavelength circles $S_w$ are obtained.
  • Step 2. For $r = 2$ to $R$
      (2a) For each demand $d \in D_{1 \cap r}$, keep the same assignment as in the first traffic requirement.
      (2b) For each demand $d \in D_{r \setminus 1}$, if there exists a subwavelength circle $s \in S_w$ such that $d$ can be assigned to $s$ while the number of the associated nodes of $s$ remains the same , then assign $d$ to $s$, update the demand assignment for $s$, and delete $d$ from $D_{r \setminus 1}$.
      (2c) For each demand $d \in D_{r \setminus 1}$, if there exists a subwavelength circle $s \in S_w$ such that $d$ can be assigned to $s$ while the number of the associated nodes of $s$ is increased only by one, then assign $d$ to $s$, update the demand assignment for $s$, and delete $d$ from $D_{r \setminus 1}$.
      (2d) For each demand $d \in D_{r \setminus 1}$, if there exists a subwavelength circle $s \in S_w$ such that $d$ can be assigned to $s$ (in this case the number of the associated nodes of $s$ is increased by two), then assign $d$ to $s$, update the demand assignment for $s$, and delete $d$ from $D_{r \setminus 1}$.
      (2e) Select a demand $d \in D_{r \setminus 1}$. Add a new subwavelength circle $s$ to $S_w$ and assign $d$ to $s$ (say using the shortest distance to determine its direction).
      (2g) Repeat Steps (2b)–(2e) until $D_{r \setminus 1}$ becomes empty.
  • Step 3. Apply Algorithm 1 to combine subwavelength circles in $S_w$ into wavelengths.
```

C. *Simulated Annealing Method*

We can further improve the heuristic method developed in the previous subsection by combining it with the simulated annealing method. The simulated annealing method was first applied to the traffic grooming problem in [18]. In our case, it works as follows.

Algorithm 3 (Simulated Annealing Algorithm)

```
• Step 1. Use Algorithm 1 or 2 to obtain an initial solution.
• Step 2.
    (2a) Randomly select two wavelengths in the current solution, and randomly select two subwavelengths, one from each of the two wavelengths.
    (2b) Swap the two subwavelength circles (or portions of the two subwavelength circles) to obtain a new solution.
    (2c) Calculate $\Delta$cost, which is the difference in costs between the new solution and the current solution.
    (2d) If $\Delta$cost $< 0$, then accept the current solution; otherwise accept the current solution with probability $e^{-\Delta \text{cost}/T}$, where $T$ is the parameter (temperature) of the simulated annealing method.
    (2e) If necessary, decrease $T$ (at certain rate), and repeat Step 2.
```

IV. NUMERICAL RESULTS

In this section, we present three sets of numerical results to validate the method we proposed in the previous section. All ILPs and MILPs were solved by use of CPLEX 7.0 on a Pentium III 450-MHz PC with 384M memory.

Example 1. In this example, we focus on $R = 1$ and set $c_2 = 0$ in (ILP$_2^{\mathrm{b}}$) in obtaining the subwavelength circles to be used as input for Algorithm 1. We compare Algorithm 1 with (MILP$_1^{\mathrm{u}}$) or (ILP$_1^{\mathrm{b}}$). The following five different cases are considered:

Case 1. UPSR with $g = 4$ and all-to-all uniform traffic (i.e., there is one and only one traffic demand between every pair of nodes). For this network, it can be shown (see [11]) that the minimum number of ADMs required is $N(N-1)/2$. Also, as pointed in the previous section, for UPSR each subwavelength circle contains only one demand. The results obtained based on Algorithm 1 and (MILP$_1^{\mathrm{u}}$) along with the optimal solutions are given in Table 1. In all the cases except $N = 5, 6$, we forced the CPLEX program to terminate if it was still running after certain amount of time (otherwise it would take very long time – a few hours, even days – for the CPLEX program to terminate by itself) and therefore the best solution obtained at termination may not be optimal. For each N, the best solution found when the CPLEX program was terminated is provided in Table 1, along with the termination time (in seconds). For example, for $N = 11$, the best solution found by the CPLEX program is 56 when it was terminated at 600 seconds. We mark the time with * if it is the time when the CPLEX program stopped by itself, in which case the corresponding solution must be optimal. For example, for $N = 6$, the CPLEX program terminated in 20 seconds with the optimal solution 15. It is clear from Table 1 that Algorithm 1 is much faster and its results are comparable with those obtained based on (MILP$_1^{\mathrm{u}}$) and are within 10% of the optimal solutions.

Case 2. UPSR with $g = 4$ and arbitrary traffic which was randomly generated. The results for Algorithm 1 and (MILP$_1^{\mathrm{u}}$) are provided in Table 2. In this case, Algorithm 1 performs consistently better than (MILP$_1^{\mathrm{u}}$) and it requires much shorter run time.

Case 3. BLSR with $g = 4$ and all-to-all uniform traffic. The results for Algorithm 1 and (ILP$_1^{\mathrm{b}}$) are provided in Table 3. Again, the results are similar to those of Case 2.

Case 4. BLSR with $g = 4$ and arbitrary traffic which was randomly generated. The results for Algorithm 1 and (ILP$_1^{\mathrm{b}}$) are provided in Table 4. We have the similar results as those in Case 2.

Case 5. BLSR with $g = 16$ and arbitrary traffic which was randomly generated. The results for Algorithm 1 and (ILP$_1^{\mathrm{b}}$) are provided in Table 5. The results are similar.

Example 2. We consider BLSR with $R = 1$,

$g = 4$, and randomly generated traffic. For this example, we compare the results obtained from

(a) Algorithm 1 with $c_2 = 0$, i.e., the subwavelength circles used as its input are generated from (ILP$^{\mathrm{b}}_2$) with $c_2 = 0$;
(b) Algorithm 1 with $c_1 = 50$ and $c_2 = 1$;
(c) Algorithm 3 with the solution obtained from (a) as its initial solution;
(b) Algorithm 3 with the solution obtained from (b) as its initial solution.

The numerical results are given in Table 6. The run time in this example is also in seconds. It is clear from the results in Table 6 that we can obtain better results by choosing $c_2 > 0$ in (ILP$^{\mathrm{b}}_2$), though the run time is longer. Also, further improvement can be made by combining our heuristic method with the simulated annealing method.

Example 3. In this example, we consider BLSR with $g = 4$ and $R \geq 1$. Traffic demands are randomly generated. We use $c_2 = 0$ in (ILP$^{\mathrm{b}}_2$). We consider three different cases depending on how much common traffic demands shared among R different traffic requirements:

- **Case 1.** 90% common traffic demands;
- **Case 2.** 60% common traffic demands;
- **Case 3.** 30% common traffic demands;

The numerical results based on Algorithm 2 and (ILP$^{\mathrm{b}}_1$) are provided in Tables 7-9. In this example, the run time is in minutes. It is quite clear from the results in Tables 7-9 that Algorithm 2 performs quite well comparing with (ILP$^{\mathrm{b}}_1$). We also note that in some instances (ILP$^{\mathrm{b}}_1$) does not even produce feasible solution when the CPLEX program was terminated (e.g., $N = 14$, $R = 4, 5$ in Table 7).

V. CONCLUSION

In this paper, we studied the traffic grooming problem for WDM rings with dynamic traffic. Both bidirectional and uni-directional rings were considered. We developed a heuristic method in combination with the ILP formulation. The basic idea of our method is to first find a solution with minimum number of wavelengths based on which we then construct a solution with as few ADMs as possible. Our numerical results demonstrated that the method works quite well.

One important future research direction is to consider the traffic grooming problem with stochastic traffic. As we mentioned earlier, this problem has not been studied in the literature yet, however it is of great practical importance since traffic demands are often unpredictable (stochastic) in many applications.

REFERENCES

[1] D. Banerjee and B. Mukherjee, “A practical approach for routing and wavelength assignment in large wavelength-routed optical networks,” *IEEE Journal on Selected Areas in Communications,* Vol. 14, No. 5, pp. 903-908, 1996.

[2] R. Barry and P. Humblet, “Models of blocking probability in all-optical networks with and without wavelength changes,” *IEEE Journal on Selected Areas in Communications,* Vol. 14, No. 5, pp. 858-865, 1996.

[3] R. Berry and E. Modiano, “Reducing electronic multiplexing costs in SONET/WDM rings with dynamic changing traffic,” *IEEE Journal on Selected Areas in Communications,* Vol. 18, No. 10, pp. 1961-1971, 2000.

[4] A. Chiu and E. Modiano, “Reducing electronic multiplexing costs in unidirectional SONET/WDM ring networks via efficient traffic grooming,” in *Proceedings of Globecom,* (IEEE, Sydney, Australia, November 1998), Vol. 1, pp. 322-327.

[5] A. Chiu and E. Modiano, “Traffic grooming algorithms for reducing electronic multiplexing costs in WDM ring networks,” *Journal of Lightwave Technology,* Vol. 18, No. 1, pp. 2-12, 2000.

[6] R. Dutta and G. Rouskas, “On optimal traffic grooming in WDM rings,” in *Proc. of SIGMETRICS/PERFORMANCE 2001,* (ACM, Cambridge, MA, June 2001), pp. 164-174.

[7] O. Gerstel, G. Sasaki, S. Kutten, “Worst-case analysis of dynamic wavelength allocation in optical networks,” *IEEE/ACM Transactions on Networking,* Vol. 7, No. 6, pp. 833-845, 1999.

[8] O. Gerstel, P. Lin, and G. Sasaki, “Wavelength assignment in a WDM ring to minimize the cost of embedded SONET rings,” in *Proc. Infocom,* (IEEE, San Francisco, CA, April 1998), Vol. 1, pp. 94-101.

[9] O. Gerstel, P. Lin, and G. Sasaki, “Combined WDM and SONET network design,” in *Proc. Infocom,* (IEEE, New York, NY, April 1999), Vol. pp. 734-743.

[10] O. Gerstel, R. Ramaswami, and G. Sasaki, “Cost-effective traffic grooming in WDM rings,” *IEEE/ACM Transactions on Networking,* Vol. 5, No. 5, pp. 618-630, 2000.

[11] J.Q. Hu, “Optimal traffic grooming for wavelength-division-multiplexing rings with all-to-all uniform traffic,” *Journal of Optical Networks,* Vol. 1, pp. 32-42, 2002. (http://www.osa-jon.org/abstract.cfm?URI=JON-1-1-32)

[12] J.Q. Hu, “Traffic grooming in WDM ring networks: A linear programming solution,” to appear in *Journal of Optical Networks,* 2002.

[13] R. Ramaswami and K. Sivarajan, *Optical Networks: A Practical Perspective,* Morgan Kaufmann Publishers, 1998.

[14] G. Sasaki, O. Gerstel, and R. Ramaswami, “A WDM ring network for incremental traffic,” in *Proceedings of The Thirty-Sixth Annual Allerton Conference on Communication, Control, and Computing,* Monticello, IL, September 23-25, 1998.

[15] J. Simmons, E. Goldstein, and A. Saleh, “On the value of wavelength add/drop in WDM rings with uniform traffic,” in *Proc. Optical Fiber Communication Conference and Exhibit,* (Optical Society of America, San Jose, CA, February 1998), pp. 361-362.

[16] J. Simmons, E. Goldstein, and A. Saleh, “Quantifying the benefits of wavelength add-drop WDM rings with distance independent and dependent traffic,” *Journal of Lightwave Technology,* Vol. 17, No. 1, pp. 48-57, 1999.

[17] S. Subramaniam, A. Somani, M. Azizoglu, and R. Barry, “A performance model for wavelength conversion with non-Poisson traffic,” in *Proc. Infocom,* (IEEE, Kobe, Japan, April 1997), Vol. 2, pp. 499-506.

[18] J. Wang, W. Cho, V. Vemuri, and B. Mukherjee, “Improved approaches for cost-effective traffic grooming in WDM ring networks: ILP formulations and single-hop and multihop connections,” *Journal of Lightwave Technology,* Vol. 19, No. 11, pp. 1645-1653, 2001.

[19] X. Zhang and C. Qiao, “An effective and comprehensive solution to traffic grooming and wavelength assignment in WDM rings,” in *SPIE Proc. All-Optical Networking: Architecture, Control, and Management Issues,* J. M. Senior and C. M. Qiao, Eds., Vol. 3531, pp. 221-232, Boston, MA, November 1998.

N	Opt.	Algo. 1		MILP_1^u	
		Sol.	Time (sec)	Sol.	Time (sec)
5	10	11	1	10	1*
6	15	16	1	15	20*
7	21	23	1	21	60
8	28	29	1	28	60
9	36	38	2	36	60
10	45	48	2	45	600
11	55	60	2	56	1200
12	66	71	3	67	1200
13	78	81	3	80	1200
14	91	96	4	93	1800
15	105	112	4	110	3000

TABLE I

Case 1 in Example 1 (UPSR with $g = 4$ and all-to-all uniform traffic)

N	Algo. 1		MILP_1^u	
	Sol.	Time (sec)	Sol.	Time (sec)
5	14	1	14	60
6	23	1	24	60
7	33	1	34	60
8	43	2	45	60
9	58	2	62	60
10	78	3	81	300
11	89	3	95	300
12	101	4	111	600
13	131	4	139	600
14	151	5	163	600
15	172	5	192	600

TABLE II

Case 2 in Example 1 (UPSR with $g = 4$ and arbitrary traffic)

N	Algo. 1		ILP_1^b	
	Sol.	Time (sec)	Sol.	Time (sec)
6	9	2	9	30
7	12	2	12	60
8	18	2	16	60
9	20	5	18	60
10	27	12	25	300
11	30	13	31	300
12	38	13	42	300
13	44	15	49	300
14	50	19	56	300
15	57	25	62	300

TABLE III

Case 3 in Example 1 (BLSR with $g = 4$ and all-to-all uniform traffic)

N	Algo. 1		ILP$^b_1$	
	Sol.	Time (sec)	Sol.	Time (sec)
6	14	4	14	60
7	32	12	28	60
8	32	12	34	60
9	46	15	43	60
10	68	24	64	300
11	73	35	81	300
12	78	35	89	300
13	97	46	112	600
14	110	46	123	600
15	126	49	161	600

TABLE IV

Case 4 in Example 1 (BLSR with $g = 4$ and arbitrary traffic)

N	Algo. 1		ILP$^b_1$	
	Sol.	Time (sec)	Sol.	Time (sec)
6	6	1	6	60
7	13	3	12	60
8	14	3	17	60
9	16	3	19	60
10	24	8	26	300
11	21	7	29	300
12	30	8	37	300
13	40	12	46	300
14	45	14	51	300
15	48	13	53	300

TABLE V

Case 5 in Example 1 (BLSR with $g = 16$ and arbitrary traffic)

N	R	Algo. 2		ILP$^b_1$	
		Sol.	Time (min)	Sol.	Time (min)
6	1	10	1	10	10
	2	10	1	10	10
	3	10	1	10	10
	4	10	1	12	20
	5	11	1	11	60
9	1	22	1	25	10
	2	24	1	28	10
	3	25	1	28	10
	4	25	1	30	30
	5	25	1	31	60
12	1	49	2	49	60
	2	49	2	51	60
	3	51	2	49	60
	4	51	2	59	60
	5	51	2	57	60
14	1	63	4	51	60
	2	65	4	71	60
	3	67	4	94	60
	4	70	4	None	90
	5	70	4	None	90

TABLE VII

Case 1 in Example 3 (90% common traffic)

N	Algo. 1				Algo. 3				ILP$^b_1$	
	$c_2 = 0$		$c_1 = 50, c_2 = 1$		$c_2 = 0$		$c_1 = 50, c_2 = 1$			
	Sol.	Time (sec)	Sol.	Time (sec)	Sol.	Time (sec)	Sol.	Time (sec)	Sol.	Time (sec)
8	25	12	23	20	24	13	22	23	23	60
9	28	10	28	23	25	12	26	29	27	60
10	30	12	28	28	27	15	27	36	32	300
11	40	20	37	48	40	26	37	57	37	300
12	53	28	45	67	52	38	45	78	48	600
13	69	35	59	86	67	45	59	100	62	600
14	70	32	64	105	69	50	62	123	71	600
15	90	38	87	143	89	56	85	160	84	600

TABLE VI

Example 2 (BLSR with $g = 4$ and arbitrary traffic)

N	R	Algo. 2		ILP$_1^b$	
		Sol.	Time (min)	Sol.	Time (min)
6	1	10	1	10	10
	2	10	1	10	15
	3	10	1	11	15
	4	10	1	11	15
	5	11	1	13	15
9	1	22	1	25	15
	2	25	1	30	15
	3	26	1	29	15
	4	26	1	35	30
	5	30	1	29	30
12	1	49	2	49	30
	2	54	2	68	240
	3	60	2	92	240
	4	63	2	89	300
	5	67	2	None	300
14	1	63	4	51	60
	2	69	4	76	200
	3	72	4	76	200
	4	76	4	None	300
	5	76	4	None	300

TABLE VIII

CASE 2 IN EXAMPLE 3 (60% COMMON TRAFFIC)

N	R	Algo. 2		ILP$_1^b$	
		Sol.	Time (min)	Sol.	Time (min)
6	1	10	1	10	30
	2	11	1	11	30
	3	11	1	12	30
	4	12	1	12	30
	5	13	1	14	30
9	1	22	1	25	30
	2	29	1	34	30
	3	32	1	38	30
	4	36	1	38	30
	5	36	1	40	30
12	1	49	2	49	60
	2	57	2	75	300
	3	70	2	90	300
	4	72	2	94	300
	5	76	2	None	300
14	1	63	4	51	60
	2	79	4	86	300
	3	82	4	96	300
	4	88	4	None	300
	5	92	4	None	300

TABLE IX

CASE 3 IN EXAMPLE 3 (30% COMMON TRAFFIC)